Computer algebra independent integration tests

Summer 2022 edition

1-Algebraic-functions/1.2-Trinomial-products/1.2.3-General/47-1.2.3.3-d+e-x^n-^q-a+b-x^n+c-x^-2-n-^p

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Contents

Chapter 1

Introduction

Local contents

This report gives the result of running the computer algebra independent integration test.

The download section in the appendix contains links to download the problems in plain text format used for all CAS systems.

The number of integrals in this report is [96]. This is test number [47].

1.1 Listing of CAS systems tested

The following are the CAS systems tested:

1. Mathematica 13.1 (June 29, 2022) on windows 10.
2. Rubi 4.16.1 (Dec 19, 2018) on Mathematica 13.0.1 on windows 10.
3. Maple 2022.1 (June 1, 2022) on windows 10.
4. Maxima 5.46 (April 13, 2022) using Lisp SBCL 2.1.11.debian on Linux via sagemath 9.6.
5. Fricas 1.3.8 (June 21, 2022) based on sbcl 2.1.11.debian on Linux via sagemath 9.6.
6. Giac/Xcas 1.9.0-13 (July 3, 2022) on Linux via sagemath 9.6.
7. Sympy 1.10.1 (March 20, 2022) Using Python 3.10.4 on Linux.
8. Mupad using Matlab 2021a with Symbolic Math Toolbox Version 8.7 on windows 10.

Maxima and Fricas and Giac are called using Sagemath. This was done using Sagemath `integrate` command by changing the name of the algorithm to use the different CAS systems.

Sympy was called directly from Python.

1.2 Results

Important note: A number of problems in this test suite have no antiderivative in closed form. This means the antiderivative of these integrals can not be expressed in terms of elementary, special functions or `Hypergeometric2F1` functions. `RootSum` and `RootOf` are not allowed.

If a CAS returns the above integral unevaluated within the time limit, then the result is counted as passed and assigned an A grade.

However, if CAS times out, then it is assigned an F grade even if the integral is not integrable, as this implies CAS could not determine that the integral is not integrable in the time limit.

If a CAS returns an antiderivative to such an integral, it is assigned an A grade automatically and this special result is listed in the introduction section of each individual test report to make it easy to identify as this can be important result to investigate.

The results given in in the table below reflects the above.

System	% solved	% Failed
Rubi	100.00 (96)	0.00 (0)
Mathematica	95.83 (92)	4.17 (4)
Maple	51.04 (49)	48.96 (47)
Mupad	51.04 (49)	48.96 (47)
Fricas	50.00 (48)	50.00 (48)
Sympy	44.79 (43)	55.21 (53)
Giac	38.54 (37)	61.46 (59)
Maxima	17.71 (17)	82.29 (79)

Table 1.1: Percentage solved for each CAS

The table below gives additional break down of the grading of quality of the antiderivatives generated by each CAS. The grading is given using the letters A,B,C and F with A being the best quality. The grading is accomplished by comparing the antiderivative generated with the optimal antiderivatives included in the test suite. The following table describes the meaning of these grades.

grade	description
A	Integral was solved and antiderivative is optimal in quality and leaf size.
B	Integral was solved and antiderivative is optimal in quality but leaf size is larger than twice the optimal antiderivatives leaf size.
C	Integral was solved and antiderivative is non-optimal in quality. This can be due to one or more of the following reasons 1. antiderivative contains a hypergeometric function and the optimal antiderivative does not. 2. antiderivative contains a special function and the optimal antiderivative does not. 3. antiderivative contains the imaginary unit and the optimal antiderivative does not.
F	Integral was not solved. Either the integral was returned unevaluated within the time limit, or it timed out, or CAS hanged or crashed or an exception was raised.

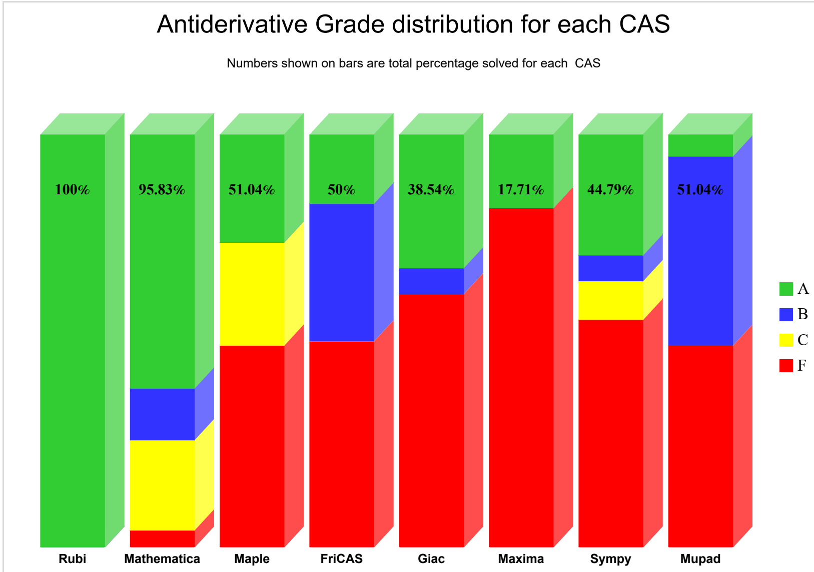
Table 1.2: Description of grading applied to integration result

Grading is implemented for all CAS systems. Based on the above, the following table summarizes the grading for this test suite.

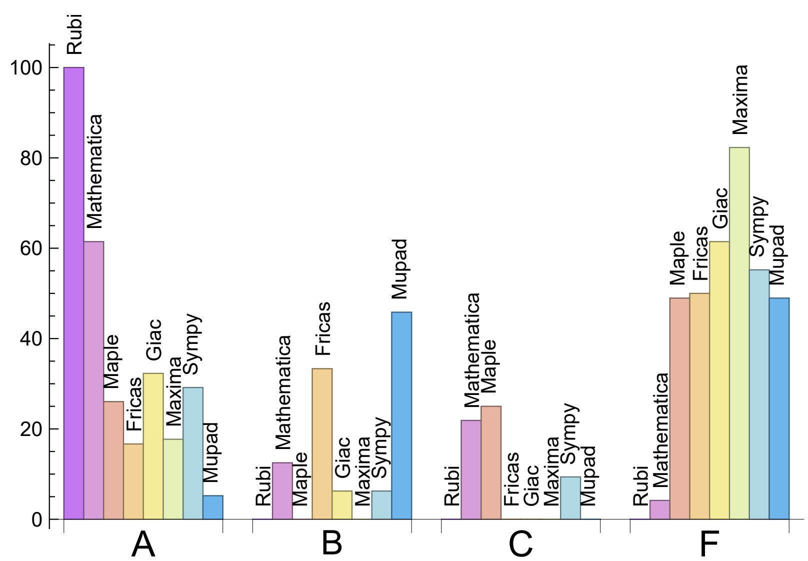
System	% A grade	% B grade	% C grade	% F grade
Rubi	100.00	0.00	0.00	0.00
Mathematica	61.46	12.50	21.88	4.17
Giac	32.29	6.25	0.00	61.46
Sympy	29.17	6.25	9.38	55.21
Maple	26.04	0.00	25.00	48.96
Maxima	17.71	0.00	0.00	82.29
Fricas	16.67	33.33	0.00	50.00
Mupad	N/A	45.83	0.00	48.96

Table 1.3: Antiderivative Grade distribution of each CAS

The following is a Bar chart illustration of the data in the above table.



The figure below compares the CAS systems for each grade level.



The following table shows the distribution of the different types of failure for each CAS. There are 3 types of reasons why it can fail. The first is when CAS returns back the input within the time limit, which means it could not solve it. This the typical normal failure **F** .

The second is due to time out. CAS could not solve the integral within the 3 minutes time limit which is assigned **F(-1)**.

The third is due to an exception generated. Assigned **F(-2)**. This most likely indicates an interface problem between sagemath and the CAS (applicable only to FriCAS, Maxima and

Giac) or it could be an indication of an internal error in CAS. This type of error requires more investigations to determine the cause.

System	Number failed	Percentage normal failure	Percentage timeout failure	Percentage exception failure
Rubi	0	0.00 %	0.00 %	0.00 %
Mathematica	4	100.00 %	0.00 %	0.00 %
Maple	47	100.00 %	0.00 %	0.00 %
Fricas	48	87.50 %	2.08 %	10.42 %
Giac	59	91.53 %	1.69 %	6.78 %
Maxima	79	98.73 %	0.00 %	1.27 %
Sympy	53	24.53 %	50.94 %	24.53 %
Mupad	47	100.00 %	0.00 %	0.00 %

Table 1.4: Failure statistics for each CAS

1.3 Time and leaf size Performance

The table below summarizes the performance of each CAS system in terms of time used and leaf size of results.

Mean size is the average leaf size produced by the CAS (before any normalization). The Normalized mean is relative to the mean size of the optimal anti-derivative given in the input files.

For example, if CAS has **Normalized mean** of 3, then the mean size of its leaf size is 3 times as large as the mean size of the optimal leaf size.

Median size is value of leaf size where half the values are larger than this and half are smaller (before any normalization). i.e. The Middle value.

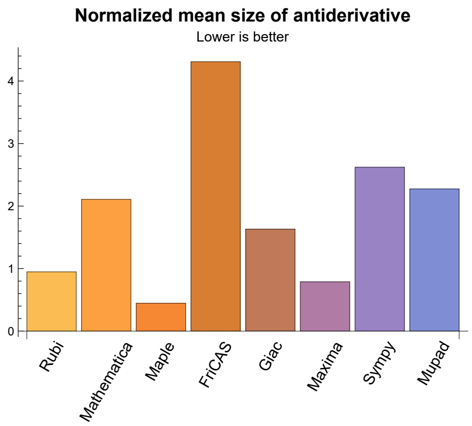
Similarly the **Normalized median** is relative to the median leaf size of the optimal.

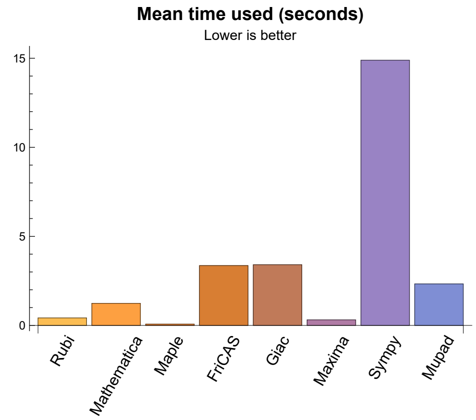
For example, if a CAS has Normalized median of 1.2, then its median is 1.2 as large as the median leaf size of the optimal.

System	Mean time (sec)	Mean size	Normalized mean	Median size	Normalized median
Rubi	0.42	361.82	0.95	288.00	1.00
Mathematica	1.24	2064.77	2.11	153.00	0.92
Maple	0.07	79.35	0.45	52.00	0.25
Maxima	0.31	121.59	0.79	72.00	0.94
Fricas	3.36	1518.48	4.31	436.50	2.21
Sympy	14.89	415.74	2.62	112.00	0.52
Giac	3.41	332.00	1.63	147.00	0.85
Mupad	2.33	819.20	2.27	269.00	1.48

Table 1.5: Time and leaf size performance for each CAS

The following are bar charts for the normalized leafsize and time used from the above table.


Normalized mean size of antiderivative
Lower is better
0
1
2
3
4
Rubi
Mathematica
Maple
FriCAS
Giac
Maxima
Sympy
Mupad


Mean time used (seconds)
Lower is better
0
5
10
15
Rubi
Mathematica
Maple
FriCAS
Giac
Maxima
Sympy
Mupad

1.4 list of integrals that has no closed form antiderivative

$\{59, 90, 94, 95, 96\}$

1.5 List of integrals solved by CAS but has no known antiderivative

Rubi {}

Mathematica {}

Maple {}

Maxima {}

Fricas {}

Sympy {}

Giac {}

Mupad {}

1.6 list of integrals solved by CAS but failed verification

The following are integrals solved by CAS but the verification phase failed to verify the anti-derivative produced is correct. This does not mean necessarily that the anti-derivative is wrong, as additional methods of verification might be needed, or more time is needed (3 minutes time limit was used). These integrals are listed here to make it easier to do further investigation to determine why it was not possible to verify the result produced.

Rubi $\{\}$

Mathematica $\{79, 83, 84, 86, 89\}$

Maple Verification phase not implemented yet.

Maxima Verification phase not implemented yet.

Fricas Verification phase not implemented yet.

Sympy Verification phase not implemented yet.

Giac Verification phase not implemented yet.

Mupad Verification phase not implemented yet.

1.7 Timing

The command `AbsoluteTiming[]` was used in Mathematica to obtain the elapsed time for each integrate call. In Maple, the command `Usage` was used as in the following example

```
cpu_time := Usage(assign ('result_of_int',int(expr,x)),output='realtime'
```

For all other CAS systems, the elapsed time to complete each integral was found by taking the difference between the time after the call completed from the time before the call was made. This was done using Python's `time.time()` call.

All elapsed times shown are in seconds. A time limit of 3 CPU minutes was used for each integral. If the integrate command did not complete within this time limit, the integral was aborted and considered to have failed and assigned an F grade. The time used by failed integrals due to time out was not counted in the final statistics.

1.8 Verification

A verification phase was applied on the result of integration for `Rubi` and `Mathematica`.

Future version of this report will implement verification for the other CAS systems. For the integrals whose result was not run through a verification phase, it is assumed that the antiderivative was correct.

Verification phase also had 3 minutes time out. An integral whose result was not verified could still be correct, but further investigation is needed on those integrals. These integrals were marked in the summary table below and also in each integral separate section so they are easy to identify and locate.

1.9 Important notes about some of the results

1.9.1 Important note about Maxima results

Since tests were run in a batch mode, and using an automated script, then any integral where Maxima needed an interactive response from the user to answer a question during the evaluation of the integral will fail.

The exception raised is `ValueError`. Therefore Maxima results is lower than what would result if Maxima was run directly and each question was answered correctly.

The percentage of such failures were not counted for each test file, but for an example, for the `Timofeev` test file, there were about 14 such integrals out of total 705, or about 2 percent. This percentage can be higher or lower depending on the specific input test file.

Such integrals can be identified by looking at the output of the integration in each section for Maxima. The exception message will indicate the cause of error.

Maxima integrate was run using SageMath with the following settings set by default

```
'besselexpand : true'
'display2d : false'
'domain : complex'
'keepfloat : true'
'load(to_poly_solve)'
'load(simplify_sum)'
'load(abs_integrate)' 'load(diag)'
```

SageMath automatic loading of Maxima `abs_integrate` was found to cause some problems. So the following code was added to disable this effect.

```
from sage.interfaces.maxima_lib import maxima_lib
maxima_lib.set('extra_definite_integration_methods', '[]')
```

```
maxima_lib.set('extra_integration_methods', '[]')
```

See `https://ask.sagemath.org/question/43088/integrate-results-that-are-different-from-using-maxima/` for reference.

1.9.2 Important note about FriCAS result

There were few integrals which failed due to SageMath interface and not because FriCAS system could not do the integration.

These will fail With error `Exception raised: NotImplementedError`.

The number of such cases seems to be very small. About 1 or 2 percent of all integrals. These can be identified by looking at the exception message given in the result.

1.9.3 Important note about finding leaf size of antiderivative

For Mathematica, Rubi, and Maple, the builtin system function `LeafSize` was used to find the leaf size of each antiderivative.

The other CAS systems (SageMath and Sympy) do not have special builtin function for this purpose at this time. Therefore the leaf size for Fricas and Sympy antiderivative was determined using the following function, thanks to user `slelievre` at `https://ask.sagemath.org/question/57123/could-we-have-a-leaf_count-function-in-base-sagemath/`

```
def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)
```

For Sympy, which was called directly from Python, the following code was used to obtain the leafsize of its result

```
try:
  # 1.7 is a fudge factor since it is low side from actual leaf count
  leafCount = round(1.7*count_ops(anti))

  except Exception as ee:
        leafCount =1
```

1.9.4 Important note about Mupad results

Matlab's symbolic toolbox does not have a leaf count function to measure the size of the antiderivative. Maple was used to determine the leaf size of Mupad output by post processing Mupad result.

Currently no grading of the antiderivative for Mupad is implemented. If it can integrate the problem, it was assigned a B grade automatically as a placeholder. In the future, when grading function is implemented for Mupad, the tests will be rerun again.

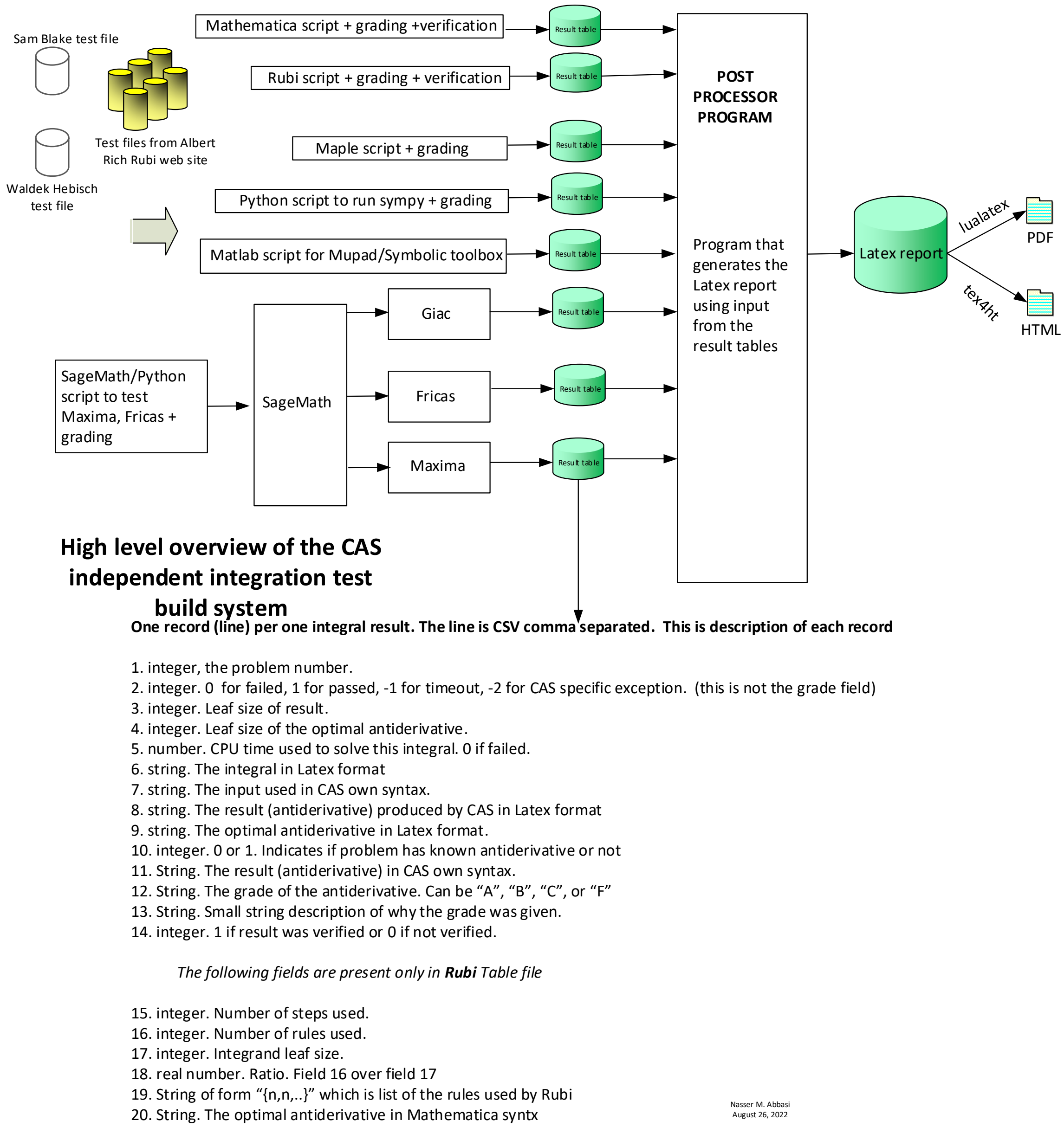
The following is an example of using Matlab's symbolic toolbox (Mupad) to solve an integral

```
integrand = evalin(symengine,'cos(x)*sin(x)')
the_variable = evalin(symengine,'x')
anti = int(integrand,the_variable)
```

Which gives `sin(x)^2/2`

1.10 Design of the test system

The following diagram gives a high level view of the current test build system.



Chapter 2

detailed summary tables of results

Local contents

2.1 List of integrals sorted by grade for each CAS

Local contents

2.1.1 Rubi

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96 }

B grade: { }

C grade: { }

F grade: { }

2.1.2 Mathematica

A grade: { 1, 2, 3, 4, 11, 13, 15, 16, 19, 22, 24, 26, 27, 30, 34, 35, 36, 37, 38, 40, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 59, 60, 61, 62, 66, 67, 68, 69, 70, 71, 72, 73, 77, 85, 87, 88, 90, 91, 92, 93, 94, 95, 96 }

B grade: { 74, 75, 76, 78, 79, 80, 81, 82, 83, 84, 86, 89 }

C grade: { 5, 6, 7, 8, 9, 10, 12, 14, 17, 18, 20, 21, 23, 25, 28, 29, 31, 32, 33, 39, 41 }

F grade: { 58, 63, 64, 65 }

2.1.3 Maple

A grade: { 1, 2, 11, 12, 15, 16, 19, 22, 23, 26, 27, 30, 34, 35, 36, 37, 38, 59, 66, 67, 68, 90, 94, 95, 96 }

B grade: { }

C grade: { 3, 4, 5, 6, 7, 8, 9, 10, 13, 14, 17, 18, 20, 21, 24, 25, 28, 29, 31, 32, 33, 39, 40, 41 }

F grade: { 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 60, 61, 62, 63, 64, 65, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 91, 92, 93 }

2.1.4 Maxima

A grade: { 1, 2, 11, 15, 22, 26, 34, 36, 38, 59, 66, 67, 68, 90, 94, 95, 96 }

B grade: { }

C grade: { }

F grade: { 3, 4, 5, 6, 7, 8, 9, 10, 12, 13, 14, 16, 17, 18, 19, 20, 21, 23, 24, 25, 27, 28, 29, 30, 31, 32, 33, 35, 37, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 60, 61, 62, 63, 64, 65, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 91, 92, 93 }

2.1.5 FriCAS

A grade: { 11, 12, 14, 22, 23, 26, 31, 32, 33, 34, 35, 59, 90, 94, 95, 96 }

B grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 13, 15, 16, 17, 18, 19, 20, 21, 24, 25, 27, 28, 29, 30, 36, 37, 38, 40, 41, 66, 67, 68 }

C grade: { }

F grade: { 39, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 60, 61, 62, 63, 64, 65, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 91, 92, 93 }

2.1.6 Sympy

A grade: { 1, 2, 5, 6, 7, 8, 9, 10, 11, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 24, 25, 27, 28, 29, 30, 31, 36, 38 }

B grade: { 26, 34, 35, 66, 67, 68 }

C grade: { 12, 23, 42, 43, 44, 47, 50, 61, 62 }

F grade: { 3, 4, 32, 33, 37, 39, 40, 41, 45, 46, 48, 49, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 63, 64, 65, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96 }

2.1.7 Giac

A grade: { 1, 2, 3, 10, 11, 12, 13, 14, 15, 16, 19, 21, 22, 23, 24, 25, 27, 30, 31, 32, 33, 34, 35, 36, 38, 40, 59, 90, 94, 95, 96 }

B grade: { 4, 26, 37, 66, 67, 68 }

C grade: { }

F grade: { 5, 6, 7, 8, 9, 17, 18, 20, 28, 29, 39, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 60, 61, 62, 63, 64, 65, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 91, 92, 93 }

2.1.8 Mupad

A grade: { 59, 90, 94, 95, 96 }

B grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 66, 67, 68 }

C grade: { }

F grade: { 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 60, 61, 62, 63, 64, 65, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 91, 92, 93 }

2.2 Detailed conclusion table per each integral for all CAS systems

Detailed conclusion table per each integral is given by table below. The elapsed time is in seconds. For failed result it is given as F(-1) if the failure was due to timeout. It is given as F(-2) if the failure was due to an exception being raised, which could indicate a bug in the system. If the failure was due to integral not being evaluated within the time limit, then it is given just an F.

In this table,the column `N.S.` in the table below, which stands for **normalized size** is defined as $\frac{\text{antiderivative leaf size}}{\text{optimal antiderivative leaf size}}$. To help make the table fit, **`Mathematica`** was abbreviated to **`MMA`**.

Problem 1	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	305	305	334	329	287	3602	165	288	1331
N.S.	1	1.00	1.10	1.08	0.94	11.81	0.54	0.94	4.36
time (sec)	N/A	0.170	0.068	0.213	0.484	1.269	9.318	3.108	1.542

Problem 2	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	323	323	337	386	319	3566	168	308	1293
N.S.	1	1.00	1.04	1.20	0.99	11.04	0.52	0.95	4.00
time (sec)	N/A	0.133	0.074	0.214	0.481	1.384	9.422	3.460	2.973

Problem 3	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	754	754	534	34	0	3580	0	601	2510
N.S.	1	1.00	0.71	0.05	0.00	4.75	0.00	0.80	3.33
time (sec)	N/A	0.823	0.470	0.191	0.000	2.020	0.000	3.786	2.780

Problem 4	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	329	329	425	36	0	3551	0	633	2438
N.S.	1	1.00	1.29	0.11	0.00	10.79	0.00	1.92	7.41
time (sec)	N/A	0.142	0.090	0.206	0.000	2.022	0.000	3.936	2.719

Problem 5	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	B	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	791	791	67	53	0	3072	136	0	2500
N.S.	1	1.00	0.08	0.07	0.00	3.88	0.17	0.00	3.16
time (sec)	N/A	0.585	0.031	0.063	0.000	0.513	18.754	0.000	3.825

Problem 6	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	B	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	791	791	67	53	0	3072	136	0	2500
N.S.	1	1.00	0.08	0.07	0.00	3.88	0.17	0.00	3.16
time (sec)	N/A	0.558	0.025	0.075	0.000	0.461	6.612	0.000	4.030

Problem 7	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	B	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	349	349	69	57	0	3059	136	0	2500
N.S.	1	1.00	0.20	0.16	0.00	8.77	0.39	0.00	7.16
time (sec)	N/A	0.287	0.030	0.052	0.000	0.429	19.024	0.000	4.035

Problem 8	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	B	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	751	751	69	55	0	3059	136	0	2500
N.S.	1	1.00	0.09	0.07	0.00	4.07	0.18	0.00	3.33
time (sec)	N/A	0.560	0.027	0.067	0.000	0.407	6.663	0.000	4.204

Problem 9	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	B	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	411	411	55	42	0	1443	75	0	2500
N.S.	1	1.00	0.13	0.10	0.00	3.51	0.18	0.00	6.08
time (sec)	N/A	0.195	0.017	0.070	0.000	0.415	1.967	0.000	3.683

Problem 10	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	451	451	55	42	0	973	24	239	459
N.S.	1	1.00	0.12	0.09	0.00	2.16	0.05	0.53	1.02
time (sec)	N/A	0.274	0.011	0.023	0.000	0.371	0.881	4.569	0.176

Problem 11	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	85	85	64	52	72	100	73	72	33
N.S.	1	1.00	0.75	0.61	0.85	1.18	0.86	0.85	0.39
time (sec)	N/A	0.032	0.014	0.028	0.497	0.364	0.054	3.589	1.562

Problem 12	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	A	C	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	140	140	135	109	0	215	190	108	95
N.S.	1	1.00	0.96	0.78	0.00	1.54	1.36	0.77	0.68
time (sec)	N/A	0.065	0.115	0.051	0.000	0.364	0.358	3.290	0.144

Problem 13	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	347	347	258	27	0	1001	19	247	311
N.S.	1	1.00	0.74	0.08	0.00	2.88	0.05	0.71	0.90
time (sec)	N/A	0.167	0.130	0.237	0.000	0.388	1.137	3.015	2.284

Problem 14	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	331	331	55	42	0	369	20	245	145
N.S.	1	1.00	0.17	0.13	0.00	1.11	0.06	0.74	0.44
time (sec)	N/A	0.167	0.011	0.025	0.000	0.388	1.305	3.409	0.225

Problem 15	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	27	27	31	42	27	43	26	29	21
N.S.	1	1.00	1.15	1.56	1.00	1.59	0.96	1.07	0.78
time (sec)	N/A	0.005	0.011	0.025	0.486	0.362	0.056	4.660	0.047

Problem 16	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	131	131	131	96	0	247	49	147	269
N.S.	1	1.00	1.00	0.73	0.00	1.89	0.37	1.12	2.05
time (sec)	N/A	0.062	0.055	0.074	0.000	0.345	0.685	4.577	0.200

Problem 17	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	B	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	157	157	53	40	0	331	24	0	399
N.S.	1	1.00	0.34	0.25	0.00	2.11	0.15	0.00	2.54
time (sec)	N/A	0.061	0.010	0.026	0.000	0.362	0.078	0.000	1.721

Problem 18	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	B	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	171	171	55	42	0	574	24	0	483
N.S.	1	1.00	0.32	0.25	0.00	3.36	0.14	0.00	2.82
time (sec)	N/A	0.109	0.010	0.023	0.000	0.357	0.078	0.000	1.758

Problem 19	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	117	117	111	78	0	181	49	123	233
N.S.	1	1.00	0.95	0.67	0.00	1.55	0.42	1.05	1.99
time (sec)	N/A	0.038	0.032	0.070	0.000	0.372	0.677	3.858	0.190

Problem 20	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	B	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	511	511	57	44	0	1443	76	0	2500
N.S.	1	1.00	0.11	0.09	0.00	2.82	0.15	0.00	4.89
time (sec)	N/A	0.255	0.017	0.026	0.000	0.367	2.104	0.000	3.743

Problem 21	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	411	411	57	44	0	896	26	223	447
N.S.	1	1.00	0.14	0.11	0.00	2.18	0.06	0.54	1.09
time (sec)	N/A	0.222	0.019	0.026	0.000	0.394	0.910	3.704	1.677

Problem 22	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	97	97	90	63	82	131	82	82	44
N.S.	1	1.00	0.93	0.65	0.85	1.35	0.85	0.85	0.45
time (sec)	N/A	0.041	0.041	0.020	0.527	0.332	0.064	3.998	1.616

Problem 23	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	A	C	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	140	140	129	109	0	142	148	108	109
N.S.	1	1.00	0.92	0.78	0.00	1.01	1.06	0.77	0.78
time (sec)	N/A	0.069	0.100	0.045	0.000	0.364	0.317	3.213	0.185

Problem 24	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	347	347	257	29	0	1001	20	247	312
N.S.	1	1.00	0.74	0.08	0.00	2.88	0.06	0.71	0.90
time (sec)	N/A	0.180	0.102	0.183	0.000	0.424	1.115	4.773	1.956

Problem 25	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	355	355	57	44	0	719	26	253	208
N.S.	1	1.00	0.16	0.12	0.00	2.03	0.07	0.71	0.59
time (sec)	N/A	0.192	0.013	0.026	0.000	0.382	1.357	4.414	1.666

Problem 26	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	13	13	25	10	17	17	17	19	9
N.S.	1	1.00	1.92	0.77	1.31	1.31	1.31	1.46	0.69
time (sec)	N/A	0.003	0.004	0.022	0.485	0.332	0.047	5.844	0.025

Problem 27	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	129	129	129	110	0	255	51	147	269
N.S.	1	1.00	1.00	0.85	0.00	1.98	0.40	1.14	2.09
time (sec)	N/A	0.079	0.046	0.053	0.000	0.387	0.696	3.438	1.709

Problem 28	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	B	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	165	165	55	42	0	302	26	0	399
N.S.	1	1.00	0.33	0.25	0.00	1.83	0.16	0.00	2.42
time (sec)	N/A	0.068	0.009	0.024	0.000	0.362	0.080	0.000	0.181

Problem 29	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	B	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	169	169	57	44	0	546	26	0	483
N.S.	1	1.00	0.34	0.26	0.00	3.23	0.15	0.00	2.86
time (sec)	N/A	0.103	0.009	0.025	0.000	0.383	0.081	0.000	1.787

Problem 30	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	125	125	114	90	0	199	51	135	245
N.S.	1	1.00	0.91	0.72	0.00	1.59	0.41	1.08	1.96
time (sec)	N/A	0.045	0.032	0.046	0.000	0.377	0.688	3.757	0.199

Problem 31	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	135	135	71	47	0	153	163	107	133
N.S.	1	1.00	0.53	0.35	0.00	1.13	1.21	0.79	0.99
time (sec)	N/A	0.089	0.023	0.111	0.000	0.416	0.499	3.852	2.235

Problem 32	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	A	F(-2)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	164	164	72	62	0	158	0	123	1
N.S.	1	1.00	0.44	0.38	0.00	0.96	0.00	0.75	0.01
time (sec)	N/A	0.064	0.024	0.098	0.000	0.339	0.000	4.068	2.190

Problem 33	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	A	F(-2)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	180	180	89	62	0	181	0	131	1
N.S.	1	1.00	0.49	0.34	0.00	1.01	0.00	0.73	0.01
time (sec)	N/A	0.088	0.031	0.089	0.000	0.369	0.000	3.951	2.230

Problem 34	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	49	49	49	42	43	110	112	43	39
N.S.	1	1.00	1.00	0.86	0.88	2.24	2.29	0.88	0.80
time (sec)	N/A	0.021	0.017	0.024	0.498	0.343	0.134	4.525	1.594

Problem 35	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	86	86	86	90	0	295	423	85	127
N.S.	1	1.00	1.00	1.05	0.00	3.43	4.92	0.99	1.48
time (sec)	N/A	0.056	0.061	0.056	0.000	0.385	0.778	4.023	1.772

Problem 36	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	253	253	293	211	244	726	109	247	555
N.S.	1	1.00	1.16	0.83	0.96	2.87	0.43	0.98	2.19
time (sec)	N/A	0.138	0.060	0.044	0.497	0.358	0.372	3.595	0.313

Problem 37	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	208	208	251	212	0	2524	0	3183	2500
N.S.	1	1.00	1.21	1.02	0.00	12.13	0.00	15.30	12.02
time (sec)	N/A	0.375	0.115	0.046	0.000	0.448	0.000	4.643	2.854

Problem 38	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	311	311	346	339	300	3543	167	295	1308
N.S.	1	1.00	1.11	1.09	0.96	11.39	0.54	0.95	4.21
time (sec)	N/A	0.199	0.078	0.091	0.507	1.454	5.928	4.538	3.100

Problem 39	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F(-1)	F(-1)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	716	716	88	67	0	0	0	0	2500
N.S.	1	1.00	0.12	0.09	0.00	0.00	0.00	0.00	3.49
time (sec)	N/A	1.075	0.040	0.226	0.000	0.000	0.000	0.000	29.420

Problem 40	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	753	753	551	45	0	3552	0	647	2520
N.S.	1	1.00	0.73	0.06	0.00	4.72	0.00	0.86	3.35
time (sec)	N/A	0.987	0.621	0.035	0.000	2.056	0.000	3.651	1.220

Problem 41	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	B	F(-1)	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	433	433	88	67	0	22075	0	0	2500
N.S.	1	1.00	0.20	0.15	0.00	50.98	0.00	0.00	5.77
time (sec)	N/A	0.572	0.052	0.054	0.000	137.292	0.000	0.000	9.242

Problem 42	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	141	141	128	0	0	0	337	0	-1
N.S.	1	1.00	0.91	0.00	0.00	0.00	2.39	0.00	-0.01
time (sec)	N/A	0.100	0.425	0.008	0.000	0.000	4.342	0.000	0.000

Problem 43	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	107	107	107	0	0	0	207	0	-1
N.S.	1	1.00	1.00	0.00	0.00	0.00	1.93	0.00	-0.01
time (sec)	N/A	0.069	0.169	0.007	0.000	0.000	3.169	0.000	0.000

Problem 44	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	83	83	82	0	0	0	153	0	-1
N.S.	1	1.00	0.99	0.00	0.00	0.00	1.84	0.00	-0.01
time (sec)	N/A	0.019	0.059	0.040	0.000	0.000	2.379	0.000	0.000

Problem 45	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	152	152	131	0	0	0	0	0	-1
N.S.	1	1.00	0.86	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.079	0.136	0.076	0.000	0.000	0.000	0.000	0.000

Problem 46	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	205	205	188	0	0	0	0	0	-1
N.S.	1	1.00	0.92	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.121	0.393	0.019	0.000	0.000	0.000	0.000	0.000

Problem 47	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	81	81	80	0	0	0	158	0	-1
N.S.	1	1.00	0.99	0.00	0.00	0.00	1.95	0.00	-0.01
time (sec)	N/A	0.021	0.082	0.041	0.000	0.000	2.455	0.000	0.000

Problem 48	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	288	288	165	0	0	0	0	0	-1
N.S.	1	1.00	0.57	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.170	0.372	0.011	0.000	0.000	0.000	0.000	0.000

Problem 49	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	203	203	142	0	0	0	0	0	-1
N.S.	1	1.00	0.70	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.112	0.284	0.011	0.000	0.000	0.000	0.000	0.000

Problem 50	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	134	134	137	0	0	0	784	0	-1
N.S.	1	1.00	1.02	0.00	0.00	0.00	5.85	0.00	-0.01
time (sec)	N/A	0.037	0.113	0.011	0.000	0.000	180.695	0.000	0.000

Problem 51	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	333	333	245	0	0	0	0	0	-1
N.S.	1	1.00	0.74	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.155	0.350	0.020	0.000	0.000	0.000	0.000	0.000

Problem 52	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	410	410	495	0	0	0	0	0	-1
N.S.	1	1.00	1.21	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.262	0.673	0.024	0.000	0.000	0.000	0.000	0.000

Problem 53	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	424	424	252	0	0	0	0	0	-1
N.S.	1	1.00	0.59	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.265	1.005	0.020	0.000	0.000	0.000	0.000	0.000

Problem 54	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	272	272	212	0	0	0	0	0	-1
N.S.	1	1.00	0.78	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.172	0.700	0.019	0.000	0.000	0.000	0.000	0.000

Problem 55	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	184	184	164	0	0	0	0	0	-1
N.S.	1	1.00	0.89	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.071	0.234	0.019	0.000	0.000	0.000	0.000	0.000

Problem 56	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	582	582	1031	0	0	0	0	0	-1
N.S.	1	1.00	1.77	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.295	0.855	0.038	0.000	0.000	0.000	0.000	0.000

Problem 57	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	701	701	1241	0	0	0	0	0	-1
N.S.	1	1.00	1.77	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.481	1.555	0.045	0.000	0.000	0.000	0.000	0.000

Problem 58	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F	F	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	171	171	0	0	0	0	0	0	-1
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.117	0.155	0.068	0.000	0.000	0.000	0.000	0.000

Problem 59	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	24	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.005	0.139	0.081	0.000	0.000	0.000	0.000	0.000

Problem 60	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	299	299	213	0	0	0	0	0	-1
N.S.	1	1.00	0.71	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.111	0.238	0.052	0.000	0.000	0.000	0.000	0.000

Problem 61	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	C	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	217	217	171	0	0	0	177	0	-1
N.S.	1	1.00	0.79	0.00	0.00	0.00	0.82	0.00	-0.00
time (sec)	N/A	0.072	0.146	0.050	0.000	0.000	214.766	0.000	0.000

Problem 62	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	135	135	110	0	0	0	114	0	-1
N.S.	1	1.00	0.81	0.00	0.00	0.00	0.84	0.00	-0.01
time (sec)	N/A	0.044	0.074	0.050	0.000	0.000	129.601	0.000	0.000

Problem 63	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F	F	F(-2)	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	167	167	0	0	0	0	0	0	-1
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.098	0.086	0.090	0.000	0.000	0.000	0.000	0.000

Problem 64	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F	F	F(-1)	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	261	261	0	0	0	0	0	0	-1
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.168	0.107	0.067	0.000	0.000	0.000	0.000	0.000

Problem 65	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F	F	F(-1)	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	357	357	0	0	0	0	0	0	-1
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.226	0.256	0.068	0.000	0.000	0.000	0.000	0.000

Problem 66	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	62	62	57	60	82	145	656	207	59
N.S.	1	1.00	0.92	0.97	1.32	2.34	10.58	3.34	0.95
time (sec)	N/A	0.028	0.169	0.018	0.292	0.349	0.359	4.175	1.662

Problem 67	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	132	132	123	128	208	504	3128	828	131
N.S.	1	1.00	0.93	0.97	1.58	3.82	23.70	6.27	0.99
time (sec)	N/A	0.074	0.751	0.029	0.280	0.372	1.829	3.282	1.711

Problem 68	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	218	218	205	212	386	1232	9190	2134	227
N.S.	1	1.00	0.94	0.97	1.77	5.65	42.16	9.79	1.04
time (sec)	N/A	0.138	3.857	0.027	0.290	0.362	8.414	3.362	1.850

Problem 69	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	308	308	455	0	0	0	0	0	-1
N.S.	1	1.00	1.48	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.464	2.667	0.012	0.000	0.000	0.000	0.000	0.000

Problem 70	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	224	224	348	0	0	0	0	0	-1
N.S.	1	1.00	1.55	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.315	0.744	0.009	0.000	0.000	0.000	0.000	0.000

Problem 71	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	154	154	279	0	0	0	0	0	-1
N.S.	1	1.00	1.81	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.083	0.371	0.013	0.000	0.000	0.000	0.000	0.000

Problem 72	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	243	243	379	0	0	0	0	0	-1
N.S.	1	1.00	1.56	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.293	0.979	0.049	0.000	0.000	0.000	0.000	0.000

Problem 73	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	368	368	564	0	0	0	0	0	-1
N.S.	1	1.00	1.53	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.463	5.316	0.023	0.000	0.000	0.000	0.000	0.000

Problem 74	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	552	552	2482	0	0	0	0	0	-1
N.S.	1	1.00	4.50	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.677	5.539	0.033	0.000	0.000	0.000	0.000	0.000

Problem 75	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	750	750	5537	0	0	0	0	0	-1
N.S.	1	1.00	7.38	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.973	7.259	0.016	0.000	0.000	0.000	0.000	0.000

Problem 76	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	543	543	2980	0	0	0	0	0	-1
N.S.	1	1.00	5.49	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.251	3.483	0.016	0.000	0.000	0.000	0.000	0.000

Problem 77	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	362	328	603	0	0	0	0	0	-1
N.S.	1	0.91	1.67	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.425	3.864	0.015	0.000	0.000	0.000	0.000	0.000

Problem 78	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	726	726	11767	0	0	0	0	0	-1
N.S.	1	1.00	16.21	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.273	6.804	0.026	0.000	0.000	0.000	0.000	0.000

Problem 79	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F	F(-2)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	1129	1129	16855	0	0	0	0	0	-1
N.S.	1	1.00	14.93	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	2.179	7.374	0.036	0.000	0.000	0.000	0.000	0.000

Problem 80	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	1707	1707	13018	0	0	0	0	0	-1
N.S.	1	1.00	7.63	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	3.309	7.928	0.037	0.000	0.000	0.000	0.000	0.000

Problem 81	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	1191	1191	10910	0	0	0	0	0	-1
N.S.	1	1.00	9.16	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	2.545	7.406	0.035	0.000	0.000	0.000	0.000	0.000

Problem 82	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	713	713	8593	0	0	0	0	0	-1
N.S.	1	1.00	12.05	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.139	6.772	0.036	0.000	0.000	0.000	0.000	0.000

Problem 83	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F	F(-2)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	1708	1708	43535	0	0	0	0	0	-1
N.S.	1	1.00	25.49	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	3.346	7.824	0.057	0.000	0.000	0.000	0.000	0.000

Problem 84	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F	F(-2)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	2446	2446	56566	0	0	0	0	0	-1
N.S.	1	1.00	23.13	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	5.754	8.895	0.073	0.000	0.000	0.000	0.000	0.000

Problem 85	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	292	292	424	0	0	0	0	0	-1
N.S.	1	1.00	1.45	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.244	1.067	0.003	0.000	0.000	0.000	0.000	0.000

Problem 86	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F(-2)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	294	294	690	0	0	0	0	0	-1
N.S.	1	1.00	2.35	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.229	3.269	0.003	0.000	0.000	0.000	0.000	0.000

Problem 87	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	292	292	245	0	0	0	0	0	-1
N.S.	1	1.00	0.84	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.219	0.321	0.008	0.000	0.000	0.000	0.000	0.000

Problem 88	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	298	298	414	0	0	0	0	0	-1
N.S.	1	1.00	1.39	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.235	1.846	0.009	0.000	0.000	0.000	0.000	0.000

Problem 89	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F(-2)	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	298	298	701	0	0	0	0	0	-1
N.S.	1	1.00	2.35	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.226	5.640	0.007	0.000	0.000	0.000	0.000	0.000

Problem 90	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	29	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.03
time (sec)	N/A	0.007	0.264	0.053	0.000	0.000	0.000	0.000	0.000

Problem 91	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	606	606	438	0	0	0	0	0	-1
N.S.	1	1.00	0.72	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.412	0.772	0.021	0.000	0.000	0.000	0.000	0.000

Problem 92	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	447	447	338	0	0	0	0	0	-1
N.S.	1	1.00	0.76	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.296	0.538	0.022	0.000	0.000	0.000	0.000	0.000

Problem 93	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	288	288	243	0	0	0	0	0	-1
N.S.	1	1.00	0.84	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.190	0.356	0.029	0.000	0.000	0.000	0.000	0.000

Problem 94	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-2)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	29	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.03
time (sec)	N/A	0.008	0.200	0.069	0.000	0.000	0.000	0.000	0.000

Problem 95	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-2)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	29	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.03
time (sec)	N/A	0.007	0.227	0.040	0.000	0.000	0.000	0.000	0.000

Problem 96	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	29	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.03
time (sec)	N/A	0.007	0.605	0.040	0.000	0.000	0.000	0.000	0.000

2.3 Detailed conclusion table specific for Rubi results

The following table is specific to Rubi. It gives additional statistics for each integral. the column **steps** is the number of steps used by Rubi to obtain the antiderivative. The **rules** column is the number of unique rules used. The **integrand size** column is the leaf size of the integrand. Finally the ratio $\frac{\text{number of rules}}{\text{integrand size}}$ is given. The larger this ratio is, the harder the integral was to solve. In this test, problem number [33] had the largest ratio of [33]

Table 2.1: Rubi specific breakdown of results for each integral

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1	A	12	8	1.00	17	0.471
2	A	13	7	1.00	18	0.389
3	A	19	6	1.00	17	0.353
4	A	13	10	1.00	18	0.556
5	A	19	6	1.00	26	0.231
6	A	19	6	1.00	26	0.231
7	A	7	4	1.00	27	0.148
8	A	19	6	1.00	27	0.222
9	A	19	6	1.00	18	0.333
10	A	19	7	1.00	18	0.389
11	A	10	7	1.00	18	0.389
12	A	19	6	1.00	16	0.375
13	A	19	6	1.00	13	0.462
14	A	19	6	1.00	18	0.333
15	A	5	5	1.00	18	0.278
16	A	7	4	1.00	18	0.222
17	A	7	4	1.00	18	0.222
18	A	7	4	1.00	18	0.222
19	A	7	4	1.00	18	0.222
20	A	19	6	1.00	20	0.300
21	A	19	7	1.00	20	0.350
22	A	11	8	1.00	20	0.400
23	A	19	6	1.00	18	0.333
24	A	19	6	1.00	15	0.400
25	A	19	6	1.00	20	0.300

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
26	A	5	5	1.00	20	0.250
27	A	7	4	1.00	20	0.200
28	A	7	4	1.00	20	0.200
29	A	7	4	1.00	20	0.200
30	A	7	4	1.00	20	0.200
31	A	9	6	1.00	25	0.240
32	A	9	6	1.00	26	0.231
33	A	9	6	1.00	33	0.182
34	A	5	5	1.00	17	0.294
35	A	6	6	1.00	22	0.273
36	A	11	8	1.00	17	0.471
37	A	5	4	1.00	22	0.182
38	A	14	10	1.00	17	0.588
39	A	15	9	1.00	22	0.409
40	A	21	8	1.00	17	0.471
41	A	9	6	1.00	22	0.273
42	A	5	4	1.00	21	0.190
43	A	5	4	1.00	21	0.190
44	A	3	3	1.00	19	0.158
45	A	6	4	1.00	21	0.190
46	A	7	4	1.00	21	0.190
47	A	3	3	1.00	20	0.150
48	A	9	5	1.00	21	0.238
49	A	7	5	1.00	21	0.238
50	A	4	4	1.00	19	0.210
51	A	10	5	1.00	21	0.238
52	A	11	5	1.00	21	0.238
53	A	11	5	1.00	21	0.238
54	A	8	5	1.00	21	0.238
55	A	5	4	1.00	19	0.210
56	A	15	5	1.00	21	0.238
57	A	16	5	1.00	21	0.238
58	A	6	5	1.00	23	0.217
59	A	0	0	0.00	0	0.000
60	A	10	5	1.00	21	0.238

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
61	A	8	5	1.00	21	0.238
62	A	6	5	1.00	19	0.263
63	A	6	5	1.00	21	0.238
64	A	8	5	1.00	21	0.238
65	A	10	5	1.00	21	0.238
66	A	2	1	1.00	22	0.045
67	A	2	1	1.00	24	0.042
68	A	2	1	1.00	24	0.042
69	A	5	3	1.00	26	0.115
70	A	5	3	1.00	26	0.115
71	A	3	2	1.00	24	0.083
72	A	6	3	1.00	26	0.115
73	A	7	3	1.00	26	0.115
74	A	8	3	1.00	26	0.115
75	A	9	4	1.00	26	0.154
76	A	9	5	1.00	26	0.192
77	A	4	3	0.91	24	0.125
78	A	10	4	1.00	26	0.154
79	A	11	4	1.00	26	0.154
80	A	11	4	1.00	26	0.154
81	A	11	5	1.00	26	0.192
82	A	5	3	1.00	24	0.125
83	A	15	4	1.00	26	0.154
84	A	16	4	1.00	26	0.154
85	A	6	5	1.00	26	0.192
86	A	6	5	1.00	26	0.192
87	A	6	5	1.00	26	0.192
88	A	6	5	1.00	26	0.192
89	A	6	5	1.00	26	0.192
90	A	0	0	0.00	0	0.000
91	A	10	5	1.00	26	0.192
92	A	8	5	1.00	26	0.192
93	A	6	5	1.00	24	0.208
94	A	0	0	0.00	0	0.000
95	A	0	0	0.00	0	0.000

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
96	A	0	0	0.00	0	0.000

Chapter 3

Listing of integrals

Local contents

3.1 $\int \frac{d+ex^3}{a+cx^6} \, dx$

Optimal. Leaf size=305

$$\frac{d\tan^{-1}\left(\frac{\sqrt[6]{c}\,x}{\sqrt[6]{a}}\right)}{3a^{5/6}\sqrt[6]{c}}-\frac{\left(\sqrt{c}\,d+\sqrt{3}\,\sqrt{a}\,e\right)\tan^{-1}\left(\sqrt{3}-\frac{2\sqrt[6]{c}\,x}{\sqrt[6]{a}}\right)}{6a^{5/6}c^{2/3}}+\frac{\left(\sqrt{c}\,d-\sqrt{3}\,\sqrt{a}\,e\right)\tan^{-1}\left(\sqrt{3}+\frac{2\sqrt[6]{c}\,x}{\sqrt[6]{a}}\right.}{6a^{5/6}c^{2/3}}$$

```
[Out] 1/3*d*arctan(c^(1/6)*x/a^(1/6))/a^(5/6)/c^(1/6)-1/6*e*ln(a^(1/3)+c^(1/3)*x^
2)/a^(1/3)/c^(2/3)+1/6*arctan(2*c^(1/6)*x/a^(1/6)+3^(1/2))*(-e*3^(1/2)*a^(1
/2)+d*c^(1/2))/a^(5/6)/c^(2/3)+1/6*arctan(2*c^(1/6)*x/a^(1/6)-3^(1/2))*(e*3
^(1/2)*a^(1/2)+d*c^(1/2))/a^(5/6)/c^(2/3)-1/12*ln(a^(1/3)+c^(1/3)*x^2-a^(1/
6)*c^(1/6)*x*3^(1/2))*(-e*a^(1/2)+d*3^(1/2)*c^(1/2))/a^(5/6)/c^(2/3)+1/12*l
n(a^(1/3)+c^(1/3)*x^2+a^(1/6)*c^(1/6)*x*3^(1/2))*(e*a^(1/2)+d*3^(1/2)*c^(1/
2))/a^(5/6)/c^(2/3)
```

Rubi [A]

time = 0.17, antiderivative size = 305, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 8, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.471$, Rules used = {1430, 649, 209, 266, 648, 631, 210, 642}

$$-\frac{\text{ArcTan}\left(\sqrt{3}-\frac{2\sqrt[6]{c}\,x}{\sqrt[6]{a}}\right)\left(\sqrt{3}\,\sqrt{a}\,e+\sqrt{c}\,d\right)}{6a^{5/6}c^{2/3}}+\frac{\text{ArcTan}\left(\frac{2\sqrt[6]{c}\,x}{\sqrt[6]{a}}+\sqrt{3}\right)\left(\sqrt{c}\,d-\sqrt{3}\,\sqrt{a}\,e\right)}{6a^{5/6}c^{2/3}}+\frac{d\text{ArcTan}\left(\frac{\sqrt[6]{c}\,x}{\sqrt[6]{a}}\right)}{3a^{5/6}\sqrt[6]{c}}-\frac{\left(\sqrt{3}\,\sqrt{c}\,d-\sqrt{a}\,e\right)\log\left(-\sqrt{3}\,\sqrt[6]{a}\,\sqrt[6]{c}\,x+\sqrt[3]{a}+\sqrt[3]{c}\,x^2\right)}{12a^{5/6}c^{2/3}}+\frac{\left(\sqrt{a}\,e+\sqrt{3}\,\sqrt{c}\,d\right)\log\left(\sqrt{3}\,\sqrt[6]{a}\,\sqrt[6]{c}\,x+\sqrt[3]{a}+\sqrt[3]{c}\,x^2\right)}{12a^{5/6}c^{2/3}}-\frac{e\log\left(\sqrt[3]{a}+\sqrt[3]{c}\,x^2\right)}{6\sqrt[3]{a}\,c^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x^3)/(a + c*x^6),x]
```

```
[Out] (d*ArcTan[(c^(1/6)*x)/a^(1/6)])/(3*a^(5/6)*c^(1/6)) - ((Sqrt[c]*d + Sqrt[3]
*Sqrt[a]*e)*ArcTan[Sqrt[3] - (2*c^(1/6)*x)/a^(1/6)])/(6*a^(5/6)*c^(2/3)) +
((Sqrt[c]*d - Sqrt[3]*Sqrt[a]*e)*ArcTan[Sqrt[3] + (2*c^(1/6)*x)/a^(1/6)])/(
6*a^(5/6)*c^(2/3)) - (e*Log[a^(1/3) + c^(1/3)*x^2])/(6*a^(1/3)*c^(2/3)) - (
(Sqrt[3]*Sqrt[c]*d - Sqrt[a]*e)*Log[a^(1/3) - Sqrt[3]*a^(1/6)*c^(1/6)*x + c
^(1/3)*x^2])/(12*a^(5/6)*c^(2/3)) + ((Sqrt[3]*Sqrt[c]*d + Sqrt[a]*e)*Log[a^
(1/3) + Sqrt[3]*a^(1/6)*c^(1/6)*x + c^(1/3)*x^2])/(12*a^(5/6)*c^(2/3))
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 649

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[(-a)*c]
```

Rule 1430

```
Int[((d_) + (e_.)*(x_)^3)/((a_) + (c_.)*(x_)^6), x_Symbol] :> With[{q = Rt[
c/a, 6]}, Dist[1/(3*a*q^2), Int[(q^2*d - e*x)/(1 + q^2*x^2), x], x] + (Dist
[1/(6*a*q^2), Int[(2*q^2*d - (Sqrt[3]*q^3*d - e)*x)/(1 - Sqrt[3]*q*x + q^2*
x^2), x], x] + Dist[1/(6*a*q^2), Int[(2*q^2*d + (Sqrt[3]*q^3*d + e)*x)/(1 +
 Sqrt[3]*q*x + q^2*x^2), x], x])] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 +
a*e^2, 0] && PosQ[c/a]
```

Rubi steps

$$\int \frac{d+ex^3}{a+cx^6}\,dx = \frac{\int \frac{\frac{2\sqrt[3]{c}\,d}{\sqrt[3]{a}}-\left(\frac{\sqrt{3}\,\sqrt{c}\,d}{\sqrt{a}}-e\right)x}{1-\frac{\sqrt{3}\,\sqrt[6]{c}\,x}{\sqrt[6]{a}}+\frac{\sqrt[3]{c}\,x^2}{\sqrt[3]{a}}}\,dx}{6a^{2/3}\sqrt[3]{c}} + \frac{\int \frac{\frac{2\sqrt[3]{c}\,d}{\sqrt[3]{a}}+\left(\frac{\sqrt{3}\,\sqrt{c}\,d}{\sqrt{a}}+e\right)x}{1+\frac{\sqrt{3}\,\sqrt[6]{c}\,x}{\sqrt[6]{a}}+\frac{\sqrt[3]{c}\,x^2}{\sqrt[3]{a}}}\,dx}{6a^{2/3}\sqrt[3]{c}} + \frac{\int \frac{\frac{\sqrt[3]{c}\,d}{\sqrt[3]{a}}-ex}{1+\frac{\sqrt[3]{c}\,x^2}{\sqrt[3]{a}}}\,dx}{3a^{2/3}\sqrt[3]{c}}$$

$$= \frac{d\int \frac{1}{1+\frac{\sqrt[3]{c}\,x^2}{\sqrt[3]{a}}}\,dx}{3a} - \frac{e\int \frac{x}{1+\frac{\sqrt[3]{c}\,x^2}{\sqrt[3]{a}}}\,dx}{3a^{2/3}\sqrt[3]{c}} - \frac{\left(\sqrt{3}\,\sqrt{c}\,d-\sqrt{a}\,e\right)\int \frac{-\frac{\sqrt{3}\,\sqrt[6]{c}}{\sqrt[6]{a}}+\frac{2\sqrt[3]{c}\,x}{\sqrt[3]{a}}}{1-\frac{\sqrt{3}\,\sqrt[6]{c}\,x}{\sqrt[6]{a}}+\frac{\sqrt[3]{c}\,x^2}{\sqrt[3]{a}}}\,dx}{12a^{5/6}c^{2/3}} + \frac{(\sqrt{3}}{}$$

$$= \frac{d\tan^{-1}\left(\frac{\sqrt[6]{c}\,x}{\sqrt[6]{a}}\right)}{3a^{5/6}\sqrt[6]{c}} - \frac{e\log\left(\sqrt[3]{a}+\sqrt[3]{c}\,x^2\right)}{6\sqrt[3]{a}\,c^{2/3}} - \frac{\left(\sqrt{3}\,\sqrt{c}\,d-\sqrt{a}\,e\right)\log\left(\sqrt[3]{a}-\sqrt{3}\,\sqrt[6]{a}\,\sqrt[6]{c}\,x\right.}{12a^{5/6}c^{2/3}}$$

$$= \frac{d\tan^{-1}\left(\frac{\sqrt[6]{c}\,x}{\sqrt[6]{a}}\right)}{3a^{5/6}\sqrt[6]{c}} - \frac{\left(\sqrt{c}\,d+\sqrt{3}\,\sqrt{a}\,e\right)\tan^{-1}\left(\sqrt{3}-\frac{2\sqrt[6]{c}\,x}{\sqrt[6]{a}}\right)}{6a^{5/6}c^{2/3}} + \frac{\left(\sqrt{c}\,d-\sqrt{3}\,\sqrt{a}\,e\right)\mathrm{ta}}{6a^{5/6}}$$

Mathematica [A]
time = 0.07, size = 334, normalized size = 1.10

$$\frac{d\tan^{-1}\left(\frac{\sqrt[6]{c}\,x}{\sqrt[6]{a}}\right)}{3a^{5/6}\sqrt[6]{c}} + \frac{\left(\sqrt[6]{a}\,\sqrt{c}\,d+\sqrt{3}\,a^{2/3}e\right)\tan^{-1}\left(\frac{-\sqrt{3}\,\sqrt[6]{a}+2\sqrt[6]{c}\,x}{\sqrt[6]{a}}\right)}{6ac^{2/3}} + \frac{\left(\sqrt[6]{a}\,\sqrt{c}\,d-\sqrt{3}\,a^{2/3}e\right)\tan^{-1}\left(\frac{\sqrt{3}\,\sqrt[6]{a}+2\sqrt[6]{c}\,x}{\sqrt[6]{a}}\right)}{6ac^{2/3}} - \frac{e\log\left(\sqrt[3]{a}+\sqrt[3]{c}\,x^2\right)}{6\sqrt[3]{a}\,c^{2/3}} - \frac{\left(\sqrt{3}\,\sqrt[6]{a}\,\sqrt{c}\,d-a^{2/3}e\right)\log\left(\sqrt[3]{a}-\sqrt{3}\,\sqrt[6]{a}\,\sqrt[6]{c}\,x+\sqrt[3]{c}\,x^2\right)}{12ac^{2/3}} - \frac{\left(-\sqrt{3}\,\sqrt[6]{a}\,\sqrt{c}\,d-a^{2/3}e\right)\log\left(\sqrt[3]{a}+\sqrt{3}\,\sqrt[6]{a}\,\sqrt[6]{c}\,x+\sqrt[3]{c}\,x^2\right)}{12ac^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x^3)/(a + c*x^6),x]
```

```
[Out] (d*ArcTan[(c^(1/6)*x)/a^(1/6)])/(3*a^(5/6)*c^(1/6)) + ((a^(1/6)*Sqrt[c]*d +
 Sqrt[3]*a^(2/3)*e)*ArcTan[(-(Sqrt[3]*a^(1/6)) + 2*c^(1/6)*x)/a^(1/6)])/(6*
a*c^(2/3)) + ((a^(1/6)*Sqrt[c]*d - Sqrt[3]*a^(2/3)*e)*ArcTan[(Sqrt[3]*a^(1/
6) + 2*c^(1/6)*x)/a^(1/6)])/(6*a*c^(2/3)) - (e*Log[a^(1/3) + c^(1/3)*x^2])/
(6*a^(1/3)*c^(2/3)) - ((Sqrt[3]*a^(1/6)*Sqrt[c]*d - a^(2/3)*e)*Log[a^(1/3)
- Sqrt[3]*a^(1/6)*c^(1/6)*x + c^(1/3)*x^2])/(12*a*c^(2/3)) - ((-(Sqrt[3]*a^
(1/6)*Sqrt[c]*d) - a^(2/3)*e)*Log[a^(1/3) + Sqrt[3]*a^(1/6)*c^(1/6)*x + c^(
1/3)*x^2])/(12*a*c^(2/3))
```

Maple [A]
time = 0.21, size = 329, normalized size = 1.08

method	result

risch	$\frac{\sum_{\_R=\mathrm{RootOf}\left(\_Z^{6}c+a\right)}\frac{\left(\_R^{3}e+d\right)\ln\left(x-\_R\right)}{\_R^{5}}}{6c}$
default	$\frac{c\left(\frac{a}{c}\right)^{\frac{7}{6}}\ln\left(x^{2}+\sqrt{3}\,\left(\frac{a}{c}\right)^{\frac{1}{6}}x+\left(\frac{a}{c}\right)^{\frac{1}{3}}\right)\sqrt{3}\,d}{12a^{2}}+\frac{\left(\frac{a}{c}\right)^{\frac{2}{3}}\ln\left(x^{2}+\sqrt{3}\,\left(\frac{a}{c}\right)^{\frac{1}{6}}x+\left(\frac{a}{c}\right)^{\frac{1}{3}}\right)e}{12a}+\frac{\left(\frac{a}{c}\right)^{\frac{1}{6}}\arctan\left(\frac{2x}{\left(\frac{a}{c}\right)^{\frac{1}{6}}}+\sqrt{3}\right)d}{6a}-\frac{\left(\frac{a}{c}\right)^{\frac{2}{3}}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^3+d)/(c*x^6+a),x,method=_RETURNVERBOSE)
```

```
[Out] 1/12*c*(a/c)^(7/6)/a^2*ln(x^2+3^(1/2)*(a/c)^(1/6)*x+(a/c)^(1/3))*3^(1/2)*d+
1/12*(a/c)^(2/3)/a*ln(x^2+3^(1/2)*(a/c)^(1/6)*x+(a/c)^(1/3))*e+1/6*(a/c)^(1
/6)/a*arctan(2*x/(a/c)^(1/6)+3^(1/2))*d-1/6*(a/c)^(2/3)/a*arctan(2*x/(a/c)^
(1/6)+3^(1/2))*3^(1/2)*e-1/6*(a/c)^(2/3)/a*e*ln(x^2+(a/c)^(1/3))+1/3*(a/c)^
(1/6)/a*d*arctan(x/(a/c)^(1/6))+1/12/a*ln(x^2-3^(1/2)*(a/c)^(1/6)*x+(a/c)^(
1/3))*(a/c)^(2/3)*e-1/12/a*ln(x^2-3^(1/2)*(a/c)^(1/6)*x+(a/c)^(1/3))*3^(1/2
)*(a/c)^(1/6)*d+1/6/a*(a/c)^(2/3)*arctan(2*x/(a/c)^(1/6)-3^(1/2))*3^(1/2)*e
+1/6/a*(a/c)^(1/6)*arctan(2*x/(a/c)^(1/6)-3^(1/2))*d
```

Maxima [A]
time = 0.48, size = 287, normalized size = 0.94

$$-\frac{e\log\left(c^{\frac{1}{3}}x^{2}+a^{\frac{1}{3}}\right)}{6\,a^{\frac{1}{3}}c^{\frac{2}{3}}}+\frac{d\arctan\left(\frac{c^{\frac{1}{3}}x}{\sqrt{a^{\frac{1}{3}}c^{\frac{1}{3}}}}\right)}{3\,a^{\frac{2}{3}}\sqrt{a^{\frac{1}{3}}c^{\frac{1}{3}}}}+\frac{\left(\sqrt{3}\,a^{\frac{1}{6}}\sqrt{c}\,d+a^{\frac{2}{3}}e\right)\log\left(c^{\frac{1}{3}}x^{2}+\sqrt{3}\,a^{\frac{1}{6}}c^{\frac{1}{6}}x+a^{\frac{1}{3}}\right)}{12\,ac^{\frac{2}{3}}}-\frac{\left(\sqrt{3}\,a^{\frac{1}{6}}\sqrt{c}\,d-a^{\frac{2}{3}}e\right)\log\left(c^{\frac{1}{3}}x^{2}-\sqrt{3}\,a^{\frac{1}{6}}c^{\frac{1}{6}}x+a^{\frac{1}{3}}\right)}{12\,ac^{\frac{2}{3}}}-\frac{\left(\sqrt{3}\,a^{\frac{5}{6}}c^{\frac{1}{6}}e-a^{\frac{1}{3}}c^{\frac{2}{3}}d\right)\arctan\left(\frac{2\,c^{\frac{1}{3}}x+\sqrt{3}\,a^{\frac{1}{6}}c^{\frac{1}{6}}}{\sqrt{a^{\frac{1}{3}}c^{\frac{1}{3}}}}\right)}{6\,ac^{\frac{2}{3}}\sqrt{a^{\frac{1}{3}}c^{\frac{1}{3}}}}+\frac{\left(\sqrt{3}\,a^{\frac{5}{6}}c^{\frac{1}{6}}e+a^{\frac{1}{3}}c^{\frac{2}{3}}d\right)\arctan\left(\frac{2\,c^{\frac{1}{3}}x-\sqrt{3}\,a^{\frac{1}{6}}c^{\frac{1}{6}}}{\sqrt{a^{\frac{1}{3}}c^{\frac{1}{3}}}}\right)}{6\,ac^{\frac{2}{3}}\sqrt{a^{\frac{1}{3}}c^{\frac{1}{3}}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^3+d)/(c*x^6+a),x, algorithm="maxima")
```

```
[Out] -1/6*e*log(c^(1/3)*x^2 + a^(1/3))/(a^(1/3)*c^(2/3)) + 1/3*d*arctan(c^(1/3)*
x/sqrt(a^(1/3)*c^(1/3)))/(a^(2/3)*sqrt(a^(1/3)*c^(1/3))) + 1/12*(sqrt(3)*a^
(1/6)*sqrt(c)*d + a^(2/3)*e)*log(c^(1/3)*x^2 + sqrt(3)*a^(1/6)*c^(1/6)*x +
a^(1/3))/(a*c^(2/3)) - 1/12*(sqrt(3)*a^(1/6)*sqrt(c)*d - a^(2/3)*e)*log(c^(
1/3)*x^2 - sqrt(3)*a^(1/6)*c^(1/6)*x + a^(1/3))/(a*c^(2/3)) - 1/6*(sqrt(3)*
a^(5/6)*c^(1/6)*e - a^(1/3)*c^(2/3)*d)*arctan((2*c^(1/3)*x + sqrt(3)*a^(1/6
)*c^(1/6))/sqrt(a^(1/3)*c^(1/3)))/(a*c^(2/3)*sqrt(a^(1/3)*c^(1/3))) + 1/6*(
sqrt(3)*a^(5/6)*c^(1/6)*e + a^(1/3)*c^(2/3)*d)*arctan((2*c^(1/3)*x - sqrt(3
)*a^(1/6)*c^(1/6))/sqrt(a^(1/3)*c^(1/3)))/(a*c^(2/3)*sqrt(a^(1/3)*c^(1/3)))
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 3602 vs. $2(212) = 424$.
time = 1.27, size = 3602, normalized size = 11.81

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^3+d)/(c*x^6+a),x, algorithm="fricas")

[Out] 1/3*sqrt(3)*((a^2*c^2*sqrt(-(c^2*d^6 - 6*a*c*d^4*e^2 + 9*a^2*d^2*e^4)/(a^5*
c^3)) + 3*c*d^2*e - a*e^3)/(a^2*c^2))^(1/3)*arctan(-1/3*(2*sqrt(c^4*d^10*x^
2 - 4*a*c^3*d^8*x^2*e^2 - 2*a^2*c^2*d^6*x^2*e^4 + 12*a^3*c*d^4*x^2*e^6 + 9*
a^4*d^2*x^2*e^8 + (a^2*c^4*d^8 - 7*a^3*c^3*d^6*e^2 + 15*a^4*c^2*d^4*e^4 - 9
*a^5*c*d^2*e^6 + 2*(a^5*c^4*d^4*e - 3*a^6*c^3*d^2*e^3)*sqrt(-(c^2*d^6 - 6*a
*c*d^4*e^2 + 9*a^2*d^2*e^4)/(a^5*c^3)))*((a^2*c^2*sqrt(-(c^2*d^6 - 6*a*c*d^
4*e^2 + 9*a^2*d^2*e^4)/(a^5*c^3)) + 3*c*d^2*e - a*e^3)/(a^2*c^2))^(2/3) - (
a*c^4*d^9*x - 5*a^2*c^3*d^7*x*e^2 + 3*a^3*c^2*d^5*x*e^4 + 9*a^4*c*d^3*x*e^6
 + (a^4*c^4*d^5*x*e - 2*a^5*c^3*d^3*x*e^3 - 3*a^6*c^2*d*x*e^5)*sqrt(-(c^2*d
^6 - 6*a*c*d^4*e^2 + 9*a^2*d^2*e^4)/(a^5*c^3)))*((a^2*c^2*sqrt(-(c^2*d^6 -
6*a*c*d^4*e^2 + 9*a^2*d^2*e^4)/(a^5*c^3)) + 3*c*d^2*e - a*e^3)/(a^2*c^2))^(
1/3))*(sqrt(3)*(a^4*c^4*d^2 - a^5*c^3*e^2)*sqrt(-(c^2*d^6 - 6*a*c*d^4*e^2 +
 9*a^2*d^2*e^4)/(a^5*c^3)) - 2*sqrt(3)*(a^2*c^3*d^4*e - 3*a^3*c^2*d^2*e^3))
*((a^2*c^2*sqrt(-(c^2*d^6 - 6*a*c*d^4*e^2 + 9*a^2*d^2*e^4)/(a^5*c^3)) + 3*c
*d^2*e - a*e^3)/(a^2*c^2))^(2/3) + 2*(sqrt(3)*(a^4*c^6*d^7*x - 3*a^5*c^5*d^
5*x*e^2 - a^6*c^4*d^3*x*e^4 + 3*a^7*c^3*d*x*e^6)*sqrt(-(c^2*d^6 - 6*a*c*d^4
*e^2 + 9*a^2*d^2*e^4)/(a^5*c^3)) - 2*sqrt(3)*(a^2*c^5*d^9*x*e - 5*a^3*c^4*d
^7*x*e^3 + 3*a^4*c^3*d^5*x*e^5 + 9*a^5*c^2*d^3*x*e^7))*((a^2*c^2*sqrt(-(c^2
*d^6 - 6*a*c*d^4*e^2 + 9*a^2*d^2*e^4)/(a^5*c^3)) + 3*c*d^2*e - a*e^3)/(a^2*
c^2))^(2/3) + sqrt(3)*(c^5*d^12 - 3*a*c^4*d^10*e^2 - 6*a^2*c^3*d^8*e^4 + 10
*a^3*c^2*d^6*e^6 + 21*a^4*c*d^4*e^8 + 9*a^5*d^2*e^10))/(c^5*d^12 - 3*a*c^4*
d^10*e^2 - 6*a^2*c^3*d^8*e^4 + 10*a^3*c^2*d^6*e^6 + 21*a^4*c*d^4*e^8 + 9*a^
5*d^2*e^10)) - 1/3*sqrt(3)*(-(a^2*c^2*sqrt(-(c^2*d^6 - 6*a*c*d^4*e^2 + 9*a^
2*d^2*e^4)/(a^5*c^3)) - 3*c*d^2*e + a*e^3)/(a^2*c^2))^(1/3)*arctan(-1/3*(2*
sqrt(c^4*d^10*x^2 - 4*a*c^3*d^8*x^2*e^2 - 2*a^2*c^2*d^6*x^2*e^4 + 12*a^3*c*
d^4*x^2*e^6 + 9*a^4*d^2*x^2*e^8 + (a^2*c^4*d^8 - 7*a^3*c^3*d^6*e^2 + 15*a^4
*c^2*d^4*e^4 - 9*a^5*c*d^2*e^6 - 2*(a^5*c^4*d^4*e - 3*a^6*c^3*d^2*e^3)*sqrt
(-(c^2*d^6 - 6*a*c*d^4*e^2 + 9*a^2*d^2*e^4)/(a^5*c^3)))*(-(a^2*c^2*sqrt(-(c
^2*d^6 - 6*a*c*d^4*e^2 + 9*a^2*d^2*e^4)/(a^5*c^3)) - 3*c*d^2*e + a*e^3)/(a^
2*c^2))^(2/3) - (a*c^4*d^9*x - 5*a^2*c^3*d^7*x*e^2 + 3*a^3*c^2*d^5*x*e^4 +
9*a^4*c*d^3*x*e^6 - (a^4*c^4*d^5*x*e - 2*a^5*c^3*d^3*x*e^3 - 3*a^6*c^2*d*x*
e^5)*sqrt(-(c^2*d^6 - 6*a*c*d^4*e^2 + 9*a^2*d^2*e^4)/(a^5*c^3)))*(-(a^2*c^2
*sqrt(-(c^2*d^6 - 6*a*c*d^4*e^2 + 9*a^2*d^2*e^4)/(a^5*c^3)) - 3*c*d^2*e + a
*e^3)/(a^2*c^2))^(1/3))*(sqrt(3)*(a^4*c^4*d^2 - a^5*c^3*e^2)*sqrt(-(c^2*d^6
 - 6*a*c*d^4*e^2 + 9*a^2*d^2*e^4)/(a^5*c^3)) + 2*sqrt(3)*(a^2*c^3*d^4*e - 3
*a^3*c^2*d^2*e^3))*(-(a^2*c^2*sqrt(-(c^2*d^6 - 6*a*c*d^4*e^2 + 9*a^2*d^2*e^
4)/(a^5*c^3)) - 3*c*d^2*e + a*e^3)/(a^2*c^2))^(2/3) + 2*(sqrt(3)*(a^4*c^6*d
^7*x - 3*a^5*c^5*d^5*x*e^2 - a^6*c^4*d^3*x*e^4 + 3*a^7*c^3*d*x*e^6)*sqrt(-(
c^2*d^6 - 6*a*c*d^4*e^2 + 9*a^2*d^2*e^4)/(a^5*c^3)) + 2*sqrt(3)*(a^2*c^5*d^
9*x*e - 5*a^3*c^4*d^7*x*e^3 + 3*a^4*c^3*d^5*x*e^5 + 9*a^5*c^2*d^3*x*e^7))*(
-(a^2*c^2*sqrt(-(c^2*d^6 - 6*a*c*d^4*e^2 + 9*a^2*d^2*e^4)/(a^5*c^3)) - 3*c*
d^2*e + a*e^3)/(a^2*c^2))^(2/3) - sqrt(3)*(c^5*d^12 - 3*a*c^4*d^10*e^2 - 6*
a^2*c^3*d^8*e^4 + 10*a^3*c^2*d^6*e^6 + 21*a^4*c*d^4*e^8 + 9*a^5*d^2*e^10))/
```

```
(c^5*d^12 - 3*a*c^4*d^10*e^2 - 6*a^2*c^3*d^8*e^4 + 10*a^3*c^2*d^6*e^6 + 21*
a^4*c*d^4*e^8 + 9*a^5*d^2*e^10)) - 1/12*((a^2*c^2*sqrt(-(c^2*d^6 - 6*a*c*d^
4*e^2 + 9*a^2*d^2*e^4)/(a^5*c^3)) + 3*c*d^2*e - a*e^3)/(a^2*c^2))^(1/3)*log
(c^4*d^10*x^2 - 4*a*c^3*d^8*x^2*e^2 - 2*a^2*c^2*d^6*x^2*e^4 + 12*a^3*c*d^4*
x^2*e^6 + 9*a^4*d^2*x^2*e^8 + (a^2*c^4*d^8 - 7*a^3*c^3*d^6*e^2 + 15*a^4*c^2
*d^4*e^4 - 9*a^5*c*d^2*e^6 + 2*(a^5*c^4*d^4*e - 3*a^6*c^3*d^2*e^3)*sqrt(-(c
^2*d^6 - 6*a*c*d^4*e^2 + 9*a^2*d^2*e^4)/(a^5*c^3)))*((a^2*c^2*sqrt(-(c^2*d^
6 - 6*a*c*d^4*e^2 + 9*a^2*d^2*e^4)/(a^5*c^3)) + 3*c*d^2*e - a*e^3)/(a^2*c^2
))^(2/3) - (a*c^4*d^9*x - 5*a^2*c^3*d^7*x*e^2 + 3*a^3*c^2*d^5*x*e^4 + 9*a^4
*c*d^3*x*e^6 + (a^4*c^4*d^5*x*e - 2*a^5*c^3*d^3*x*e^3 - 3*a^6*c^2*d*x*e^5)*
sqrt(-(c^2*d^6 - 6*a*c*d^4*e^2 + 9*a^2*d^2*e^4)/(a^5*c^3)))*((a^2*c^2*sqrt(
-(c^2*d^6 - 6*a*c*d^4*e^2 + 9*a^2*d^2*e^4)/(a^5*c^3)) + 3*c*d^2*e - a*e^3)/
(a^2*c^2))^(1/3)) - 1/12*(-(a^2*c^2*sqrt(-(c^2*d^6 - 6*a*c*d^4*e^2 + 9*a^2*
d^2*e^4)/(a^5*c^3)) - 3*c*d^2*e + a*e^3)/(a^2*c^2))^(1/3)*log(c^4*d^10*x^2
- 4*a*c^3*d^8*x^2*e^2 - 2*a^2*c^2*d^6*x^2*e^4 + 12*a^3*c*d^4*x^2*e^6 + 9*a^
4*d^2*x^2*e^8 + (a^2*c^4*d^8 - 7*a^3*c^3*d^6*e^2 + 15*a^4*c^2*d^4*e^4 - 9*a
^5*c*d^2*e^6 - 2*(a^5*c^4*d^4*e - 3*a^6*c^3*d^2*e^3)*sqrt(-(c^2*d^6 - 6*a*c
*d^4*e^2 + 9*a^2*d^2*e^4)/(a^5*c^3)))*(-(a^2*c^2*sqrt(-(c^2*d^6 - 6*a*c*d^4
*e^2 + 9*a^2*d^2*e^4)/(a^5*c^3)) - 3*c*d^2*e + a*e^3)/(a^2*c^2))^(2/3) - (a
*c^4*d^9*x - 5*a^2*c^3*d^7*x*e^2 + 3*a^3*c^2*d^5*x*e^4 + 9*a^4*c*d^3*x*e^6
- (a^4*c^4*d^5*x*e - 2*a^5*c^3*d^3*x*e^3 - 3*a^6*c^2*d*x*e^5)*sqrt(-(c^2*d^
6 - 6*a*c*d^4*e^2 + 9*a^2*d^2*e^4)/(a^5*c^3)))*(-(a^2*c^2*sqrt(-(c^2*d^6 -
6*a*c*d^4*e^2 + 9*a^2*d^2*e^4)/(a^5*c^3)) - 3*c*d^2*e + a*e^3)/(a^2*c^2))^(
1/3)) + 1/6*((a^2*c^2*sqrt(-(c^2*d^6 - 6*a*c*d^...
```

Sympy [A]
time = 9.32, size = 165, normalized size = 0.54

$$\operatorname{RootSum}\left(46656t^6a^5c^4 + t^3\cdot\left(432a^4c^2e^3 - 1296a^3c^3d^2e\right) + a^3e^6 + 3a^2cd^2e^4 + 3ac^2d^4e^2 + c^3d^6, \left(t \mapsto t\log\left(x + \frac{-1296t^4a^4c^2e - 6ta^3e^4 + 36ta^2cd^2e^2 - 6tac^2d^4}{3a^2de^4 + 2acd^3e^2 - c^2d^5}\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**3+d)/(c*x**6+a),x)
```

```
[Out] RootSum(46656*_t**6*a**5*c**4 + _t**3*(432*a**4*c**2*e**3 - 1296*a**3*c**3*
d**2*e) + a**3*e**6 + 3*a**2*c*d**2*e**4 + 3*a*c**2*d**4*e**2 + c**3*d**6,
Lambda(_t, _t*log(x + (-1296*_t**4*a**4*c**2*e - 6*_t*a**3*e**4 + 36*_t*a**
2*c*d**2*e**2 - 6*_t*a*c**2*d**4)/(3*a**2*d*e**4 + 2*a*c*d**3*e**2 - c**2*d
**5))))
```

Giac [A]
time = 3.11, size = 288, normalized size = 0.94

$$-\frac{|c|e\log\left(x^2 + \left(\frac{a}{c}\right)^{\frac{1}{3}}\right)}{6\,\left(ac^5\right)^{\frac{1}{3}}} + \frac{\left(ac^5\right)^{\frac{1}{6}}d\arctan\left(\frac{x}{\left(\frac{a}{c}\right)^{\frac{1}{6}}}\right)}{3\,ac} + \frac{\left(\left(ac^5\right)^{\frac{1}{6}}c^3d - \sqrt{3}\left(ac^5\right)^{\frac{2}{3}}e\right)\arctan\left(\frac{2\,x + \sqrt{3}\left(\frac{a}{c}\right)^{\frac{1}{6}}}{\left(\frac{a}{c}\right)^{\frac{1}{6}}}\right)}{6\,ac^4} + \frac{\left(\left(ac^5\right)^{\frac{1}{6}}c^3d + \sqrt{3}\left(ac^5\right)^{\frac{2}{3}}e\right)\arctan\left(\frac{2\,x - \sqrt{3}\left(\frac{a}{c}\right)^{\frac{1}{6}}}{\left(\frac{a}{c}\right)^{\frac{1}{6}}}\right)}{6\,ac^4} + \frac{\left(\sqrt{3}\left(ac^5\right)^{\frac{1}{6}}c^3d + \left(ac^5\right)^{\frac{2}{3}}e\right)\log\left(x^2 + \sqrt{3}\,x\left(\frac{a}{c}\right)^{\frac{1}{6}} + \left(\frac{a}{c}\right)^{\frac{1}{3}}\right)}{12\,ac^4} - \frac{\left(\sqrt{3}\left(ac^5\right)^{\frac{1}{6}}c^3d - \left(ac^5\right)^{\frac{2}{3}}e\right)\log\left(x^2 - \sqrt{3}\,x\left(\frac{a}{c}\right)^{\frac{1}{6}} + \left(\frac{a}{c}\right)^{\frac{1}{3}}\right)}{12\,ac^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^3+d)/(c*x^6+a),x, algorithm="giac")
```

```
[Out] -1/6*abs(c)*e*log(x^2 + (a/c)^(1/3))/(a*c^5)^(1/3) + 1/3*(a*c^5)^(1/6)*d*ar
ctan(x/(a/c)^(1/6))/(a*c) + 1/6*((a*c^5)^(1/6)*c^3*d - sqrt(3)*(a*c^5)^(2/3
)*e)*arctan((2*x + sqrt(3)*(a/c)^(1/6))/(a/c)^(1/6))/(a*c^4) + 1/6*((a*c^5)
^(1/6)*c^3*d + sqrt(3)*(a*c^5)^(2/3)*e)*arctan((2*x - sqrt(3)*(a/c)^(1/6))/
(a/c)^(1/6))/(a*c^4) + 1/12*(sqrt(3)*(a*c^5)^(1/6)*c^3*d + (a*c^5)^(2/3)*e)
*log(x^2 + sqrt(3)*x*(a/c)^(1/6) + (a/c)^(1/3))/(a*c^4) - 1/12*(sqrt(3)*(a*
c^5)^(1/6)*c^3*d - (a*c^5)^(2/3)*e)*log(x^2 - sqrt(3)*x*(a/c)^(1/6) + (a/c)
^(1/3))/(a*c^4)
```

Mupad [B]
time = 1.54, size = 1331, normalized size = 4.36

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x^3)/(a + c*x^6),x)
```

```
[Out] log(a^3*c^3*(-(a^4*c^2*e^3 + c*d^3*(-a^5*c^5)^(1/2) - 3*a^3*c^3*d^2*e - 3*a
*d*e^2*(-a^5*c^5)^(1/2))/(a^5*c^4))^(1/3) + e*x*(-a^5*c^5)^(1/2) + a^2*c^3*
d*x)*(-(a^4*c^2*e^3 + c*d^3*(-a^5*c^5)^(1/2) - 3*a^3*c^3*d^2*e - 3*a*d*e^2*
(-a^5*c^5)^(1/2))/(216*a^5*c^4))^(1/3) + log(a^3*c^3*(-(a^4*c^2*e^3 - c*d^3
*(-a^5*c^5)^(1/2) - 3*a^3*c^3*d^2*e + 3*a*d*e^2*(-a^5*c^5)^(1/2))/(a^5*c^4)
)^(1/3) - e*x*(-a^5*c^5)^(1/2) + a^2*c^3*d*x)*(-(a^4*c^2*e^3 - c*d^3*(-a^5*
c^5)^(1/2) - 3*a^3*c^3*d^2*e + 3*a*d*e^2*(-a^5*c^5)^(1/2))/(216*a^5*c^4))^(
1/3) - log(a^3*c^3*(-(a^4*c^2*e^3 + c*d^3*(-a^5*c^5)^(1/2) - 3*a^3*c^3*d^2*
e - 3*a*d*e^2*(-a^5*c^5)^(1/2))/(a^5*c^4))^(1/3) - 2*e*x*(-a^5*c^5)^(1/2) +
 3^(1/2)*a^3*c^3*(-(a^4*c^2*e^3 + c*d^3*(-a^5*c^5)^(1/2) - 3*a^3*c^3*d^2*e
- 3*a*d*e^2*(-a^5*c^5)^(1/2))/(a^5*c^4))^(1/3)*1i - 2*a^2*c^3*d*x)*((3^(1/2
)*1i)/2 + 1/2)*(-(a^4*c^2*e^3 + c*d^3*(-a^5*c^5)^(1/2) - 3*a^3*c^3*d^2*e -
3*a*d*e^2*(-a^5*c^5)^(1/2))/(216*a^5*c^4))^(1/3) + log(e*x*(-a^5*c^5)^(1/2)
 - (a^3*c^3*(-(a^4*c^2*e^3 + c*d^3*(-a^5*c^5)^(1/2) - 3*a^3*c^3*d^2*e - 3*a
*d*e^2*(-a^5*c^5)^(1/2))/(a^5*c^4))^(1/3))/2 + (3^(1/2)*a^3*c^3*(-(a^4*c^2*
e^3 + c*d^3*(-a^5*c^5)^(1/2) - 3*a^3*c^3*d^2*e - 3*a*d*e^2*(-a^5*c^5)^(1/2)
)/(a^5*c^4))^(1/3)*1i)/2 + a^2*c^3*d*x)*((3^(1/2)*1i)/2 - 1/2)*(-(a^4*c^2*e
^3 + c*d^3*(-a^5*c^5)^(1/2) - 3*a^3*c^3*d^2*e - 3*a*d*e^2*(-a^5*c^5)^(1/2))
/(216*a^5*c^4))^(1/3) + log(a^3*c^3*(-(a^4*c^2*e^3 - c*d^3*(-a^5*c^5)^(1/2)
 - 3*a^3*c^3*d^2*e + 3*a*d*e^2*(-a^5*c^5)^(1/2))/(a^5*c^4))^(1/3) + 2*e*x*(
-a^5*c^5)^(1/2) - 3^(1/2)*a^3*c^3*(-(a^4*c^2*e^3 - c*d^3*(-a^5*c^5)^(1/2) -
 3*a^3*c^3*d^2*e + 3*a*d*e^2*(-a^5*c^5)^(1/2))/(a^5*c^4))^(1/3)*1i - 2*a^2*
c^3*d*x)*((3^(1/2)*1i)/2 - 1/2)*(-(a^4*c^2*e^3 - c*d^3*(-a^5*c^5)^(1/2) - 3
*a^3*c^3*d^2*e + 3*a*d*e^2*(-a^5*c^5)^(1/2))/(216*a^5*c^4))^(1/3) - log(a^3
*c^3*(-(a^4*c^2*e^3 - c*d^3*(-a^5*c^5)^(1/2) - 3*a^3*c^3*d^2*e + 3*a*d*e^2*
(-a^5*c^5)^(1/2))/(a^5*c^4))^(1/3) + 2*e*x*(-a^5*c^5)^(1/2) + 3^(1/2)*a^3*c
^3*(-(a^4*c^2*e^3 - c*d^3*(-a^5*c^5)^(1/2) - 3*a^3*c^3*d^2*e + 3*a*d*e^2*(-
a^5*c^5)^(1/2))/(a^5*c^4))^(1/3)*1i - 2*a^2*c^3*d*x)*((3^(1/2)*1i)/2 + 1/2)
```

```
*(-(a^4*c^2*e^3 - c*d^3*(-a^5*c^5)^(1/2) - 3*a^3*c^3*d^2*e + 3*a*d*e^2*(-a^
5*c^5)^(1/2))/(216*a^5*c^4))^(1/3)
```

3.2 $\int \frac{d+ex^3}{a-cx^6} dx$

Optimal. Leaf size=323

$$-\frac{\left(d-\frac{\sqrt{a}\,e}{\sqrt{c}}\right)\tan^{-1}\left(\frac{\sqrt[6]{a}-2\sqrt[6]{c}\,x}{\sqrt{3}\,\sqrt[6]{a}}\right)}{2\sqrt{3}\,a^{5/6}\sqrt[6]{c}}+\frac{\left(\sqrt{c}\,d+\sqrt{a}\,e\right)\tan^{-1}\left(\frac{\sqrt[6]{a}+2\sqrt[6]{c}\,x}{\sqrt{3}\,\sqrt[6]{a}}\right)}{2\sqrt{3}\,a^{5/6}c^{2/3}}-\frac{\left(\sqrt{c}\,d+\sqrt{a}\,e\right)\log\left(\sqrt[6]{a}-\right.}{6a^{5/6}c^{2/3}}$$

```
[Out] 1/6*ln(a^(1/6)+c^(1/6)*x)*(d-e*a^(1/2)/c^(1/2))/a^(5/6)/c^(1/6)-1/12*ln(a^(
1/3)-a^(1/6)*c^(1/6)*x+c^(1/3)*x^2)*(d-e*a^(1/2)/c^(1/2))/a^(5/6)/c^(1/6)-1
/6*arctan(1/3*(a^(1/6)-2*c^(1/6)*x)/a^(1/6)*3^(1/2))*(d-e*a^(1/2)/c^(1/2))/
a^(5/6)/c^(1/6)*3^(1/2)-1/6*ln(a^(1/6)-c^(1/6)*x)*(e*a^(1/2)+d*c^(1/2))/a^(
5/6)/c^(2/3)+1/12*ln(a^(1/3)+a^(1/6)*c^(1/6)*x+c^(1/3)*x^2)*(e*a^(1/2)+d*c^
(1/2))/a^(5/6)/c^(2/3)+1/6*arctan(1/3*(a^(1/6)+2*c^(1/6)*x)/a^(1/6)*3^(1/2)
)*(e*a^(1/2)+d*c^(1/2))/a^(5/6)/c^(2/3)*3^(1/2)
```

Rubi [A]
time = 0.13, antiderivative size = 323, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 7, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.389$, Rules used = {1431, 206, 31, 648, 631, 210, 642}

$$\frac{\text{ArcTan}\left(\frac{\sqrt[6]{a}+2\sqrt[6]{c}\,x}{\sqrt{3}\,\sqrt[6]{a}}\right)\left(\sqrt{a}\,e+\sqrt{c}\,d\right)}{2\sqrt{3}\,a^{5/6}c^{2/3}}-\frac{\text{ArcTan}\left(\frac{\sqrt[6]{a}-2\sqrt[6]{c}\,x}{\sqrt{3}\,\sqrt[6]{a}}\right)\left(d-\frac{\sqrt{a}\,e}{\sqrt{c}}\right)}{2\sqrt{3}\,a^{5/6}\sqrt[6]{c}}+\frac{\left(\sqrt{a}\,e+\sqrt{c}\,d\right)\log\left(\sqrt[6]{a}\,\sqrt[6]{c}\,x+\sqrt[3]{a}+\sqrt[3]{c}\,x^2\right)}{12a^{5/6}c^{2/3}}-\frac{\left(\sqrt{a}\,e+\sqrt{c}\,d\right)\log\left(\sqrt[6]{a}-\sqrt[6]{c}\,x\right)}{6a^{5/6}c^{2/3}}-\frac{\left(d-\frac{\sqrt{a}\,e}{\sqrt{c}}\right)\log\left(-\sqrt[6]{a}\,\sqrt[6]{c}\,x+\sqrt[3]{a}+\sqrt[3]{c}\,x^2\right)}{12a^{5/6}\sqrt[6]{c}}+\frac{\left(d-\frac{\sqrt{a}\,e}{\sqrt{c}}\right)\log\left(\sqrt[6]{a}+\sqrt[6]{c}\,x\right)}{6a^{5/6}\sqrt[6]{c}}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x^3)/(a - c*x^6),x]
```

```
[Out] -1/2*((d - (Sqrt[a]*e)/Sqrt[c])*ArcTan[(a^(1/6) - 2*c^(1/6)*x)/(Sqrt[3]*a^(
1/6))])/(Sqrt[3]*a^(5/6)*c^(1/6)) + ((Sqrt[c]*d + Sqrt[a]*e)*ArcTan[(a^(1/6
) + 2*c^(1/6)*x)/(Sqrt[3]*a^(1/6))])/(2*Sqrt[3]*a^(5/6)*c^(2/3)) - ((Sqrt[c
]*d + Sqrt[a]*e)*Log[a^(1/6) - c^(1/6)*x])/(6*a^(5/6)*c^(2/3)) + ((d - (Sqr
t[a]*e)/Sqrt[c])*Log[a^(1/6) + c^(1/6)*x])/(6*a^(5/6)*c^(1/6)) - ((d - (Sqr
t[a]*e)/Sqrt[c])*Log[a^(1/3) - a^(1/6)*c^(1/6)*x + c^(1/3)*x^2])/(12*a^(5/6
)*c^(1/6)) + ((Sqrt[c]*d + Sqrt[a]*e)*Log[a^(1/3) + a^(1/6)*c^(1/6)*x + c^(
1/3)*x^2])/(12*a^(5/6)*c^(2/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
```

```
reeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1431

```
Int[((d_) + (e_.)*(x_)^(n_))/((a_) + (c_.)*(x_)^(n2_)), x_Symbol] :> With[{
q = Rt[-a/c, 2]}, Dist[(d + e*q)/2, Int[1/(a + c*q*x^n), x], x] + Dist[(d -
 e*q)/2, Int[1/(a - c*q*x^n), x], x]] /; FreeQ[{a, c, d, e, n}, x] && EqQ[n
2, 2*n] && NeQ[c*d^2 + a*e^2, 0] && NegQ[a*c] && IntegerQ[n]
```

Rubi steps

$$\int \frac{d+ex^3}{a-cx^6}\,dx = \frac{1}{2}\left(d-\frac{\sqrt{a}\,e}{\sqrt{c}}\right)\int \frac{1}{a+\sqrt{a}\,\sqrt{c}\,x^3}\,dx + \frac{1}{2}\left(d+\frac{\sqrt{a}\,e}{\sqrt{c}}\right)\int \frac{1}{a-\sqrt{a}\,\sqrt{c}\,x^3}\,dx$$

$$= \frac{\left(d-\frac{\sqrt{a}\,e}{\sqrt{c}}\right)\int \frac{1}{\sqrt[3]{a}+\sqrt[6]{a}\,\sqrt[6]{c}\,x}\,dx}{6a^{2/3}} + \frac{\left(d-\frac{\sqrt{a}\,e}{\sqrt{c}}\right)\int \frac{2\sqrt[3]{a}-\sqrt[6]{a}\,\sqrt[6]{c}\,x}{a^{2/3}-\sqrt{a}\,\sqrt[6]{c}\,x+\sqrt[3]{a}\,\sqrt[3]{c}\,x^2}\,dx}{6a^{2/3}} + \frac{\Big(d+ \cdots}{}$$

$$= -\frac{\left(\sqrt{c}\,d+\sqrt{a}\,e\right)\log\left(\sqrt[6]{a}-\sqrt[6]{c}\,x\right)}{6a^{5/6}c^{2/3}} + \frac{\left(d-\frac{\sqrt{a}\,e}{\sqrt{c}}\right)\log\left(\sqrt[6]{a}+\sqrt[6]{c}\,x\right)}{6a^{5/6}\sqrt[6]{c}} + \frac{\left(\sqrt{c}\,d+\sqrt{a}\,e\right) \cdots}{}$$

$$= -\frac{\left(\sqrt{c}\,d+\sqrt{a}\,e\right)\log\left(\sqrt[6]{a}-\sqrt[6]{c}\,x\right)}{6a^{5/6}c^{2/3}} + \frac{\left(d-\frac{\sqrt{a}\,e}{\sqrt{c}}\right)\log\left(\sqrt[6]{a}+\sqrt[6]{c}\,x\right)}{6a^{5/6}\sqrt[6]{c}} - \frac{\left(d-\frac{\sqrt{a}\,e}{\sqrt{c}}\right)\log \cdots}{}$$

$$= -\frac{\left(d-\frac{\sqrt{a}\,e}{\sqrt{c}}\right)\tan^{-1}\left(\frac{\sqrt[6]{a}-2\sqrt[6]{c}\,x}{\sqrt{3}\,\sqrt[6]{a}}\right)}{2\sqrt{3}\,a^{5/6}\sqrt[6]{c}} + \frac{\left(\sqrt{c}\,d+\sqrt{a}\,e\right)\tan^{-1}\left(\frac{\sqrt[6]{a}+2\sqrt[6]{c}\,x}{\sqrt{3}\,\sqrt[6]{a}}\right)}{2\sqrt{3}\,a^{5/6}c^{2/3}} - \frac{\left(\sqrt{c}\,d+ \cdots\right.}{}$$

Mathematica [A]
time = 0.07, size = 337, normalized size = 1.04

$$\frac{-2\sqrt{3}\left(\sqrt{c}\,d-\sqrt{a}\,e\right)\tan^{-1}\left(\frac{1-\frac{2\sqrt[6]{c}\,x}{\sqrt[6]{a}}}{\sqrt{3}}\right)+2\sqrt{3}\left(\sqrt{c}\,d+\sqrt{a}\,e\right)\tan^{-1}\left(\frac{1+\frac{2\sqrt[6]{c}\,x}{\sqrt[6]{a}}}{\sqrt{3}}\right)-2\sqrt{c}\,d\log\left(\sqrt[6]{a}-\sqrt[6]{c}\,x\right)-2\sqrt{a}\,e\log\left(\sqrt[6]{a}-\sqrt[6]{c}\,x\right)+2\sqrt{c}\,d\log\left(\sqrt[6]{a}+\sqrt[6]{c}\,x\right)-2\sqrt{a}\,e\log\left(\sqrt[6]{a}+\sqrt[6]{c}\,x\right)-\sqrt{c}\,d\log\left(\sqrt[3]{a}-\sqrt[6]{a}\sqrt[6]{c}\,x+\sqrt[3]{c}\,x^2\right)+\sqrt{a}\,e\log\left(\sqrt[3]{a}-\sqrt[6]{a}\sqrt[6]{c}\,x+\sqrt[3]{c}\,x^2\right)+\sqrt{c}\,d\log\left(\sqrt[3]{a}+\sqrt[6]{a}\sqrt[6]{c}\,x+\sqrt[3]{c}\,x^2\right)+\sqrt{a}\,e\log\left(\sqrt[3]{a}+\sqrt[6]{a}\sqrt[6]{c}\,x+\sqrt[3]{c}\,x^2\right)}{12a^{5/6}c^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x^3)/(a - c*x^6),x]
```

```
[Out] (-2*Sqrt[3]*(Sqrt[c]*d - Sqrt[a]*e)*ArcTan[(1 - (2*c^(1/6)*x)/a^(1/6))/Sqrt
[3]] + 2*Sqrt[3]*(Sqrt[c]*d + Sqrt[a]*e)*ArcTan[(1 + (2*c^(1/6)*x)/a^(1/6))
/Sqrt[3]] - 2*Sqrt[c]*d*Log[a^(1/6) - c^(1/6)*x] - 2*Sqrt[a]*e*Log[a^(1/6)
- c^(1/6)*x] + 2*Sqrt[c]*d*Log[a^(1/6) + c^(1/6)*x] - 2*Sqrt[a]*e*Log[a^(1/
6) + c^(1/6)*x] - Sqrt[c]*d*Log[a^(1/3) - a^(1/6)*c^(1/6)*x + c^(1/3)*x^2]
+ Sqrt[a]*e*Log[a^(1/3) - a^(1/6)*c^(1/6)*x + c^(1/3)*x^2] + Sqrt[c]*d*Log[
a^(1/3) + a^(1/6)*c^(1/6)*x + c^(1/3)*x^2] + Sqrt[a]*e*Log[a^(1/3) + a^(1/6
)*c^(1/6)*x + c^(1/3)*x^2])/(12*a^(5/6)*c^(2/3))
```

Maple [A]
time = 0.21, size = 386, normalized size = 1.20

method	result
risch	$-\frac{\sum_{\_R=\mathrm{RootOf}\left(\_Z^6 c-a\right)}\frac{\left(\_R^3 e+d\right)\ln\left(x-\_R\right)}{\_R^5}}{6c}$

default	$-\frac{\ln\left(x+\left(\frac{a}{c}\right)^{\frac{1}{6}}\right)e}{6c\left(\frac{a}{c}\right)^{\frac{1}{3}}}+\frac{\ln\left(x+\left(\frac{a}{c}\right)^{\frac{1}{6}}\right)d}{6c\left(\frac{a}{c}\right)^{\frac{5}{6}}}-\frac{\ln\left(-x+\left(\frac{a}{c}\right)^{\frac{1}{6}}\right)e}{6c\left(\frac{a}{c}\right)^{\frac{1}{3}}}-\frac{\ln\left(-x+\left(\frac{a}{c}\right)^{\frac{1}{6}}\right)d}{6c\left(\frac{a}{c}\right)^{\frac{5}{6}}}+\frac{e\left(\frac{a}{c}\right)^{\frac{2}{3}}\ln\left(x^{2}+\left(\frac{a}{c}\right)^{\frac{1}{6}}x+\left(\frac{a}{c}\right)^{\frac{1}{3}}\right)}{12a}+\frac{e\left(\frac{a}{c}\right)^{\frac{2}{3}}\sqrt{3}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^3+d)/(-c*x^6+a),x,method=_RETURNVERBOSE)
```

```
[Out] -1/6/c/(a/c)^(1/3)*ln(x+(a/c)^(1/6))*e+1/6/c/(a/c)^(5/6)*ln(x+(a/c)^(1/6))*
d-1/6/c/(a/c)^(1/3)*ln(-x+(a/c)^(1/6))*e-1/6/c/(a/c)^(5/6)*ln(-x+(a/c)^(1/6
))*d+1/12/a*e*(a/c)^(2/3)*ln(x^2+(a/c)^(1/6)*x+(a/c)^(1/3))+1/6/a*e*(a/c)^(
2/3)*3^(1/2)*arctan(2/3*x*3^(1/2)/(a/c)^(1/6)+1/3*3^(1/2))+1/12/a*d*(a/c)^(
1/6)*ln(x^2+(a/c)^(1/6)*x+(a/c)^(1/3))+1/6/a*d*(a/c)^(1/6)*3^(1/2)*arctan(2
/3*x*3^(1/2)/(a/c)^(1/6)+1/3*3^(1/2))+1/12*(a/c)^(2/3)/a*ln((a/c)^(1/6)*x-x
^2-(a/c)^(1/3))*e-1/12*(a/c)^(1/6)/a*ln((a/c)^(1/6)*x-x^2-(a/c)^(1/3))*d-1/
6*(a/c)^(2/3)/a*3^(1/2)*e*arctan(-1/3*3^(1/2)+2/3*x*3^(1/2)/(a/c)^(1/6))+1/
6*(a/c)^(1/6)/a*3^(1/2)*d*arctan(-1/3*3^(1/2)+2/3*x*3^(1/2)/(a/c)^(1/6))
```

<u>Maxima</u> [**A**]
time = 0.48, size = 319, normalized size = 0.99

$$\frac{\sqrt{3}\left(\sqrt{c}d+\sqrt{a}e\right)\arctan\left(\frac{\sqrt{3}\left(2x+\left(\frac{\sqrt{a}}{\sqrt{c}}\right)^{\frac{1}{3}}\right)}{3\left(\frac{\sqrt{a}}{\sqrt{c}}\right)^{\frac{1}{3}}}\right)}{6\sqrt{a}c\left(\frac{\sqrt{a}}{\sqrt{c}}\right)^{\frac{2}{3}}}+\frac{\sqrt{3}\left(\sqrt{c}d-\sqrt{a}e\right)\arctan\left(\frac{\sqrt{3}\left(2x-\left(\frac{\sqrt{a}}{\sqrt{c}}\right)^{\frac{1}{3}}\right)}{3\left(\frac{\sqrt{a}}{\sqrt{c}}\right)^{\frac{1}{3}}}\right)}{6\sqrt{a}c\left(\frac{\sqrt{a}}{\sqrt{c}}\right)^{\frac{2}{3}}}+\frac{\left(\sqrt{c}d+\sqrt{a}e\right)\log\left(x^{2}+x\left(\frac{\sqrt{a}}{\sqrt{c}}\right)^{\frac{1}{3}}+\left(\frac{\sqrt{a}}{\sqrt{c}}\right)^{\frac{2}{3}}\right)}{12\sqrt{a}c\left(\frac{\sqrt{a}}{\sqrt{c}}\right)^{\frac{2}{3}}}-\frac{\left(\sqrt{c}d-\sqrt{a}e\right)\log\left(x^{2}-x\left(\frac{\sqrt{a}}{\sqrt{c}}\right)^{\frac{1}{3}}+\left(\frac{\sqrt{a}}{\sqrt{c}}\right)^{\frac{2}{3}}\right)}{12\sqrt{a}c\left(\frac{\sqrt{a}}{\sqrt{c}}\right)^{\frac{2}{3}}}+\frac{\left(\sqrt{c}d-\sqrt{a}e\right)\log\left(x+\left(\frac{\sqrt{a}}{\sqrt{c}}\right)^{\frac{1}{3}}\right)}{6\sqrt{a}c\left(\frac{\sqrt{a}}{\sqrt{c}}\right)^{\frac{2}{3}}}-\frac{\left(\sqrt{c}d+\sqrt{a}e\right)\log\left(x-\left(\frac{\sqrt{a}}{\sqrt{c}}\right)^{\frac{1}{3}}\right)}{6\sqrt{a}c\left(\frac{\sqrt{a}}{\sqrt{c}}\right)^{\frac{2}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^3+d)/(-c*x^6+a),x, algorithm="maxima")
```

```
[Out] 1/6*sqrt(3)*(sqrt(c)*d + sqrt(a)*e)*arctan(1/3*sqrt(3)*(2*x + (sqrt(a)/sqrt
(c))^(1/3))/(sqrt(a)/sqrt(c))^(1/3))/(sqrt(a)*c*(sqrt(a)/sqrt(c))^(2/3)) +
1/6*sqrt(3)*(sqrt(c)*d - sqrt(a)*e)*arctan(1/3*sqrt(3)*(2*x - (sqrt(a)/sqrt
(c))^(1/3))/(sqrt(a)/sqrt(c))^(1/3))/(sqrt(a)*c*(sqrt(a)/sqrt(c))^(2/3)) +
1/12*(sqrt(c)*d + sqrt(a)*e)*log(x^2 + x*(sqrt(a)/sqrt(c))^(1/3) + (sqrt(a)
/sqrt(c))^(2/3))/(sqrt(a)*c*(sqrt(a)/sqrt(c))^(2/3)) - 1/12*(sqrt(c)*d - sq
rt(a)*e)*log(x^2 - x*(sqrt(a)/sqrt(c))^(1/3) + (sqrt(a)/sqrt(c))^(2/3))/(sq
rt(a)*c*(sqrt(a)/sqrt(c))^(2/3)) + 1/6*(sqrt(c)*d - sqrt(a)*e)*log(x + (sqr
t(a)/sqrt(c))^(1/3))/(sqrt(a)*c*(sqrt(a)/sqrt(c))^(2/3)) - 1/6*(sqrt(c)*d +
 sqrt(a)*e)*log(x - (sqrt(a)/sqrt(c))^(1/3))/(sqrt(a)*c*(sqrt(a)/sqrt(c))^(
2/3))
```

<u>Fricas</u> [**B**] Leaf count of result is larger than twice the leaf count of optimal. 3566 vs. 2(229) = 458.
time = 1.38, size = 3566, normalized size = 11.04

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^3+d)/(-c*x^6+a),x, algorithm="fricas")

[Out] 1/3*sqrt(3)*(-(a^2*c^2*sqrt((c^2*d^6 + 6*a*c*d^4*e^2 + 9*a^2*d^2*e^4)/(a^5*
c^3)) + 3*c*d^2*e + a*e^3)/(a^2*c^2))^(1/3)*arctan(-1/3*(2*sqrt(c^4*d^10*x^
2 + 4*a*c^3*d^8*x^2*e^2 - 2*a^2*c^2*d^6*x^2*e^4 - 12*a^3*c*d^4*x^2*e^6 + 9*
a^4*d^2*x^2*e^8 + (a^2*c^4*d^8 + 7*a^3*c^3*d^6*e^2 + 15*a^4*c^2*d^4*e^4 + 9
*a^5*c*d^2*e^6 - 2*(a^5*c^4*d^4*e + 3*a^6*c^3*d^2*e^3)*sqrt((c^2*d^6 + 6*a*
c*d^4*e^2 + 9*a^2*d^2*e^4)/(a^5*c^3)))*(-(a^2*c^2*sqrt((c^2*d^6 + 6*a*c*d^4
*e^2 + 9*a^2*d^2*e^4)/(a^5*c^3)) + 3*c*d^2*e + a*e^3)/(a^2*c^2))^(2/3) - (a
*c^4*d^9*x + 5*a^2*c^3*d^7*x*e^2 + 3*a^3*c^2*d^5*x*e^4 - 9*a^4*c*d^3*x*e^6
- (a^4*c^4*d^5*x*e + 2*a^5*c^3*d^3*x*e^3 - 3*a^6*c^2*d*x*e^5)*sqrt((c^2*d^6
 + 6*a*c*d^4*e^2 + 9*a^2*d^2*e^4)/(a^5*c^3)))*(-(a^2*c^2*sqrt((c^2*d^6 + 6*
a*c*d^4*e^2 + 9*a^2*d^2*e^4)/(a^5*c^3)) + 3*c*d^2*e + a*e^3)/(a^2*c^2))^(1/
3))*(sqrt(3)*(a^4*c^4*d^2 + a^5*c^3*e^2)*sqrt((c^2*d^6 + 6*a*c*d^4*e^2 + 9*
a^2*d^2*e^4)/(a^5*c^3)) - 2*sqrt(3)*(a^2*c^3*d^4*e + 3*a^3*c^2*d^2*e^3))*(-
(a^2*c^2*sqrt((c^2*d^6 + 6*a*c*d^4*e^2 + 9*a^2*d^2*e^4)/(a^5*c^3)) + 3*c*d^
2*e + a*e^3)/(a^2*c^2))^(2/3) + 2*(sqrt(3)*(a^4*c^6*d^7*x + 3*a^5*c^5*d^5*x
*e^2 - a^6*c^4*d^3*x*e^4 - 3*a^7*c^3*d*x*e^6)*sqrt((c^2*d^6 + 6*a*c*d^4*e^2
 + 9*a^2*d^2*e^4)/(a^5*c^3)) - 2*sqrt(3)*(a^2*c^5*d^9*x*e + 5*a^3*c^4*d^7*x
*e^3 + 3*a^4*c^3*d^5*x*e^5 - 9*a^5*c^2*d^3*x*e^7))*(-(a^2*c^2*sqrt((c^2*d^6
 + 6*a*c*d^4*e^2 + 9*a^2*d^2*e^4)/(a^5*c^3)) + 3*c*d^2*e + a*e^3)/(a^2*c^2)
)^(2/3) + sqrt(3)*(c^5*d^12 + 3*a*c^4*d^10*e^2 - 6*a^2*c^3*d^8*e^4 - 10*a^3
*c^2*d^6*e^6 + 21*a^4*c*d^4*e^8 - 9*a^5*d^2*e^10))/(c^5*d^12 + 3*a*c^4*d^10
*e^2 - 6*a^2*c^3*d^8*e^4 - 10*a^3*c^2*d^6*e^6 + 21*a^4*c*d^4*e^8 - 9*a^5*d^
2*e^10)) - 1/3*sqrt(3)*((a^2*c^2*sqrt((c^2*d^6 + 6*a*c*d^4*e^2 + 9*a^2*d^2*
e^4)/(a^5*c^3)) - 3*c*d^2*e - a*e^3)/(a^2*c^2))^(1/3)*arctan(-1/3*(2*sqrt(c
^4*d^10*x^2 + 4*a*c^3*d^8*x^2*e^2 - 2*a^2*c^2*d^6*x^2*e^4 - 12*a^3*c*d^4*x^
2*e^6 + 9*a^4*d^2*x^2*e^8 + (a^2*c^4*d^8 + 7*a^3*c^3*d^6*e^2 + 15*a^4*c^2*d
^4*e^4 + 9*a^5*c*d^2*e^6 + 2*(a^5*c^4*d^4*e + 3*a^6*c^3*d^2*e^3)*sqrt((c^2*
d^6 + 6*a*c*d^4*e^2 + 9*a^2*d^2*e^4)/(a^5*c^3)))*((a^2*c^2*sqrt((c^2*d^6 +
6*a*c*d^4*e^2 + 9*a^2*d^2*e^4)/(a^5*c^3)) - 3*c*d^2*e - a*e^3)/(a^2*c^2))^(
2/3) - (a*c^4*d^9*x + 5*a^2*c^3*d^7*x*e^2 + 3*a^3*c^2*d^5*x*e^4 - 9*a^4*c*d
^3*x*e^6 + (a^4*c^4*d^5*x*e + 2*a^5*c^3*d^3*x*e^3 - 3*a^6*c^2*d*x*e^5)*sqrt
((c^2*d^6 + 6*a*c*d^4*e^2 + 9*a^2*d^2*e^4)/(a^5*c^3)))*((a^2*c^2*sqrt((c^2*
d^6 + 6*a*c*d^4*e^2 + 9*a^2*d^2*e^4)/(a^5*c^3)) - 3*c*d^2*e - a*e^3)/(a^2*c
^2))^(1/3))*(sqrt(3)*(a^4*c^4*d^2 + a^5*c^3*e^2)*sqrt((c^2*d^6 + 6*a*c*d^4*
e^2 + 9*a^2*d^2*e^4)/(a^5*c^3)) + 2*sqrt(3)*(a^2*c^3*d^4*e + 3*a^3*c^2*d^2*
e^3))*((a^2*c^2*sqrt((c^2*d^6 + 6*a*c*d^4*e^2 + 9*a^2*d^2*e^4)/(a^5*c^3)) -
 3*c*d^2*e - a*e^3)/(a^2*c^2))^(2/3) + 2*(sqrt(3)*(a^4*c^6*d^7*x + 3*a^5*c^
5*d^5*x*e^2 - a^6*c^4*d^3*x*e^4 - 3*a^7*c^3*d*x*e^6)*sqrt((c^2*d^6 + 6*a*c*
d^4*e^2 + 9*a^2*d^2*e^4)/(a^5*c^3)) + 2*sqrt(3)*(a^2*c^5*d^9*x*e + 5*a^3*c^
4*d^7*x*e^3 + 3*a^4*c^3*d^5*x*e^5 - 9*a^5*c^2*d^3*x*e^7))*((a^2*c^2*sqrt((c
^2*d^6 + 6*a*c*d^4*e^2 + 9*a^2*d^2*e^4)/(a^5*c^3)) - 3*c*d^2*e - a*e^3)/(a^
2*c^2))^(2/3) - sqrt(3)*(c^5*d^12 + 3*a*c^4*d^10*e^2 - 6*a^2*c^3*d^8*e^4 -
10*a^3*c^2*d^6*e^6 + 21*a^4*c*d^4*e^8 - 9*a^5*d^2*e^10))/(c^5*d^12 + 3*a*c^
```

```
4*d^10*e^2 - 6*a^2*c^3*d^8*e^4 - 10*a^3*c^2*d^6*e^6 + 21*a^4*c*d^4*e^8 - 9*
a^5*d^2*e^10)) - 1/12*(-(a^2*c^2*sqrt((c^2*d^6 + 6*a*c*d^4*e^2 + 9*a^2*d^2*
e^4)/(a^5*c^3)) + 3*c*d^2*e + a*e^3)/(a^2*c^2))^(1/3)*log(c^4*d^10*x^2 + 4*
a*c^3*d^8*x^2*e^2 - 2*a^2*c^2*d^6*x^2*e^4 - 12*a^3*c*d^4*x^2*e^6 + 9*a^4*d^
2*x^2*e^8 + (a^2*c^4*d^8 + 7*a^3*c^3*d^6*e^2 + 15*a^4*c^2*d^4*e^4 + 9*a^5*c
*d^2*e^6 - 2*(a^5*c^4*d^4*e + 3*a^6*c^3*d^2*e^3)*sqrt((c^2*d^6 + 6*a*c*d^4*
e^2 + 9*a^2*d^2*e^4)/(a^5*c^3)))*(-(a^2*c^2*sqrt((c^2*d^6 + 6*a*c*d^4*e^2 +
 9*a^2*d^2*e^4)/(a^5*c^3)) + 3*c*d^2*e + a*e^3)/(a^2*c^2))^(2/3) - (a*c^4*d
^9*x + 5*a^2*c^3*d^7*x*e^2 + 3*a^3*c^2*d^5*x*e^4 - 9*a^4*c*d^3*x*e^6 - (a^4
*c^4*d^5*x*e + 2*a^5*c^3*d^3*x*e^3 - 3*a^6*c^2*d*x*e^5)*sqrt((c^2*d^6 + 6*a
*c*d^4*e^2 + 9*a^2*d^2*e^4)/(a^5*c^3)))*(-(a^2*c^2*sqrt((c^2*d^6 + 6*a*c*d^
4*e^2 + 9*a^2*d^2*e^4)/(a^5*c^3)) + 3*c*d^2*e + a*e^3)/(a^2*c^2))^(1/3)) -
1/12*((a^2*c^2*sqrt((c^2*d^6 + 6*a*c*d^4*e^2 + 9*a^2*d^2*e^4)/(a^5*c^3)) -
3*c*d^2*e - a*e^3)/(a^2*c^2))^(1/3)*log(c^4*d^10*x^2 + 4*a*c^3*d^8*x^2*e^2
- 2*a^2*c^2*d^6*x^2*e^4 - 12*a^3*c*d^4*x^2*e^6 + 9*a^4*d^2*x^2*e^8 + (a^2*c
^4*d^8 + 7*a^3*c^3*d^6*e^2 + 15*a^4*c^2*d^4*e^4 + 9*a^5*c*d^2*e^6 + 2*(a^5*
c^4*d^4*e + 3*a^6*c^3*d^2*e^3)*sqrt((c^2*d^6 + 6*a*c*d^4*e^2 + 9*a^2*d^2*e^
4)/(a^5*c^3)))*((a^2*c^2*sqrt((c^2*d^6 + 6*a*c*d^4*e^2 + 9*a^2*d^2*e^4)/(a^
5*c^3)) - 3*c*d^2*e - a*e^3)/(a^2*c^2))^(2/3) - (a*c^4*d^9*x + 5*a^2*c^3*d^
7*x*e^2 + 3*a^3*c^2*d^5*x*e^4 - 9*a^4*c*d^3*x*e^6 + (a^4*c^4*d^5*x*e + 2*a^
5*c^3*d^3*x*e^3 - 3*a^6*c^2*d*x*e^5)*sqrt((c^2*d^6 + 6*a*c*d^4*e^2 + 9*a^2*
d^2*e^4)/(a^5*c^3)))*((a^2*c^2*sqrt((c^2*d^6 + 6*a*c*d^4*e^2 + 9*a^2*d^2*e^
4)/(a^5*c^3)) - 3*c*d^2*e - a*e^3)/(a^2*c^2))^(1/3)) + 1/6*(-(a^2*c^2*sqrt(
(c^2*d^6 + 6*a*c*d^4*e^2 + 9*a^2*d^2*e^4)/(a^5*...
```

Sympy [A]
time = 9.42, size = 168, normalized size = 0.52

$$-\operatorname{RootSum}\left(46656t^6a^5c^4 + t^3\left(-432a^4c^2e^3 - 1296a^3c^3d^2e\right) + a^3e^6 - 3a^2cd^2e^4 + 3ac^2d^4e^2 - c^3d^6, \left(t \mapsto t\log\left(x + \frac{-1296t^4a^4c^2e + 6ta^3e^4 + 36ta^2cd^2e^2 + 6tac^2d^4}{3a^2de^4 - 2acd^3e^2 - c^2d^5}\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**3+d)/(-c*x**6+a),x)
```

```
[Out] -RootSum(46656*_t**6*a**5*c**4 + _t**3*(-432*a**4*c**2*e**3 - 1296*a**3*c**
3*d**2*e) + a**3*e**6 - 3*a**2*c*d**2*e**4 + 3*a*c**2*d**4*e**2 - c**3*d**6
, Lambda(_t, _t*log(x + (-1296*_t**4*a**4*c**2*e + 6*_t*a**3*e**4 + 36*_t*a
**2*c*d**2*e**2 + 6*_t*a*c**2*d**4)/(3*a**2*d*e**4 - 2*a*c*d**3*e**2 - c**2
*d**5))))
```

Giac [A]
time = 3.46, size = 308, normalized size = 0.95

$$\frac{|c|e\log\left(x^2 + \left(-\frac{a}{c}\right)^{\frac{1}{3}}\right)}{6\left(-ac^5\right)^{\frac{1}{3}}} + \frac{\left(-ac^5\right)^{\frac{1}{6}} d\arctan\left(\frac{x}{\left(-\frac{a}{c}\right)^{\frac{1}{6}}}\right)}{3ac} + \frac{\left(\left(-ac^5\right)^{\frac{1}{6}} c^3d - \sqrt{3}\left(-ac^5\right)^{\frac{2}{3}} e\right)\arctan\left(\frac{2x + \sqrt{3}\left(-\frac{a}{c}\right)^{\frac{1}{6}}}{\left(-\frac{a}{c}\right)^{\frac{1}{6}}}\right)}{6ac^4} + \frac{\left(\left(-ac^5\right)^{\frac{1}{6}} c^3d + \sqrt{3}\left(-ac^5\right)^{\frac{2}{3}} e\right)\arctan\left(\frac{2x - \sqrt{3}\left(-\frac{a}{c}\right)^{\frac{1}{6}}}{\left(-\frac{a}{c}\right)^{\frac{1}{6}}}\right)}{6ac^4} + \frac{\left(\sqrt{3}\left(-ac^5\right)^{\frac{1}{6}} c^3d + \left(-ac^5\right)^{\frac{2}{3}} e\right)\log\left(x^2 + \sqrt{3}x\left(-\frac{a}{c}\right)^{\frac{1}{6}} + \left(-\frac{a}{c}\right)^{\frac{1}{3}}\right)}{12ac^4} - \frac{\left(\sqrt{3}\left(-ac^5\right)^{\frac{1}{6}} c^3d - \left(-ac^5\right)^{\frac{2}{3}} e\right)\log\left(x^2 - \sqrt{3}x\left(-\frac{a}{c}\right)^{\frac{1}{6}} + \left(-\frac{a}{c}\right)^{\frac{1}{3}}\right)}{12ac^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^3+d)/(-c*x^6+a),x, algorithm="giac")
```

```
[Out] 1/6*abs(c)*e*log(x^2 + (-a/c)^(1/3))/(-a*c^5)^(1/3) + 1/3*(-a*c^5)^(1/6)*d*
arctan(x/(-a/c)^(1/6))/(a*c) + 1/6*((-a*c^5)^(1/6)*c^3*d - sqrt(3)*(-a*c^5)
^(2/3)*e)*arctan((2*x + sqrt(3)*(-a/c)^(1/6))/(-a/c)^(1/6))/(a*c^4) + 1/6*(
(-a*c^5)^(1/6)*c^3*d + sqrt(3)*(-a*c^5)^(2/3)*e)*arctan((2*x - sqrt(3)*(-a/
c)^(1/6))/(-a/c)^(1/6))/(a*c^4) + 1/12*(sqrt(3)*(-a*c^5)^(1/6)*c^3*d + (-a*
c^5)^(2/3)*e)*log(x^2 + sqrt(3)*x*(-a/c)^(1/6) + (-a/c)^(1/3))/(a*c^4) - 1/
12*(sqrt(3)*(-a*c^5)^(1/6)*c^3*d - (-a*c^5)^(2/3)*e)*log(x^2 - sqrt(3)*x*(-
a/c)^(1/6) + (-a/c)^(1/3))/(a*c^4)
```

Mupad [B]
time = 2.97, size = 1293, normalized size = 4.00

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x^3)/(a - c*x^6),x)
```

```
[Out] log(a^3*c^3*(-(a^4*c^2*e^3 + c*d^3*(a^5*c^5)^(1/2) + 3*a^3*c^3*d^2*e + 3*a*
d*e^2*(a^5*c^5)^(1/2))/(a^5*c^4))^(1/3) + e*x*(a^5*c^5)^(1/2) + a^2*c^3*d*x
)*(-(a^4*c^2*e^3 + c*d^3*(a^5*c^5)^(1/2) + 3*a^3*c^3*d^2*e + 3*a*d*e^2*(a^5
*c^5)^(1/2))/(216*a^5*c^4))^(1/3) + log(a^3*c^3*(-(a^4*c^2*e^3 - c*d^3*(a^5
*c^5)^(1/2) + 3*a^3*c^3*d^2*e - 3*a*d*e^2*(a^5*c^5)^(1/2))/(a^5*c^4))^(1/3)
 - e*x*(a^5*c^5)^(1/2) + a^2*c^3*d*x)*(-(a^4*c^2*e^3 - c*d^3*(a^5*c^5)^(1/2
) + 3*a^3*c^3*d^2*e - 3*a*d*e^2*(a^5*c^5)^(1/2))/(216*a^5*c^4))^(1/3) - log
(a^3*c^3*(-(a^4*c^2*e^3 + c*d^3*(a^5*c^5)^(1/2) + 3*a^3*c^3*d^2*e + 3*a*d*e
^2*(a^5*c^5)^(1/2))/(a^5*c^4))^(1/3) - 2*e*x*(a^5*c^5)^(1/2) + 3^(1/2)*a^3*
c^3*(-(a^4*c^2*e^3 + c*d^3*(a^5*c^5)^(1/2) + 3*a^3*c^3*d^2*e + 3*a*d*e^2*(a
^5*c^5)^(1/2))/(a^5*c^4))^(1/3)*1i - 2*a^2*c^3*d*x)*((3^(1/2)*1i)/2 + 1/2)*
(-(a^4*c^2*e^3 + c*d^3*(a^5*c^5)^(1/2) + 3*a^3*c^3*d^2*e + 3*a*d*e^2*(a^5*c
^5)^(1/2))/(216*a^5*c^4))^(1/3) + log(e*x*(a^5*c^5)^(1/2) - (a^3*c^3*(-(a^4
*c^2*e^3 + c*d^3*(a^5*c^5)^(1/2) + 3*a^3*c^3*d^2*e + 3*a*d*e^2*(a^5*c^5)^(1
/2))/(a^5*c^4))^(1/3))/2 + (3^(1/2)*a^3*c^3*(-(a^4*c^2*e^3 + c*d^3*(a^5*c^5
)^(1/2) + 3*a^3*c^3*d^2*e + 3*a*d*e^2*(a^5*c^5)^(1/2))/(a^5*c^4))^(1/3)*1i)
/2 + a^2*c^3*d*x)*((3^(1/2)*1i)/2 - 1/2)*(-(a^4*c^2*e^3 + c*d^3*(a^5*c^5)^(
1/2) + 3*a^3*c^3*d^2*e + 3*a*d*e^2*(a^5*c^5)^(1/2))/(216*a^5*c^4))^(1/3) +
log(a^3*c^3*(-(a^4*c^2*e^3 - c*d^3*(a^5*c^5)^(1/2) + 3*a^3*c^3*d^2*e - 3*a*
d*e^2*(a^5*c^5)^(1/2))/(a^5*c^4))^(1/3) + 2*e*x*(a^5*c^5)^(1/2) - 3^(1/2)*a
^3*c^3*(-(a^4*c^2*e^3 - c*d^3*(a^5*c^5)^(1/2) + 3*a^3*c^3*d^2*e - 3*a*d*e^2
*(a^5*c^5)^(1/2))/(a^5*c^4))^(1/3)*1i - 2*a^2*c^3*d*x)*((3^(1/2)*1i)/2 - 1/
2)*(-(a^4*c^2*e^3 - c*d^3*(a^5*c^5)^(1/2) + 3*a^3*c^3*d^2*e - 3*a*d*e^2*(a^
5*c^5)^(1/2))/(216*a^5*c^4))^(1/3) - log(a^3*c^3*(-(a^4*c^2*e^3 - c*d^3*(a^
5*c^5)^(1/2) + 3*a^3*c^3*d^2*e - 3*a*d*e^2*(a^5*c^5)^(1/2))/(a^5*c^4))^(1/3
) + 2*e*x*(a^5*c^5)^(1/2) + 3^(1/2)*a^3*c^3*(-(a^4*c^2*e^3 - c*d^3*(a^5*c^5
)^(1/2) + 3*a^3*c^3*d^2*e - 3*a*d*e^2*(a^5*c^5)^(1/2))/(a^5*c^4))^(1/3)*1i
- 2*a^2*c^3*d*x)*((3^(1/2)*1i)/2 + 1/2)*(-(a^4*c^2*e^3 - c*d^3*(a^5*c^5)^(1
/2) + 3*a^3*c^3*d^2*e - 3*a*d*e^2*(a^5*c^5)^(1/2))/(216*a^5*c^4))^(1/3)
```

3.3 $\int \frac{d+ex^4}{a+cx^8}\,dx$

Optimal. Leaf size=754

$$-\frac{\sqrt{2-\sqrt{2}}\left(\left(1+\sqrt{2}\right)\sqrt{c}\,d-\sqrt{a}\,e\right)\tan^{-1}\left(\frac{\sqrt{2-\sqrt{2}}\,\sqrt[8]{a}-2\sqrt[8]{c}\,x}{\sqrt{2+\sqrt{2}}\,\sqrt[8]{a}}\right)}{8a^{7/8}c^{5/8}}+\frac{\sqrt{2+\sqrt{2}}\left(\left(1-\sqrt{2}\right)\sqrt{c}\,d\right.}{}$$

```
[Out] -1/8*arctan((-2*c^(1/8)*x+a^(1/8)*(2-2^(1/2))^(1/2))/a^(1/8)/(2+2^(1/2))^(1
/2))*(-e*a^(1/2)+d*(1+2^(1/2))*c^(1/2))*(2-2^(1/2))^(1/2)/a^(7/8)/c^(5/8)+1
/8*arctan((2*c^(1/8)*x+a^(1/8)*(2-2^(1/2))^(1/2))/a^(1/8)/(2+2^(1/2))^(1/2)
)*(-e*a^(1/2)+d*(1+2^(1/2))*c^(1/2))*(2-2^(1/2))^(1/2)/a^(7/8)/c^(5/8)+1/4*
arctan((-2*c^(1/8)*x+a^(1/8)*(2+2^(1/2))^(1/2))/a^(1/8)/(2-2^(1/2))^(1/2))*
(-e*a^(1/2)+d*(1-2^(1/2))*c^(1/2))/a^(7/8)/c^(5/8)/(4-2*2^(1/2))^(1/2)-1/4*
arctan((2*c^(1/8)*x+a^(1/8)*(2+2^(1/2))^(1/2))/a^(1/8)/(2-2^(1/2))^(1/2))*(
-e*a^(1/2)+d*(1-2^(1/2))*c^(1/2))/a^(7/8)/c^(5/8)/(4-2*2^(1/2))^(1/2)+1/8*l
n(a^(1/4)+c^(1/4)*x^2-a^(1/8)*c^(1/8)*x*(2-2^(1/2))^(1/2))*(-e*a^(1/2)+d*(1
-2^(1/2))*c^(1/2))/a^(7/8)/c^(5/8)/(4-2*2^(1/2))^(1/2)-1/8*ln(a^(1/4)+c^(1/
4)*x^2+a^(1/8)*c^(1/8)*x*(2-2^(1/2))^(1/2))*(-e*a^(1/2)+d*(1-2^(1/2))*c^(1/
2))/a^(7/8)/c^(5/8)/(4-2*2^(1/2))^(1/2)+1/8*ln(a^(1/4)+c^(1/4)*x^2+a^(1/8)*
c^(1/8)*x*(2+2^(1/2))^(1/2))*(d+d*2^(1/2)-e*a^(1/2)/c^(1/2))/a^(7/8)/c^(1/8
)/(4+2*2^(1/2))^(1/2)-1/8*ln(a^(1/4)+c^(1/4)*x^2-a^(1/8)*c^(1/8)*x*(2+2^(1/
2))^(1/2))*(-e*a^(1/2)+d*(1+2^(1/2))*c^(1/2))/a^(7/8)/c^(5/8)/(4+2*2^(1/2))
^(1/2)
```

Rubi [A]
time = 0.82, antiderivative size = 754, normalized size of antiderivative = 1.00, number of steps used = 19, number of rules used = 6, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.353$, Rules used = {1429, 1183, 648, 632, 210, 642}

Antiderivative was successfully verified.

```
[In] Int[(d + e*x^4)/(a + c*x^8),x]
```

```
[Out] -1/8*(Sqrt[2 - Sqrt[2]]*((1 + Sqrt[2])*Sqrt[c]*d - Sqrt[a]*e)*ArcTan[(Sqrt[
2 - Sqrt[2]]*a^(1/8) - 2*c^(1/8)*x)/(Sqrt[2 + Sqrt[2]]*a^(1/8))])/(a^(7/8)*
c^(5/8)) + (Sqrt[2 + Sqrt[2]]*((1 - Sqrt[2])*Sqrt[c]*d - Sqrt[a]*e)*ArcTan[
(Sqrt[2 + Sqrt[2]]*a^(1/8) - 2*c^(1/8)*x)/(Sqrt[2 - Sqrt[2]]*a^(1/8))])/(8*
a^(7/8)*c^(5/8)) + (Sqrt[2 - Sqrt[2]]*((1 + Sqrt[2])*Sqrt[c]*d - Sqrt[a]*e)
*ArcTan[(Sqrt[2 - Sqrt[2]]*a^(1/8) + 2*c^(1/8)*x)/(Sqrt[2 + Sqrt[2]]*a^(1/8
))])/(8*a^(7/8)*c^(5/8)) - (Sqrt[2 + Sqrt[2]]*((1 - Sqrt[2])*Sqrt[c]*d - Sq
```

```
rt[a]*e)*ArcTan[(Sqrt[2 + Sqrt[2]]*a^(1/8) + 2*c^(1/8)*x)/(Sqrt[2 - Sqrt[2]
]*a^(1/8))])/(8*a^(7/8)*c^(5/8)) + (((1 - Sqrt[2])*Sqrt[c]*d - Sqrt[a]*e)*L
og[a^(1/4) - Sqrt[2 - Sqrt[2]]*a^(1/8)*c^(1/8)*x + c^(1/4)*x^2])/(8*Sqrt[2*
(2 - Sqrt[2])]*a^(7/8)*c^(5/8)) - (((1 - Sqrt[2])*Sqrt[c]*d - Sqrt[a]*e)*Lo
g[a^(1/4) + Sqrt[2 - Sqrt[2]]*a^(1/8)*c^(1/8)*x + c^(1/4)*x^2])/(8*Sqrt[2*(
2 - Sqrt[2])]*a^(7/8)*c^(5/8)) - (((1 + Sqrt[2])*Sqrt[c]*d - Sqrt[a]*e)*Log
[a^(1/4) - Sqrt[2 + Sqrt[2]]*a^(1/8)*c^(1/8)*x + c^(1/4)*x^2])/(8*Sqrt[2*(2
 + Sqrt[2])]*a^(7/8)*c^(5/8)) + ((d + Sqrt[2]*d - (Sqrt[a]*e)/Sqrt[c])*Log[
a^(1/4) + Sqrt[2 + Sqrt[2]]*a^(1/8)*c^(1/8)*x + c^(1/4)*x^2])/(8*Sqrt[2*(2
+ Sqrt[2])]*a^(7/8)*c^(1/8))
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1183

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[a/c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Dist[1/(2*c*q*r), Int
[(d*r - (d - e*q)*x)/(q - r*x + x^2), x], x] + Dist[1/(2*c*q*r), Int[(d*r +
 (d - e*q)*x)/(q + r*x + x^2), x], x]]] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NegQ[b^2 - 4*a*c]
```

Rule 1429

```
Int[((d_) + (e_.)*(x_)^(n_))/((a_) + (c_.)*(x_)^(n2_)), x_Symbol] :> With[{
q = Rt[a/c, 4]}, Dist[1/(2*Sqrt[2]*c*q^3), Int[(Sqrt[2]*d*q - (d - e*q^2)*x
```

```
^(n/2))/(q^2 - Sqrt[2]*q*x^(n/2) + x^n), x], x] + Dist[1/(2*Sqrt[2]*c*q^3),
 Int[(Sqrt[2]*d*q + (d - e*q^2)*x^(n/2))/(q^2 + Sqrt[2]*q*x^(n/2) + x^n), x
], x]] /; FreeQ[{a, c, d, e}, x] && EqQ[n2, 2*n] && NeQ[c*d^2 + a*e^2, 0] &
& NeQ[c*d^2 - a*e^2, 0] && IGtQ[n/2, 0] && PosQ[a*c]
```

Rubi steps

$$\int \frac{d+ex^4}{a+cx^8}\,dx = \frac{\int \frac{\frac{\sqrt{2}\sqrt[4]{a}\,d}{\sqrt[4]{c}}+\left(-d+\frac{\sqrt{a}\,e}{\sqrt{c}}\right)x^2}{\frac{\sqrt{a}}{\sqrt{c}}-\frac{\sqrt{2}\sqrt[4]{a}\,x^2}{\sqrt[4]{c}}+x^4}\,dx}{2\sqrt{2}\,a^{3/4}\sqrt[4]{c}} + \frac{\int \frac{\frac{\sqrt{2}\sqrt[4]{a}\,d}{\sqrt[4]{c}}+\left(d-\frac{\sqrt{a}\,e}{\sqrt{c}}\right)x^2}{\frac{\sqrt{a}}{\sqrt{c}}+\frac{\sqrt{2}\sqrt[4]{a}\,x^2}{\sqrt[4]{c}}+x^4}\,dx}{2\sqrt{2}\,a^{3/4}\sqrt[4]{c}}$$

$$= \frac{\sqrt[8]{c}\int \frac{\frac{\sqrt{2\left(2-\sqrt{2}\right)}\,a^{3/8}d}{c^{3/8}}-\left(\frac{\sqrt{2}\sqrt[4]{a}\,d}{\sqrt[4]{c}}-\frac{\sqrt[4]{a}\left(d-\frac{\sqrt{a}\,e}{\sqrt{c}}\right)}{\sqrt[4]{c}}\right)x}{\frac{\sqrt[4]{a}}{\sqrt[4]{c}}-\frac{\sqrt{2-\sqrt{2}}\,\sqrt[8]{a}\,x}{\sqrt[8]{c}}+x^2}\,dx}{4\sqrt{2\left(2-\sqrt{2}\right)}\,a^{9/8}} + \frac{\sqrt[8]{c}\int \frac{\frac{\sqrt{2\left(2-\sqrt{2}\right)}\,a^{3/8}d}{c^{3/8}}+\cdots}{\frac{\sqrt[4]{a}}{\sqrt[4]{c}}+\cdots}}{4\sqrt{2\left(2\cdots\right.}}$$

$$= -\frac{\left(\left(1-\sqrt{2}\right)\sqrt{c}\,d-\sqrt{a}\,e\right)\int \frac{1}{\frac{\sqrt[4]{a}}{\sqrt[4]{c}}-\frac{\sqrt{2+\sqrt{2}}\,\sqrt[8]{a}\,x}{\sqrt[8]{c}}+x^2}\,dx}{8\sqrt{2}\,a^{3/4}c^{3/4}} - \frac{\left(\left(1-\sqrt{2}\right)\sqrt{c}\,d-\sqrt{a}\,e\cdots\right.}{8\sqrt{\cdots}}$$

$$= \frac{\left(\left(1-\sqrt{2}\right)\sqrt{c}\,d-\sqrt{a}\,e\right)\log\left(\sqrt[4]{a}-\sqrt{2-\sqrt{2}}\,\sqrt[8]{a}\,\sqrt[8]{c}\,x+\sqrt[4]{c}\,x^2\right)}{8\sqrt{2\left(2-\sqrt{2}\right)}\,a^{7/8}c^{5/8}} - \frac{\left(\left(1-\sqrt{2}\right)\sqrt{\cdots}\right.}{\cdots}$$

$$= -\frac{\left(\left(1+\sqrt{2}\right)\sqrt{c}\,d-\sqrt{a}\,e\right)\tan^{-1}\left(\frac{\sqrt{2-\sqrt{2}}\,\sqrt[8]{a}-2\sqrt[8]{c}\,x}{\sqrt{2+\sqrt{2}}\,\sqrt[8]{a}}\right)}{4\sqrt{2\left(2+\sqrt{2}\right)}\,a^{7/8}c^{5/8}} + \frac{\left(\left(1-\sqrt{2}\right)\sqrt{c}\,d-\sqrt{a}\cdots\right.}{4\sqrt{2\left(\cdots\right.}}$$

Mathematica [A]

time = 0.47, size = 534, normalized size = 0.71

$$\frac{-2\sqrt[8]{a}\tan^{-1}\left(\cot\left(\frac{\pi}{8}\right)-\frac{\sqrt[8]{c}\,x\csc\left(\frac{\pi}{8}\right)}{\sqrt[8]{a}}\right)\left(\sqrt{a}\,e\cos\left(\frac{\pi}{8}\right)+\sqrt{c}\,d\sin\left(\frac{\pi}{8}\right)\right)+2\sqrt[8]{a}\tan^{-1}\left(\cot\left(\frac{\pi}{8}\right)+\frac{\sqrt[8]{c}\,x\csc\left(\frac{\pi}{8}\right)}{\sqrt[8]{a}}\right)\left(\sqrt{a}\,e\cos\left(\frac{\pi}{8}\right)+\sqrt{c}\,d\sin\left(\frac{\pi}{8}\right)\right)-\sqrt[8]{a}\log\left(\sqrt[4]{a}+\sqrt[4]{c}\,x^2-2\sqrt[8]{a}\sqrt[8]{c}\,x\sin\left(\frac{\pi}{8}\right)\right)\left(\sqrt{a}\,e\cos\left(\frac{\pi}{8}\right)+\sqrt{c}\,d\sin\left(\frac{\pi}{8}\right)\right)+\sqrt[8]{a}\log\left(\sqrt[4]{a}+\sqrt[4]{c}\,x^2+2\sqrt[8]{a}\sqrt[8]{c}\,x\sin\left(\frac{\pi}{8}\right)\right)\left(\sqrt{a}\,e\cos\left(\frac{\pi}{8}\right)+\sqrt{c}\,d\sin\left(\frac{\pi}{8}\right)\right)+\sqrt[8]{a}\log\left(\sqrt[4]{a}+\sqrt[4]{c}\,x^2-2\sqrt[8]{a}\sqrt[8]{c}\,x\cos\left(\frac{\pi}{8}\right)\right)\left(-\sqrt{c}\,d\cos\left(\frac{\pi}{8}\right)+\sqrt{a}\,e\sin\left(\frac{\pi}{8}\right)\right)-\sqrt[8]{a}\log\left(\sqrt[4]{a}+\sqrt[4]{c}\,x^2+2\sqrt[8]{a}\sqrt[8]{c}\,x\cos\left(\frac{\pi}{8}\right)\right)\left(-\sqrt{c}\,d\cos\left(\frac{\pi}{8}\right)+\sqrt{a}\,e\sin\left(\frac{\pi}{8}\right)\right)+2\tan^{-1}\left(\frac{\sqrt[8]{c}\,x\sec\left(\frac{\pi}{8}\right)}{\sqrt[8]{a}}-\tan\left(\frac{\pi}{8}\right)\right)\left(\sqrt[8]{a}\sqrt{c}\,d\cos\left(\frac{\pi}{8}\right)-a^{5/8}e\sin\left(\frac{\pi}{8}\right)\right)+2\tan^{-1}\left(\frac{\sqrt[8]{c}\,x\sec\left(\frac{\pi}{8}\right)}{\sqrt[8]{a}}+\tan\left(\frac{\pi}{8}\right)\right)\left(\sqrt[8]{a}\sqrt{c}\,d\cos\left(\frac{\pi}{8}\right)-a^{5/8}e\sin\left(\frac{\pi}{8}\right)\right)}{8ac^{5/8}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x^4)/(a + c*x^8),x]
```

```
[Out] (-2*a^(1/8)*ArcTan[Cot[Pi/8] - (c^(1/8)*x*Csc[Pi/8])/a^(1/8)]*(Sqrt[a]*e*Co
s[Pi/8] + Sqrt[c]*d*Sin[Pi/8]) + 2*a^(1/8)*ArcTan[Cot[Pi/8] + (c^(1/8)*x*Cs
c[Pi/8])/a^(1/8)]*(Sqrt[a]*e*Cos[Pi/8] + Sqrt[c]*d*Sin[Pi/8]) - a^(1/8)*Log
[a^(1/4) + c^(1/4)*x^2 - 2*a^(1/8)*c^(1/8)*x*Sin[Pi/8]]*(Sqrt[a]*e*Cos[Pi/8
] + Sqrt[c]*d*Sin[Pi/8]) + a^(1/8)*Log[a^(1/4) + c^(1/4)*x^2 + 2*a^(1/8)*c^
(1/8)*x*Sin[Pi/8]]*(Sqrt[a]*e*Cos[Pi/8] + Sqrt[c]*d*Sin[Pi/8]) + a^(1/8)*Lo
g[a^(1/4) + c^(1/4)*x^2 - 2*a^(1/8)*c^(1/8)*x*Cos[Pi/8]]*(-(Sqrt[c]*d*Cos[P
i/8]) + Sqrt[a]*e*Sin[Pi/8]) - a^(1/8)*Log[a^(1/4) + c^(1/4)*x^2 + 2*a^(1/8
)*c^(1/8)*x*Cos[Pi/8]]*(-(Sqrt[c]*d*Cos[Pi/8]) + Sqrt[a]*e*Sin[Pi/8]) + 2*A
rcTan[(c^(1/8)*x*Sec[Pi/8])/a^(1/8) - Tan[Pi/8]]*(a^(1/8)*Sqrt[c]*d*Cos[Pi/
8] - a^(5/8)*e*Sin[Pi/8]) + 2*ArcTan[(c^(1/8)*x*Sec[Pi/8])/a^(1/8) + Tan[Pi
/8]]*(a^(1/8)*Sqrt[c]*d*Cos[Pi/8] - a^(5/8)*e*Sin[Pi/8]))/(8*a*c^(5/8))
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 0.19, size = 34, normalized size = 0.05

method	result	size
default	$\frac{\sum_{\_R=\mathrm{RootOf}\left(c\_Z^8+a\right)}\frac{\left(\_R^4e+d\right)\ln\left(x-\_R\right)}{\_R^7}}{8c}$	34
risch	$\frac{\sum_{\_R=\mathrm{RootOf}\left(c\_Z^8+a\right)}\frac{\left(\_R^4e+d\right)\ln\left(x-\_R\right)}{\_R^7}}{8c}$	34

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^4+d)/(c*x^8+a),x,method=_RETURNVERBOSE)
```

```
[Out] 1/8/c*sum((_R^4*e+d)/_R^7*ln(x-_R),_R=RootOf(_Z^8*c+a))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^4+d)/(c*x^8+a),x, algorithm="maxima")
```

```
[Out] integrate((x^4*e + d)/(c*x^8 + a), x)
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 3580 vs. $2(522) = 1044$.
time = 2.02, size = 3580, normalized size = 4.75

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^4+d)/(c*x^8+a),x, algorithm="fricas")
```

```
[Out] -1/2*((a^3*c^2*sqrt(-(c^4*d^8 - 12*a*c^3*d^6*e^2 + 38*a^2*c^2*d^4*e^4 - 12*
a^3*c*d^2*e^6 + a^4*e^8)/(a^7*c^5)) - 4*c*d^3*e + 4*a*d*e^3)/(a^3*c^2))^(1/
4)*arctan(-(sqrt(c^6*d^12*x^2 - 10*a*c^5*d^10*x^2*e^2 + 15*a^2*c^4*d^8*x^2*
e^4 + 52*a^3*c^3*d^6*x^2*e^6 + 15*a^4*c^2*d^4*x^2*e^8 - 10*a^5*c*d^2*x^2*e^
10 + a^6*x^2*e^12 + (a^2*c^6*d^10 - 13*a^3*c^5*d^8*e^2 + 50*a^4*c^4*d^6*e^4
 - 50*a^5*c^3*d^4*e^6 + 13*a^6*c^2*d^2*e^8 - a^7*c*e^10 - 2*(a^6*c^6*d^5*e
- 6*a^7*c^5*d^3*e^3 + a^8*c^4*d*e^5)*sqrt(-(c^4*d^8 - 12*a*c^3*d^6*e^2 + 38
*a^2*c^2*d^4*e^4 - 12*a^3*c*d^2*e^6 + a^4*e^8)/(a^7*c^5)))*sqrt((a^3*c^2*sq
rt(-(c^4*d^8 - 12*a*c^3*d^6*e^2 + 38*a^2*c^2*d^4*e^4 - 12*a^3*c*d^2*e^6 + a
^4*e^8)/(a^7*c^5)) - 4*c*d^3*e + 4*a*d*e^3)/(a^3*c^2)))*(3*a^3*c^5*d^6*e -
19*a^4*c^4*d^4*e^3 + 9*a^5*c^3*d^2*e^5 - a^6*c^2*e^7 + (a^6*c^6*d^3 - 3*a^7
*c^5*d*e^2)*sqrt(-(c^4*d^8 - 12*a*c^3*d^6*e^2 + 38*a^2*c^2*d^4*e^4 - 12*a^3
*c*d^2*e^6 + a^4*e^8)/(a^7*c^5)))*sqrt((a^3*c^2*sqrt(-(c^4*d^8 - 12*a*c^3*d
^6*e^2 + 38*a^2*c^2*d^4*e^4 - 12*a^3*c*d^2*e^6 + a^4*e^8)/(a^7*c^5)) - 4*c*
d^3*e + 4*a*d*e^3)/(a^3*c^2)) - (3*a^3*c^8*d^12*x*e - 34*a^4*c^7*d^10*x*e^3
 + 89*a^5*c^6*d^8*x*e^5 + 52*a^6*c^5*d^6*x*e^7 - 59*a^7*c^4*d^4*x*e^9 + 14*
a^8*c^3*d^2*x*e^11 - a^9*c^2*x*e^13 + (a^6*c^9*d^9*x - 8*a^7*c^8*d^7*x*e^2
+ 10*a^8*c^7*d^5*x*e^4 + 16*a^9*c^6*d^3*x*e^6 - 3*a^10*c^5*d*x*e^8)*sqrt(-(
c^4*d^8 - 12*a*c^3*d^6*e^2 + 38*a^2*c^2*d^4*e^4 - 12*a^3*c*d^2*e^6 + a^4*e^
8)/(a^7*c^5)))*sqrt((a^3*c^2*sqrt(-(c^4*d^8 - 12*a*c^3*d^6*e^2 + 38*a^2*c^2
*d^4*e^4 - 12*a^3*c*d^2*e^6 + a^4*e^8)/(a^7*c^5)) - 4*c*d^3*e + 4*a*d*e^3)/
(a^3*c^2)))*((a^3*c^2*sqrt(-(c^4*d^8 - 12*a*c^3*d^6*e^2 + 38*a^2*c^2*d^4*e^
4 - 12*a^3*c*d^2*e^6 + a^4*e^8)/(a^7*c^5)) - 4*c*d^3*e + 4*a*d*e^3)/(a^3*c^
2))^(1/4)/(c^8*d^16 - 8*a*c^7*d^14*e^2 - 4*a^2*c^6*d^12*e^4 + 72*a^3*c^5*d^
10*e^6 + 134*a^4*c^4*d^8*e^8 + 72*a^5*c^3*d^6*e^10 - 4*a^6*c^2*d^4*e^12 - 8
*a^7*c*d^2*e^14 + a^8*e^16)) + 1/2*(-(a^3*c^2*sqrt(-(c^4*d^8 - 12*a*c^3*d^6
*e^2 + 38*a^2*c^2*d^4*e^4 - 12*a^3*c*d^2*e^6 + a^4*e^8)/(a^7*c^5)) + 4*c*d^
3*e - 4*a*d*e^3)/(a^3*c^2))^(1/4)*arctan((sqrt(c^6*d^12*x^2 - 10*a*c^5*d^10
*x^2*e^2 + 15*a^2*c^4*d^8*x^2*e^4 + 52*a^3*c^3*d^6*x^2*e^6 + 15*a^4*c^2*d^4
*x^2*e^8 - 10*a^5*c*d^2*x^2*e^10 + a^6*x^2*e^12 + (a^2*c^6*d^10 - 13*a^3*c^
5*d^8*e^2 + 50*a^4*c^4*d^6*e^4 - 50*a^5*c^3*d^4*e^6 + 13*a^6*c^2*d^2*e^8 -
a^7*c*e^10 + 2*(a^6*c^6*d^5*e - 6*a^7*c^5*d^3*e^3 + a^8*c^4*d*e^5)*sqrt(-(c
^4*d^8 - 12*a*c^3*d^6*e^2 + 38*a^2*c^2*d^4*e^4 - 12*a^3*c*d^2*e^6 + a^4*e^8
)/(a^7*c^5)))*sqrt(-(a^3*c^2*sqrt(-(c^4*d^8 - 12*a*c^3*d^6*e^2 + 38*a^2*c^2
*d^4*e^4 - 12*a^3*c*d^2*e^6 + a^4*e^8)/(a^7*c^5)) + 4*c*d^3*e - 4*a*d*e^3)/
(a^3*c^2)))*(3*a^3*c^5*d^6*e - 19*a^4*c^4*d^4*e^3 + 9*a^5*c^3*d^2*e^5 - a^6
```

```
*c^2*e^7 - (a^6*c^6*d^3 - 3*a^7*c^5*d*e^2)*sqrt(-(c^4*d^8 - 12*a*c^3*d^6*e^
2 + 38*a^2*c^2*d^4*e^4 - 12*a^3*c*d^2*e^6 + a^4*e^8)/(a^7*c^5)))*(-(a^3*c^2
*sqrt(-(c^4*d^8 - 12*a*c^3*d^6*e^2 + 38*a^2*c^2*d^4*e^4 - 12*a^3*c*d^2*e^6
+ a^4*e^8)/(a^7*c^5)) + 4*c*d^3*e - 4*a*d*e^3)/(a^3*c^2))^(3/4) - (3*a^3*c^
8*d^12*x*e - 34*a^4*c^7*d^10*x*e^3 + 89*a^5*c^6*d^8*x*e^5 + 52*a^6*c^5*d^6*
x*e^7 - 59*a^7*c^4*d^4*x*e^9 + 14*a^8*c^3*d^2*x*e^11 - a^9*c^2*x*e^13 - (a^
6*c^9*d^9*x - 8*a^7*c^8*d^7*x*e^2 + 10*a^8*c^7*d^5*x*e^4 + 16*a^9*c^6*d^3*x
*e^6 - 3*a^10*c^5*d*x*e^8)*sqrt(-(c^4*d^8 - 12*a*c^3*d^6*e^2 + 38*a^2*c^2*d
^4*e^4 - 12*a^3*c*d^2*e^6 + a^4*e^8)/(a^7*c^5)))*(-(a^3*c^2*sqrt(-(c^4*d^8
- 12*a*c^3*d^6*e^2 + 38*a^2*c^2*d^4*e^4 - 12*a^3*c*d^2*e^6 + a^4*e^8)/(a^7*
c^5)) + 4*c*d^3*e - 4*a*d*e^3)/(a^3*c^2))^(3/4))/(c^8*d^16 - 8*a*c^7*d^14*e
^2 - 4*a^2*c^6*d^12*e^4 + 72*a^3*c^5*d^10*e^6 + 134*a^4*c^4*d^8*e^8 + 72*a^
5*c^3*d^6*e^10 - 4*a^6*c^2*d^4*e^12 - 8*a^7*c*d^2*e^14 + a^8*e^16)) + 1/8*(
-(a^3*c^2*sqrt(-(c^4*d^8 - 12*a*c^3*d^6*e^2 + 38*a^2*c^2*d^4*e^4 - 12*a^3*c
*d^2*e^6 + a^4*e^8)/(a^7*c^5)) + 4*c*d^3*e - 4*a*d*e^3)/(a^3*c^2))^(1/4)*lo
g(c^3*d^6*x - 5*a*c^2*d^4*x*e^2 - 5*a^2*c*d^2*x*e^4 + a^3*x*e^6 + (a^5*c^3*
sqrt(-(c^4*d^8 - 12*a*c^3*d^6*e^2 + 38*a^2*c^2*d^4*e^4 - 12*a^3*c*d^2*e^6 +
 a^4*e^8)/(a^7*c^5))*e + a*c^3*d^5 - 6*a^2*c^2*d^3*e^2 + a^3*c*d*e^4)*(-(a^
3*c^2*sqrt(-(c^4*d^8 - 12*a*c^3*d^6*e^2 + 38*a^2*c^2*d^4*e^4 - 12*a^3*c*d^2
*e^6 + a^4*e^8)/(a^7*c^5)) + 4*c*d^3*e - 4*a*d*e^3)/(a^3*c^2))^(1/4)) - 1/8
*(-(a^3*c^2*sqrt(-(c^4*d^8 - 12*a*c^3*d^6*e^2 + 38*a^2*c^2*d^4*e^4 - 12*a^3
*c*d^2*e^6 + a^4*e^8)/(a^7*c^5)) + 4*c*d^3*e - 4*a*d*e^3)/(a^3*c^2))^(1/4)*
log(c^3*d^6*x - 5*a*c^2*d^4*x*e^2 - 5*a^2*c*d^2*x*e^4 + a^3*x*e^6 - (a^5*c^
3*sqrt(-(c^4*d^8 - 12*a*c^3*d^6*e^2 + 38*a^2*c^2*d^4*e^4 - 12*a^3*c*d^2*e^6
 + a^4*e^8)/(a^7*c^5))*e + a*c^3*d^5 - 6*a^2*c^2*d^3*e^2 + a^3*c*d*e^4)*(-(
a^3*c^2*sqrt(-(c^4*d^8 - 12*a*c^3*d^6*e^2 + 38*a^2*c^2*d^4*e^4 - 12*a^3*c*d
^2*e^6 + a^4*e^8)/(a^7*c^5)) + 4*c*d^3*e - 4*a*d*e^3)/(a^3*c^2))^(1/4)) - 1
/8*((a^3*c^2*sqrt(-(c^4*d^8 - 12*a*c^3*d^6*e^2 + 38*a^2*c^2*d^4*e^4 - 12*a^
3*c*d^2*e^6 + a^4*e^8)/(a^7*c^5)) - 4*c*d^3*e + 4*a*d*e^3)/(a^3*c^2))^(1/4)
*log(c^3*d^6*x - 5*a*c^2*d^4*x*e^2 - 5*a^2*c*d^2*x*e^4 + a^3*x*e^6 + (a^5*c
^3*sqrt(-(c^4*d^8 - 12*a*c^3*d^6*e^2 + 38*a^2*c...
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

$$\text{Timed out}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**4+d)/(c*x**8+a),x)
```

```
[Out] Timed out
```

Giac [**A**]
time = 3.79, size = 601, normalized size = 0.80

$$-\frac{\left(\sqrt{-\sqrt{2}+2}\left(\frac{a}{c}\right)^{\frac{5}{8}}e-d\sqrt{\sqrt{2}+2}\left(\frac{a}{c}\right)^{\frac{1}{8}}\right)\arctan\left(\frac{2x+\sqrt{-\sqrt{2}+2}\left(\frac{a}{c}\right)^{\frac{1}{8}}}{\sqrt{\sqrt{2}+2}\left(\frac{a}{c}\right)^{\frac{1}{8}}}\right)}{8a}-\frac{\left(\sqrt{-\sqrt{2}+2}\left(\frac{a}{c}\right)^{\frac{5}{8}}e-d\sqrt{\sqrt{2}+2}\left(\frac{a}{c}\right)^{\frac{1}{8}}\right)\arctan\left(\frac{2x-\sqrt{-\sqrt{2}+2}\left(\frac{a}{c}\right)^{\frac{1}{8}}}{\sqrt{\sqrt{2}+2}\left(\frac{a}{c}\right)^{\frac{1}{8}}}\right)}{8a}+\frac{\left(\sqrt{\sqrt{2}+2}\left(\frac{a}{c}\right)^{\frac{5}{8}}e+d\sqrt{-\sqrt{2}+2}\left(\frac{a}{c}\right)^{\frac{1}{8}}\right)\arctan\left(\frac{2x+\sqrt{\sqrt{2}+2}\left(\frac{a}{c}\right)^{\frac{1}{8}}}{\sqrt{-\sqrt{2}+2}\left(\frac{a}{c}\right)^{\frac{1}{8}}}\right)}{8a}+\frac{\left(\sqrt{\sqrt{2}+2}\left(\frac{a}{c}\right)^{\frac{5}{8}}e+d\sqrt{-\sqrt{2}+2}\left(\frac{a}{c}\right)^{\frac{1}{8}}\right)\arctan\left(\frac{2x-\sqrt{\sqrt{2}+2}\left(\frac{a}{c}\right)^{\frac{1}{8}}}{\sqrt{-\sqrt{2}+2}\left(\frac{a}{c}\right)^{\frac{1}{8}}}\right)}{8a}-\frac{\left(\sqrt{-\sqrt{2}+2}\left(\frac{a}{c}\right)^{\frac{5}{8}}e-d\sqrt{\sqrt{2}+2}\left(\frac{a}{c}\right)^{\frac{1}{8}}\right)\log\left(x^2+x\sqrt{\sqrt{2}+2}\left(\frac{a}{c}\right)^{\frac{1}{8}}+\left(\frac{a}{c}\right)^{\frac{1}{4}}\right)}{16a}+\frac{\left(\sqrt{-\sqrt{2}+2}\left(\frac{a}{c}\right)^{\frac{5}{8}}e-d\sqrt{\sqrt{2}+2}\left(\frac{a}{c}\right)^{\frac{1}{8}}\right)\log\left(x^2-x\sqrt{\sqrt{2}+2}\left(\frac{a}{c}\right)^{\frac{1}{8}}+\left(\frac{a}{c}\right)^{\frac{1}{4}}\right)}{16a}+\frac{\left(\sqrt{\sqrt{2}+2}\left(\frac{a}{c}\right)^{\frac{5}{8}}e+d\sqrt{-\sqrt{2}+2}\left(\frac{a}{c}\right)^{\frac{1}{8}}\right)\log\left(x^2+x\sqrt{-\sqrt{2}+2}\left(\frac{a}{c}\right)^{\frac{1}{8}}+\left(\frac{a}{c}\right)^{\frac{1}{4}}\right)}{16a}-\frac{\left(\sqrt{\sqrt{2}+2}\left(\frac{a}{c}\right)^{\frac{5}{8}}e+d\sqrt{-\sqrt{2}+2}\left(\frac{a}{c}\right)^{\frac{1}{8}}\right)\log\left(x^2-x\sqrt{-\sqrt{2}+2}\left(\frac{a}{c}\right)^{\frac{1}{8}}+\left(\frac{a}{c}\right)^{\frac{1}{4}}\right)}{16a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^4+d)/(c*x^8+a),x, algorithm="giac")
```

```
[Out] -1/8*(sqrt(-sqrt(2) + 2)*(a/c)^(5/8)*e - d*sqrt(sqrt(2) + 2)*(a/c)^(1/8))*a
rctan((2*x + sqrt(-sqrt(2) + 2)*(a/c)^(1/8))/(sqrt(sqrt(2) + 2)*(a/c)^(1/8)
))/a - 1/8*(sqrt(-sqrt(2) + 2)*(a/c)^(5/8)*e - d*sqrt(sqrt(2) + 2)*(a/c)^(1
/8))*arctan((2*x - sqrt(-sqrt(2) + 2)*(a/c)^(1/8))/(sqrt(sqrt(2) + 2)*(a/c)
^(1/8)))/a + 1/8*(sqrt(sqrt(2) + 2)*(a/c)^(5/8)*e + d*sqrt(-sqrt(2) + 2)*(a
/c)^(1/8))*arctan((2*x + sqrt(sqrt(2) + 2)*(a/c)^(1/8))/(sqrt(-sqrt(2) + 2)
*(a/c)^(1/8)))/a + 1/8*(sqrt(sqrt(2) + 2)*(a/c)^(5/8)*e + d*sqrt(-sqrt(2) +
 2)*(a/c)^(1/8))*arctan((2*x - sqrt(sqrt(2) + 2)*(a/c)^(1/8))/(sqrt(-sqrt(2
) + 2)*(a/c)^(1/8)))/a - 1/16*(sqrt(-sqrt(2) + 2)*(a/c)^(5/8)*e - d*sqrt(sq
rt(2) + 2)*(a/c)^(1/8))*log(x^2 + x*sqrt(sqrt(2) + 2)*(a/c)^(1/8) + (a/c)^(
1/4))/a + 1/16*(sqrt(-sqrt(2) + 2)*(a/c)^(5/8)*e - d*sqrt(sqrt(2) + 2)*(a/c
)^(1/8))*log(x^2 - x*sqrt(sqrt(2) + 2)*(a/c)^(1/8) + (a/c)^(1/4))/a + 1/16*
(sqrt(sqrt(2) + 2)*(a/c)^(5/8)*e + d*sqrt(-sqrt(2) + 2)*(a/c)^(1/8))*log(x^
2 + x*sqrt(-sqrt(2) + 2)*(a/c)^(1/8) + (a/c)^(1/4))/a - 1/16*(sqrt(sqrt(2)
+ 2)*(a/c)^(5/8)*e + d*sqrt(-sqrt(2) + 2)*(a/c)^(1/8))*log(x^2 - x*sqrt(-sq
rt(2) + 2)*(a/c)^(1/8) + (a/c)^(1/4))/a
```

Mupad [B]
time = 2.78, size = 2510, normalized size = 3.33

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x^4)/(a + c*x^8),x)
```

```
[Out] (atan((c^3*d^6*x - a^3*e^6*x + a*c^2*d^4*e^2*x - a^2*c*d^2*e^4*x + (2*d*e*x
*(a^2*e^4*(-a^7*c^5)^(1/2) + c^2*d^4*(-a^7*c^5)^(1/2) - 4*a^4*c^4*d^3*e + 4
*a^5*c^3*d*e^3 - 6*a*c*d^2*e^2*(-a^7*c^5)^(1/2)))/(a^3*c^2))/(a*c^3*d^5*((a
^2*e^4*(-a^7*c^5)^(1/2) + c^2*d^4*(-a^7*c^5)^(1/2) - 4*a^4*c^4*d^3*e + 4*a^
5*c^3*d*e^3 - 6*a*c*d^2*e^2*(-a^7*c^5)^(1/2))/(a^7*c^5))^(1/4) + a^5*c^3*e*
((a^2*e^4*(-a^7*c^5)^(1/2) + c^2*d^4*(-a^7*c^5)^(1/2) - 4*a^4*c^4*d^3*e + 4
*a^5*c^3*d*e^3 - 6*a*c*d^2*e^2*(-a^7*c^5)^(1/2))/(a^7*c^5))^(5/4) - 2*a^2*c
^2*d^3*e^2*((a^2*e^4*(-a^7*c^5)^(1/2) + c^2*d^4*(-a^7*c^5)^(1/2) - 4*a^4*c^
4*d^3*e + 4*a^5*c^3*d*e^3 - 6*a*c*d^2*e^2*(-a^7*c^5)^(1/2))/(a^7*c^5))^(1/4
) - 3*a^3*c*d*e^4*((a^2*e^4*(-a^7*c^5)^(1/2) + c^2*d^4*(-a^7*c^5)^(1/2) - 4
*a^4*c^4*d^3*e + 4*a^5*c^3*d*e^3 - 6*a*c*d^2*e^2*(-a^7*c^5)^(1/2))/(a^7*c^5
))^(1/4)))*((a^2*e^4*(-a^7*c^5)^(1/2) + c^2*d^4*(-a^7*c^5)^(1/2) - 4*a^4*c^
4*d^3*e + 4*a^5*c^3*d*e^3 - 6*a*c*d^2*e^2*(-a^7*c^5)^(1/2))/(a^7*c^5))^(1/4
))/4 - (atan((a^3*e^6*x - c^3*d^6*x - a*c^2*d^4*e^2*x + a^2*c*d^2*e^4*x + (
2*d*e*x*(a^2*e^4*(-a^7*c^5)^(1/2) + c^2*d^4*(-a^7*c^5)^(1/2) + 4*a^4*c^4*d^
3*e - 4*a^5*c^3*d*e^3 - 6*a*c*d^2*e^2*(-a^7*c^5)^(1/2)))/(a^3*c^2))/(a*c^3*
d^5*(-(a^2*e^4*(-a^7*c^5)^(1/2) + c^2*d^4*(-a^7*c^5)^(1/2) + 4*a^4*c^4*d^3*
```

```
e - 4*a^5*c^3*d*e^3 - 6*a*c*d^2*e^2*(-a^7*c^5)^(1/2))/(a^7*c^5))^(1/4) + a^
5*c^3*e*(-(a^2*e^4*(-a^7*c^5)^(1/2) + c^2*d^4*(-a^7*c^5)^(1/2) + 4*a^4*c^4*
d^3*e - 4*a^5*c^3*d*e^3 - 6*a*c*d^2*e^2*(-a^7*c^5)^(1/2))/(a^7*c^5))^(5/4)
- 2*a^2*c^2*d^3*e^2*(-(a^2*e^4*(-a^7*c^5)^(1/2) + c^2*d^4*(-a^7*c^5)^(1/2)
+ 4*a^4*c^4*d^3*e - 4*a^5*c^3*d*e^3 - 6*a*c*d^2*e^2*(-a^7*c^5)^(1/2))/(a^7*
c^5))^(1/4) - 3*a^3*c*d*e^4*(-(a^2*e^4*(-a^7*c^5)^(1/2) + c^2*d^4*(-a^7*c^5
)^(1/2) + 4*a^4*c^4*d^3*e - 4*a^5*c^3*d*e^3 - 6*a*c*d^2*e^2*(-a^7*c^5)^(1/2
))/(a^7*c^5))^(1/4)))*(-(a^2*e^4*(-a^7*c^5)^(1/2) + c^2*d^4*(-a^7*c^5)^(1/2
) + 4*a^4*c^4*d^3*e - 4*a^5*c^3*d*e^3 - 6*a*c*d^2*e^2*(-a^7*c^5)^(1/2))/(a^
7*c^5))^(1/4))/4 - atan((c^3*d^6*x*1i - a^3*e^6*x*1i + a*c^2*d^4*e^2*x*1i -
 a^2*c*d^2*e^4*x*1i + (d*e*x*(a^2*e^4*(-a^7*c^5)^(1/2) + c^2*d^4*(-a^7*c^5)
^(1/2) - 4*a^4*c^4*d^3*e + 4*a^5*c^3*d*e^3 - 6*a*c*d^2*e^2*(-a^7*c^5)^(1/2)
)*2i)/(a^3*c^2))/(a*c^3*d^5*((a^2*e^4*(-a^7*c^5)^(1/2) + c^2*d^4*(-a^7*c^5)
^(1/2) - 4*a^4*c^4*d^3*e + 4*a^5*c^3*d*e^3 - 6*a*c*d^2*e^2*(-a^7*c^5)^(1/2)
)/(a^7*c^5))^(1/4) + a^5*c^3*e*((a^2*e^4*(-a^7*c^5)^(1/2) + c^2*d^4*(-a^7*c
^5)^(1/2) - 4*a^4*c^4*d^3*e + 4*a^5*c^3*d*e^3 - 6*a*c*d^2*e^2*(-a^7*c^5)^(1
/2))/(a^7*c^5))^(5/4) - 2*a^2*c^2*d^3*e^2*((a^2*e^4*(-a^7*c^5)^(1/2) + c^2*
d^4*(-a^7*c^5)^(1/2) - 4*a^4*c^4*d^3*e + 4*a^5*c^3*d*e^3 - 6*a*c*d^2*e^2*(-
a^7*c^5)^(1/2))/(a^7*c^5))^(1/4) - 3*a^3*c*d*e^4*((a^2*e^4*(-a^7*c^5)^(1/2)
 + c^2*d^4*(-a^7*c^5)^(1/2) - 4*a^4*c^4*d^3*e + 4*a^5*c^3*d*e^3 - 6*a*c*d^2
*e^2*(-a^7*c^5)^(1/2))/(a^7*c^5))^(1/4)))*((a^2*e^4*(-a^7*c^5)^(1/2) + c^2*
d^4*(-a^7*c^5)^(1/2) - 4*a^4*c^4*d^3*e + 4*a^5*c^3*d*e^3 - 6*a*c*d^2*e^2*(-
a^7*c^5)^(1/2))/(4096*a^7*c^5))^(1/4)*2i + atan((a^3*e^6*x*1i - c^3*d^6*x*1
i - a*c^2*d^4*e^2*x*1i + a^2*c*d^2*e^4*x*1i + (d*e*x*(a^2*e^4*(-a^7*c^5)^(1
/2) + c^2*d^4*(-a^7*c^5)^(1/2) + 4*a^4*c^4*d^3*e - 4*a^5*c^3*d*e^3 - 6*a*c*
d^2*e^2*(-a^7*c^5)^(1/2))*2i)/(a^3*c^2))/(a*c^3*d^5*(-(a^2*e^4*(-a^7*c^5)^(
1/2) + c^2*d^4*(-a^7*c^5)^(1/2) + 4*a^4*c^4*d^3*e - 4*a^5*c^3*d*e^3 - 6*a*c
*d^2*e^2*(-a^7*c^5)^(1/2))/(a^7*c^5))^(1/4) + a^5*c^3*e*(-(a^2*e^4*(-a^7*c^
5)^(1/2) + c^2*d^4*(-a^7*c^5)^(1/2) + 4*a^4*c^4*d^3*e - 4*a^5*c^3*d*e^3 - 6
*a*c*d^2*e^2*(-a^7*c^5)^(1/2))/(a^7*c^5))^(5/4) - 2*a^2*c^2*d^3*e^2*(-(a^2*
e^4*(-a^7*c^5)^(1/2) + c^2*d^4*(-a^7*c^5)^(1/2) + 4*a^4*c^4*d^3*e - 4*a^5*c
^3*d*e^3 - 6*a*c*d^2*e^2*(-a^7*c^5)^(1/2))/(a^7*c^5))^(1/4) - 3*a^3*c*d*e^4
*(-(a^2*e^4*(-a^7*c^5)^(1/2) + c^2*d^4*(-a^7*c^5)^(1/2) + 4*a^4*c^4*d^3*e -
 4*a^5*c^3*d*e^3 - 6*a*c*d^2*e^2*(-a^7*c^5)^(1/2))/(a^7*c^5))^(1/4)))*(-(a^
2*e^4*(-a^7*c^5)^(1/2) + c^2*d^4*(-a^7*c^5)^(1/2) + 4*a^4*c^4*d^3*e - 4*a^5
*c^3*d*e^3 - 6*a*c*d^2*e^2*(-a^7*c^5)^(1/2))/(4096*a^7*c^5))^(1/4)*2i
```

3.4 $\int \frac{d+ex^4}{a-cx^8}\,dx$

Optimal. Leaf size=329

$$\frac{\left(\sqrt{c}\,d+\sqrt{a}\,e\right)\tan^{-1}\left(\frac{\sqrt[8]{c}\,x}{\sqrt[8]{a}}\right)}{4a^{7/8}c^{5/8}}-\frac{\left(d-\frac{\sqrt{a}\,e}{\sqrt{c}}\right)\tan^{-1}\left(1-\frac{\sqrt{2}\,\sqrt[8]{c}\,x}{\sqrt[8]{a}}\right)}{4\sqrt{2}\,a^{7/8}\sqrt[8]{c}}+\frac{\left(d-\frac{\sqrt{a}\,e}{\sqrt{c}}\right)\tan^{-1}\left(1+\frac{\sqrt{2}\,\sqrt[8]{c}\,x}{\sqrt[8]{a}}\right)}{4\sqrt{2}\,a^{7/8}\sqrt[8]{c}}$$

```
[Out] 1/8*arctan(-1+c^(1/8)*x*2^(1/2)/a^(1/8))*(d-e*a^(1/2)/c^(1/2))/a^(7/8)/c^(1
/8)*2^(1/2)+1/8*arctan(1+c^(1/8)*x*2^(1/2)/a^(1/8))*(d-e*a^(1/2)/c^(1/2))/a
^(7/8)/c^(1/8)*2^(1/2)-1/16*ln(a^(1/4)+c^(1/4)*x^2-a^(1/8)*c^(1/8)*x*2^(1/2
))*(d-e*a^(1/2)/c^(1/2))/a^(7/8)/c^(1/8)*2^(1/2)+1/16*ln(a^(1/4)+c^(1/4)*x^
2+a^(1/8)*c^(1/8)*x*2^(1/2))*(d-e*a^(1/2)/c^(1/2))/a^(7/8)/c^(1/8)*2^(1/2)+
1/4*arctan(c^(1/8)*x/a^(1/8))*(e*a^(1/2)+d*c^(1/2))/a^(7/8)/c^(5/8)+1/4*arc
tanh(c^(1/8)*x/a^(1/8))*(e*a^(1/2)+d*c^(1/2))/a^(7/8)/c^(5/8)
```

Rubi [A]
time = 0.14, antiderivative size = 329, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 10, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.556$, Rules used = {1431, 218, 214, 211, 217, 1179, 642, 1176, 631, 210}

$$\frac{\text{ArcTan}\left(\frac{\sqrt[8]{c}\,x}{\sqrt[8]{a}}\right)\left(\sqrt{a}\,e+\sqrt{c}\,d\right)}{4a^{7/8}c^{5/8}}-\frac{\text{ArcTan}\left(1-\frac{\sqrt{2}\,\sqrt[8]{c}\,x}{\sqrt[8]{a}}\right)\left(d-\frac{\sqrt{a}\,e}{\sqrt{c}}\right)}{4\sqrt{2}\,a^{7/8}\sqrt[8]{c}}+\frac{\text{ArcTan}\left(\frac{\sqrt{2}\,\sqrt[8]{c}\,x}{\sqrt[8]{a}}+1\right)\left(d-\frac{\sqrt{a}\,e}{\sqrt{c}}\right)}{4\sqrt{2}\,a^{7/8}\sqrt[8]{c}}+\frac{\left(\sqrt{a}\,e+\sqrt{c}\,d\right)\tanh^{-1}\left(\frac{\sqrt[8]{c}\,x}{\sqrt[8]{a}}\right)}{4a^{7/8}c^{5/8}}-\frac{\left(d-\frac{\sqrt{a}\,e}{\sqrt{c}}\right)\log\left(-\sqrt{2}\,\sqrt[8]{a}\,\sqrt[8]{c}\,x+\sqrt[4]{a}+\sqrt[4]{c}\,x^2\right)}{8\sqrt{2}\,a^{7/8}\sqrt[8]{c}}+\frac{\left(d-\frac{\sqrt{a}\,e}{\sqrt{c}}\right)\log\left(\sqrt{2}\,\sqrt[8]{a}\,\sqrt[8]{c}\,x+\sqrt[4]{a}+\sqrt[4]{c}\,x^2\right)}{8\sqrt{2}\,a^{7/8}\sqrt[8]{c}}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x^4)/(a - c*x^8),x]
```

```
[Out] ((Sqrt[c]*d + Sqrt[a]*e)*ArcTan[(c^(1/8)*x)/a^(1/8)])/(4*a^(7/8)*c^(5/8)) -
 ((d - (Sqrt[a]*e)/Sqrt[c])*ArcTan[1 - (Sqrt[2]*c^(1/8)*x)/a^(1/8)])/(4*Sqr
t[2]*a^(7/8)*c^(1/8)) + ((d - (Sqrt[a]*e)/Sqrt[c])*ArcTan[1 + (Sqrt[2]*c^(1
/8)*x)/a^(1/8)])/(4*Sqrt[2]*a^(7/8)*c^(1/8)) + ((Sqrt[c]*d + Sqrt[a]*e)*Arc
Tanh[(c^(1/8)*x)/a^(1/8)])/(4*a^(7/8)*c^(5/8)) - ((d - (Sqrt[a]*e)/Sqrt[c])
*Log[a^(1/4) - Sqrt[2]*a^(1/8)*c^(1/8)*x + c^(1/4)*x^2])/(8*Sqrt[2]*a^(7/8)
*c^(1/8)) + ((d - (Sqrt[a]*e)/Sqrt[c])*Log[a^(1/4) + Sqrt[2]*a^(1/8)*c^(1/8
)*x + c^(1/4)*x^2])/(8*Sqrt[2]*a^(7/8)*c^(1/8))
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 217

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 218

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x], x]
+ Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a/b
, 0]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1431

```
Int[((d_) + (e_.)*(x_)^(n_))/((a_) + (c_.)*(x_)^(n2_)), x_Symbol] :> With[{
q = Rt[-a/c, 2]}, Dist[(d + e*q)/2, Int[1/(a + c*q*x^n), x], x] + Dist[(d -
 e*q)/2, Int[1/(a - c*q*x^n), x], x]] /; FreeQ[{a, c, d, e, n}, x] && EqQ[n
2, 2*n] && NeQ[c*d^2 + a*e^2, 0] && NegQ[a*c] && IntegerQ[n]
```

Rubi steps

$$\int \frac{d+ex^4}{a-cx^8}\,dx = \frac{1}{2}\left(d-\frac{\sqrt{a}\,e}{\sqrt{c}}\right)\int \frac{1}{a+\sqrt{a}\,\sqrt{c}\,x^4}\,dx + \frac{1}{2}\left(d+\frac{\sqrt{a}\,e}{\sqrt{c}}\right)\int \frac{1}{a-\sqrt{a}\,\sqrt{c}\,x^4}\,dx$$

$$= \frac{\left(d-\frac{\sqrt{a}\,e}{\sqrt{c}}\right)\int \frac{\sqrt[4]{a}-\sqrt[4]{c}\,x^2}{a+\sqrt{a}\,\sqrt{c}\,x^4}\,dx}{4\sqrt[4]{a}} + \frac{\left(d-\frac{\sqrt{a}\,e}{\sqrt{c}}\right)\int \frac{\sqrt[4]{a}+\sqrt[4]{c}\,x^2}{a+\sqrt{a}\,\sqrt{c}\,x^4}\,dx}{4\sqrt[4]{a}} + \frac{\left(d+\frac{\sqrt{a}\,e}{\sqrt{c}}\right)\int \frac{1}{\sqrt[4]{a}-\sqrt[4]{}}}{4a^{3/4}}$$

$$= \frac{\left(\sqrt{c}\,d+\sqrt{a}\,e\right)\tan^{-1}\left(\frac{\sqrt[8]{c}\,x}{\sqrt[8]{a}}\right)}{4a^{7/8}c^{5/8}} + \frac{\left(\sqrt{c}\,d+\sqrt{a}\,e\right)\tanh^{-1}\left(\frac{\sqrt[8]{c}\,x}{\sqrt[8]{a}}\right)}{4a^{7/8}c^{5/8}} + \frac{\left(d-\frac{\sqrt{a}\,e}{\sqrt{c}}\right)\int \frac{}{\frac{\sqrt[4]{a}}{\sqrt[4]{c}}-}}{8a^{3/4}\sqrt[4]{}}$$

$$= \frac{\left(\sqrt{c}\,d+\sqrt{a}\,e\right)\tan^{-1}\left(\frac{\sqrt[8]{c}\,x}{\sqrt[8]{a}}\right)}{4a^{7/8}c^{5/8}} + \frac{\left(\sqrt{c}\,d+\sqrt{a}\,e\right)\tanh^{-1}\left(\frac{\sqrt[8]{c}\,x}{\sqrt[8]{a}}\right)}{4a^{7/8}c^{5/8}} - \frac{\left(d-\frac{\sqrt{a}\,e}{\sqrt{c}}\right)\log\left(\sqrt[4]{a}\right.}{8\sqrt{}}$$

$$= \frac{\left(\sqrt{c}\,d+\sqrt{a}\,e\right)\tan^{-1}\left(\frac{\sqrt[8]{c}\,x}{\sqrt[8]{a}}\right)}{4a^{7/8}c^{5/8}} - \frac{\left(d-\frac{\sqrt{a}\,e}{\sqrt{c}}\right)\tan^{-1}\left(1-\frac{\sqrt{2}\,\sqrt[8]{c}\,x}{\sqrt[8]{a}}\right)}{4\sqrt{2}\,a^{7/8}\sqrt[8]{c}} + \frac{\left(d-\frac{\sqrt{a}\,e}{\sqrt{c}}\right)\tan}{4\sqrt{2}}$$

Mathematica [A]
time = 0.09, size = 425, normalized size = 1.29

$$\frac{\left(\sqrt[8]{a}\,\sqrt{c}\,d+a^{5/8}e\right)\tan^{-1}\left(\frac{\sqrt[8]{c}\,x}{\sqrt[8]{a}}\right)}{4ac^{5/8}} - \frac{\left(-\sqrt[8]{a}\,\sqrt{c}\,d+a^{5/8}e\right)\tan^{-1}\left(\frac{-\sqrt{2}\,\sqrt[8]{a}+2\sqrt[8]{c}\,x}{\sqrt{2}\,\sqrt[8]{a}}\right)}{4\sqrt{2}\,ac^{5/8}} - \frac{\left(-\sqrt[8]{a}\,\sqrt{c}\,d+a^{5/8}e\right)\tan^{-1}\left(\frac{\sqrt{2}\,\sqrt[8]{a}+2\sqrt[8]{c}\,x}{\sqrt{2}\,\sqrt[8]{a}}\right)}{4\sqrt{2}\,ac^{5/8}} - \frac{\left(\sqrt[8]{a}\,\sqrt{c}\,d+a^{5/8}e\right)\log\left(\sqrt[8]{a}-\sqrt[8]{c}\,x\right)}{8ac^{5/8}} - \frac{\left(-\sqrt[8]{a}\,\sqrt{c}\,d-a^{5/8}e\right)\log\left(\sqrt[8]{a}+\sqrt[8]{c}\,x\right)}{8ac^{5/8}} + \frac{\left(-\sqrt[8]{a}\,\sqrt{c}\,d+a^{5/8}e\right)\log\left(\sqrt[4]{a}-\sqrt{2}\,\sqrt[8]{a}\,\sqrt[8]{c}\,x+\sqrt[4]{c}\,x^2\right)}{8\sqrt{2}\,ac^{5/8}} - \frac{\left(-\sqrt[8]{a}\,\sqrt{c}\,d+a^{5/8}e\right)\log\left(\sqrt[4]{a}+\sqrt{2}\,\sqrt[8]{a}\,\sqrt[8]{c}\,x+\sqrt[4]{c}\,x^2\right)}{8\sqrt{2}\,ac^{5/8}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x^4)/(a - c*x^8),x]
```

```
[Out] ((a^(1/8)*Sqrt[c]*d + a^(5/8)*e)*ArcTan[(c^(1/8)*x)/a^(1/8)])/(4*a*c^(5/8))
 - ((-(a^(1/8)*Sqrt[c]*d) + a^(5/8)*e)*ArcTan[(-(Sqrt[2]*a^(1/8)) + 2*c^(1/
8)*x)/(Sqrt[2]*a^(1/8))])/(4*Sqrt[2]*a*c^(5/8)) - ((-(a^(1/8)*Sqrt[c]*d) +
a^(5/8)*e)*ArcTan[(Sqrt[2]*a^(1/8) + 2*c^(1/8)*x)/(Sqrt[2]*a^(1/8))])/(4*Sq
rt[2]*a*c^(5/8)) - ((a^(1/8)*Sqrt[c]*d + a^(5/8)*e)*Log[a^(1/8) - c^(1/8)*x
])/(8*a*c^(5/8)) - ((-(a^(1/8)*Sqrt[c]*d) - a^(5/8)*e)*Log[a^(1/8) + c^(1/8
)*x])/(8*a*c^(5/8)) + ((-(a^(1/8)*Sqrt[c]*d) + a^(5/8)*e)*Log[a^(1/4) - Sqr
```

```
t[2]*a^(1/8)*c^(1/8)*x + c^(1/4)*x^2])/(8*Sqrt[2]*a*c^(5/8)) - ((-(a^(1/8)*
Sqrt[c]*d) + a^(5/8)*e)*Log[a^(1/4) + Sqrt[2]*a^(1/8)*c^(1/8)*x + c^(1/4)*x
^2])/(8*Sqrt[2]*a*c^(5/8))
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 0.21, size = 36, normalized size = 0.11

method	result	size
default	$-\frac{\sum_{\_R=\mathrm{RootOf}\left(c\_Z^8-a\right)}\frac{\left(\_R^4e+d\right)\ln\left(x-\_R\right)}{\_R^7}}{8c}$	36
risch	$-\frac{\sum_{\_R=\mathrm{RootOf}\left(c\_Z^8-a\right)}\frac{\left(\_R^4e+d\right)\ln\left(x-\_R\right)}{\_R^7}}{8c}$	36

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^4+d)/(-c*x^8+a),x,method=_RETURNVERBOSE)
```

```
[Out] -1/8/c*sum((_R^4*e+d)/_R^7*ln(x-_R),_R=RootOf(_Z^8*c-a))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^4+d)/(-c*x^8+a),x, algorithm="maxima")
```

```
[Out] -integrate((x^4*e + d)/(c*x^8 - a), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 3551 vs. 2(226) = 452.
time = 2.02, size = 3551, normalized size = 10.79

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^4+d)/(-c*x^8+a),x, algorithm="fricas")
```

```
[Out] 1/2*((a^3*c^2*sqrt((c^4*d^8 + 12*a*c^3*d^6*e^2 + 38*a^2*c^2*d^4*e^4 + 12*a^
3*c*d^2*e^6 + a^4*e^8)/(a^7*c^5)) + 4*c*d^3*e + 4*a*d*e^3)/(a^3*c^2))^(1/4)
*arctan(-(sqrt(c^6*d^12*x^2 + 10*a*c^5*d^10*x^2*e^2 + 15*a^2*c^4*d^8*x^2*e^
4 - 52*a^3*c^3*d^6*x^2*e^6 + 15*a^4*c^2*d^4*x^2*e^8 + 10*a^5*c*d^2*x^2*e^10
 + a^6*x^2*e^12 + (a^2*c^6*d^10 + 13*a^3*c^5*d^8*e^2 + 50*a^4*c^4*d^6*e^4 +
 50*a^5*c^3*d^4*e^6 + 13*a^6*c^2*d^2*e^8 + a^7*c*e^10 - 2*(a^6*c^6*d^5*e +
```

6*a^7*c^5*d^3*e^3 + a^8*c^4*d*e^5)*sqrt((c^4*d^8 + 12*a*c^3*d^6*e^2 + 38*a^2*c^2*d^4*e^4 + 12*a^3*c*d^2*e^6 + a^4*e^8)/(a^7*c^5)))*sqrt((a^3*c^2*sqrt((c^4*d^8 + 12*a*c^3*d^6*e^2 + 38*a^2*c^2*d^4*e^4 + 12*a^3*c*d^2*e^6 + a^4*e^8)/(a^7*c^5)) + 4*c*d^3*e + 4*a*d*e^3)/(a^3*c^2)))*(3*a^3*c^5*d^6*e + 19*a^4*c^4*d^4*e^3 + 9*a^5*c^3*d^2*e^5 + a^6*c^2*e^7 - (a^6*c^6*d^3 + 3*a^7*c^5*d*e^2)*sqrt((c^4*d^8 + 12*a*c^3*d^6*e^2 + 38*a^2*c^2*d^4*e^4 + 12*a^3*c*d^2*e^6 + a^4*e^8)/(a^7*c^5)))*sqrt((a^3*c^2*sqrt((c^4*d^8 + 12*a*c^3*d^6*e^2 + 38*a^2*c^2*d^4*e^4 + 12*a^3*c*d^2*e^6 + a^4*e^8)/(a^7*c^5)) + 4*c*d^3*e + 4*a*d*e^3)/(a^3*c^2)) + (3*a^3*c^8*d^12*x*e + 34*a^4*c^7*d^10*x*e^3 + 89*a^5*c^6*d^8*x*e^5 - 52*a^6*c^5*d^6*x*e^7 - 59*a^7*c^4*d^4*x*e^9 - 14*a^8*c^3*d^2*x*e^11 - a^9*c^2*x*e^13 - (a^6*c^9*d^9*x + 8*a^7*c^8*d^7*x*e^2 + 10*a^8*c^7*d^5*x*e^4 - 16*a^9*c^6*d^3*x*e^6 - 3*a^10*c^5*d*x*e^8)*sqrt((c^4*d^8 + 12*a*c^3*d^6*e^2 + 38*a^2*c^2*d^4*e^4 + 12*a^3*c*d^2*e^6 + a^4*e^8)/(a^7*c^5)))*sqrt((a^3*c^2*sqrt((c^4*d^8 + 12*a*c^3*d^6*e^2 + 38*a^2*c^2*d^4*e^4 + 12*a^3*c*d^2*e^6 + a^4*e^8)/(a^7*c^5)) + 4*c*d^3*e + 4*a*d*e^3)/(a^3*c^2)))*((a^3*c^2*sqrt((c^4*d^8 + 12*a*c^3*d^6*e^2 + 38*a^2*c^2*d^4*e^4 + 12*a^3*c*d^2*e^6 + a^4*e^8)/(a^7*c^5)) + 4*c*d^3*e + 4*a*d*e^3)/(a^3*c^2))^(1/4)/(c^8*d^16 + 8*a*c^7*d^14*e^2 - 4*a^2*c^6*d^12*e^4 - 72*a^3*c^5*d^10*e^6 + 134*a^4*c^4*d^8*e^8 - 72*a^5*c^3*d^6*e^10 - 4*a^6*c^2*d^4*e^12 + 8*a^7*c*d^2*e^14 + a^8*e^16)) - 1/2*(-(a^3*c^2*sqrt((c^4*d^8 + 12*a*c^3*d^6*e^2 + 38*a^2*c^2*d^4*e^4 + 12*a^3*c*d^2*e^6 + a^4*e^8)/(a^7*c^5)) - 4*c*d^3*e - 4*a*d*e^3)/(a^3*c^2))^(1/4)*arctan((sqrt(c^6*d^12*x^2 + 10*a*c^5*d^10*x^2*e^2 + 15*a^2*c^4*d^8*x^2*e^4 - 52*a^3*c^3*d^6*x^2*e^6 + 15*a^4*c^2*d^4*x^2*e^8 + 10*a^5*c*d^2*x^2*e^10 + a^6*x^2*e^12 + (a^2*c^6*d^10 + 13*a^3*c^5*d^8*e^2 + 50*a^4*c^4*d^6*e^4 + 50*a^5*c^3*d^4*e^6 + 13*a^6*c^2*d^2*e^8 + a^7*c*e^10 + 2*(a^6*c^6*d^5*e + 6*a^7*c^5*d^3*e^3 + a^8*c^4*d*e^5)*sqrt((c^4*d^8 + 12*a*c^3*d^6*e^2 + 38*a^2*c^2*d^4*e^4 + 12*a^3*c*d^2*e^6 + a^4*e^8)/(a^7*c^5)))*sqrt(-(a^3*c^2*sqrt((c^4*d^8 + 12*a*c^3*d^6*e^2 + 38*a^2*c^2*d^4*e^4 + 12*a^3*c*d^2*e^6 + a^4*e^8)/(a^7*c^5)) - 4*c*d^3*e - 4*a*d*e^3)/(a^3*c^2)))*(3*a^3*c^5*d^6*e + 19*a^4*c^4*d^4*e^3 + 9*a^5*c^3*d^2*e^5 + a^6*c^2*e^7 + (a^6*c^6*d^3 + 3*a^7*c^5*d*e^2)*sqrt((c^4*d^8 + 12*a*c^3*d^6*e^2 + 38*a^2*c^2*d^4*e^4 + 12*a^3*c*d^2*e^6 + a^4*e^8)/(a^7*c^5)))*(-(a^3*c^2*sqrt((c^4*d^8 + 12*a*c^3*d^6*e^2 + 38*a^2*c^2*d^4*e^4 + 12*a^3*c*d^2*e^6 + a^4*e^8)/(a^7*c^5)) - 4*c*d^3*e - 4*a*d*e^3)/(a^3*c^2))^(3/4) + (3*a^3*c^8*d^12*x*e + 34*a^4*c^7*d^10*x*e^3 + 89*a^5*c^6*d^8*x*e^5 - 52*a^6*c^5*d^6*x*e^7 - 59*a^7*c^4*d^4*x*e^9 - 14*a^8*c^3*d^2*x*e^11 - a^9*c^2*x*e^13 + (a^6*c^9*d^9*x + 8*a^7*c^8*d^7*x*e^2 + 10*a^8*c^7*d^5*x*e^4 - 16*a^9*c^6*d^3*x*e^6 - 3*a^10*c^5*d*x*e^8)*sqrt((c^4*d^8 + 12*a*c^3*d^6*e^2 + 38*a^2*c^2*d^4*e^4 + 12*a^3*c*d^2*e^6 + a^4*e^8)/(a^7*c^5)))*(-(a^3*c^2*sqrt((c^4*d^8 + 12*a*c^3*d^6*e^2 + 38*a^2*c^2*d^4*e^4 + 12*a^3*c*d^2*e^6 + a^4*e^8)/(a^7*c^5)) - 4*c*d^3*e - 4*a*d*e^3)/(a^3*c^2))^(3/4))/(c^8*d^16 + 8*a*c^7*d^14*e^2 - 4*a^2*c^6*d^12*e^4 - 72*a^3*c^5*d^10*e^6 + 134*a^4*c^4*d^8*e^8 - 72*a^5*c^3*d^6*e^10 - 4*a^6*c^2*d^4*e^12 + 8*a^7*c*d^2*e^14 + a^8*e^16)) + 1/8*((a^3*c^2*sqrt((c^4*d^8 + 12*a*c^3*d^6*e^2 + 38*a^2*c^2*d^4*e^4 + 12*a^3*c*d^2*e^6 + a^4*e^8)/(a^7*c^5)) + 4*c*d^3*e + 4*a*d*e^3)/(a^3*c^2))^(1/4)*log(-c^3*d^6*x - 5*a

```
*c^2*d^4*x*e^2 + 5*a^2*c*d^2*x*e^4 + a^3*x*e^6 + (a^5*c^3*sqrt((c^4*d^8 + 1
2*a*c^3*d^6*e^2 + 38*a^2*c^2*d^4*e^4 + 12*a^3*c*d^2*e^6 + a^4*e^8)/(a^7*c^5
))*e - a*c^3*d^5 - 6*a^2*c^2*d^3*e^2 - a^3*c*d*e^4)*((a^3*c^2*sqrt((c^4*d^8
 + 12*a*c^3*d^6*e^2 + 38*a^2*c^2*d^4*e^4 + 12*a^3*c*d^2*e^6 + a^4*e^8)/(a^7
*c^5)) + 4*c*d^3*e + 4*a*d*e^3)/(a^3*c^2))^(1/4)) - 1/8*((a^3*c^2*sqrt((c^4
*d^8 + 12*a*c^3*d^6*e^2 + 38*a^2*c^2*d^4*e^4 + 12*a^3*c*d^2*e^6 + a^4*e^8)/
(a^7*c^5)) + 4*c*d^3*e + 4*a*d*e^3)/(a^3*c^2))^(1/4)*log(-c^3*d^6*x - 5*a*c
^2*d^4*x*e^2 + 5*a^2*c*d^2*x*e^4 + a^3*x*e^6 - (a^5*c^3*sqrt((c^4*d^8 + 12*
a*c^3*d^6*e^2 + 38*a^2*c^2*d^4*e^4 + 12*a^3*c*d^2*e^6 + a^4*e^8)/(a^7*c^5))
*e - a*c^3*d^5 - 6*a^2*c^2*d^3*e^2 - a^3*c*d*e^4)*((a^3*c^2*sqrt((c^4*d^8 +
 12*a*c^3*d^6*e^2 + 38*a^2*c^2*d^4*e^4 + 12*a^3*c*d^2*e^6 + a^4*e^8)/(a^7*c
^5)) + 4*c*d^3*e + 4*a*d*e^3)/(a^3*c^2))^(1/4)) - 1/8*(-(a^3*c^2*sqrt((c^4*
d^8 + 12*a*c^3*d^6*e^2 + 38*a^2*c^2*d^4*e^4 + 12*a^3*c*d^2*e^6 + a^4*e^8)/(
a^7*c^5)) - 4*c*d^3*e - 4*a*d*e^3)/(a^3*c^2))^(1/4)*log(-c^3*d^6*x - 5*a*c^
2*d^4*x*e^2 + 5*a^2*c*d^2*x*e^4 + a^3*x*e^6 + (a^5*c^3*sqrt((c^4*d^8 + 12*a
*c^3*d^6*e^2 + 38*a^2*c^2*d^4*e^4 + 12*a^3*c*d^...
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

$$\text{Timed out}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**4+d)/(-c*x**8+a),x)
```

```
[Out] Timed out
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 633 vs. 2(226) = 452.
time = 3.94, size = 633, normalized size = 1.92

$$-\frac{\left(\sqrt{-\sqrt{2}+2}\left(-\frac{a}{c}\right)^{\frac{5}{8}}e-d\sqrt{\sqrt{2}+2}\left(-\frac{a}{c}\right)^{\frac{1}{8}}\right)\arctan\left(\frac{2x+\sqrt{-\sqrt{2}+2}\left(-\frac{a}{c}\right)^{\frac{1}{8}}}{\sqrt{\sqrt{2}+2}\left(-\frac{a}{c}\right)^{\frac{1}{8}}}\right)}{8a}-\frac{\left(\sqrt{-\sqrt{2}+2}\left(-\frac{a}{c}\right)^{\frac{5}{8}}e-d\sqrt{\sqrt{2}+2}\left(-\frac{a}{c}\right)^{\frac{1}{8}}\right)\arctan\left(\frac{2x-\sqrt{-\sqrt{2}+2}\left(-\frac{a}{c}\right)^{\frac{1}{8}}}{\sqrt{\sqrt{2}+2}\left(-\frac{a}{c}\right)^{\frac{1}{8}}}\right)}{8a}+\frac{\left(\sqrt{\sqrt{2}+2}\left(-\frac{a}{c}\right)^{\frac{5}{8}}e+d\sqrt{-\sqrt{2}+2}\left(-\frac{a}{c}\right)^{\frac{1}{8}}\right)\arctan\left(\frac{2x+\sqrt{\sqrt{2}+2}\left(-\frac{a}{c}\right)^{\frac{1}{8}}}{\sqrt{-\sqrt{2}+2}\left(-\frac{a}{c}\right)^{\frac{1}{8}}}\right)}{8a}+\frac{\left(\sqrt{\sqrt{2}+2}\left(-\frac{a}{c}\right)^{\frac{5}{8}}e+d\sqrt{-\sqrt{2}+2}\left(-\frac{a}{c}\right)^{\frac{1}{8}}\right)\arctan\left(\frac{2x-\sqrt{\sqrt{2}+2}\left(-\frac{a}{c}\right)^{\frac{1}{8}}}{\sqrt{-\sqrt{2}+2}\left(-\frac{a}{c}\right)^{\frac{1}{8}}}\right)}{8a}-\frac{\left(\sqrt{-\sqrt{2}+2}\left(-\frac{a}{c}\right)^{\frac{5}{8}}e-d\sqrt{\sqrt{2}+2}\left(-\frac{a}{c}\right)^{\frac{1}{8}}\right)\log\left(x^2+x\sqrt{\sqrt{2}+2}\left(-\frac{a}{c}\right)^{\frac{1}{8}}+\left(-\frac{a}{c}\right)^{\frac{1}{4}}\right)}{16a}+\frac{\left(\sqrt{-\sqrt{2}+2}\left(-\frac{a}{c}\right)^{\frac{5}{8}}e-d\sqrt{\sqrt{2}+2}\left(-\frac{a}{c}\right)^{\frac{1}{8}}\right)\log\left(x^2-x\sqrt{\sqrt{2}+2}\left(-\frac{a}{c}\right)^{\frac{1}{8}}+\left(-\frac{a}{c}\right)^{\frac{1}{4}}\right)}{16a}+\frac{\left(\sqrt{\sqrt{2}+2}\left(-\frac{a}{c}\right)^{\frac{5}{8}}e+d\sqrt{-\sqrt{2}+2}\left(-\frac{a}{c}\right)^{\frac{1}{8}}\right)\log\left(x^2+x\sqrt{-\sqrt{2}+2}\left(-\frac{a}{c}\right)^{\frac{1}{8}}+\left(-\frac{a}{c}\right)^{\frac{1}{4}}\right)}{16a}-\frac{\left(\sqrt{\sqrt{2}+2}\left(-\frac{a}{c}\right)^{\frac{5}{8}}e+d\sqrt{-\sqrt{2}+2}\left(-\frac{a}{c}\right)^{\frac{1}{8}}\right)\log\left(x^2-x\sqrt{-\sqrt{2}+2}\left(-\frac{a}{c}\right)^{\frac{1}{8}}+\left(-\frac{a}{c}\right)^{\frac{1}{4}}\right)}{16a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^4+d)/(-c*x^8+a),x, algorithm="giac")
```

```
[Out] -1/8*(sqrt(-sqrt(2) + 2)*(-a/c)^(5/8)*e - d*sqrt(sqrt(2) + 2)*(-a/c)^(1/8))
*arctan((2*x + sqrt(-sqrt(2) + 2)*(-a/c)^(1/8))/(sqrt(sqrt(2) + 2)*(-a/c)^(
1/8)))/a - 1/8*(sqrt(-sqrt(2) + 2)*(-a/c)^(5/8)*e - d*sqrt(sqrt(2) + 2)*(-a
/c)^(1/8))*arctan((2*x - sqrt(-sqrt(2) + 2)*(-a/c)^(1/8))/(sqrt(sqrt(2) + 2
)*(-a/c)^(1/8)))/a + 1/8*(sqrt(sqrt(2) + 2)*(-a/c)^(5/8)*e + d*sqrt(-sqrt(2
) + 2)*(-a/c)^(1/8))*arctan((2*x + sqrt(sqrt(2) + 2)*(-a/c)^(1/8))/(sqrt(-s
qrt(2) + 2)*(-a/c)^(1/8)))/a + 1/8*(sqrt(sqrt(2) + 2)*(-a/c)^(5/8)*e + d*sq
rt(-sqrt(2) + 2)*(-a/c)^(1/8))*arctan((2*x - sqrt(sqrt(2) + 2)*(-a/c)^(1/8)
)/(sqrt(-sqrt(2) + 2)*(-a/c)^(1/8)))/a - 1/16*(sqrt(-sqrt(2) + 2)*(-a/c)^(5
/8)*e - d*sqrt(sqrt(2) + 2)*(-a/c)^(1/8))*log(x^2 + x*sqrt(sqrt(2) + 2)*(-a
```

```
/c)^(1/8) + (-a/c)^(1/4))/a + 1/16*(sqrt(-sqrt(2) + 2)*(-a/c)^(5/8)*e - d*s
qrt(sqrt(2) + 2)*(-a/c)^(1/8))*log(x^2 - x*sqrt(sqrt(2) + 2)*(-a/c)^(1/8) +
 (-a/c)^(1/4))/a + 1/16*(sqrt(sqrt(2) + 2)*(-a/c)^(5/8)*e + d*sqrt(-sqrt(2)
 + 2)*(-a/c)^(1/8))*log(x^2 + x*sqrt(-sqrt(2) + 2)*(-a/c)^(1/8) + (-a/c)^(1
/4))/a - 1/16*(sqrt(sqrt(2) + 2)*(-a/c)^(5/8)*e + d*sqrt(-sqrt(2) + 2)*(-a/
c)^(1/8))*log(x^2 - x*sqrt(-sqrt(2) + 2)*(-a/c)^(1/8) + (-a/c)^(1/4))/a
```

Mupad [B]
time = 2.72, size = 2438, normalized size = 7.41

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x^4)/(a - c*x^8),x)
```

```
[Out] (atan((a^3*e^6*x + c^3*d^6*x - a*c^2*d^4*e^2*x - a^2*c*d^2*e^4*x + (2*d*e*x
*(a^2*e^4*(a^7*c^5)^(1/2) + c^2*d^4*(a^7*c^5)^(1/2) + 4*a^4*c^4*d^3*e + 4*a
^5*c^3*d*e^3 + 6*a*c*d^2*e^2*(a^7*c^5)^(1/2)))/(a^3*c^2))/(a*c^3*d^5*((a^2*
e^4*(a^7*c^5)^(1/2) + c^2*d^4*(a^7*c^5)^(1/2) + 4*a^4*c^4*d^3*e + 4*a^5*c^3
*d*e^3 + 6*a*c*d^2*e^2*(a^7*c^5)^(1/2))/(a^7*c^5))^(1/4) + a^5*c^3*e*((a^2*
e^4*(a^7*c^5)^(1/2) + c^2*d^4*(a^7*c^5)^(1/2) + 4*a^4*c^4*d^3*e + 4*a^5*c^3
*d*e^3 + 6*a*c*d^2*e^2*(a^7*c^5)^(1/2))/(a^7*c^5))^(5/4) + 2*a^2*c^2*d^3*e^
2*((a^2*e^4*(a^7*c^5)^(1/2) + c^2*d^4*(a^7*c^5)^(1/2) + 4*a^4*c^4*d^3*e + 4
*a^5*c^3*d*e^3 + 6*a*c*d^2*e^2*(a^7*c^5)^(1/2))/(a^7*c^5))^(1/4) - 3*a^3*c*
d*e^4*((a^2*e^4*(a^7*c^5)^(1/2) + c^2*d^4*(a^7*c^5)^(1/2) + 4*a^4*c^4*d^3*e
 + 4*a^5*c^3*d*e^3 + 6*a*c*d^2*e^2*(a^7*c^5)^(1/2))/(a^7*c^5))^(1/4)))*((a^
2*e^4*(a^7*c^5)^(1/2) + c^2*d^4*(a^7*c^5)^(1/2) + 4*a^4*c^4*d^3*e + 4*a^5*c
^3*d*e^3 + 6*a*c*d^2*e^2*(a^7*c^5)^(1/2))/(a^7*c^5))^(1/4))/4 - (atan((a*c^
2*d^4*e^2*x - c^3*d^6*x - a^3*e^6*x + a^2*c*d^2*e^4*x + (2*d*e*x*(a^2*e^4*(
a^7*c^5)^(1/2) + c^2*d^4*(a^7*c^5)^(1/2) - 4*a^4*c^4*d^3*e - 4*a^5*c^3*d*e^
3 + 6*a*c*d^2*e^2*(a^7*c^5)^(1/2)))/(a^3*c^2))/(a*c^3*d^5*(-(a^2*e^4*(a^7*c
^5)^(1/2) + c^2*d^4*(a^7*c^5)^(1/2) - 4*a^4*c^4*d^3*e - 4*a^5*c^3*d*e^3 + 6
*a*c*d^2*e^2*(a^7*c^5)^(1/2))/(a^7*c^5))^(1/4) + a^5*c^3*e*(-(a^2*e^4*(a^7*
c^5)^(1/2) + c^2*d^4*(a^7*c^5)^(1/2) - 4*a^4*c^4*d^3*e - 4*a^5*c^3*d*e^3 +
6*a*c*d^2*e^2*(a^7*c^5)^(1/2))/(a^7*c^5))^(5/4) + 2*a^2*c^2*d^3*e^2*(-(a^2*
e^4*(a^7*c^5)^(1/2) + c^2*d^4*(a^7*c^5)^(1/2) - 4*a^4*c^4*d^3*e - 4*a^5*c^3
*d*e^3 + 6*a*c*d^2*e^2*(a^7*c^5)^(1/2))/(a^7*c^5))^(1/4) - 3*a^3*c*d*e^4*(-
(a^2*e^4*(a^7*c^5)^(1/2) + c^2*d^4*(a^7*c^5)^(1/2) - 4*a^4*c^4*d^3*e - 4*a^
5*c^3*d*e^3 + 6*a*c*d^2*e^2*(a^7*c^5)^(1/2))/(a^7*c^5))^(1/4)))*(-(a^2*e^4*
(a^7*c^5)^(1/2) + c^2*d^4*(a^7*c^5)^(1/2) - 4*a^4*c^4*d^3*e - 4*a^5*c^3*d*e
^3 + 6*a*c*d^2*e^2*(a^7*c^5)^(1/2))/(a^7*c^5))^(1/4))/4 - atan((a^3*e^6*x*1
i + c^3*d^6*x*1i - a*c^2*d^4*e^2*x*1i - a^2*c*d^2*e^4*x*1i + (d*e*x*(a^2*e^
4*(a^7*c^5)^(1/2) + c^2*d^4*(a^7*c^5)^(1/2) + 4*a^4*c^4*d^3*e + 4*a^5*c^3*d
*e^3 + 6*a*c*d^2*e^2*(a^7*c^5)^(1/2))*2i)/(a^3*c^2))/(a*c^3*d^5*((a^2*e^4*(
a^7*c^5)^(1/2) + c^2*d^4*(a^7*c^5)^(1/2) + 4*a^4*c^4*d^3*e + 4*a^5*c^3*d*e^
```

```
3 + 6*a*c*d^2*e^2*(a^7*c^5)^(1/2))/(a^7*c^5))^(1/4) + a^5*c^3*e*((a^2*e^4*(
a^7*c^5)^(1/2) + c^2*d^4*(a^7*c^5)^(1/2) + 4*a^4*c^4*d^3*e + 4*a^5*c^3*d*e^
3 + 6*a*c*d^2*e^2*(a^7*c^5)^(1/2))/(a^7*c^5))^(5/4) + 2*a^2*c^2*d^3*e^2*((a
^2*e^4*(a^7*c^5)^(1/2) + c^2*d^4*(a^7*c^5)^(1/2) + 4*a^4*c^4*d^3*e + 4*a^5*
c^3*d*e^3 + 6*a*c*d^2*e^2*(a^7*c^5)^(1/2))/(a^7*c^5))^(1/4) - 3*a^3*c*d*e^4
*((a^2*e^4*(a^7*c^5)^(1/2) + c^2*d^4*(a^7*c^5)^(1/2) + 4*a^4*c^4*d^3*e + 4*
a^5*c^3*d*e^3 + 6*a*c*d^2*e^2*(a^7*c^5)^(1/2))/(a^7*c^5))^(1/4)))*((a^2*e^4
*(a^7*c^5)^(1/2) + c^2*d^4*(a^7*c^5)^(1/2) + 4*a^4*c^4*d^3*e + 4*a^5*c^3*d*
e^3 + 6*a*c*d^2*e^2*(a^7*c^5)^(1/2))/(4096*a^7*c^5))^(1/4)*2i + atan((a*c^2
*d^4*e^2*x*1i - c^3*d^6*x*1i - a^3*e^6*x*1i + a^2*c*d^2*e^4*x*1i + (d*e*x*(
a^2*e^4*(a^7*c^5)^(1/2) + c^2*d^4*(a^7*c^5)^(1/2) - 4*a^4*c^4*d^3*e - 4*a^5
*c^3*d*e^3 + 6*a*c*d^2*e^2*(a^7*c^5)^(1/2))*2i)/(a^3*c^2))/(a*c^3*d^5*(-(a^
2*e^4*(a^7*c^5)^(1/2) + c^2*d^4*(a^7*c^5)^(1/2) - 4*a^4*c^4*d^3*e - 4*a^5*c
^3*d*e^3 + 6*a*c*d^2*e^2*(a^7*c^5)^(1/2))/(a^7*c^5))^(1/4) + a^5*c^3*e*(-(a
^2*e^4*(a^7*c^5)^(1/2) + c^2*d^4*(a^7*c^5)^(1/2) - 4*a^4*c^4*d^3*e - 4*a^5*
c^3*d*e^3 + 6*a*c*d^2*e^2*(a^7*c^5)^(1/2))/(a^7*c^5))^(5/4) + 2*a^2*c^2*d^3
*e^2*(-(a^2*e^4*(a^7*c^5)^(1/2) + c^2*d^4*(a^7*c^5)^(1/2) - 4*a^4*c^4*d^3*e
 - 4*a^5*c^3*d*e^3 + 6*a*c*d^2*e^2*(a^7*c^5)^(1/2))/(a^7*c^5))^(1/4) - 3*a^
3*c*d*e^4*(-(a^2*e^4*(a^7*c^5)^(1/2) + c^2*d^4*(a^7*c^5)^(1/2) - 4*a^4*c^4*
d^3*e - 4*a^5*c^3*d*e^3 + 6*a*c*d^2*e^2*(a^7*c^5)^(1/2))/(a^7*c^5))^(1/4)))
*(-(a^2*e^4*(a^7*c^5)^(1/2) + c^2*d^4*(a^7*c^5)^(1/2) - 4*a^4*c^4*d^3*e - 4
*a^5*c^3*d*e^3 + 6*a*c*d^2*e^2*(a^7*c^5)^(1/2))/(4096*a^7*c^5))^(1/4)*2i
```

3.5 $\int \frac{d+ex^4}{d^2+bx^4+e^2x^8}\,dx$

Optimal. Leaf size=791

$$-\frac{\tan^{-1}\left(\frac{\sqrt{2\sqrt{d}\sqrt{e}-\sqrt{-b+2de}}-2\sqrt{e}\,x}{\sqrt{2\sqrt{d}\sqrt{e}+\sqrt{-b+2de}}}\right)}{4\sqrt{d}\sqrt{2\sqrt{d}\sqrt{e}+\sqrt{-b+2de}}}-\frac{\tan^{-1}\left(\frac{\sqrt{2\sqrt{d}\sqrt{e}+\sqrt{-b+2de}}-2\sqrt{e}\,x}{\sqrt{2\sqrt{d}\sqrt{e}-\sqrt{-b+2de}}}\right)}{4\sqrt{d}\sqrt{2\sqrt{d}\sqrt{e}-\sqrt{-b+2de}}}+\frac{\tan^{-1}\left(\frac{\sqrt{}}{}\right)}{4\sqrt{d}}$$

```
[Out] -1/4*arctan((-2*x*e^(1/2)+(2*d^(1/2)*e^(1/2)+(2*d*e-b)^(1/2))^(1/2))/(2*d^(
1/2)*e^(1/2)-(2*d*e-b)^(1/2))^(1/2))/d^(1/2)/(2*d^(1/2)*e^(1/2)-(2*d*e-b)^(
1/2))^(1/2)+1/4*arctan((2*x*e^(1/2)+(2*d^(1/2)*e^(1/2)+(2*d*e-b)^(1/2))^(1/
2))/(2*d^(1/2)*e^(1/2)-(2*d*e-b)^(1/2))^(1/2))/d^(1/2)/(2*d^(1/2)*e^(1/2)-(
2*d*e-b)^(1/2))^(1/2)-1/8*ln(d^(1/2)+x^2*e^(1/2)-x*(2*d^(1/2)*e^(1/2)-(2*d*
e-b)^(1/2))^(1/2))/d^(1/2)/(2*d^(1/2)*e^(1/2)-(2*d*e-b)^(1/2))^(1/2)+1/8*ln
(d^(1/2)+x^2*e^(1/2)+x*(2*d^(1/2)*e^(1/2)-(2*d*e-b)^(1/2))^(1/2))/d^(1/2)/(
2*d^(1/2)*e^(1/2)-(2*d*e-b)^(1/2))^(1/2)-1/4*arctan((-2*x*e^(1/2)+(2*d^(1/2
)*e^(1/2)-(2*d*e-b)^(1/2))^(1/2))/(2*d^(1/2)*e^(1/2)+(2*d*e-b)^(1/2))^(1/2)
)/d^(1/2)/(2*d^(1/2)*e^(1/2)+(2*d*e-b)^(1/2))^(1/2)+1/4*arctan((2*x*e^(1/2)
+(2*d^(1/2)*e^(1/2)-(2*d*e-b)^(1/2))^(1/2))/(2*d^(1/2)*e^(1/2)+(2*d*e-b)^(1
/2))^(1/2))/d^(1/2)/(2*d^(1/2)*e^(1/2)+(2*d*e-b)^(1/2))^(1/2)-1/8*ln(d^(1/2
)+x^2*e^(1/2)-x*(2*d^(1/2)*e^(1/2)+(2*d*e-b)^(1/2))^(1/2))/d^(1/2)/(2*d^(1/
2)*e^(1/2)+(2*d*e-b)^(1/2))^(1/2)+1/8*ln(d^(1/2)+x^2*e^(1/2)+x*(2*d^(1/2)*e
^(1/2)+(2*d*e-b)^(1/2))^(1/2))/d^(1/2)/(2*d^(1/2)*e^(1/2)+(2*d*e-b)^(1/2))^
(1/2)
```

Rubi [A]
time = 0.58, antiderivative size = 791, normalized size of antiderivative = 1.00, number of steps used = 19, number of rules used = 6, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {1433, 1108, 648, 632, 210, 642}

$$-\frac{\text{ArcTan}\left(\frac{\sqrt{2\sqrt{d}\sqrt{e}-\sqrt{2de-b}}-2\sqrt{e}\,x}{\sqrt{\sqrt{2de-b}+2\sqrt{d}\sqrt{e}}}\right)}{4\sqrt{d}\sqrt{\sqrt{2de-b}+2\sqrt{d}\sqrt{e}}}-\frac{\text{ArcTan}\left(\frac{\sqrt{\sqrt{2de-b}+2\sqrt{d}\sqrt{e}}-2\sqrt{e}\,x}{\sqrt{2\sqrt{d}\sqrt{e}-\sqrt{2de-b}}}\right)}{4\sqrt{d}\sqrt{2\sqrt{d}\sqrt{e}-\sqrt{2de-b}}}+\frac{\text{ArcTan}\left(\frac{\sqrt{2\sqrt{d}\sqrt{e}-\sqrt{2de-b}}+2\sqrt{e}\,x}{\sqrt{\sqrt{2de-b}+2\sqrt{d}\sqrt{e}}}\right)}{4\sqrt{d}\sqrt{\sqrt{2de-b}+2\sqrt{d}\sqrt{e}}}+\frac{\text{ArcTan}\left(\frac{\sqrt{\sqrt{2de-b}+2\sqrt{d}\sqrt{e}}+2\sqrt{e}\,x}{\sqrt{2\sqrt{d}\sqrt{e}-\sqrt{2de-b}}}\right)}{4\sqrt{d}\sqrt{2\sqrt{d}\sqrt{e}-\sqrt{2de-b}}}-\frac{\log\left(-x\sqrt{2\sqrt{d}\sqrt{e}-\sqrt{2de-b}}+\sqrt{d}+\sqrt{e}\,x^2\right)}{8\sqrt{d}\sqrt{2\sqrt{d}\sqrt{e}-\sqrt{2de-b}}}+\frac{\log\left(x\sqrt{2\sqrt{d}\sqrt{e}-\sqrt{2de-b}}+\sqrt{d}+\sqrt{e}\,x^2\right)}{8\sqrt{d}\sqrt{2\sqrt{d}\sqrt{e}-\sqrt{2de-b}}}-\frac{\log\left(-x\sqrt{\sqrt{2de-b}+2\sqrt{d}\sqrt{e}}+\sqrt{d}+\sqrt{e}\,x^2\right)}{8\sqrt{d}\sqrt{\sqrt{2de-b}+2\sqrt{d}\sqrt{e}}}+\frac{\log\left(x\sqrt{\sqrt{2de-b}+2\sqrt{d}\sqrt{e}}+\sqrt{d}+\sqrt{e}\,x^2\right)}{8\sqrt{d}\sqrt{\sqrt{2de-b}+2\sqrt{d}\sqrt{e}}}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x^4)/(d^2 + b*x^4 + e^2*x^8),x]
```

```
[Out] -1/4*ArcTan[(Sqrt[2*Sqrt[d]*Sqrt[e] - Sqrt[-b + 2*d*e]] - 2*Sqrt[e]*x)/Sqrt
[2*Sqrt[d]*Sqrt[e] + Sqrt[-b + 2*d*e]]]/(Sqrt[d]*Sqrt[2*Sqrt[d]*Sqrt[e] + S
qrt[-b + 2*d*e]]) - ArcTan[(Sqrt[2*Sqrt[d]*Sqrt[e] + Sqrt[-b + 2*d*e]] - 2*
Sqrt[e]*x)/Sqrt[2*Sqrt[d]*Sqrt[e] - Sqrt[-b + 2*d*e]]]/(4*Sqrt[d]*Sqrt[2*Sq
rt[d]*Sqrt[e] - Sqrt[-b + 2*d*e]]) + ArcTan[(Sqrt[2*Sqrt[d]*Sqrt[e] - Sqrt[
-b + 2*d*e]] + 2*Sqrt[e]*x)/Sqrt[2*Sqrt[d]*Sqrt[e] + Sqrt[-b + 2*d*e]]]/(4*
Sqrt[d]*Sqrt[2*Sqrt[d]*Sqrt[e] + Sqrt[-b + 2*d*e]]) + ArcTan[(Sqrt[2*Sqrt[d
]*Sqrt[e] + Sqrt[-b + 2*d*e]] + 2*Sqrt[e]*x)/Sqrt[2*Sqrt[d]*Sqrt[e] - Sqrt[
```

```
-b + 2*d*e]]]/(4*Sqrt[d]*Sqrt[2*Sqrt[d]*Sqrt[e] - Sqrt[-b + 2*d*e]]) - Log[
Sqrt[d] - Sqrt[2*Sqrt[d]*Sqrt[e] - Sqrt[-b + 2*d*e]]*x + Sqrt[e]*x^2]/(8*Sq
rt[d]*Sqrt[2*Sqrt[d]*Sqrt[e] - Sqrt[-b + 2*d*e]]) + Log[Sqrt[d] + Sqrt[2*Sq
rt[d]*Sqrt[e] - Sqrt[-b + 2*d*e]]*x + Sqrt[e]*x^2]/(8*Sqrt[d]*Sqrt[2*Sqrt[d
]*Sqrt[e] - Sqrt[-b + 2*d*e]]) - Log[Sqrt[d] - Sqrt[2*Sqrt[d]*Sqrt[e] + Sqr
t[-b + 2*d*e]]*x + Sqrt[e]*x^2]/(8*Sqrt[d]*Sqrt[2*Sqrt[d]*Sqrt[e] + Sqrt[-b
 + 2*d*e]]) + Log[Sqrt[d] + Sqrt[2*Sqrt[d]*Sqrt[e] + Sqrt[-b + 2*d*e]]*x +
Sqrt[e]*x^2]/(8*Sqrt[d]*Sqrt[2*Sqrt[d]*Sqrt[e] + Sqrt[-b + 2*d*e]])
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1108

```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(-1), x_Symbol] :> With[{q = Rt[a/
c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Dist[1/(2*c*q*r), Int[(r - x)/(q - r*x
 + x^2), x], x] + Dist[1/(2*c*q*r), Int[(r + x)/(q + r*x + x^2), x], x]]] /
; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && NegQ[b^2 - 4*a*c]
```

Rule 1433

```
Int[((d_) + (e_.)*(x_)^(n_))/((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_)), x
_Symbol] :> With[{q = Rt[2*(d/e) - b/c, 2]}, Dist[e/(2*c), Int[1/Simp[d/e +
 q*x^(n/2) + x^n, x], x], x] + Dist[e/(2*c), Int[1/Simp[d/e - q*x^(n/2) + x
^n, x], x], x]] /; FreeQ[{a, b, c, d, e}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4
```

```
*a*c, 0] && EqQ[c*d^2 - a*e^2, 0] && IGtQ[n/2, 0] && (GtQ[2*(d/e) - b/c, 0]
 || ( !LtQ[2*(d/e) - b/c, 0] && EqQ[d, e*Rt[a/c, 2]]))
```

Rubi steps

$$\int \frac{d+ex^4}{d^2+bx^4+e^2x^8}\,dx = \frac{\int \frac{1}{\frac{d}{e}-\frac{\sqrt{-b+2de}\,x^2}{e}+x^4}\,dx}{2e} + \frac{\int \frac{1}{\frac{d}{e}+\frac{\sqrt{-b+2de}\,x^2}{e}+x^4}\,dx}{2e}$$

$$= \frac{\int \frac{\frac{\sqrt{2\sqrt{d}\,\sqrt{e}-\sqrt{-b+2de}}}{\sqrt{e}}-x}{\frac{\sqrt{d}}{\sqrt{e}}-\frac{\sqrt{2\sqrt{d}\,\sqrt{e}-\sqrt{-b+2de}}\,x}{\sqrt{e}}+x^2}\,dx}{4\sqrt{d}\,\sqrt{2\sqrt{d}\,\sqrt{e}-\sqrt{-b+2de}}} + \frac{\int \frac{\frac{\sqrt{2\sqrt{d}\,\sqrt{e}-\sqrt{-b+2de}}}{\sqrt{e}}+x}{\frac{\sqrt{d}}{\sqrt{e}}+\frac{\sqrt{2\sqrt{d}\,\sqrt{e}-\sqrt{-b+2de}}}{\sqrt{e}}}}{4\sqrt{d}\,\sqrt{2\sqrt{d}\,\sqrt{e}-\sqrt{-b+2de}}}$$

$$= \frac{\int \frac{1}{\frac{\sqrt{d}}{\sqrt{e}}-\frac{\sqrt{2\sqrt{d}\,\sqrt{e}-\sqrt{-b+2de}}\,x}{\sqrt{e}}+x^2}\,dx}{8\sqrt{d}\,\sqrt{e}} + \frac{\int \frac{1}{\frac{\sqrt{d}}{\sqrt{e}}+\frac{\sqrt{2\sqrt{d}\,\sqrt{e}-\sqrt{-b+2de}}}{\sqrt{e}}}}{8\sqrt{d}\,\sqrt{e}}$$

$$= -\frac{\log\left(\sqrt{d}-\sqrt{2\sqrt{d}\,\sqrt{e}-\sqrt{-b+2de}}\,x+\sqrt{e}\,x^2\right)}{8\sqrt{d}\,\sqrt{2\sqrt{d}\,\sqrt{e}-\sqrt{-b+2de}}} + \frac{\log\left(\sqrt{d}+\sqrt{2\sqrt{d}\,\sqrt{e}\right.}{8\sqrt{d}\,\sqrt{2\sqrt{d}\,\sqrt{}}}$$

$$= -\frac{\tan^{-1}\left(\frac{\sqrt{2\sqrt{d}\,\sqrt{e}-\sqrt{-b+2de}}-2\sqrt{e}\,x}{\sqrt{2\sqrt{d}\,\sqrt{e}+\sqrt{-b+2de}}}\right)}{4\sqrt{d}\,\sqrt{2\sqrt{d}\,\sqrt{e}+\sqrt{-b+2de}}} - \frac{\tan^{-1}\left(\frac{\sqrt{2\sqrt{d}\,\sqrt{e}+\sqrt{-b+2}}}{\sqrt{2\sqrt{d}\,\sqrt{e}-\sqrt{-b}}}\right.}{4\sqrt{d}\,\sqrt{2\sqrt{d}\,\sqrt{e}-\sqrt{-b}}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.
time = 0.03, size = 67, normalized size = 0.08

$$\frac{1}{4}\text{RootSum}\left[d^2+b\#1^4+e^2\#1^8\&,\frac{d\log(x-\#1)+e\log(x-\#1)\#1^4}{b\#1^3+2e^2\#1^7}\&\right]$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x^4)/(d^2 + b*x^4 + e^2*x^8),x]
```

```
[Out] RootSum[d^2 + b*#1^4 + e^2*#1^8 & , (d*Log[x - #1] + e*Log[x - #1]*#1^4)/(b
*#1^3 + 2*e^2*#1^7) & ]/4
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 0.06, size = 53, normalized size = 0.07

method	result	size
default	$\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(e^{2}\_Z^{8}+\_Z^{4}b+d^{2}\right)}\frac{\left(\_R^{4}e+d\right)\ln\left(x-\_R\right)}{2\_R^{7}e^{2}+\_R^{3}b}\right)}{4}$	53
risch	$\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(e^{2}\_Z^{8}+\_Z^{4}b+d^{2}\right)}\frac{\left(\_R^{4}e+d\right)\ln\left(x-\_R\right)}{2\_R^{7}e^{2}+\_R^{3}b}\right)}{4}$	53

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^4+d)/(e^2*x^8+b*x^4+d^2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/4*sum((_R^4*e+d)/(2*_R^7*e^2+_R^3*b)*ln(x-_R),_R=RootOf(_Z^8*e^2+_Z^4*b+d
^2))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^4+d)/(e^2*x^8+b*x^4+d^2),x, algorithm="maxima")
```

```
[Out] integrate((x^4*e + d)/(x^8*e^2 + b*x^4 + d^2), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 3072 vs. 2(573) = 1146.
time = 0.51, size = 3072, normalized size = 3.88

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^4+d)/(e^2*x^8+b*x^4+d^2),x, algorithm="fricas")
```

```
[Out] -sqrt(sqrt(1/2)*sqrt(-((4*d^4*e^2 + 4*b*d^3*e + b^2*d^2)*sqrt(-(2*d*e - b)/
(8*d^7*e^3 + 12*b*d^6*e^2 + 6*b^2*d^5*e + b^3*d^4)) + b)/(4*d^4*e^2 + 4*b*d
^3*e + b^2*d^2)))*arctan(-1/2*(sqrt(1/2)*(4*d^2*e^2 + 4*b*d*e + b^2 - (8*d^
5*e^3 + 12*b*d^4*e^2 + 6*b^2*d^3*e + b^3*d^2)*sqrt(-(2*d*e - b)/(8*d^7*e^3
+ 12*b*d^6*e^2 + 6*b^2*d^5*e + b^3*d^4)))*sqrt(x^2*e^2 + 1/2*sqrt(1/2)*(2*b
*d*e + b^2 - (8*d^5*e^3 + 12*b*d^4*e^2 + 6*b^2*d^3*e + b^3*d^2)*sqrt(-(2*d*
```

```
e - b)/(8*d^7*e^3 + 12*b*d^6*e^2 + 6*b^2*d^5*e + b^3*d^4)))*sqrt(-((4*d^4*e
^2 + 4*b*d^3*e + b^2*d^2)*sqrt(-(2*d*e - b)/(8*d^7*e^3 + 12*b*d^6*e^2 + 6*b
^2*d^5*e + b^3*d^4)) + b)/(4*d^4*e^2 + 4*b*d^3*e + b^2*d^2)))*sqrt(-((4*d^4
*e^2 + 4*b*d^3*e + b^2*d^2)*sqrt(-(2*d*e - b)/(8*d^7*e^3 + 12*b*d^6*e^2 + 6
*b^2*d^5*e + b^3*d^4)) + b)/(4*d^4*e^2 + 4*b*d^3*e + b^2*d^2)) - sqrt(1/2)*
(4*d^2*x*e^3 + 4*b*d*x*e^2 + b^2*x*e - (8*d^5*x*e^4 + 12*b*d^4*x*e^3 + 6*b^
2*d^3*x*e^2 + b^3*d^2*x*e)*sqrt(-(2*d*e - b)/(8*d^7*e^3 + 12*b*d^6*e^2 + 6*
b^2*d^5*e + b^3*d^4)))*sqrt(-((4*d^4*e^2 + 4*b*d^3*e + b^2*d^2)*sqrt(-(2*d*
e - b)/(8*d^7*e^3 + 12*b*d^6*e^2 + 6*b^2*d^5*e + b^3*d^4)) + b)/(4*d^4*e^2
+ 4*b*d^3*e + b^2*d^2)))*sqrt(sqrt(1/2)*sqrt(-((4*d^4*e^2 + 4*b*d^3*e + b^2
*d^2)*sqrt(-(2*d*e - b)/(8*d^7*e^3 + 12*b*d^6*e^2 + 6*b^2*d^5*e + b^3*d^4))
 + b)/(4*d^4*e^2 + 4*b*d^3*e + b^2*d^2)))*e^(-2)) + sqrt(sqrt(1/2)*sqrt(((4
*d^4*e^2 + 4*b*d^3*e + b^2*d^2)*sqrt(-(2*d*e - b)/(8*d^7*e^3 + 12*b*d^6*e^2
 + 6*b^2*d^5*e + b^3*d^4)) - b)/(4*d^4*e^2 + 4*b*d^3*e + b^2*d^2)))*arctan(
1/2*(sqrt(1/2)*(4*d^2*e^2 + 4*b*d*e + b^2 + (8*d^5*e^3 + 12*b*d^4*e^2 + 6*b
^2*d^3*e + b^3*d^2)*sqrt(-(2*d*e - b)/(8*d^7*e^3 + 12*b*d^6*e^2 + 6*b^2*d^5
*e + b^3*d^4)))*sqrt(x^2*e^2 + 1/2*sqrt(1/2)*(2*b*d*e + b^2 + (8*d^5*e^3 +
12*b*d^4*e^2 + 6*b^2*d^3*e + b^3*d^2)*sqrt(-(2*d*e - b)/(8*d^7*e^3 + 12*b*d
^6*e^2 + 6*b^2*d^5*e + b^3*d^4)))*sqrt(((4*d^4*e^2 + 4*b*d^3*e + b^2*d^2)*s
qrt(-(2*d*e - b)/(8*d^7*e^3 + 12*b*d^6*e^2 + 6*b^2*d^5*e + b^3*d^4)) - b)/(
4*d^4*e^2 + 4*b*d^3*e + b^2*d^2)))*sqrt(sqrt(1/2)*sqrt(((4*d^4*e^2 + 4*b*d^
3*e + b^2*d^2)*sqrt(-(2*d*e - b)/(8*d^7*e^3 + 12*b*d^6*e^2 + 6*b^2*d^5*e +
b^3*d^4)) - b)/(4*d^4*e^2 + 4*b*d^3*e + b^2*d^2)))*sqrt(((4*d^4*e^2 + 4*b*d
^3*e + b^2*d^2)*sqrt(-(2*d*e - b)/(8*d^7*e^3 + 12*b*d^6*e^2 + 6*b^2*d^5*e +
 b^3*d^4)) - b)/(4*d^4*e^2 + 4*b*d^3*e + b^2*d^2)) - sqrt(1/2)*(4*d^2*x*e^3
 + 4*b*d*x*e^2 + b^2*x*e + (8*d^5*x*e^4 + 12*b*d^4*x*e^3 + 6*b^2*d^3*x*e^2
+ b^3*d^2*x*e)*sqrt(-(2*d*e - b)/(8*d^7*e^3 + 12*b*d^6*e^2 + 6*b^2*d^5*e +
b^3*d^4)))*sqrt(sqrt(1/2)*sqrt(((4*d^4*e^2 + 4*b*d^3*e + b^2*d^2)*sqrt(-(2*
d*e - b)/(8*d^7*e^3 + 12*b*d^6*e^2 + 6*b^2*d^5*e + b^3*d^4)) - b)/(4*d^4*e^
2 + 4*b*d^3*e + b^2*d^2)))*sqrt(((4*d^4*e^2 + 4*b*d^3*e + b^2*d^2)*sqrt(-(2
*d*e - b)/(8*d^7*e^3 + 12*b*d^6*e^2 + 6*b^2*d^5*e + b^3*d^4)) - b)/(4*d^4*e
^2 + 4*b*d^3*e + b^2*d^2)))*e^(-2)) + 1/4*sqrt(sqrt(1/2)*sqrt(-((4*d^4*e^2
+ 4*b*d^3*e + b^2*d^2)*sqrt(-(2*d*e - b)/(8*d^7*e^3 + 12*b*d^6*e^2 + 6*b^2*
d^5*e + b^3*d^4)) + b)/(4*d^4*e^2 + 4*b*d^3*e + b^2*d^2)))*log(x*e + 1/2*(2
*d*e - (4*d^4*e^2 + 4*b*d^3*e + b^2*d^2)*sqrt(-(2*d*e - b)/(8*d^7*e^3 + 12*
b*d^6*e^2 + 6*b^2*d^5*e + b^3*d^4)) + b)*sqrt(sqrt(1/2)*sqrt(-((4*d^4*e^2 +
 4*b*d^3*e + b^2*d^2)*sqrt(-(2*d*e - b)/(8*d^7*e^3 + 12*b*d^6*e^2 + 6*b^2*d
^5*e + b^3*d^4)) + b)/(4*d^4*e^2 + 4*b*d^3*e + b^2*d^2)))) - 1/4*sqrt(sqrt(
1/2)*sqrt(-((4*d^4*e^2 + 4*b*d^3*e + b^2*d^2)*sqrt(-(2*d*e - b)/(8*d^7*e^3
+ 12*b*d^6*e^2 + 6*b^2*d^5*e + b^3*d^4)) + b)/(4*d^4*e^2 + 4*b*d^3*e + b^2*
d^2)))*log(x*e - 1/2*(2*d*e - (4*d^4*e^2 + 4*b*d^3*e + b^2*d^2)*sqrt(-(2*d*
e - b)/(8*d^7*e^3 + 12*b*d^6*e^2 + 6*b^2*d^5*e + b^3*d^4)) + b)*sqrt(sqrt(1
/2)*sqrt(-((4*d^4*e^2 + 4*b*d^3*e + b^2*d^2)*sqrt(-(2*d*e - b)/(8*d^7*e^3 +
 12*b*d^6*e^2 + 6*b^2*d^5*e + b^3*d^4)) + b)/(4*d^4*e^2 + 4*b*d^3*e + b^2*d
^2)))) + 1/4*sqrt(sqrt(1/2)*sqrt(((4*d^4*e^2 + 4*b*d^3*e + b^2*d^2)*sqrt(-(
```

```
2*d*e - b)/(8*d^7*e^3 + 12*b*d^6*e^2 + 6*b^2*d^5*e + b^3*d^4)) - b)/(4*d^4*
e^2 + 4*b*d^3*e + b^2*d^2)))*log(x*e + 1/2*(2*d*e + (4*d^4*e^2 + 4*b*d^3*e
+ b^2*d^2)*sqrt(-(2*d*e - b)/(8*d^7*e^3 + 12*b*d^6*e^2 + 6*b^2*d^5*e + b^3*
d^4)) + b)*sqrt(sqrt(1/2)*sqrt(((4*d^4*e^2 + 4*b*d^3*e + b^2*d^2)*sqrt(-(2*
d*e - b)/(8*d^7*e^3 + 12*b*d^6*e^2 + 6*b^2*d^5*e + b^3*d^4)) - b)/(4*d^4*e^
2 + 4*b*d^3*e + b^2*d^2)))) - 1/4*sqrt(sqrt(1/2)*sqrt(((4*d^4*e^2 + 4*b*d^3
*e + b^2*d^2)*sqrt(-(2*d*e - b)/(8*d^7*e^3 + 12*b*d^6*e^2 + 6*b^2*d^5*e + b
^3*d^4)) - b)/(4*d^4*e^2 + 4*b*d^3*e + b^2*d^2)))*log(x*e - 1/2*(2*d*e + (4
*d^4*e^2 + 4*b*d^3*e + b^2*d^2)*sqrt(-(2*d*e - b)/(8*d^7*e^3 + 12*b*d^6*e^2
 + 6*b^2*d^5*e + b^3*d^4)) + b)*sqrt(sqrt(1/2)*sqrt(((4*d^4*e^2 + 4*b*d^3*e
 + b^2*d^2)*sqrt(-(2*d*e - b)/(8*d^7*e^3 + 12*b*d^6*e^2 + 6*b^2*d^5*e + b^3
*d^4)) - b)/(4*d^4*e^2 + 4*b*d^3*e + b^2*d^2))))
```

Sympy [A]
time = 18.75, size = 136, normalized size = 0.17

$$\operatorname{RootSum}\left(t^{8}\cdot\left(65536b^{4}d^{2}+524288b^{3}d^{3}e+1572864b^{2}d^{4}e^{2}+2097152bd^{5}e^{3}+1048576d^{6}e^{4}\right)+t^{4}\cdot\left(256b^{3}+1024b^{2}de+1024bd^{2}e^{2}\right)+e^{2},\left(t\mapsto t\log\left(x+\frac{1024t^{5}b^{2}d^{2}+4096t^{5}bd^{3}e+4096t^{5}d^{4}e^{2}+4tb+4tde}{e}\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**4+d)/(e**2*x**8+b*x**4+d**2),x)
```

```
[Out] RootSum(_t**8*(65536*b**4*d**2 + 524288*b**3*d**3*e + 1572864*b**2*d**4*e**
2 + 2097152*b*d**5*e**3 + 1048576*d**6*e**4) + _t**4*(256*b**3 + 1024*b**2*
d*e + 1024*b*d**2*e**2) + e**2, Lambda(_t, _t*log(x + (1024*_t**5*b**2*d**2
 + 4096*_t**5*b*d**3*e + 4096*_t**5*d**4*e**2 + 4*_t*b + 4*_t*d*e)/e)))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^4+d)/(e^2*x^8+b*x^4+d^2),x, algorithm="giac")
```

```
[Out] integrate((x^4*e + d)/(x^8*e^2 + b*x^4 + d^2), x)
```

Mupad [B]
time = 3.83, size = 2500, normalized size = 3.16

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x^4)/(b*x^4 + d^2 + e^2*x^8),x)
```

```
[Out] 2*atan(((x*(32*b*d^5*e^13 - 4*b^4*d^2*e^10 + 24*b^3*d^3*e^11 - 48*b^2*d^4*e
^12) + (-(b^3 + ((b - 2*d*e)*(b + 2*d*e)^5)^(1/2) + 4*b*d^2*e^2 + 4*b^2*d*e
```

)/(512*(b^4*d^2 + 16*d^6*e^4 + 8*b^3*d^3*e + 32*b*d^5*e^3 + 24*b^2*d^4*e^2)
))^(1/4)*((x*(65536*d^9*e^15 - 32768*b*d^8*e^14 + 1024*b^7*d^2*e^8 - 2048*b
^6*d^3*e^9 - 10240*b^5*d^4*e^10 + 20480*b^4*d^5*e^11 + 32768*b^3*d^6*e^12 -
65536*b^2*d^7*e^13) - (-(b^3 + ((b - 2*d*e)*(b + 2*d*e)^5)^(1/2) + 4*b*d^2
*e^2 + 4*b^2*d*e)/(512*(b^4*d^2 + 16*d^6*e^4 + 8*b^3*d^3*e + 32*b*d^5*e^3 +
24*b^2*d^4*e^2)))^(1/4)*(262144*d^10*e^15 - 262144*b*d^9*e^14 + 4096*b^7*d
^3*e^8 - 4096*b^6*d^4*e^9 - 49152*b^5*d^5*e^10 + 49152*b^4*d^6*e^11 + 19660
8*b^3*d^7*e^12 - 196608*b^2*d^8*e^13)*1i)*(-(b^3 + ((b - 2*d*e)*(b + 2*d*e)
^5)^(1/2) + 4*b*d^2*e^2 + 4*b^2*d*e)/(512*(b^4*d^2 + 16*d^6*e^4 + 8*b^3*d^3
*e + 32*b*d^5*e^3 + 24*b^2*d^4*e^2)))^(3/4)*1i - 256*d^7*e^14 + 256*b*d^6*e
^13 + 16*b^4*d^3*e^10 - 64*b^3*d^4*e^11)*1i)*(-(b^3 + ((b - 2*d*e)*(b + 2*d
*e)^5)^(1/2) + 4*b*d^2*e^2 + 4*b^2*d*e)/(512*(b^4*d^2 + 16*d^6*e^4 + 8*b^3*
d^3*e + 32*b*d^5*e^3 + 24*b^2*d^4*e^2)))^(1/4) + (x*(32*b*d^5*e^13 - 4*b^4*
d^2*e^10 + 24*b^3*d^3*e^11 - 48*b^2*d^4*e^12) + (-(b^3 + ((b - 2*d*e)*(b +
2*d*e)^5)^(1/2) + 4*b*d^2*e^2 + 4*b^2*d*e)/(512*(b^4*d^2 + 16*d^6*e^4 + 8*b
^3*d^3*e + 32*b*d^5*e^3 + 24*b^2*d^4*e^2)))^(1/4)*((x*(65536*d^9*e^15 - 327
68*b*d^8*e^14 + 1024*b^7*d^2*e^8 - 2048*b^6*d^3*e^9 - 10240*b^5*d^4*e^10 +
20480*b^4*d^5*e^11 + 32768*b^3*d^6*e^12 - 65536*b^2*d^7*e^13) + (-(b^3 + ((
b - 2*d*e)*(b + 2*d*e)^5)^(1/2) + 4*b*d^2*e^2 + 4*b^2*d*e)/(512*(b^4*d^2 +
16*d^6*e^4 + 8*b^3*d^3*e + 32*b*d^5*e^3 + 24*b^2*d^4*e^2)))^(1/4)*(262144*d
^10*e^15 - 262144*b*d^9*e^14 + 4096*b^7*d^3*e^8 - 4096*b^6*d^4*e^9 - 49152*
b^5*d^5*e^10 + 49152*b^4*d^6*e^11 + 196608*b^3*d^7*e^12 - 196608*b^2*d^8*e^
13)*1i)*(-(b^3 + ((b - 2*d*e)*(b + 2*d*e)^5)^(1/2) + 4*b*d^2*e^2 + 4*b^2*d*
e)/(512*(b^4*d^2 + 16*d^6*e^4 + 8*b^3*d^3*e + 32*b*d^5*e^3 + 24*b^2*d^4*e^2
)))^(3/4)*1i + 256*d^7*e^14 - 256*b*d^6*e^13 - 16*b^4*d^3*e^10 + 64*b^3*d^4
*e^11)*1i)*(-(b^3 + ((b - 2*d*e)*(b + 2*d*e)^5)^(1/2) + 4*b*d^2*e^2 + 4*b^2
*d*e)/(512*(b^4*d^2 + 16*d^6*e^4 + 8*b^3*d^3*e + 32*b*d^5*e^3 + 24*b^2*d^4*
e^2)))^(1/4))/((x*(32*b*d^5*e^13 - 4*b^4*d^2*e^10 + 24*b^3*d^3*e^11 - 48*b^
2*d^4*e^12) + (-(b^3 + ((b - 2*d*e)*(b + 2*d*e)^5)^(1/2) + 4*b*d^2*e^2 + 4*
b^2*d*e)/(512*(b^4*d^2 + 16*d^6*e^4 + 8*b^3*d^3*e + 32*b*d^5*e^3 + 24*b^2*d
^4*e^2)))^(1/4)*((x*(65536*d^9*e^15 - 32768*b*d^8*e^14 + 1024*b^7*d^2*e^8 -
2048*b^6*d^3*e^9 - 10240*b^5*d^4*e^10 + 20480*b^4*d^5*e^11 + 32768*b^3*d^6
*e^12 - 65536*b^2*d^7*e^13) - (-(b^3 + ((b - 2*d*e)*(b + 2*d*e)^5)^(1/2) +
4*b*d^2*e^2 + 4*b^2*d*e)/(512*(b^4*d^2 + 16*d^6*e^4 + 8*b^3*d^3*e + 32*b*d^
5*e^3 + 24*b^2*d^4*e^2)))^(1/4)*(262144*d^10*e^15 - 262144*b*d^9*e^14 + 409
6*b^7*d^3*e^8 - 4096*b^6*d^4*e^9 - 49152*b^5*d^5*e^10 + 49152*b^4*d^6*e^11
+ 196608*b^3*d^7*e^12 - 196608*b^2*d^8*e^13)*1i)*(-(b^3 + ((b - 2*d*e)*(b +
2*d*e)^5)^(1/2) + 4*b*d^2*e^2 + 4*b^2*d*e)/(512*(b^4*d^2 + 16*d^6*e^4 + 8*
b^3*d^3*e + 32*b*d^5*e^3 + 24*b^2*d^4*e^2)))^(3/4)*1i - 256*d^7*e^14 + 256*
b*d^6*e^13 + 16*b^4*d^3*e^10 - 64*b^3*d^4*e^11)*1i)*(-(b^3 + ((b - 2*d*e)*(
b + 2*d*e)^5)^(1/2) + 4*b*d^2*e^2 + 4*b^2*d*e)/(512*(b^4*d^2 + 16*d^6*e^4 +
8*b^3*d^3*e + 32*b*d^5*e^3 + 24*b^2*d^4*e^2)))^(1/4)*1i - (x*(32*b*d^5*e^1
3 - 4*b^4*d^2*e^10 + 24*b^3*d^3*e^11 - 48*b^2*d^4*e^12) + (-(b^3 + ((b - 2*
d*e)*(b + 2*d*e)^5)^(1/2) + 4*b*d^2*e^2 + 4*b^2*d*e)/(512*(b^4*d^2 + 16*d^6
*e^4 + 8*b^3*d^3*e + 32*b*d^5*e^3 + 24*b^2*d^4*e^2)))^(1/4)*((x*(65536*d^9*

```
e^15 - 32768*b*d^8*e^14 + 1024*b^7*d^2*e^8 - 2048*b^6*d^3*e^9 - 10240*b^5*d
^4*e^10 + 20480*b^4*d^5*e^11 + 32768*b^3*d^6*e^12 - 65536*b^2*d^7*e^13) + (
-(b^3 + ((b - 2*d*e)*(b + 2*d*e)^5)^(1/2) + 4*b*d^2*e^2 + 4*b^2*d*e)/(512*(
b^4*d^2 + 16*d^6*e^4 + 8*b^3*d^3*e + 32*b*d^5*e^3 + 24*b^2*d^4*e^2)))^(1/4)
*(262144*d^10*e^15 - 262144*b*d^9*e^14 + 4096*b^7*d^3*e^8 - 4096*b^6*d^4*e^
9 - 49152*b^5*d^5*e^10 + 49152*b^4*d^6*e^11 + 196608*b^3*d^7*e^12 - 196608*
b^2*d^8*e^13)*1i)*(-(b^3 + ((b - 2*d*e)*(b + 2*d*e)^5)^(1/2) + 4*b*d^2*e^2
+ 4*b^2*d*e)/(512*(b^4*d^2 + 16*d^6*e^4 + 8*b^3*d^3*e + 32*b*d^5*e^3 + 24*b
^2*d^4*e^2)))^(3/4)*1i + 256*d^7*e^14 - 256*b*d^6*e^13 - 16*b^4*d^3*e^10 +
64*b^3*d^4*e^11)*1i)*(-(b^3 + ((b - 2*d*e)*(b + 2*d*e)^5)^(1/2) + 4*b*d^2*e
^2 + 4*b^2*d*e)/(512*(b^4*d^2 + 16*d^6*e^4 + 8*b^3*d^3*e + 32*b*d^5*e^3 + 2
4*b^2*d^4*e^2)))^(1/4)*1i))*(-(b^3 + ((b - 2*d*e)*(b + 2*d*e)^5)^(1/2) + 4*
b*d^2*e^2 + 4*b^2*d*e)/(512*(b^4*d^2 + 16*d^6*e^4 + 8*b^3*d^3*e + 32*b*d^5*
e^3 + 24*b^2*d^4*e^2)))^(1/4) - atan(((x*(32*b*d^5*e^13 - 4*b^4*d^2*e^10 +
24*b^3*d^3*e^11 - 48*b^2*d^4*e^12) - (-(b^3 + ((b - 2*d*e)*(b + 2*d*e)^5)^(
1/2) + 4*b*d^2*e^2 + 4*b^2*d*e)/(512*(b^4*d^2 + 16*d^6*e^4 + 8*b^3*d^3*e +
32*b*d^5*e^3 + 24*b^2*d^4*e^2)))^(1/4)*((x*(65536*d^9*e^15 - 32768*b*d^8*e^
14 + 1024*b^7*d^2*e^8 - 2048*b^6*d^3*e^9 - 10240*b^5*d^4*e^10 + 20480*b^4*d
^5*e^11 + 32768*b^3*d^6*e^12 - 65536*b^2*d^7*e^13) + (-(b^3 + ((b - 2*d*e)*
(b + 2*d*e)^5)^(1/2) + 4*b*d^2*e^2 + 4*b^2*d*e)...
```

3.6 $\int \frac{d+ex^4}{d^2+fx^4+e^2x^8}\,dx$

Optimal. Leaf size=791

$$-\frac{\tan^{-1}\left(\frac{\sqrt{2\sqrt{d}\sqrt{e}-\sqrt{2de-f}}-2\sqrt{e}\,x}{\sqrt{2\sqrt{d}\sqrt{e}+\sqrt{2de-f}}}\right)}{4\sqrt{d}\sqrt{2\sqrt{d}\sqrt{e}+\sqrt{2de-f}}}-\frac{\tan^{-1}\left(\frac{\sqrt{2\sqrt{d}\sqrt{e}+\sqrt{2de-f}}-2\sqrt{e}\,x}{\sqrt{2\sqrt{d}\sqrt{e}-\sqrt{2de-f}}}\right)}{4\sqrt{d}\sqrt{2\sqrt{d}\sqrt{e}-\sqrt{2de-f}}}+\frac{\tan^{-1}\left(\frac{\sqrt{2}\cdots}{\cdots}\right)}{4\sqrt{d}\cdots}$$

[Out] -1/4*arctan((-2*x*e^(1/2)+(2*d^(1/2)*e^(1/2)+(2*d*e-f)^(1/2))^(1/2))/(2*d^(1/2)*e^(1/2)-(2*d*e-f)^(1/2))^(1/2))/d^(1/2)/(2*d^(1/2)*e^(1/2)-(2*d*e-f)^(1/2))^(1/2)+1/4*arctan((2*x*e^(1/2)+(2*d^(1/2)*e^(1/2)+(2*d*e-f)^(1/2))^(1/2))/(2*d^(1/2)*e^(1/2)-(2*d*e-f)^(1/2))^(1/2))/d^(1/2)/(2*d^(1/2)*e^(1/2)-(2*d*e-f)^(1/2))^(1/2)-1/8*ln(d^(1/2)+x^2*e^(1/2)-x*(2*d^(1/2)*e^(1/2)-(2*d*e-f)^(1/2))^(1/2))/d^(1/2)/(2*d^(1/2)*e^(1/2)-(2*d*e-f)^(1/2))^(1/2)+1/8*ln(d^(1/2)+x^2*e^(1/2)+x*(2*d^(1/2)*e^(1/2)-(2*d*e-f)^(1/2))^(1/2))/d^(1/2)/(2*d^(1/2)*e^(1/2)-(2*d*e-f)^(1/2))^(1/2)-1/4*arctan((-2*x*e^(1/2)+(2*d^(1/2)*e^(1/2)-(2*d*e-f)^(1/2))^(1/2))/(2*d^(1/2)*e^(1/2)+(2*d*e-f)^(1/2))^(1/2))/d^(1/2)/(2*d^(1/2)*e^(1/2)+(2*d*e-f)^(1/2))^(1/2)+1/4*arctan((2*x*e^(1/2)+(2*d^(1/2)*e^(1/2)-(2*d*e-f)^(1/2))^(1/2))/(2*d^(1/2)*e^(1/2)+(2*d*e-f)^(1/2))^(1/2))/d^(1/2)/(2*d^(1/2)*e^(1/2)+(2*d*e-f)^(1/2))^(1/2)-1/8*ln(d^(1/2)+x^2*e^(1/2)-x*(2*d^(1/2)*e^(1/2)+(2*d*e-f)^(1/2))^(1/2))/d^(1/2)/(2*d^(1/2)*e^(1/2)+(2*d*e-f)^(1/2))^(1/2)+1/8*ln(d^(1/2)+x^2*e^(1/2)+x*(2*d^(1/2)*e^(1/2)+(2*d*e-f)^(1/2))^(1/2))/d^(1/2)/(2*d^(1/2)*e^(1/2)+(2*d*e-f)^(1/2))^(1/2)

Rubi [A]
time = 0.56, antiderivative size = 791, normalized size of antiderivative = 1.00, number of steps used = 19, number of rules used = 6, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {1433, 1108, 648, 632, 210, 642}

$$-\frac{\text{ArcTan}\left(\frac{\sqrt{2\sqrt{d}\sqrt{e}-\sqrt{2de-f}}-2\sqrt{e}\,x}{\sqrt{\sqrt{2de-f}+2\sqrt{d}\sqrt{e}}}\right)}{4\sqrt{d}\sqrt{\sqrt{2de-f}+2\sqrt{d}\sqrt{e}}}-\frac{\text{ArcTan}\left(\frac{\sqrt{\sqrt{2de-f}+2\sqrt{d}\sqrt{e}}-2\sqrt{e}\,x}{\sqrt{2\sqrt{d}\sqrt{e}-\sqrt{2de-f}}}\right)}{4\sqrt{d}\sqrt{2\sqrt{d}\sqrt{e}-\sqrt{2de-f}}}+\frac{\text{ArcTan}\left(\frac{\sqrt{2\sqrt{d}\sqrt{e}-\sqrt{2de-f}}+2\sqrt{e}\,x}{\sqrt{\sqrt{2de-f}+2\sqrt{d}\sqrt{e}}}\right)}{4\sqrt{d}\sqrt{\sqrt{2de-f}+2\sqrt{d}\sqrt{e}}}+\frac{\text{ArcTan}\left(\frac{\sqrt{\sqrt{2de-f}+2\sqrt{d}\sqrt{e}}+2\sqrt{e}\,x}{\sqrt{2\sqrt{d}\sqrt{e}-\sqrt{2de-f}}}\right)}{4\sqrt{d}\sqrt{2\sqrt{d}\sqrt{e}-\sqrt{2de-f}}}-\frac{\log\left(-x\sqrt{2\sqrt{d}\sqrt{e}-\sqrt{2de-f}}+\sqrt{d}+\sqrt{e}\,x^2\right)}{8\sqrt{d}\sqrt{2\sqrt{d}\sqrt{e}-\sqrt{2de-f}}}+\frac{\log\left(x\sqrt{2\sqrt{d}\sqrt{e}-\sqrt{2de-f}}+\sqrt{d}+\sqrt{e}\,x^2\right)}{8\sqrt{d}\sqrt{2\sqrt{d}\sqrt{e}-\sqrt{2de-f}}}-\frac{\log\left(-x\sqrt{\sqrt{2de-f}+2\sqrt{d}\sqrt{e}}+\sqrt{d}+\sqrt{e}\,x^2\right)}{8\sqrt{d}\sqrt{\sqrt{2de-f}+2\sqrt{d}\sqrt{e}}}+\frac{\log\left(x\sqrt{\sqrt{2de-f}+2\sqrt{d}\sqrt{e}}+\sqrt{d}+\sqrt{e}\,x^2\right)}{8\sqrt{d}\sqrt{\sqrt{2de-f}+2\sqrt{d}\sqrt{e}}}$$

Antiderivative was successfully verified.

[In] Int[(d + e*x^4)/(d^2 + f*x^4 + e^2*x^8),x]

[Out] -1/4*ArcTan[(Sqrt[2*Sqrt[d]*Sqrt[e] - Sqrt[2*d*e - f]] - 2*Sqrt[e]*x)/Sqrt[2*Sqrt[d]*Sqrt[e] + Sqrt[2*d*e - f]]]/(Sqrt[d]*Sqrt[2*Sqrt[d]*Sqrt[e] + Sqrt[2*d*e - f]]) - ArcTan[(Sqrt[2*Sqrt[d]*Sqrt[e] + Sqrt[2*d*e - f]] - 2*Sqrt[e]*x)/Sqrt[2*Sqrt[d]*Sqrt[e] - Sqrt[2*d*e - f]]]/(4*Sqrt[d]*Sqrt[2*Sqrt[d]*Sqrt[e] - Sqrt[2*d*e - f]]) + ArcTan[(Sqrt[2*Sqrt[d]*Sqrt[e] - Sqrt[2*d*e - f]] + 2*Sqrt[e]*x)/Sqrt[2*Sqrt[d]*Sqrt[e] + Sqrt[2*d*e - f]]]/(4*Sqrt[d]*Sqrt[2*Sqrt[d]*Sqrt[e] + Sqrt[2*d*e - f]]) + ArcTan[(Sqrt[2*Sqrt[d]*Sqrt[e] + Sqrt[2*d*e - f]] + 2*Sqrt[e]*x)/Sqrt[2*Sqrt[d]*Sqrt[e] - Sqrt[2*d*e - f]

```
]]/(4*Sqrt[d]*Sqrt[2*Sqrt[d]*Sqrt[e] - Sqrt[2*d*e - f]]) - Log[Sqrt[d] - Sq
rt[2*Sqrt[d]*Sqrt[e] - Sqrt[2*d*e - f]]*x + Sqrt[e]*x^2]/(8*Sqrt[d]*Sqrt[2*
Sqrt[d]*Sqrt[e] - Sqrt[2*d*e - f]]) + Log[Sqrt[d] + Sqrt[2*Sqrt[d]*Sqrt[e]
- Sqrt[2*d*e - f]]*x + Sqrt[e]*x^2]/(8*Sqrt[d]*Sqrt[2*Sqrt[d]*Sqrt[e] - Sqr
t[2*d*e - f]]) - Log[Sqrt[d] - Sqrt[2*Sqrt[d]*Sqrt[e] + Sqrt[2*d*e - f]]*x
+ Sqrt[e]*x^2]/(8*Sqrt[d]*Sqrt[2*Sqrt[d]*Sqrt[e] + Sqrt[2*d*e - f]]) + Log[
Sqrt[d] + Sqrt[2*Sqrt[d]*Sqrt[e] + Sqrt[2*d*e - f]]*x + Sqrt[e]*x^2]/(8*Sqr
t[d]*Sqrt[2*Sqrt[d]*Sqrt[e] + Sqrt[2*d*e - f]])
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1108

```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(-1), x_Symbol] :> With[{q = Rt[a/
c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Dist[1/(2*c*q*r), Int[(r - x)/(q - r*x
 + x^2), x], x] + Dist[1/(2*c*q*r), Int[(r + x)/(q + r*x + x^2), x], x]]] /
; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && NegQ[b^2 - 4*a*c]
```

Rule 1433

```
Int[((d_) + (e_.)*(x_)^(n_))/((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_)), x
_Symbol] :> With[{q = Rt[2*(d/e) - b/c, 2]}, Dist[e/(2*c), Int[1/Simp[d/e +
 q*x^(n/2) + x^n, x], x], x] + Dist[e/(2*c), Int[1/Simp[d/e - q*x^(n/2) + x
^n, x], x], x]] /; FreeQ[{a, b, c, d, e}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4
```

```
*a*c, 0] && EqQ[c*d^2 - a*e^2, 0] && IGtQ[n/2, 0] && (GtQ[2*(d/e) - b/c, 0]
 || ( !LtQ[2*(d/e) - b/c, 0] && EqQ[d, e*Rt[a/c, 2]]))
```

Rubi steps

$$\int \frac{d+ex^4}{d^2+fx^4+e^2x^8}\,dx = \frac{\int \frac{1}{\frac{d}{e}-\frac{\sqrt{2de-f}\,x^2}{e}+x^4}\,dx}{2e} + \frac{\int \frac{1}{\frac{d}{e}+\frac{\sqrt{2de-f}\,x^2}{e}+x^4}\,dx}{2e}$$

$$= \frac{\int \frac{\frac{\sqrt{2\sqrt{d}\sqrt{e}-\sqrt{2de-f}}}{\sqrt{e}}-x}{\frac{\sqrt{d}}{\sqrt{e}}-\frac{\sqrt{2\sqrt{d}\sqrt{e}-\sqrt{2de-f}}\,x}{\sqrt{e}}+x^2}\,dx}{4\sqrt{d}\sqrt{2\sqrt{d}\sqrt{e}-\sqrt{2de-f}}} + \frac{\int \frac{\frac{\sqrt{2\sqrt{d}\sqrt{e}-\sqrt{2de-f}}}{\sqrt{e}}+x}{\frac{\sqrt{d}}{\sqrt{e}}+\frac{\sqrt{2\sqrt{d}\sqrt{e}-\sqrt{2de-f}}\,x}{\sqrt{e}}\cdots}}{4\sqrt{d}\sqrt{2\sqrt{d}\sqrt{e}-\sqrt{2de-f}}\cdots}$$

$$= \frac{\int \frac{1}{\frac{\sqrt{d}}{\sqrt{e}}-\frac{\sqrt{2\sqrt{d}\sqrt{e}-\sqrt{2de-f}}\,x}{\sqrt{e}}+x^2}\,dx}{8\sqrt{d}\sqrt{e}} + \frac{\int \frac{1}{\frac{\sqrt{d}}{\sqrt{e}}+\frac{\sqrt{2\sqrt{d}\sqrt{e}-\sqrt{2de-f}}\,x}{\sqrt{e}}\cdots}}{8\sqrt{d}\sqrt{e}}$$

$$= -\frac{\log\left(\sqrt{d}-\sqrt{2\sqrt{d}\sqrt{e}-\sqrt{2de-f}}\,x+\sqrt{e}\,x^2\right)}{8\sqrt{d}\sqrt{2\sqrt{d}\sqrt{e}-\sqrt{2de-f}}} + \frac{\log\left(\sqrt{d}+\sqrt{2\sqrt{d}\sqrt{e}\cdots}\right.}{8\sqrt{d}\sqrt{2\sqrt{d}\sqrt{e}\cdots}}$$

$$= -\frac{\tan^{-1}\left(\frac{\sqrt{2\sqrt{d}\sqrt{e}-\sqrt{2de-f}}-2\sqrt{e}\,x}{\sqrt{2\sqrt{d}\sqrt{e}+\sqrt{2de-f}}}\right)}{4\sqrt{d}\sqrt{2\sqrt{d}\sqrt{e}+\sqrt{2de-f}}} - \frac{\tan^{-1}\left(\frac{\sqrt{2\sqrt{d}\sqrt{e}+\sqrt{2de-\cdots}}}{\sqrt{2\sqrt{d}\sqrt{e}-\sqrt{2de\cdots}}}\right.}{4\sqrt{d}\sqrt{2\sqrt{d}\sqrt{e}-\sqrt{2de\cdots}}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.
time = 0.02, size = 67, normalized size = 0.08

$$\frac{1}{4}\text{RootSum}\left[d^2+f\#1^4+e^2\#1^8\&,\frac{d\log(x-\#1)+e\log(x-\#1)\#1^4}{f\#1^3+2e^2\#1^7}\&\right]$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x^4)/(d^2 + f*x^4 + e^2*x^8),x]
```

```
[Out] RootSum[d^2 + f*#1^4 + e^2*#1^8 & , (d*Log[x - #1] + e*Log[x - #1]*#1^4)/(f
*#1^3 + 2*e^2*#1^7) & ]/4
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 0.08, size = 53, normalized size = 0.07

method	result	size
default	$\frac{\sum_{\_R=\mathrm{RootOf}\left(e^2\_Z^8+f\_Z^4+d^2\right)}\frac{\left(\_R^4 e+d\right)\ln\left(x-\_R\right)}{2\_R^7 e^2+\_R^3 f}}{4}$	53
risch	$\frac{\sum_{\_R=\mathrm{RootOf}\left(e^2\_Z^8+f\_Z^4+d^2\right)}\frac{\left(\_R^4 e+d\right)\ln\left(x-\_R\right)}{2\_R^7 e^2+\_R^3 f}}{4}$	53

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^4+d)/(e^2*x^8+f*x^4+d^2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/4*sum((_R^4*e+d)/(2*_R^7*e^2+_R^3*f)*ln(x-_R),_R=RootOf(_Z^8*e^2+_Z^4*f+d
^2))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^4+d)/(e^2*x^8+f*x^4+d^2),x, algorithm="maxima")
```

```
[Out] integrate((x^4*e + d)/(x^8*e^2 + f*x^4 + d^2), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 3072 vs. 2(573) = 1146.
time = 0.46, size = 3072, normalized size = 3.88

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^4+d)/(e^2*x^8+f*x^4+d^2),x, algorithm="fricas")
```

```
[Out] -sqrt(sqrt(1/2)*sqrt(-((4*d^4*e^2 + 4*d^3*f*e + d^2*f^2)*sqrt(-(2*d*e - f)/
(8*d^7*e^3 + 12*d^6*f*e^2 + 6*d^5*f^2*e + d^4*f^3)) + f)/(4*d^4*e^2 + 4*d^3
*f*e + d^2*f^2)))*arctan(-1/2*(sqrt(1/2)*(4*d^2*e^2 + 4*d*f*e + f^2 - (8*d^
5*e^3 + 12*d^4*f*e^2 + 6*d^3*f^2*e + d^2*f^3)*sqrt(-(2*d*e - f)/(8*d^7*e^3
+ 12*d^6*f*e^2 + 6*d^5*f^2*e + d^4*f^3)))*sqrt(x^2*e^2 + 1/2*sqrt(1/2)*(2*d
*f*e + f^2 - (8*d^5*e^3 + 12*d^4*f*e^2 + 6*d^3*f^2*e + d^2*f^3)*sqrt(-(2*d*
```

```
e - f)/(8*d^7*e^3 + 12*d^6*f*e^2 + 6*d^5*f^2*e + d^4*f^3)))*sqrt(-((4*d^4*e
^2 + 4*d^3*f*e + d^2*f^2)*sqrt(-(2*d*e - f)/(8*d^7*e^3 + 12*d^6*f*e^2 + 6*d
^5*f^2*e + d^4*f^3)) + f)/(4*d^4*e^2 + 4*d^3*f*e + d^2*f^2)))*sqrt(-((4*d^4
*e^2 + 4*d^3*f*e + d^2*f^2)*sqrt(-(2*d*e - f)/(8*d^7*e^3 + 12*d^6*f*e^2 + 6
*d^5*f^2*e + d^4*f^3)) + f)/(4*d^4*e^2 + 4*d^3*f*e + d^2*f^2)) - sqrt(1/2)*
(4*d^2*x*e^3 + 4*d*f*x*e^2 + f^2*x*e - (8*d^5*x*e^4 + 12*d^4*f*x*e^3 + 6*d^
3*f^2*x*e^2 + d^2*f^3*x*e)*sqrt(-(2*d*e - f)/(8*d^7*e^3 + 12*d^6*f*e^2 + 6*
d^5*f^2*e + d^4*f^3)))*sqrt(-((4*d^4*e^2 + 4*d^3*f*e + d^2*f^2)*sqrt(-(2*d*
e - f)/(8*d^7*e^3 + 12*d^6*f*e^2 + 6*d^5*f^2*e + d^4*f^3)) + f)/(4*d^4*e^2
+ 4*d^3*f*e + d^2*f^2)))*sqrt(sqrt(1/2)*sqrt(-((4*d^4*e^2 + 4*d^3*f*e + d^2
*f^2)*sqrt(-(2*d*e - f)/(8*d^7*e^3 + 12*d^6*f*e^2 + 6*d^5*f^2*e + d^4*f^3))
 + f)/(4*d^4*e^2 + 4*d^3*f*e + d^2*f^2)))*e^(-2)) + sqrt(sqrt(1/2)*sqrt(((4
*d^4*e^2 + 4*d^3*f*e + d^2*f^2)*sqrt(-(2*d*e - f)/(8*d^7*e^3 + 12*d^6*f*e^2
 + 6*d^5*f^2*e + d^4*f^3)) - f)/(4*d^4*e^2 + 4*d^3*f*e + d^2*f^2)))*arctan(
1/2*(sqrt(1/2)*(4*d^2*e^2 + 4*d*f*e + f^2 + (8*d^5*e^3 + 12*d^4*f*e^2 + 6*d
^3*f^2*e + d^2*f^3)*sqrt(-(2*d*e - f)/(8*d^7*e^3 + 12*d^6*f*e^2 + 6*d^5*f^2
*e + d^4*f^3)))*sqrt(x^2*e^2 + 1/2*sqrt(1/2)*(2*d*f*e + f^2 + (8*d^5*e^3 +
12*d^4*f*e^2 + 6*d^3*f^2*e + d^2*f^3)*sqrt(-(2*d*e - f)/(8*d^7*e^3 + 12*d^6
*f*e^2 + 6*d^5*f^2*e + d^4*f^3)))*sqrt(((4*d^4*e^2 + 4*d^3*f*e + d^2*f^2)*s
qrt(-(2*d*e - f)/(8*d^7*e^3 + 12*d^6*f*e^2 + 6*d^5*f^2*e + d^4*f^3)) - f)/(
4*d^4*e^2 + 4*d^3*f*e + d^2*f^2)))*sqrt(sqrt(1/2)*sqrt(((4*d^4*e^2 + 4*d^3*
f*e + d^2*f^2)*sqrt(-(2*d*e - f)/(8*d^7*e^3 + 12*d^6*f*e^2 + 6*d^5*f^2*e +
d^4*f^3)) - f)/(4*d^4*e^2 + 4*d^3*f*e + d^2*f^2)))*sqrt(((4*d^4*e^2 + 4*d^3
*f*e + d^2*f^2)*sqrt(-(2*d*e - f)/(8*d^7*e^3 + 12*d^6*f*e^2 + 6*d^5*f^2*e +
 d^4*f^3)) - f)/(4*d^4*e^2 + 4*d^3*f*e + d^2*f^2)) - sqrt(1/2)*(4*d^2*x*e^3
 + 4*d*f*x*e^2 + f^2*x*e + (8*d^5*x*e^4 + 12*d^4*f*x*e^3 + 6*d^3*f^2*x*e^2
+ d^2*f^3*x*e)*sqrt(-(2*d*e - f)/(8*d^7*e^3 + 12*d^6*f*e^2 + 6*d^5*f^2*e +
d^4*f^3)))*sqrt(sqrt(1/2)*sqrt(((4*d^4*e^2 + 4*d^3*f*e + d^2*f^2)*sqrt(-(2*
d*e - f)/(8*d^7*e^3 + 12*d^6*f*e^2 + 6*d^5*f^2*e + d^4*f^3)) - f)/(4*d^4*e^
2 + 4*d^3*f*e + d^2*f^2)))*sqrt(((4*d^4*e^2 + 4*d^3*f*e + d^2*f^2)*sqrt(-(2
*d*e - f)/(8*d^7*e^3 + 12*d^6*f*e^2 + 6*d^5*f^2*e + d^4*f^3)) - f)/(4*d^4*e
^2 + 4*d^3*f*e + d^2*f^2)))*e^(-2)) + 1/4*sqrt(sqrt(1/2)*sqrt(-((4*d^4*e^2
+ 4*d^3*f*e + d^2*f^2)*sqrt(-(2*d*e - f)/(8*d^7*e^3 + 12*d^6*f*e^2 + 6*d^5*
f^2*e + d^4*f^3)) + f)/(4*d^4*e^2 + 4*d^3*f*e + d^2*f^2)))*log(x*e + 1/2*(2
*d*e - (4*d^4*e^2 + 4*d^3*f*e + d^2*f^2)*sqrt(-(2*d*e - f)/(8*d^7*e^3 + 12*
d^6*f*e^2 + 6*d^5*f^2*e + d^4*f^3)) + f)*sqrt(sqrt(1/2)*sqrt(-((4*d^4*e^2 +
 4*d^3*f*e + d^2*f^2)*sqrt(-(2*d*e - f)/(8*d^7*e^3 + 12*d^6*f*e^2 + 6*d^5*f
^2*e + d^4*f^3)) + f)/(4*d^4*e^2 + 4*d^3*f*e + d^2*f^2)))) - 1/4*sqrt(sqrt(
1/2)*sqrt(-((4*d^4*e^2 + 4*d^3*f*e + d^2*f^2)*sqrt(-(2*d*e - f)/(8*d^7*e^3
+ 12*d^6*f*e^2 + 6*d^5*f^2*e + d^4*f^3)) + f)/(4*d^4*e^2 + 4*d^3*f*e + d^2*
f^2)))*log(x*e - 1/2*(2*d*e - (4*d^4*e^2 + 4*d^3*f*e + d^2*f^2)*sqrt(-(2*d*
e - f)/(8*d^7*e^3 + 12*d^6*f*e^2 + 6*d^5*f^2*e + d^4*f^3)) + f)*sqrt(sqrt(1
/2)*sqrt(-((4*d^4*e^2 + 4*d^3*f*e + d^2*f^2)*sqrt(-(2*d*e - f)/(8*d^7*e^3 +
 12*d^6*f*e^2 + 6*d^5*f^2*e + d^4*f^3)) + f)/(4*d^4*e^2 + 4*d^3*f*e + d^2*f
^2)))) + 1/4*sqrt(sqrt(1/2)*sqrt(((4*d^4*e^2 + 4*d^3*f*e + d^2*f^2)*sqrt(-(
```

```
2*d*e - f)/(8*d^7*e^3 + 12*d^6*f*e^2 + 6*d^5*f^2*e + d^4*f^3)) - f)/(4*d^4*
e^2 + 4*d^3*f*e + d^2*f^2)))*log(x*e + 1/2*(2*d*e + (4*d^4*e^2 + 4*d^3*f*e
+ d^2*f^2)*sqrt(-(2*d*e - f)/(8*d^7*e^3 + 12*d^6*f*e^2 + 6*d^5*f^2*e + d^4*
f^3)) + f)*sqrt(sqrt(1/2)*sqrt(((4*d^4*e^2 + 4*d^3*f*e + d^2*f^2)*sqrt(-(2*
d*e - f)/(8*d^7*e^3 + 12*d^6*f*e^2 + 6*d^5*f^2*e + d^4*f^3)) - f)/(4*d^4*e^
2 + 4*d^3*f*e + d^2*f^2)))) - 1/4*sqrt(sqrt(1/2)*sqrt(((4*d^4*e^2 + 4*d^3*f
*e + d^2*f^2)*sqrt(-(2*d*e - f)/(8*d^7*e^3 + 12*d^6*f*e^2 + 6*d^5*f^2*e + d
^4*f^3)) - f)/(4*d^4*e^2 + 4*d^3*f*e + d^2*f^2)))*log(x*e - 1/2*(2*d*e + (4
*d^4*e^2 + 4*d^3*f*e + d^2*f^2)*sqrt(-(2*d*e - f)/(8*d^7*e^3 + 12*d^6*f*e^2
 + 6*d^5*f^2*e + d^4*f^3)) + f)*sqrt(sqrt(1/2)*sqrt(((4*d^4*e^2 + 4*d^3*f*e
 + d^2*f^2)*sqrt(-(2*d*e - f)/(8*d^7*e^3 + 12*d^6*f*e^2 + 6*d^5*f^2*e + d^4
*f^3)) - f)/(4*d^4*e^2 + 4*d^3*f*e + d^2*f^2))))
```

Sympy [A]
time = 6.61, size = 136, normalized size = 0.17

$$\mathrm{RootSum}\left(t^8\cdot\left(1048576d^6e^4+2097152d^5e^3f+1572864d^4e^2f^2+524288d^3ef^3+65536d^2f^4\right)+t^4\cdot\left(1024d^2e^2f+1024def^2+256f^3\right)+e^2,\left(t\mapsto t\log\left(x+\frac{4096t^5d^4e^2+4096t^5d^3ef+1024t^5d^2f^2+4tde+4tf}{e}\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**4+d)/(e**2*x**8+f*x**4+d**2),x)
```

```
[Out] RootSum(_t**8*(1048576*d**6*e**4 + 2097152*d**5*e**3*f + 1572864*d**4*e**2*
f**2 + 524288*d**3*e*f**3 + 65536*d**2*f**4) + _t**4*(1024*d**2*e**2*f + 10
24*d*e*f**2 + 256*f**3) + e**2, Lambda(_t, _t*log(x + (4096*_t**5*d**4*e**2
 + 4096*_t**5*d**3*e*f + 1024*_t**5*d**2*f**2 + 4*_t*d*e + 4*_t*f)/e)))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^4+d)/(e^2*x^8+f*x^4+d^2),x, algorithm="giac")
```

```
[Out] integrate((x^4*e + d)/(x^8*e^2 + f*x^4 + d^2), x)
```

Mupad [B]
time = 4.03, size = 2500, normalized size = 3.16

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x^4)/(f*x^4 + d^2 + e^2*x^8),x)
```

```
[Out] 2*atan((((-(f^3 + ((f - 2*d*e)*(f + 2*d*e)^5)^(1/2) + 4*d^2*e^2*f + 4*d*e*f
^2)/(512*(16*d^6*e^4 + d^2*f^4 + 8*d^3*e*f^3 + 32*d^5*e^3*f + 24*d^4*e^2*f^
```

2)))^(1/4)*((x*(65536*d^9*e^15 - 32768*d^8*e^14*f + 1024*d^2*e^8*f^7 - 2048
*d^3*e^9*f^6 - 10240*d^4*e^10*f^5 + 20480*d^5*e^11*f^4 + 32768*d^6*e^12*f^3
 - 65536*d^7*e^13*f^2) - (-(f^3 + ((f - 2*d*e)*(f + 2*d*e)^5)^(1/2) + 4*d^2
*e^2*f + 4*d*e*f^2)/(512*(16*d^6*e^4 + d^2*f^4 + 8*d^3*e*f^3 + 32*d^5*e^3*f
 + 24*d^4*e^2*f^2)))^(1/4)*(262144*d^10*e^15 - 262144*d^9*e^14*f + 4096*d^3
*e^8*f^7 - 4096*d^4*e^9*f^6 - 49152*d^5*e^10*f^5 + 49152*d^6*e^11*f^4 + 196
608*d^7*e^12*f^3 - 196608*d^8*e^13*f^2)*1i)*(-(f^3 + ((f - 2*d*e)*(f + 2*d*
e)^5)^(1/2) + 4*d^2*e^2*f + 4*d*e*f^2)/(512*(16*d^6*e^4 + d^2*f^4 + 8*d^3*e
*f^3 + 32*d^5*e^3*f + 24*d^4*e^2*f^2)))^(3/4)*1i - 256*d^7*e^14 + 256*d^6*e
^13*f + 16*d^3*e^10*f^4 - 64*d^4*e^11*f^3)*1i + x*(32*d^5*e^13*f - 4*d^2*e^
10*f^4 + 24*d^3*e^11*f^3 - 48*d^4*e^12*f^2))*(-(f^3 + ((f - 2*d*e)*(f + 2*d
*e)^5)^(1/2) + 4*d^2*e^2*f + 4*d*e*f^2)/(512*(16*d^6*e^4 + d^2*f^4 + 8*d^3*
e*f^3 + 32*d^5*e^3*f + 24*d^4*e^2*f^2)))^(1/4) + ((-(f^3 + ((f - 2*d*e)*(f
+ 2*d*e)^5)^(1/2) + 4*d^2*e^2*f + 4*d*e*f^2)/(512*(16*d^6*e^4 + d^2*f^4 + 8
*d^3*e*f^3 + 32*d^5*e^3*f + 24*d^4*e^2*f^2)))^(1/4)*((x*(65536*d^9*e^15 - 3
2768*d^8*e^14*f + 1024*d^2*e^8*f^7 - 2048*d^3*e^9*f^6 - 10240*d^4*e^10*f^5
+ 20480*d^5*e^11*f^4 + 32768*d^6*e^12*f^3 - 65536*d^7*e^13*f^2) + (-(f^3 +
((f - 2*d*e)*(f + 2*d*e)^5)^(1/2) + 4*d^2*e^2*f + 4*d*e*f^2)/(512*(16*d^6*e
^4 + d^2*f^4 + 8*d^3*e*f^3 + 32*d^5*e^3*f + 24*d^4*e^2*f^2)))^(1/4)*(262144
*d^10*e^15 - 262144*d^9*e^14*f + 4096*d^3*e^8*f^7 - 4096*d^4*e^9*f^6 - 4915
2*d^5*e^10*f^5 + 49152*d^6*e^11*f^4 + 196608*d^7*e^12*f^3 - 196608*d^8*e^13
*f^2)*1i)*(-(f^3 + ((f - 2*d*e)*(f + 2*d*e)^5)^(1/2) + 4*d^2*e^2*f + 4*d*e*
f^2)/(512*(16*d^6*e^4 + d^2*f^4 + 8*d^3*e*f^3 + 32*d^5*e^3*f + 24*d^4*e^2*f
^2)))^(3/4)*1i + 256*d^7*e^14 - 256*d^6*e^13*f - 16*d^3*e^10*f^4 + 64*d^4*e
^11*f^3)*1i + x*(32*d^5*e^13*f - 4*d^2*e^10*f^4 + 24*d^3*e^11*f^3 - 48*d^4*
e^12*f^2))*(-(f^3 + ((f - 2*d*e)*(f + 2*d*e)^5)^(1/2) + 4*d^2*e^2*f + 4*d*e
f^2)/(512(16*d^6*e^4 + d^2*f^4 + 8*d^3*e*f^3 + 32*d^5*e^3*f + 24*d^4*e^2*
f^2)))^(1/4))/(((-(f^3 + ((f - 2*d*e)*(f + 2*d*e)^5)^(1/2) + 4*d^2*e^2*f +
4*d*e*f^2)/(512*(16*d^6*e^4 + d^2*f^4 + 8*d^3*e*f^3 + 32*d^5*e^3*f + 24*d^4
*e^2*f^2)))^(1/4)*((x*(65536*d^9*e^15 - 32768*d^8*e^14*f + 1024*d^2*e^8*f^7
 - 2048*d^3*e^9*f^6 - 10240*d^4*e^10*f^5 + 20480*d^5*e^11*f^4 + 32768*d^6*e
^12*f^3 - 65536*d^7*e^13*f^2) - (-(f^3 + ((f - 2*d*e)*(f + 2*d*e)^5)^(1/2)
+ 4*d^2*e^2*f + 4*d*e*f^2)/(512*(16*d^6*e^4 + d^2*f^4 + 8*d^3*e*f^3 + 32*d^
5*e^3*f + 24*d^4*e^2*f^2)))^(1/4)*(262144*d^10*e^15 - 262144*d^9*e^14*f + 4
096*d^3*e^8*f^7 - 4096*d^4*e^9*f^6 - 49152*d^5*e^10*f^5 + 49152*d^6*e^11*f^
4 + 196608*d^7*e^12*f^3 - 196608*d^8*e^13*f^2)*1i)*(-(f^3 + ((f - 2*d*e)*(f
 + 2*d*e)^5)^(1/2) + 4*d^2*e^2*f + 4*d*e*f^2)/(512*(16*d^6*e^4 + d^2*f^4 +
8*d^3*e*f^3 + 32*d^5*e^3*f + 24*d^4*e^2*f^2)))^(3/4)*1i - 256*d^7*e^14 + 25
6*d^6*e^13*f + 16*d^3*e^10*f^4 - 64*d^4*e^11*f^3)*1i + x*(32*d^5*e^13*f - 4
*d^2*e^10*f^4 + 24*d^3*e^11*f^3 - 48*d^4*e^12*f^2))*(-(f^3 + ((f - 2*d*e)*(
f + 2*d*e)^5)^(1/2) + 4*d^2*e^2*f + 4*d*e*f^2)/(512*(16*d^6*e^4 + d^2*f^4 +
 8*d^3*e*f^3 + 32*d^5*e^3*f + 24*d^4*e^2*f^2)))^(1/4)*1i - ((-(f^3 + ((f -
2*d*e)*(f + 2*d*e)^5)^(1/2) + 4*d^2*e^2*f + 4*d*e*f^2)/(512*(16*d^6*e^4 + d
^2*f^4 + 8*d^3*e*f^3 + 32*d^5*e^3*f + 24*d^4*e^2*f^2)))^(1/4)*((x*(65536*d^
9*e^15 - 32768*d^8*e^14*f + 1024*d^2*e^8*f^7 - 2048*d^3*e^9*f^6 - 10240*d^4

```
*e^10*f^5 + 20480*d^5*e^11*f^4 + 32768*d^6*e^12*f^3 - 65536*d^7*e^13*f^2) +
 (-(f^3 + ((f - 2*d*e)*(f + 2*d*e)^5)^(1/2) + 4*d^2*e^2*f + 4*d*e*f^2)/(512
*(16*d^6*e^4 + d^2*f^4 + 8*d^3*e*f^3 + 32*d^5*e^3*f + 24*d^4*e^2*f^2)))^(1/
4)*(262144*d^10*e^15 - 262144*d^9*e^14*f + 4096*d^3*e^8*f^7 - 4096*d^4*e^9*
f^6 - 49152*d^5*e^10*f^5 + 49152*d^6*e^11*f^4 + 196608*d^7*e^12*f^3 - 19660
8*d^8*e^13*f^2)*1i)*(-(f^3 + ((f - 2*d*e)*(f + 2*d*e)^5)^(1/2) + 4*d^2*e^2*
f + 4*d*e*f^2)/(512*(16*d^6*e^4 + d^2*f^4 + 8*d^3*e*f^3 + 32*d^5*e^3*f + 24
*d^4*e^2*f^2)))^(3/4)*1i + 256*d^7*e^14 - 256*d^6*e^13*f - 16*d^3*e^10*f^4
+ 64*d^4*e^11*f^3)*1i + x*(32*d^5*e^13*f - 4*d^2*e^10*f^4 + 24*d^3*e^11*f^3
 - 48*d^4*e^12*f^2))*(-(f^3 + ((f - 2*d*e)*(f + 2*d*e)^5)^(1/2) + 4*d^2*e^2
*f + 4*d*e*f^2)/(512*(16*d^6*e^4 + d^2*f^4 + 8*d^3*e*f^3 + 32*d^5*e^3*f + 2
4*d^4*e^2*f^2)))^(1/4)*1i))*(-(f^3 + ((f - 2*d*e)*(f + 2*d*e)^5)^(1/2) + 4*
d^2*e^2*f + 4*d*e*f^2)/(512*(16*d^6*e^4 + d^2*f^4 + 8*d^3*e*f^3 + 32*d^5*e^
3*f + 24*d^4*e^2*f^2)))^(1/4) - atan((((-(f^3 + ((f - 2*d*e)*(f + 2*d*e)^5)
^(1/2) + 4*d^2*e^2*f + 4*d*e*f^2)/(512*(16*d^6*e^4 + d^2*f^4 + 8*d^3*e*f^3
+ 32*d^5*e^3*f + 24*d^4*e^2*f^2)))^(1/4)*((x*(65536*d^9*e^15 - 32768*d^8*e^
14*f + 1024*d^2*e^8*f^7 - 2048*d^3*e^9*f^6 - 10240*d^4*e^10*f^5 + 20480*d^5
*e^11*f^4 + 32768*d^6*e^12*f^3 - 65536*d^7*e^13*f^2) + (-(f^3 + ((f - 2*d*e
)*(f + 2*d*e)^5)^(1/2) + 4*d^2*e^2*f + 4*d*e*f^2)/(512*(16*d^6*e^4 + d^2*f^
4 + 8*d^3*e*f^3 + 32*d^5*e^3*f + 24*d^4*e^2*f^2...
```

3.7 $\int \frac{d+ex^4}{d^2-bx^4+e^2x^8}\,dx$

Optimal. Leaf size=349

$$-\frac{\sqrt{e}\tan^{-1}\left(\frac{\sqrt{2}\sqrt{e}\,x}{\sqrt{\sqrt{b-2de}-\sqrt{b+2de}}}\right)}{\sqrt{2}\sqrt{b-2de}\sqrt{\sqrt{b-2de}-\sqrt{b+2de}}}-\frac{\sqrt{e}\tan^{-1}\left(\frac{\sqrt{2}\sqrt{e}\,x}{\sqrt{\sqrt{b-2de}+\sqrt{b+2de}}}\right)}{\sqrt{2}\sqrt{b-2de}\sqrt{\sqrt{b-2de}+\sqrt{b+2de}}}-\frac{\sqrt{e}\tanh^{-1}\Bigg(\cdots}{\sqrt{2}\sqrt{b-2de}\,\cdots}$$

```
[Out] -1/2*arctan(x*2^(1/2)*e^(1/2)/((-2*d*e+b)^(1/2)-(2*d*e+b)^(1/2))^(1/2))*e^(
1/2)*2^(1/2)/(-2*d*e+b)^(1/2)/((-2*d*e+b)^(1/2)-(2*d*e+b)^(1/2))^(1/2)-1/2*
arctanh(x*2^(1/2)*e^(1/2)/((-2*d*e+b)^(1/2)-(2*d*e+b)^(1/2))^(1/2))*e^(1/2)
*2^(1/2)/(-2*d*e+b)^(1/2)/((-2*d*e+b)^(1/2)-(2*d*e+b)^(1/2))^(1/2)-1/2*arct
an(x*2^(1/2)*e^(1/2)/((-2*d*e+b)^(1/2)+(2*d*e+b)^(1/2))^(1/2))*e^(1/2)*2^(1
/2)/(-2*d*e+b)^(1/2)/((-2*d*e+b)^(1/2)+(2*d*e+b)^(1/2))^(1/2)-1/2*arctanh(x
*2^(1/2)*e^(1/2)/((-2*d*e+b)^(1/2)+(2*d*e+b)^(1/2))^(1/2))*e^(1/2)*2^(1/2)/
(-2*d*e+b)^(1/2)/((-2*d*e+b)^(1/2)+(2*d*e+b)^(1/2))^(1/2)
```

Rubi [A]
time = 0.29, antiderivative size = 349, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 4, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.148$, Rules used = {1433, 1107, 213, 209}

$$-\frac{\sqrt{e}\,\text{ArcTan}\left(\frac{\sqrt{2}\sqrt{e}\,x}{\sqrt{\sqrt{b-2de}-\sqrt{b+2de}}}\right)}{\sqrt{2}\sqrt{b-2de}\sqrt{\sqrt{b-2de}-\sqrt{b+2de}}}-\frac{\sqrt{e}\,\text{ArcTan}\left(\frac{\sqrt{2}\sqrt{e}\,x}{\sqrt{\sqrt{b-2de}+\sqrt{b+2de}}}\right)}{\sqrt{2}\sqrt{b-2de}\sqrt{\sqrt{b-2de}+\sqrt{b+2de}}}-\frac{\sqrt{e}\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{e}\,x}{\sqrt{\sqrt{b-2de}-\sqrt{b+2de}}}\right)}{\sqrt{2}\sqrt{b-2de}\sqrt{\sqrt{b-2de}-\sqrt{b+2de}}}-\frac{\sqrt{e}\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{e}\,x}{\sqrt{\sqrt{b-2de}+\sqrt{b+2de}}}\right)}{\sqrt{2}\sqrt{b-2de}\sqrt{\sqrt{b-2de}+\sqrt{b+2de}}}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x^4)/(d^2 - b*x^4 + e^2*x^8),x]
```

```
[Out] -((Sqrt[e]*ArcTan[(Sqrt[2]*Sqrt[e]*x)/Sqrt[Sqrt[b - 2*d*e] - Sqrt[b + 2*d*e
]]])/(Sqrt[2]*Sqrt[b - 2*d*e]*Sqrt[Sqrt[b - 2*d*e] - Sqrt[b + 2*d*e]])) - (
Sqrt[e]*ArcTan[(Sqrt[2]*Sqrt[e]*x)/Sqrt[Sqrt[b - 2*d*e] + Sqrt[b + 2*d*e]]]
)/(Sqrt[2]*Sqrt[b - 2*d*e]*Sqrt[Sqrt[b - 2*d*e] + Sqrt[b + 2*d*e]]) - (Sqrt
[e]*ArcTanh[(Sqrt[2]*Sqrt[e]*x)/Sqrt[Sqrt[b - 2*d*e] - Sqrt[b + 2*d*e]]])/(
Sqrt[2]*Sqrt[b - 2*d*e]*Sqrt[Sqrt[b - 2*d*e] - Sqrt[b + 2*d*e]]) - (Sqrt[e]
*ArcTanh[(Sqrt[2]*Sqrt[e]*x)/Sqrt[Sqrt[b - 2*d*e] + Sqrt[b + 2*d*e]]])/(Sqr
t[2]*Sqrt[b - 2*d*e]*Sqrt[Sqrt[b - 2*d*e] + Sqrt[b + 2*d*e]])
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 1107

```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(-1), x_Symbol] :> With[{q = Rt[b^
2 - 4*a*c, 2]}, Dist[c/q, Int[1/(b/2 - q/2 + c*x^2), x], x] - Dist[c/q, Int
[1/(b/2 + q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c,
0] && PosQ[b^2 - 4*a*c]
```

Rule 1433

```
Int[((d_) + (e_.)*(x_)^(n_))/((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_)), x
_Symbol] :> With[{q = Rt[2*(d/e) - b/c, 2]}, Dist[e/(2*c), Int[1/Simp[d/e +
 q*x^(n/2) + x^n, x], x], x] + Dist[e/(2*c), Int[1/Simp[d/e - q*x^(n/2) + x
^n, x], x], x]] /; FreeQ[{a, b, c, d, e}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4
*a*c, 0] && EqQ[c*d^2 - a*e^2, 0] && IGtQ[n/2, 0] && (GtQ[2*(d/e) - b/c, 0]
 || ( !LtQ[2*(d/e) - b/c, 0] && EqQ[d, e*Rt[a/c, 2]]))
```

Rubi steps

$$
\begin{aligned}
\int \frac{d+ex^4}{d^2-bx^4+e^2x^8}\,dx &= \frac{\int \frac{1}{\frac{d}{e}-\frac{\sqrt{b+2de}\,x^2}{e}+x^4}\,dx}{2e}+\frac{\int \frac{1}{\frac{d}{e}+\frac{\sqrt{b+2de}\,x^2}{e}+x^4}\,dx}{2e}\\
&= \frac{\int \frac{1}{-\frac{\sqrt{b-2de}}{2e}-\frac{\sqrt{b+2de}}{2e}+x^2}\,dx}{2\sqrt{b-2de}}-\frac{\int \frac{1}{\frac{\sqrt{b-2de}}{2e}-\frac{\sqrt{b+2de}}{2e}+x^2}\,dx}{2\sqrt{b-2de}}+\frac{\int \frac{1}{-\frac{\sqrt{b-2}}{2e}\cdots}}{2\cdots}\\
&= -\frac{\sqrt{e}\tan^{-1}\left(\frac{\sqrt{2}\sqrt{e}\,x}{\sqrt{\sqrt{b-2de}-\sqrt{b+2de}}}\right)}{\sqrt{2}\sqrt{b-2de}\sqrt{\sqrt{b-2de}-\sqrt{b+2de}}}-\frac{\sqrt{e}\tan^{-1}\left(\frac{\sqrt{2}\sqrt{e}\,x}{\sqrt{\sqrt{b-2de}+\sqrt{b+\cdots}}}\right)}{\sqrt{2}\sqrt{b-2de}\sqrt{\sqrt{b-2de}+\sqrt{b\cdots}}}
\end{aligned}
$$

Mathematica [C] Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.
time = 0.03, size = 69, normalized size = 0.20

$$\frac{1}{4}\text{RootSum}\left[d^2-b\#1^4+e^2\#1^8\&,\frac{d\log(x-\#1)+e\log(x-\#1)\#1^4}{-b\#1^3+2e^2\#1^7}\&\right]$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x^4)/(d^2 - b*x^4 + e^2*x^8),x]
```

```
[Out] RootSum[d^2 - b*#1^4 + e^2*#1^8 & , (d*Log[x - #1] + e*Log[x - #1]*#1^4)/(-
(b*#1^3) + 2*e^2*#1^7) & ]/4
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 0.05, size = 57, normalized size = 0.16

method	result	size
default	$\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(e^{2}\_Z^{8}-\_Z^{4}b+d^{2}\right)}\frac{\left(-\_R^{4}e-d\right)\ln\left(x-\_R\right)}{-2\_R^{7}e^{2}+\_R^{3}b}\right)}{4}$	57
risch	$\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(e^{2}\_Z^{8}-\_Z^{4}b+d^{2}\right)}\frac{\left(-\_R^{4}e-d\right)\ln\left(x-\_R\right)}{-2\_R^{7}e^{2}+\_R^{3}b}\right)}{4}$	57

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^4+d)/(e^2*x^8-b*x^4+d^2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/4*sum((-_R^4*e-d)/(-2*_R^7*e^2+_R^3*b)*ln(x-_R),_R=RootOf(_Z^8*e^2-_Z^4*b
+d^2))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^4+d)/(e^2*x^8-b*x^4+d^2),x, algorithm="maxima")
```

```
[Out] integrate((x^4*e + d)/(x^8*e^2 - b*x^4 + d^2), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 3059 vs. 2(273) = 546.
time = 0.43, size = 3059, normalized size = 8.77

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^4+d)/(e^2*x^8-b*x^4+d^2),x, algorithm="fricas")
```

```
[Out] -sqrt(sqrt(1/2)*sqrt(-((4*d^4*e^2 - 4*b*d^3*e + b^2*d^2)*sqrt(-(2*d*e + b)/
(8*d^7*e^3 - 12*b*d^6*e^2 + 6*b^2*d^5*e - b^3*d^4)) - b)/(4*d^4*e^2 - 4*b*d
^3*e + b^2*d^2)))*arctan(-1/2*(sqrt(1/2)*(4*d^2*e^2 - 4*b*d*e + b^2 - (8*d^
5*e^3 - 12*b*d^4*e^2 + 6*b^2*d^3*e - b^3*d^2)*sqrt(-(2*d*e + b)/(8*d^7*e^3
```

- 12*b*d^6*e^2 + 6*b^2*d^5*e - b^3*d^4)))*sqrt(x^2*e^2 - 1/2*sqrt(1/2)*(2*b
*d*e - b^2 + (8*d^5*e^3 - 12*b*d^4*e^2 + 6*b^2*d^3*e - b^3*d^2)*sqrt(-(2*d*
e + b)/(8*d^7*e^3 - 12*b*d^6*e^2 + 6*b^2*d^5*e - b^3*d^4)))*sqrt(-((4*d^4*e
^2 - 4*b*d^3*e + b^2*d^2)*sqrt(-(2*d*e + b)/(8*d^7*e^3 - 12*b*d^6*e^2 + 6*b
^2*d^5*e - b^3*d^4)) - b)/(4*d^4*e^2 - 4*b*d^3*e + b^2*d^2)))*sqrt(-((4*d^4
*e^2 - 4*b*d^3*e + b^2*d^2)*sqrt(-(2*d*e + b)/(8*d^7*e^3 - 12*b*d^6*e^2 + 6
*b^2*d^5*e - b^3*d^4)) - b)/(4*d^4*e^2 - 4*b*d^3*e + b^2*d^2)) - sqrt(1/2)*
(4*d^2*x*e^3 - 4*b*d*x*e^2 + b^2*x*e - (8*d^5*x*e^4 - 12*b*d^4*x*e^3 + 6*b^
2*d^3*x*e^2 - b^3*d^2*x*e)*sqrt(-(2*d*e + b)/(8*d^7*e^3 - 12*b*d^6*e^2 + 6*
b^2*d^5*e - b^3*d^4)))*sqrt(-((4*d^4*e^2 - 4*b*d^3*e + b^2*d^2)*sqrt(-(2*d*
e + b)/(8*d^7*e^3 - 12*b*d^6*e^2 + 6*b^2*d^5*e - b^3*d^4)) - b)/(4*d^4*e^2
- 4*b*d^3*e + b^2*d^2)))*sqrt(sqrt(1/2)*sqrt(-((4*d^4*e^2 - 4*b*d^3*e + b^2
*d^2)*sqrt(-(2*d*e + b)/(8*d^7*e^3 - 12*b*d^6*e^2 + 6*b^2*d^5*e - b^3*d^4))
 - b)/(4*d^4*e^2 - 4*b*d^3*e + b^2*d^2)))*e^(-2)) + sqrt(sqrt(1/2)*sqrt(((4
*d^4*e^2 - 4*b*d^3*e + b^2*d^2)*sqrt(-(2*d*e + b)/(8*d^7*e^3 - 12*b*d^6*e^2
 + 6*b^2*d^5*e - b^3*d^4)) + b)/(4*d^4*e^2 - 4*b*d^3*e + b^2*d^2)))*arctan(
1/2*(sqrt(1/2)*(4*d^2*e^2 - 4*b*d*e + b^2 + (8*d^5*e^3 - 12*b*d^4*e^2 + 6*b
^2*d^3*e - b^3*d^2)*sqrt(-(2*d*e + b)/(8*d^7*e^3 - 12*b*d^6*e^2 + 6*b^2*d^5
*e - b^3*d^4)))*sqrt(x^2*e^2 - 1/2*sqrt(1/2)*(2*b*d*e - b^2 - (8*d^5*e^3 -
12*b*d^4*e^2 + 6*b^2*d^3*e - b^3*d^2)*sqrt(-(2*d*e + b)/(8*d^7*e^3 - 12*b*d
^6*e^2 + 6*b^2*d^5*e - b^3*d^4)))*sqrt(((4*d^4*e^2 - 4*b*d^3*e + b^2*d^2)*s
qrt(-(2*d*e + b)/(8*d^7*e^3 - 12*b*d^6*e^2 + 6*b^2*d^5*e - b^3*d^4)) + b)/(
4*d^4*e^2 - 4*b*d^3*e + b^2*d^2)))*sqrt(sqrt(1/2)*sqrt(((4*d^4*e^2 - 4*b*d^
3*e + b^2*d^2)*sqrt(-(2*d*e + b)/(8*d^7*e^3 - 12*b*d^6*e^2 + 6*b^2*d^5*e -
b^3*d^4)) + b)/(4*d^4*e^2 - 4*b*d^3*e + b^2*d^2)))*sqrt(((4*d^4*e^2 - 4*b*d
^3*e + b^2*d^2)*sqrt(-(2*d*e + b)/(8*d^7*e^3 - 12*b*d^6*e^2 + 6*b^2*d^5*e -
 b^3*d^4)) + b)/(4*d^4*e^2 - 4*b*d^3*e + b^2*d^2)) - sqrt(1/2)*(4*d^2*x*e^3
 - 4*b*d*x*e^2 + b^2*x*e + (8*d^5*x*e^4 - 12*b*d^4*x*e^3 + 6*b^2*d^3*x*e^2
- b^3*d^2*x*e)*sqrt(-(2*d*e + b)/(8*d^7*e^3 - 12*b*d^6*e^2 + 6*b^2*d^5*e -
b^3*d^4)))*sqrt(sqrt(1/2)*sqrt(((4*d^4*e^2 - 4*b*d^3*e + b^2*d^2)*sqrt(-(2*
d*e + b)/(8*d^7*e^3 - 12*b*d^6*e^2 + 6*b^2*d^5*e - b^3*d^4)) + b)/(4*d^4*e^
2 - 4*b*d^3*e + b^2*d^2)))*sqrt(((4*d^4*e^2 - 4*b*d^3*e + b^2*d^2)*sqrt(-(2
*d*e + b)/(8*d^7*e^3 - 12*b*d^6*e^2 + 6*b^2*d^5*e - b^3*d^4)) + b)/(4*d^4*e
^2 - 4*b*d^3*e + b^2*d^2)))*e^(-2)) + 1/4*sqrt(sqrt(1/2)*sqrt(((4*d^4*e^2 -
 4*b*d^3*e + b^2*d^2)*sqrt(-(2*d*e + b)/(8*d^7*e^3 - 12*b*d^6*e^2 + 6*b^2*d
^5*e - b^3*d^4)) + b)/(4*d^4*e^2 - 4*b*d^3*e + b^2*d^2)))*log(x*e + 1/2*(2*
d*e + (4*d^4*e^2 - 4*b*d^3*e + b^2*d^2)*sqrt(-(2*d*e + b)/(8*d^7*e^3 - 12*b
*d^6*e^2 + 6*b^2*d^5*e - b^3*d^4)) - b)*sqrt(sqrt(1/2)*sqrt(((4*d^4*e^2 - 4
*b*d^3*e + b^2*d^2)*sqrt(-(2*d*e + b)/(8*d^7*e^3 - 12*b*d^6*e^2 + 6*b^2*d^5
*e - b^3*d^4)) + b)/(4*d^4*e^2 - 4*b*d^3*e + b^2*d^2)))) - 1/4*sqrt(sqrt(1/
2)*sqrt(((4*d^4*e^2 - 4*b*d^3*e + b^2*d^2)*sqrt(-(2*d*e + b)/(8*d^7*e^3 - 1
2*b*d^6*e^2 + 6*b^2*d^5*e - b^3*d^4)) + b)/(4*d^4*e^2 - 4*b*d^3*e + b^2*d^2
)))*log(x*e - 1/2*(2*d*e + (4*d^4*e^2 - 4*b*d^3*e + b^2*d^2)*sqrt(-(2*d*e +
 b)/(8*d^7*e^3 - 12*b*d^6*e^2 + 6*b^2*d^5*e - b^3*d^4)) - b)*sqrt(sqrt(1/2)
*sqrt(((4*d^4*e^2 - 4*b*d^3*e + b^2*d^2)*sqrt(-(2*d*e + b)/(8*d^7*e^3 - 12*

```
b*d^6*e^2 + 6*b^2*d^5*e - b^3*d^4)) + b)/(4*d^4*e^2 - 4*b*d^3*e + b^2*d^2))
)) + 1/4*sqrt(sqrt(1/2)*sqrt(-((4*d^4*e^2 - 4*b*d^3*e + b^2*d^2)*sqrt(-(2*d
*e + b)/(8*d^7*e^3 - 12*b*d^6*e^2 + 6*b^2*d^5*e - b^3*d^4)) - b)/(4*d^4*e^2
 - 4*b*d^3*e + b^2*d^2)))*log(x*e + 1/2*(2*d*e - (4*d^4*e^2 - 4*b*d^3*e + b
^2*d^2)*sqrt(-(2*d*e + b)/(8*d^7*e^3 - 12*b*d^6*e^2 + 6*b^2*d^5*e - b^3*d^4
)) - b)*sqrt(sqrt(1/2)*sqrt(-((4*d^4*e^2 - 4*b*d^3*e + b^2*d^2)*sqrt(-(2*d*
e + b)/(8*d^7*e^3 - 12*b*d^6*e^2 + 6*b^2*d^5*e - b^3*d^4)) - b)/(4*d^4*e^2
- 4*b*d^3*e + b^2*d^2)))) - 1/4*sqrt(sqrt(1/2)*sqrt(-((4*d^4*e^2 - 4*b*d^3*
e + b^2*d^2)*sqrt(-(2*d*e + b)/(8*d^7*e^3 - 12*b*d^6*e^2 + 6*b^2*d^5*e - b^
3*d^4)) - b)/(4*d^4*e^2 - 4*b*d^3*e + b^2*d^2)))*log(x*e - 1/2*(2*d*e - (4*
d^4*e^2 - 4*b*d^3*e + b^2*d^2)*sqrt(-(2*d*e + b)/(8*d^7*e^3 - 12*b*d^6*e^2
+ 6*b^2*d^5*e - b^3*d^4)) - b)*sqrt(sqrt(1/2)*sqrt(-((4*d^4*e^2 - 4*b*d^3*e
 + b^2*d^2)*sqrt(-(2*d*e + b)/(8*d^7*e^3 - 12*b*d^6*e^2 + 6*b^2*d^5*e - b^3
*d^4)) - b)/(4*d^4*e^2 - 4*b*d^3*e + b^2*d^2))))
```

Sympy [A]
time = 19.02, size = 136, normalized size = 0.39

$$\mathrm{RootSum}\left(t^8\cdot\left(65536b^4d^2-524288b^3d^3e+1572864b^2d^4e^2-2097152bd^5e^3+1048576d^6e^4\right)+t^4\left(-256b^3+1024b^2de-1024bd^2e^2\right)+e^2,\left(t\mapsto t\log\left(x+\frac{1024t^5b^2d^2-4096t^5bd^3e+4096t^5d^4e^2-4tb+4tde}{e}\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**4+d)/(e**2*x**8-b*x**4+d**2),x)
```

```
[Out] RootSum(_t**8*(65536*b**4*d**2 - 524288*b**3*d**3*e + 1572864*b**2*d**4*e**
2 - 2097152*b*d**5*e**3 + 1048576*d**6*e**4) + _t**4*(-256*b**3 + 1024*b**2
*d*e - 1024*b*d**2*e**2) + e**2, Lambda(_t, _t*log(x + (1024*_t**5*b**2*d**
2 - 4096*_t**5*b*d**3*e + 4096*_t**5*d**4*e**2 - 4*_t*b + 4*_t*d*e)/e)))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^4+d)/(e^2*x^8-b*x^4+d^2),x, algorithm="giac")
```

```
[Out] integrate((x^4*e + d)/(x^8*e^2 - b*x^4 + d^2), x)
```

Mupad [B]
time = 4.03, size = 2500, normalized size = 7.16

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x^4)/(d^2 - b*x^4 + e^2*x^8),x)
```

[Out] 2*atan(((x*(32*b*d^5*e^13 + 4*b^4*d^2*e^10 + 24*b^3*d^3*e^11 + 48*b^2*d^4*e
^12) - ((b^3 + ((b - 2*d*e)^5*(b + 2*d*e))^(1/2) + 4*b*d^2*e^2 - 4*b^2*d*e)
/(512*(b^4*d^2 + 16*d^6*e^4 - 8*b^3*d^3*e - 32*b*d^5*e^3 + 24*b^2*d^4*e^2))
)^(1/4)*((x*(65536*d^9*e^15 + 32768*b*d^8*e^14 - 1024*b^7*d^2*e^8 - 2048*b^
6*d^3*e^9 + 10240*b^5*d^4*e^10 + 20480*b^4*d^5*e^11 - 32768*b^3*d^6*e^12 -
65536*b^2*d^7*e^13) - ((b^3 + ((b - 2*d*e)^5*(b + 2*d*e))^(1/2) + 4*b*d^2*e
^2 - 4*b^2*d*e)/(512*(b^4*d^2 + 16*d^6*e^4 - 8*b^3*d^3*e - 32*b*d^5*e^3 + 2
4*b^2*d^4*e^2)))^(1/4)*(262144*d^10*e^15 + 262144*b*d^9*e^14 - 4096*b^7*d^3
*e^8 - 4096*b^6*d^4*e^9 + 49152*b^5*d^5*e^10 + 49152*b^4*d^6*e^11 - 196608*
b^3*d^7*e^12 - 196608*b^2*d^8*e^13)*1i)*((b^3 + ((b - 2*d*e)^5*(b + 2*d*e))
^(1/2) + 4*b*d^2*e^2 - 4*b^2*d*e)/(512*(b^4*d^2 + 16*d^6*e^4 - 8*b^3*d^3*e
- 32*b*d^5*e^3 + 24*b^2*d^4*e^2)))^(3/4)*1i - 256*d^7*e^14 - 256*b*d^6*e^13
+ 16*b^4*d^3*e^10 + 64*b^3*d^4*e^11)*1i)*((b^3 + ((b - 2*d*e)^5*(b + 2*d*e
))^(1/2) + 4*b*d^2*e^2 - 4*b^2*d*e)/(512*(b^4*d^2 + 16*d^6*e^4 - 8*b^3*d^3*
e - 32*b*d^5*e^3 + 24*b^2*d^4*e^2)))^(1/4) + (x*(32*b*d^5*e^13 + 4*b^4*d^2*
e^10 + 24*b^3*d^3*e^11 + 48*b^2*d^4*e^12) - ((b^3 + ((b - 2*d*e)^5*(b + 2*d
*e))^(1/2) + 4*b*d^2*e^2 - 4*b^2*d*e)/(512*(b^4*d^2 + 16*d^6*e^4 - 8*b^3*d^
3*e - 32*b*d^5*e^3 + 24*b^2*d^4*e^2)))^(1/4)*((x*(65536*d^9*e^15 + 32768*b*
d^8*e^14 - 1024*b^7*d^2*e^8 - 2048*b^6*d^3*e^9 + 10240*b^5*d^4*e^10 + 20480
*b^4*d^5*e^11 - 32768*b^3*d^6*e^12 - 65536*b^2*d^7*e^13) + ((b^3 + ((b - 2*
d*e)^5*(b + 2*d*e))^(1/2) + 4*b*d^2*e^2 - 4*b^2*d*e)/(512*(b^4*d^2 + 16*d^6
*e^4 - 8*b^3*d^3*e - 32*b*d^5*e^3 + 24*b^2*d^4*e^2)))^(1/4)*(262144*d^10*e^
15 + 262144*b*d^9*e^14 - 4096*b^7*d^3*e^8 - 4096*b^6*d^4*e^9 + 49152*b^5*d^
5*e^10 + 49152*b^4*d^6*e^11 - 196608*b^3*d^7*e^12 - 196608*b^2*d^8*e^13)*1i
)*((b^3 + ((b - 2*d*e)^5*(b + 2*d*e))^(1/2) + 4*b*d^2*e^2 - 4*b^2*d*e)/(512
*(b^4*d^2 + 16*d^6*e^4 - 8*b^3*d^3*e - 32*b*d^5*e^3 + 24*b^2*d^4*e^2)))^(3/
4)*1i + 256*d^7*e^14 + 256*b*d^6*e^13 - 16*b^4*d^3*e^10 - 64*b^3*d^4*e^11)*
1i)*((b^3 + ((b - 2*d*e)^5*(b + 2*d*e))^(1/2) + 4*b*d^2*e^2 - 4*b^2*d*e)/(5
12*(b^4*d^2 + 16*d^6*e^4 - 8*b^3*d^3*e - 32*b*d^5*e^3 + 24*b^2*d^4*e^2)))^(
1/4))/((x*(32*b*d^5*e^13 + 4*b^4*d^2*e^10 + 24*b^3*d^3*e^11 + 48*b^2*d^4*e^
12) - ((b^3 + ((b - 2*d*e)^5*(b + 2*d*e))^(1/2) + 4*b*d^2*e^2 - 4*b^2*d*e)/
(512*(b^4*d^2 + 16*d^6*e^4 - 8*b^3*d^3*e - 32*b*d^5*e^3 + 24*b^2*d^4*e^2)))
^(1/4)*((x*(65536*d^9*e^15 + 32768*b*d^8*e^14 - 1024*b^7*d^2*e^8 - 2048*b^6
*d^3*e^9 + 10240*b^5*d^4*e^10 + 20480*b^4*d^5*e^11 - 32768*b^3*d^6*e^12 - 6
5536*b^2*d^7*e^13) - ((b^3 + ((b - 2*d*e)^5*(b + 2*d*e))^(1/2) + 4*b*d^2*e^
2 - 4*b^2*d*e)/(512*(b^4*d^2 + 16*d^6*e^4 - 8*b^3*d^3*e - 32*b*d^5*e^3 + 24
*b^2*d^4*e^2)))^(1/4)*(262144*d^10*e^15 + 262144*b*d^9*e^14 - 4096*b^7*d^3*
e^8 - 4096*b^6*d^4*e^9 + 49152*b^5*d^5*e^10 + 49152*b^4*d^6*e^11 - 196608*b
^3*d^7*e^12 - 196608*b^2*d^8*e^13)*1i)*((b^3 + ((b - 2*d*e)^5*(b + 2*d*e))^
(1/2) + 4*b*d^2*e^2 - 4*b^2*d*e)/(512*(b^4*d^2 + 16*d^6*e^4 - 8*b^3*d^3*e -
32*b*d^5*e^3 + 24*b^2*d^4*e^2)))^(3/4)*1i - 256*d^7*e^14 - 256*b*d^6*e^13
+ 16*b^4*d^3*e^10 + 64*b^3*d^4*e^11)*1i)*((b^3 + ((b - 2*d*e)^5*(b + 2*d*e)
)^(1/2) + 4*b*d^2*e^2 - 4*b^2*d*e)/(512*(b^4*d^2 + 16*d^6*e^4 - 8*b^3*d^3*e
- 32*b*d^5*e^3 + 24*b^2*d^4*e^2)))^(1/4)*1i - (x*(32*b*d^5*e^13 + 4*b^4*d^
2*e^10 + 24*b^3*d^3*e^11 + 48*b^2*d^4*e^12) - ((b^3 + ((b - 2*d*e)^5*(b + 2

```
*d*e))^(1/2) + 4*b*d^2*e^2 - 4*b^2*d*e)/(512*(b^4*d^2 + 16*d^6*e^4 - 8*b^3*
d^3*e - 32*b*d^5*e^3 + 24*b^2*d^4*e^2)))^(1/4)*((x*(65536*d^9*e^15 + 32768*
b*d^8*e^14 - 1024*b^7*d^2*e^8 - 2048*b^6*d^3*e^9 + 10240*b^5*d^4*e^10 + 204
80*b^4*d^5*e^11 - 32768*b^3*d^6*e^12 - 65536*b^2*d^7*e^13) + ((b^3 + ((b -
2*d*e)^5*(b + 2*d*e))^(1/2) + 4*b*d^2*e^2 - 4*b^2*d*e)/(512*(b^4*d^2 + 16*d
^6*e^4 - 8*b^3*d^3*e - 32*b*d^5*e^3 + 24*b^2*d^4*e^2)))^(1/4)*(262144*d^10*
e^15 + 262144*b*d^9*e^14 - 4096*b^7*d^3*e^8 - 4096*b^6*d^4*e^9 + 49152*b^5*
d^5*e^10 + 49152*b^4*d^6*e^11 - 196608*b^3*d^7*e^12 - 196608*b^2*d^8*e^13)*
1i)*((b^3 + ((b - 2*d*e)^5*(b + 2*d*e))^(1/2) + 4*b*d^2*e^2 - 4*b^2*d*e)/(5
12*(b^4*d^2 + 16*d^6*e^4 - 8*b^3*d^3*e - 32*b*d^5*e^3 + 24*b^2*d^4*e^2)))^(
3/4)*1i + 256*d^7*e^14 + 256*b*d^6*e^13 - 16*b^4*d^3*e^10 - 64*b^3*d^4*e^11
)*1i)*((b^3 + ((b - 2*d*e)^5*(b + 2*d*e))^(1/2) + 4*b*d^2*e^2 - 4*b^2*d*e)/
(512*(b^4*d^2 + 16*d^6*e^4 - 8*b^3*d^3*e - 32*b*d^5*e^3 + 24*b^2*d^4*e^2)))
^(1/4)*1i))*((b^3 + ((b - 2*d*e)^5*(b + 2*d*e))^(1/2) + 4*b*d^2*e^2 - 4*b^2
*d*e)/(512*(b^4*d^2 + 16*d^6*e^4 - 8*b^3*d^3*e - 32*b*d^5*e^3 + 24*b^2*d^4*
e^2)))^(1/4) - atan(((x*(32*b*d^5*e^13 + 4*b^4*d^2*e^10 + 24*b^3*d^3*e^11 +
 48*b^2*d^4*e^12) + ((b^3 + ((b - 2*d*e)^5*(b + 2*d*e))^(1/2) + 4*b*d^2*e^2
 - 4*b^2*d*e)/(512*(b^4*d^2 + 16*d^6*e^4 - 8*b^3*d^3*e - 32*b*d^5*e^3 + 24*
b^2*d^4*e^2)))^(1/4)*((x*(65536*d^9*e^15 + 32768*b*d^8*e^14 - 1024*b^7*d^2*
e^8 - 2048*b^6*d^3*e^9 + 10240*b^5*d^4*e^10 + 20480*b^4*d^5*e^11 - 32768*b^
3*d^6*e^12 - 65536*b^2*d^7*e^13) + ((b^3 + ((b - 2*d*e)^5*(b + 2*d*e))^(1/2
) + 4*b*d^2*e^2 - 4*b^2*d*e)/(512*(b^4*d^2 + 16...
```

3.8 $\int \frac{d+ex^4}{d^2-fx^4+e^2x^8} \, dx$

Optimal. Leaf size=751

$$-\frac{\tan^{-1}\left(\frac{\sqrt{2\sqrt{d}\sqrt{e}-\sqrt{2de+f}}-2\sqrt{e}\,x}{\sqrt{2\sqrt{d}\sqrt{e}+\sqrt{2de+f}}}\right)}{4\sqrt{d}\sqrt{2\sqrt{d}\sqrt{e}+\sqrt{2de+f}}}-\frac{\tan^{-1}\left(\frac{\sqrt{2\sqrt{d}\sqrt{e}+\sqrt{2de+f}}-2\sqrt{e}\,x}{\sqrt{2\sqrt{d}\sqrt{e}-\sqrt{2de+f}}}\right)}{4\sqrt{d}\sqrt{2\sqrt{d}\sqrt{e}-\sqrt{2de+f}}}+\frac{\tan^{-1}\left(\sqrt{\ldots}\right.}{4\sqrt{d}\ldots}$$

```
[Out] -1/4*arctan((-2*x*e^(1/2)+(2*d^(1/2)*e^(1/2)+(2*d*e+f)^(1/2))^(1/2))/(2*d^(
1/2)*e^(1/2)-(2*d*e+f)^(1/2))^(1/2))/d^(1/2)/(2*d^(1/2)*e^(1/2)-(2*d*e+f)^(
1/2))^(1/2)+1/4*arctan((2*x*e^(1/2)+(2*d^(1/2)*e^(1/2)+(2*d*e+f)^(1/2))^(1/
2))/(2*d^(1/2)*e^(1/2)-(2*d*e+f)^(1/2))^(1/2))/d^(1/2)/(2*d^(1/2)*e^(1/2)-(
2*d*e+f)^(1/2))^(1/2)-1/8*ln(d^(1/2)+x^2*e^(1/2)-x*(2*d^(1/2)*e^(1/2)-(2*d*
e+f)^(1/2))^(1/2))/d^(1/2)/(2*d^(1/2)*e^(1/2)-(2*d*e+f)^(1/2))^(1/2)+1/8*ln
(d^(1/2)+x^2*e^(1/2)+x*(2*d^(1/2)*e^(1/2)-(2*d*e+f)^(1/2))^(1/2))/d^(1/2)/(
2*d^(1/2)*e^(1/2)-(2*d*e+f)^(1/2))^(1/2)-1/4*arctan((-2*x*e^(1/2)+(2*d^(1/2
)*e^(1/2)-(2*d*e+f)^(1/2))^(1/2))/(2*d^(1/2)*e^(1/2)+(2*d*e+f)^(1/2))^(1/2)
)/d^(1/2)/(2*d^(1/2)*e^(1/2)+(2*d*e+f)^(1/2))^(1/2)+1/4*arctan((2*x*e^(1/2)
+(2*d^(1/2)*e^(1/2)-(2*d*e+f)^(1/2))^(1/2))/(2*d^(1/2)*e^(1/2)+(2*d*e+f)^(1
/2))^(1/2))/d^(1/2)/(2*d^(1/2)*e^(1/2)+(2*d*e+f)^(1/2))^(1/2)-1/8*ln(d^(1/2
)+x^2*e^(1/2)-x*(2*d^(1/2)*e^(1/2)+(2*d*e+f)^(1/2))^(1/2))/d^(1/2)/(2*d^(1/
2)*e^(1/2)+(2*d*e+f)^(1/2))^(1/2)+1/8*ln(d^(1/2)+x^2*e^(1/2)+x*(2*d^(1/2)*e
^(1/2)+(2*d*e+f)^(1/2))^(1/2))/d^(1/2)/(2*d^(1/2)*e^(1/2)+(2*d*e+f)^(1/2))^
(1/2)
```

Rubi [A]

time = 0.56, antiderivative size = 751, normalized size of antiderivative = 1.00, number of steps used = 19, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {1433, 1108, 648, 632, 210, 642}

$$-\frac{\text{ArcTan}\left(\frac{\sqrt{2\sqrt{d}\sqrt{e}-\sqrt{2de+f}}-2\sqrt{e}\,x}{\sqrt{\sqrt{2de+f}+2\sqrt{d}\sqrt{e}}}\right)}{4\sqrt{d}\sqrt{\sqrt{2de+f}+2\sqrt{d}\sqrt{e}}}-\frac{\text{ArcTan}\left(\frac{\sqrt{\sqrt{2de+f}+2\sqrt{d}\sqrt{e}}-2\sqrt{e}\,x}{\sqrt{2\sqrt{d}\sqrt{e}-\sqrt{2de+f}}}\right)}{4\sqrt{d}\sqrt{2\sqrt{d}\sqrt{e}-\sqrt{2de+f}}}+\frac{\text{ArcTan}\left(\frac{\sqrt{2\sqrt{d}\sqrt{e}-\sqrt{2de+f}}+2\sqrt{e}\,x}{\sqrt{\sqrt{2de+f}+2\sqrt{d}\sqrt{e}}}\right)}{4\sqrt{d}\sqrt{\sqrt{2de+f}+2\sqrt{d}\sqrt{e}}}+\frac{\text{ArcTan}\left(\frac{\sqrt{\sqrt{2de+f}+2\sqrt{d}\sqrt{e}}+2\sqrt{e}\,x}{\sqrt{2\sqrt{d}\sqrt{e}-\sqrt{2de+f}}}\right)}{4\sqrt{d}\sqrt{2\sqrt{d}\sqrt{e}-\sqrt{2de+f}}}-\frac{\log\left(-x\sqrt{2\sqrt{d}\sqrt{e}-\sqrt{2de+f}}+\sqrt{d}+\sqrt{e}\,x^2\right)}{8\sqrt{d}\sqrt{2\sqrt{d}\sqrt{e}-\sqrt{2de+f}}}+\frac{\log\left(x\sqrt{2\sqrt{d}\sqrt{e}-\sqrt{2de+f}}+\sqrt{d}+\sqrt{e}\,x^2\right)}{8\sqrt{d}\sqrt{2\sqrt{d}\sqrt{e}-\sqrt{2de+f}}}-\frac{\log\left(-x\sqrt{\sqrt{2de+f}+2\sqrt{d}\sqrt{e}}+\sqrt{d}+\sqrt{e}\,x^2\right)}{8\sqrt{d}\sqrt{\sqrt{2de+f}+2\sqrt{d}\sqrt{e}}}+\frac{\log\left(x\sqrt{\sqrt{2de+f}+2\sqrt{d}\sqrt{e}}+\sqrt{d}+\sqrt{e}\,x^2\right)}{8\sqrt{d}\sqrt{\sqrt{2de+f}+2\sqrt{d}\sqrt{e}}}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x^4)/(d^2 - f*x^4 + e^2*x^8),x]
```

```
[Out] -1/4*ArcTan[(Sqrt[2*Sqrt[d]*Sqrt[e] - Sqrt[2*d*e + f]] - 2*Sqrt[e]*x)/Sqrt[
2*Sqrt[d]*Sqrt[e] + Sqrt[2*d*e + f]]]/(Sqrt[d]*Sqrt[2*Sqrt[d]*Sqrt[e] + Sqr
t[2*d*e + f]]) - ArcTan[(Sqrt[2*Sqrt[d]*Sqrt[e] + Sqrt[2*d*e + f]] - 2*Sqrt
[e]*x)/Sqrt[2*Sqrt[d]*Sqrt[e] - Sqrt[2*d*e + f]]]/(4*Sqrt[d]*Sqrt[2*Sqrt[d]
*Sqrt[e] - Sqrt[2*d*e + f]]) + ArcTan[(Sqrt[2*Sqrt[d]*Sqrt[e] - Sqrt[2*d*e
+ f]] + 2*Sqrt[e]*x)/Sqrt[2*Sqrt[d]*Sqrt[e] + Sqrt[2*d*e + f]]]/(4*Sqrt[d]*
Sqrt[2*Sqrt[d]*Sqrt[e] + Sqrt[2*d*e + f]]) + ArcTan[(Sqrt[2*Sqrt[d]*Sqrt[e]
 + Sqrt[2*d*e + f]] + 2*Sqrt[e]*x)/Sqrt[2*Sqrt[d]*Sqrt[e] - Sqrt[2*d*e + f]
```

```
]]/(4*Sqrt[d]*Sqrt[2*Sqrt[d]*Sqrt[e] - Sqrt[2*d*e + f]]) - Log[Sqrt[d] - Sq
rt[2*Sqrt[d]*Sqrt[e] - Sqrt[2*d*e + f]]*x + Sqrt[e]*x^2]/(8*Sqrt[d]*Sqrt[2*
Sqrt[d]*Sqrt[e] - Sqrt[2*d*e + f]]) + Log[Sqrt[d] + Sqrt[2*Sqrt[d]*Sqrt[e]
- Sqrt[2*d*e + f]]*x + Sqrt[e]*x^2]/(8*Sqrt[d]*Sqrt[2*Sqrt[d]*Sqrt[e] - Sqr
t[2*d*e + f]]) - Log[Sqrt[d] - Sqrt[2*Sqrt[d]*Sqrt[e] + Sqrt[2*d*e + f]]*x
+ Sqrt[e]*x^2]/(8*Sqrt[d]*Sqrt[2*Sqrt[d]*Sqrt[e] + Sqrt[2*d*e + f]]) + Log[
Sqrt[d] + Sqrt[2*Sqrt[d]*Sqrt[e] + Sqrt[2*d*e + f]]*x + Sqrt[e]*x^2]/(8*Sqr
t[d]*Sqrt[2*Sqrt[d]*Sqrt[e] + Sqrt[2*d*e + f]])
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1108

```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(-1), x_Symbol] :> With[{q = Rt[a/
c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Dist[1/(2*c*q*r), Int[(r - x)/(q - r*x
 + x^2), x], x] + Dist[1/(2*c*q*r), Int[(r + x)/(q + r*x + x^2), x], x]]] /
; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && NegQ[b^2 - 4*a*c]
```

Rule 1433

```
Int[((d_) + (e_.)*(x_)^(n_))/((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_)), x
_Symbol] :> With[{q = Rt[2*(d/e) - b/c, 2]}, Dist[e/(2*c), Int[1/Simp[d/e +
 q*x^(n/2) + x^n, x], x], x] + Dist[e/(2*c), Int[1/Simp[d/e - q*x^(n/2) + x
^n, x], x], x]] /; FreeQ[{a, b, c, d, e}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4
```

```
*a*c, 0] && EqQ[c*d^2 - a*e^2, 0] && IGtQ[n/2, 0] && (GtQ[2*(d/e) - b/c, 0]
 || ( !LtQ[2*(d/e) - b/c, 0] && EqQ[d, e*Rt[a/c, 2]]))
```

Rubi steps

$$\int \frac{d+ex^4}{d^2-fx^4+e^2x^8}\,dx = \frac{\int \frac{1}{\frac{d}{e}-\frac{\sqrt{2de+f}\,x^2}{e}+x^4}\,dx}{2e} + \frac{\int \frac{1}{\frac{d}{e}+\frac{\sqrt{2de+f}\,x^2}{e}+x^4}\,dx}{2e}$$

$$= \frac{\int \frac{\frac{\sqrt{2\sqrt{d}\,\sqrt{e}-\sqrt{2de+f}}}{\sqrt{e}}-x}{\frac{\sqrt{d}}{\sqrt{e}}-\frac{\sqrt{2\sqrt{d}\,\sqrt{e}-\sqrt{2de+f}}\,x}{\sqrt{e}}+x^2}\,dx}{4\sqrt{d}\,\sqrt{2\sqrt{d}\,\sqrt{e}-\sqrt{2de+f}}} + \frac{\int \frac{\frac{\sqrt{2\sqrt{d}\,\sqrt{e}-\sqrt{2de+f}}}{\sqrt{e}}+x \cdots}{\frac{\sqrt{d}}{\sqrt{e}}+\frac{\sqrt{2\sqrt{d}\,\sqrt{e}-\sqrt{2de+f}}\,\cdots}{\sqrt{e}}\cdots}}{4\sqrt{d}\,\sqrt{2\sqrt{d}\,\sqrt{e}-\sqrt{2de+\cdots}}}$$

$$= \frac{\int \frac{1}{\frac{\sqrt{d}}{\sqrt{e}}-\frac{\sqrt{2\sqrt{d}\,\sqrt{e}-\sqrt{2de+f}}\,x}{\sqrt{e}}+x^2}\,dx}{8\sqrt{d}\,\sqrt{e}} + \frac{\int \frac{1}{\frac{\sqrt{d}}{\sqrt{e}}+\frac{\sqrt{2\sqrt{d}\,\sqrt{e}-\sqrt{2de+f}}\,\cdots}{\sqrt{e}}\cdots}}{8\sqrt{d}\,\sqrt{e}}$$

$$= -\frac{\log\left(\sqrt{d}-\sqrt{2\sqrt{d}\,\sqrt{e}-\sqrt{2de+f}}\,x+\sqrt{e}\,x^2\right)}{8\sqrt{d}\,\sqrt{2\sqrt{d}\,\sqrt{e}-\sqrt{2de+f}}} + \frac{\log\left(\sqrt{d}+\sqrt{2\sqrt{d}\,\sqrt{e}\cdots}\right.}{8\sqrt{d}\,\sqrt{2\sqrt{d}\,\cdots}}$$

$$= -\frac{\tan^{-1}\left(\frac{\sqrt{2\sqrt{d}\,\sqrt{e}-\sqrt{2de+f}}-2\sqrt{e}\,x}{\sqrt{2\sqrt{d}\,\sqrt{e}+\sqrt{2de+f}}}\right)}{4\sqrt{d}\,\sqrt{2\sqrt{d}\,\sqrt{e}+\sqrt{2de+f}}} - \frac{\tan^{-1}\left(\frac{\sqrt{2\sqrt{d}\,\sqrt{e}+\sqrt{2de+\cdots}}}{\sqrt{2\sqrt{d}\,\sqrt{e}-\sqrt{2\cdots}}}\right.}{4\sqrt{d}\,\sqrt{2\sqrt{d}\,\sqrt{e}-\sqrt{2d\cdots}}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.
time = 0.03, size = 69, normalized size = 0.09

$$\frac{1}{4}\text{RootSum}\left[d^2-f\#1^4+e^2\#1^8\&,\frac{d\log(x-\#1)+e\log(x-\#1)\#1^4}{-f\#1^3+2e^2\#1^7}\&\right]$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x^4)/(d^2 - f*x^4 + e^2*x^8),x]
```

[Out] RootSum[d^2 - f*#1^4 + e^2*#1^8 & , (d*Log[x - #1] + e*Log[x - #1]*#1^4)/(-(f*#1^3) + 2*e^2*#1^7) &]/4

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 0.07, size = 55, normalized size = 0.07

method	result	size
default	$\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(e^{2}\_Z^{8}-f\_Z^{4}+d^{2}\right)}\frac{\left(\_R^{4}e+d\right)\ln\left(x-\_R\right)}{2\_R^{7}e^{2}-\_R^{3}f}\right)}{4}$	55
risch	$\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(e^{2}\_Z^{8}-f\_Z^{4}+d^{2}\right)}\frac{\left(\_R^{4}e+d\right)\ln\left(x-\_R\right)}{2\_R^{7}e^{2}-\_R^{3}f}\right)}{4}$	55

Verification of antiderivative is not currently implemented for this CAS.

[In] int((e*x^4+d)/(e^2*x^8-f*x^4+d^2),x,method=_RETURNVERBOSE)

[Out] 1/4*sum((_R^4*e+d)/(2*_R^7*e^2-_R^3*f)*ln(x-_R),_R=RootOf(_Z^8*e^2-_Z^4*f+d^2))

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x^4+d)/(e^2*x^8-f*x^4+d^2),x, algorithm="maxima")

[Out] integrate((x^4*e + d)/(x^8*e^2 - f*x^4 + d^2), x)

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 3059 vs. 2(533) = 1066.
time = 0.41, size = 3059, normalized size = 4.07

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x^4+d)/(e^2*x^8-f*x^4+d^2),x, algorithm="fricas")

[Out] -sqrt(sqrt(1/2)*sqrt(-((4*d^4*e^2 - 4*d^3*f*e + d^2*f^2)*sqrt(-(2*d*e + f)/(8*d^7*e^3 - 12*d^6*f*e^2 + 6*d^5*f^2*e - d^4*f^3)) - f)/(4*d^4*e^2 - 4*d^3*f*e + d^2*f^2)))*arctan(-1/2*(sqrt(1/2)*(4*d^2*e^2 - 4*d*f*e + f^2 - (8*d^5*e^3 - 12*d^4*f*e^2 + 6*d^3*f^2*e - d^2*f^3)*sqrt(-(2*d*e + f)/(8*d^7*e^3 - 12*d^6*f*e^2 + 6*d^5*f^2*e - d^4*f^3)))*sqrt(x^2*e^2 - 1/2*sqrt(1/2)*(2*d*f*e - f^2 + (8*d^5*e^3 - 12*d^4*f*e^2 + 6*d^3*f^2*e - d^2*f^3)*sqrt(-(2*d*

```
e + f)/(8*d^7*e^3 - 12*d^6*f*e^2 + 6*d^5*f^2*e - d^4*f^3)))*sqrt(-((4*d^4*e
^2 - 4*d^3*f*e + d^2*f^2)*sqrt(-(2*d*e + f)/(8*d^7*e^3 - 12*d^6*f*e^2 + 6*d
^5*f^2*e - d^4*f^3)) - f)/(4*d^4*e^2 - 4*d^3*f*e + d^2*f^2)))*sqrt(-((4*d^4
*e^2 - 4*d^3*f*e + d^2*f^2)*sqrt(-(2*d*e + f)/(8*d^7*e^3 - 12*d^6*f*e^2 + 6
*d^5*f^2*e - d^4*f^3)) - f)/(4*d^4*e^2 - 4*d^3*f*e + d^2*f^2)) - sqrt(1/2)*
(4*d^2*x*e^3 - 4*d*f*x*e^2 + f^2*x*e - (8*d^5*x*e^4 - 12*d^4*f*x*e^3 + 6*d^
3*f^2*x*e^2 - d^2*f^3*x*e)*sqrt(-(2*d*e + f)/(8*d^7*e^3 - 12*d^6*f*e^2 + 6*
d^5*f^2*e - d^4*f^3)))*sqrt(-((4*d^4*e^2 - 4*d^3*f*e + d^2*f^2)*sqrt(-(2*d*
e + f)/(8*d^7*e^3 - 12*d^6*f*e^2 + 6*d^5*f^2*e - d^4*f^3)) - f)/(4*d^4*e^2
- 4*d^3*f*e + d^2*f^2)))*sqrt(sqrt(1/2)*sqrt(-((4*d^4*e^2 - 4*d^3*f*e + d^2
*f^2)*sqrt(-(2*d*e + f)/(8*d^7*e^3 - 12*d^6*f*e^2 + 6*d^5*f^2*e - d^4*f^3))
 - f)/(4*d^4*e^2 - 4*d^3*f*e + d^2*f^2)))*e^(-2)) + sqrt(sqrt(1/2)*sqrt(((4
*d^4*e^2 - 4*d^3*f*e + d^2*f^2)*sqrt(-(2*d*e + f)/(8*d^7*e^3 - 12*d^6*f*e^2
 + 6*d^5*f^2*e - d^4*f^3)) + f)/(4*d^4*e^2 - 4*d^3*f*e + d^2*f^2)))*arctan(
1/2*(sqrt(1/2)*(4*d^2*e^2 - 4*d*f*e + f^2 + (8*d^5*e^3 - 12*d^4*f*e^2 + 6*d
^3*f^2*e - d^2*f^3)*sqrt(-(2*d*e + f)/(8*d^7*e^3 - 12*d^6*f*e^2 + 6*d^5*f^2
*e - d^4*f^3)))*sqrt(x^2*e^2 - 1/2*sqrt(1/2)*(2*d*f*e - f^2 - (8*d^5*e^3 -
12*d^4*f*e^2 + 6*d^3*f^2*e - d^2*f^3)*sqrt(-(2*d*e + f)/(8*d^7*e^3 - 12*d^6
*f*e^2 + 6*d^5*f^2*e - d^4*f^3)))*sqrt(((4*d^4*e^2 - 4*d^3*f*e + d^2*f^2)*s
qrt(-(2*d*e + f)/(8*d^7*e^3 - 12*d^6*f*e^2 + 6*d^5*f^2*e - d^4*f^3)) + f)/(
4*d^4*e^2 - 4*d^3*f*e + d^2*f^2)))*sqrt(sqrt(1/2)*sqrt(((4*d^4*e^2 - 4*d^3*
f*e + d^2*f^2)*sqrt(-(2*d*e + f)/(8*d^7*e^3 - 12*d^6*f*e^2 + 6*d^5*f^2*e -
d^4*f^3)) + f)/(4*d^4*e^2 - 4*d^3*f*e + d^2*f^2)))*sqrt(((4*d^4*e^2 - 4*d^3
*f*e + d^2*f^2)*sqrt(-(2*d*e + f)/(8*d^7*e^3 - 12*d^6*f*e^2 + 6*d^5*f^2*e -
 d^4*f^3)) + f)/(4*d^4*e^2 - 4*d^3*f*e + d^2*f^2)) - sqrt(1/2)*(4*d^2*x*e^3
 - 4*d*f*x*e^2 + f^2*x*e + (8*d^5*x*e^4 - 12*d^4*f*x*e^3 + 6*d^3*f^2*x*e^2
- d^2*f^3*x*e)*sqrt(-(2*d*e + f)/(8*d^7*e^3 - 12*d^6*f*e^2 + 6*d^5*f^2*e -
d^4*f^3)))*sqrt(sqrt(1/2)*sqrt(((4*d^4*e^2 - 4*d^3*f*e + d^2*f^2)*sqrt(-(2*
d*e + f)/(8*d^7*e^3 - 12*d^6*f*e^2 + 6*d^5*f^2*e - d^4*f^3)) + f)/(4*d^4*e^
2 - 4*d^3*f*e + d^2*f^2)))*sqrt(((4*d^4*e^2 - 4*d^3*f*e + d^2*f^2)*sqrt(-(2
*d*e + f)/(8*d^7*e^3 - 12*d^6*f*e^2 + 6*d^5*f^2*e - d^4*f^3)) + f)/(4*d^4*e
^2 - 4*d^3*f*e + d^2*f^2)))*e^(-2)) + 1/4*sqrt(sqrt(1/2)*sqrt(((4*d^4*e^2 -
 4*d^3*f*e + d^2*f^2)*sqrt(-(2*d*e + f)/(8*d^7*e^3 - 12*d^6*f*e^2 + 6*d^5*f
^2*e - d^4*f^3)) + f)/(4*d^4*e^2 - 4*d^3*f*e + d^2*f^2)))*log(x*e + 1/2*(2*
d*e + (4*d^4*e^2 - 4*d^3*f*e + d^2*f^2)*sqrt(-(2*d*e + f)/(8*d^7*e^3 - 12*d
^6*f*e^2 + 6*d^5*f^2*e - d^4*f^3)) - f)*sqrt(sqrt(1/2)*sqrt(((4*d^4*e^2 - 4
*d^3*f*e + d^2*f^2)*sqrt(-(2*d*e + f)/(8*d^7*e^3 - 12*d^6*f*e^2 + 6*d^5*f^2
*e - d^4*f^3)) + f)/(4*d^4*e^2 - 4*d^3*f*e + d^2*f^2)))) - 1/4*sqrt(sqrt(1/
2)*sqrt(((4*d^4*e^2 - 4*d^3*f*e + d^2*f^2)*sqrt(-(2*d*e + f)/(8*d^7*e^3 - 1
2*d^6*f*e^2 + 6*d^5*f^2*e - d^4*f^3)) + f)/(4*d^4*e^2 - 4*d^3*f*e + d^2*f^2
)))*log(x*e - 1/2*(2*d*e + (4*d^4*e^2 - 4*d^3*f*e + d^2*f^2)*sqrt(-(2*d*e +
 f)/(8*d^7*e^3 - 12*d^6*f*e^2 + 6*d^5*f^2*e - d^4*f^3)) - f)*sqrt(sqrt(1/2)
*sqrt(((4*d^4*e^2 - 4*d^3*f*e + d^2*f^2)*sqrt(-(2*d*e + f)/(8*d^7*e^3 - 12*
d^6*f*e^2 + 6*d^5*f^2*e - d^4*f^3)) + f)/(4*d^4*e^2 - 4*d^3*f*e + d^2*f^2))
)) + 1/4*sqrt(sqrt(1/2)*sqrt(-((4*d^4*e^2 - 4*d^3*f*e + d^2*f^2)*sqrt(-(2*d
```

```
*e + f)/(8*d^7*e^3 - 12*d^6*f*e^2 + 6*d^5*f^2*e - d^4*f^3)) - f)/(4*d^4*e^2
 - 4*d^3*f*e + d^2*f^2)))*log(x*e + 1/2*(2*d*e - (4*d^4*e^2 - 4*d^3*f*e + d
^2*f^2)*sqrt(-(2*d*e + f)/(8*d^7*e^3 - 12*d^6*f*e^2 + 6*d^5*f^2*e - d^4*f^3
)) - f)*sqrt(sqrt(1/2)*sqrt(-((4*d^4*e^2 - 4*d^3*f*e + d^2*f^2)*sqrt(-(2*d*
e + f)/(8*d^7*e^3 - 12*d^6*f*e^2 + 6*d^5*f^2*e - d^4*f^3)) - f)/(4*d^4*e^2
- 4*d^3*f*e + d^2*f^2)))) - 1/4*sqrt(sqrt(1/2)*sqrt(-((4*d^4*e^2 - 4*d^3*f*
e + d^2*f^2)*sqrt(-(2*d*e + f)/(8*d^7*e^3 - 12*d^6*f*e^2 + 6*d^5*f^2*e - d^
4*f^3)) - f)/(4*d^4*e^2 - 4*d^3*f*e + d^2*f^2)))*log(x*e - 1/2*(2*d*e - (4*
d^4*e^2 - 4*d^3*f*e + d^2*f^2)*sqrt(-(2*d*e + f)/(8*d^7*e^3 - 12*d^6*f*e^2
+ 6*d^5*f^2*e - d^4*f^3)) - f)*sqrt(sqrt(1/2)*sqrt(-((4*d^4*e^2 - 4*d^3*f*e
 + d^2*f^2)*sqrt(-(2*d*e + f)/(8*d^7*e^3 - 12*d^6*f*e^2 + 6*d^5*f^2*e - d^4
*f^3)) - f)/(4*d^4*e^2 - 4*d^3*f*e + d^2*f^2))))
```

Sympy [A]
time = 6.66, size = 136, normalized size = 0.18

$$\operatorname{RootSum}\left(t^{8}\cdot\left(1048576d^{6}e^{4}-2097152d^{5}e^{3}f+1572864d^{4}e^{2}f^{2}-524288d^{3}ef^{3}+65536d^{2}f^{4}\right)+t^{4}\left(-1024d^{2}e^{2}f+1024def^{2}-256f^{3}\right)+e^{2},\left(t\mapsto t\log\left(x+\frac{4096t^{5}d^{4}e^{2}-4096t^{5}d^{3}ef+1024t^{5}d^{2}f^{2}+4tde-4tf}{e}\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**4+d)/(e**2*x**8-f*x**4+d**2),x)
```

```
[Out] RootSum(_t**8*(1048576*d**6*e**4 - 2097152*d**5*e**3*f + 1572864*d**4*e**2*
f**2 - 524288*d**3*e*f**3 + 65536*d**2*f**4) + _t**4*(-1024*d**2*e**2*f + 1
024*d*e*f**2 - 256*f**3) + e**2, Lambda(_t, _t*log(x + (4096*_t**5*d**4*e**
2 - 4096*_t**5*d**3*e*f + 1024*_t**5*d**2*f**2 + 4*_t*d*e - 4*_t*f)/e)))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^4+d)/(e^2*x^8-f*x^4+d^2),x, algorithm="giac")
```

```
[Out] integrate((x^4*e + d)/(x^8*e^2 - f*x^4 + d^2), x)
```

Mupad [B]
time = 4.20, size = 2500, normalized size = 3.33

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x^4)/(d^2 - f*x^4 + e^2*x^8),x)
```

```
[Out] 2*atan(((((f^3 + ((f - 2*d*e)^5*(f + 2*d*e))^(1/2) + 4*d^2*e^2*f - 4*d*e*f^
2)/(512*(16*d^6*e^4 + d^2*f^4 - 8*d^3*e*f^3 - 32*d^5*e^3*f + 24*d^4*e^2*f^2
```

)))^(1/4)*((x*(65536*d^9*e^15 + 32768*d^8*e^14*f - 1024*d^2*e^8*f^7 - 2048*
d^3*e^9*f^6 + 10240*d^4*e^10*f^5 + 20480*d^5*e^11*f^4 - 32768*d^6*e^12*f^3
- 65536*d^7*e^13*f^2) - ((f^3 + ((f - 2*d*e)^5*(f + 2*d*e))^(1/2) + 4*d^2*e
^2*f - 4*d*e*f^2)/(512*(16*d^6*e^4 + d^2*f^4 - 8*d^3*e*f^3 - 32*d^5*e^3*f +
24*d^4*e^2*f^2)))^(1/4)*(262144*d^10*e^15 + 262144*d^9*e^14*f - 4096*d^3*e
^8*f^7 - 4096*d^4*e^9*f^6 + 49152*d^5*e^10*f^5 + 49152*d^6*e^11*f^4 - 19660
8*d^7*e^12*f^3 - 196608*d^8*e^13*f^2)*1i)*((f^3 + ((f - 2*d*e)^5*(f + 2*d*e
))^(1/2) + 4*d^2*e^2*f - 4*d*e*f^2)/(512*(16*d^6*e^4 + d^2*f^4 - 8*d^3*e*f^
3 - 32*d^5*e^3*f + 24*d^4*e^2*f^2)))^(3/4)*1i - 256*d^7*e^14 - 256*d^6*e^13
*f + 16*d^3*e^10*f^4 + 64*d^4*e^11*f^3)*1i - x*(32*d^5*e^13*f + 4*d^2*e^10*
f^4 + 24*d^3*e^11*f^3 + 48*d^4*e^12*f^2))*((f^3 + ((f - 2*d*e)^5*(f + 2*d*e
))^(1/2) + 4*d^2*e^2*f - 4*d*e*f^2)/(512*(16*d^6*e^4 + d^2*f^4 - 8*d^3*e*f^
3 - 32*d^5*e^3*f + 24*d^4*e^2*f^2)))^(1/4) + (((f^3 + ((f - 2*d*e)^5*(f + 2
*d*e))^(1/2) + 4*d^2*e^2*f - 4*d*e*f^2)/(512*(16*d^6*e^4 + d^2*f^4 - 8*d^3*
e*f^3 - 32*d^5*e^3*f + 24*d^4*e^2*f^2)))^(1/4)*((x*(65536*d^9*e^15 + 32768*
d^8*e^14*f - 1024*d^2*e^8*f^7 - 2048*d^3*e^9*f^6 + 10240*d^4*e^10*f^5 + 204
80*d^5*e^11*f^4 - 32768*d^6*e^12*f^3 - 65536*d^7*e^13*f^2) + ((f^3 + ((f -
2*d*e)^5*(f + 2*d*e))^(1/2) + 4*d^2*e^2*f - 4*d*e*f^2)/(512*(16*d^6*e^4 + d
^2*f^4 - 8*d^3*e*f^3 - 32*d^5*e^3*f + 24*d^4*e^2*f^2)))^(1/4)*(262144*d^10*
e^15 + 262144*d^9*e^14*f - 4096*d^3*e^8*f^7 - 4096*d^4*e^9*f^6 + 49152*d^5*
e^10*f^5 + 49152*d^6*e^11*f^4 - 196608*d^7*e^12*f^3 - 196608*d^8*e^13*f^2)*
1i)*((f^3 + ((f - 2*d*e)^5*(f + 2*d*e))^(1/2) + 4*d^2*e^2*f - 4*d*e*f^2)/(5
12*(16*d^6*e^4 + d^2*f^4 - 8*d^3*e*f^3 - 32*d^5*e^3*f + 24*d^4*e^2*f^2)))^(
3/4)*1i + 256*d^7*e^14 + 256*d^6*e^13*f - 16*d^3*e^10*f^4 - 64*d^4*e^11*f^3
)*1i - x*(32*d^5*e^13*f + 4*d^2*e^10*f^4 + 24*d^3*e^11*f^3 + 48*d^4*e^12*f^
2))*((f^3 + ((f - 2*d*e)^5*(f + 2*d*e))^(1/2) + 4*d^2*e^2*f - 4*d*e*f^2)/(5
12*(16*d^6*e^4 + d^2*f^4 - 8*d^3*e*f^3 - 32*d^5*e^3*f + 24*d^4*e^2*f^2)))^(
1/4))/((((f^3 + ((f - 2*d*e)^5*(f + 2*d*e))^(1/2) + 4*d^2*e^2*f - 4*d*e*f^2
)/(512*(16*d^6*e^4 + d^2*f^4 - 8*d^3*e*f^3 - 32*d^5*e^3*f + 24*d^4*e^2*f^2)
))^(1/4)*((x*(65536*d^9*e^15 + 32768*d^8*e^14*f - 1024*d^2*e^8*f^7 - 2048*d
^3*e^9*f^6 + 10240*d^4*e^10*f^5 + 20480*d^5*e^11*f^4 - 32768*d^6*e^12*f^3 -
65536*d^7*e^13*f^2) - ((f^3 + ((f - 2*d*e)^5*(f + 2*d*e))^(1/2) + 4*d^2*e^
2*f - 4*d*e*f^2)/(512*(16*d^6*e^4 + d^2*f^4 - 8*d^3*e*f^3 - 32*d^5*e^3*f +
24*d^4*e^2*f^2)))^(1/4)*(262144*d^10*e^15 + 262144*d^9*e^14*f - 4096*d^3*e^
8*f^7 - 4096*d^4*e^9*f^6 + 49152*d^5*e^10*f^5 + 49152*d^6*e^11*f^4 - 196608
*d^7*e^12*f^3 - 196608*d^8*e^13*f^2)*1i)*((f^3 + ((f - 2*d*e)^5*(f + 2*d*e)
)^(1/2) + 4*d^2*e^2*f - 4*d*e*f^2)/(512*(16*d^6*e^4 + d^2*f^4 - 8*d^3*e*f^3
- 32*d^5*e^3*f + 24*d^4*e^2*f^2)))^(3/4)*1i - 256*d^7*e^14 - 256*d^6*e^13*
f + 16*d^3*e^10*f^4 + 64*d^4*e^11*f^3)*1i - x*(32*d^5*e^13*f + 4*d^2*e^10*f
^4 + 24*d^3*e^11*f^3 + 48*d^4*e^12*f^2))*((f^3 + ((f - 2*d*e)^5*(f + 2*d*e)
)^(1/2) + 4*d^2*e^2*f - 4*d*e*f^2)/(512*(16*d^6*e^4 + d^2*f^4 - 8*d^3*e*f^3
- 32*d^5*e^3*f + 24*d^4*e^2*f^2)))^(1/4)*1i - (((f^3 + ((f - 2*d*e)^5*(f +
2*d*e))^(1/2) + 4*d^2*e^2*f - 4*d*e*f^2)/(512*(16*d^6*e^4 + d^2*f^4 - 8*d^
3*e*f^3 - 32*d^5*e^3*f + 24*d^4*e^2*f^2)))^(1/4)*((x*(65536*d^9*e^15 + 3276
8*d^8*e^14*f - 1024*d^2*e^8*f^7 - 2048*d^3*e^9*f^6 + 10240*d^4*e^10*f^5 + 2

```
0480*d^5*e^11*f^4 - 32768*d^6*e^12*f^3 - 65536*d^7*e^13*f^2) + ((f^3 + ((f
- 2*d*e)^5*(f + 2*d*e))^(1/2) + 4*d^2*e^2*f - 4*d*e*f^2)/(512*(16*d^6*e^4 +
 d^2*f^4 - 8*d^3*e*f^3 - 32*d^5*e^3*f + 24*d^4*e^2*f^2)))^(1/4)*(262144*d^1
0*e^15 + 262144*d^9*e^14*f - 4096*d^3*e^8*f^7 - 4096*d^4*e^9*f^6 + 49152*d^
5*e^10*f^5 + 49152*d^6*e^11*f^4 - 196608*d^7*e^12*f^3 - 196608*d^8*e^13*f^2
)*1i)*((f^3 + ((f - 2*d*e)^5*(f + 2*d*e))^(1/2) + 4*d^2*e^2*f - 4*d*e*f^2)/
(512*(16*d^6*e^4 + d^2*f^4 - 8*d^3*e*f^3 - 32*d^5*e^3*f + 24*d^4*e^2*f^2)))
^(3/4)*1i + 256*d^7*e^14 + 256*d^6*e^13*f - 16*d^3*e^10*f^4 - 64*d^4*e^11*f
^3)*1i - x*(32*d^5*e^13*f + 4*d^2*e^10*f^4 + 24*d^3*e^11*f^3 + 48*d^4*e^12*
f^2))*((f^3 + ((f - 2*d*e)^5*(f + 2*d*e))^(1/2) + 4*d^2*e^2*f - 4*d*e*f^2)/
(512*(16*d^6*e^4 + d^2*f^4 - 8*d^3*e*f^3 - 32*d^5*e^3*f + 24*d^4*e^2*f^2)))
^(1/4)*1i))*((f^3 + ((f - 2*d*e)^5*(f + 2*d*e))^(1/2) + 4*d^2*e^2*f - 4*d*e
*f^2)/(512*(16*d^6*e^4 + d^2*f^4 - 8*d^3*e*f^3 - 32*d^5*e^3*f + 24*d^4*e^2*
f^2)))^(1/4) - atan(((((f^3 + ((f - 2*d*e)^5*(f + 2*d*e))^(1/2) + 4*d^2*e^2
*f - 4*d*e*f^2)/(512*(16*d^6*e^4 + d^2*f^4 - 8*d^3*e*f^3 - 32*d^5*e^3*f + 2
4*d^4*e^2*f^2)))^(1/4)*((x*(65536*d^9*e^15 + 32768*d^8*e^14*f - 1024*d^2*e^
8*f^7 - 2048*d^3*e^9*f^6 + 10240*d^4*e^10*f^5 + 20480*d^5*e^11*f^4 - 32768*
d^6*e^12*f^3 - 65536*d^7*e^13*f^2) + ((f^3 + ((f - 2*d*e)^5*(f + 2*d*e))^(1
/2) + 4*d^2*e^2*f - 4*d*e*f^2)/(512*(16*d^6*e^4 + d^2*f^4 - 8*d^3*e*f^3 - 3
2*d^5*e^3*f + 24*d^4*e^2*f^2)))^(1/4)*(262144*d...
```

3.9 $\int \frac{1+x^4}{1+bx^4+x^8}\,dx$

Optimal. Leaf size=411

$$-\frac{\tan^{-1}\left(\frac{\sqrt{2-\sqrt{2-b}}-2x}{\sqrt{2+\sqrt{2-b}}}\right)}{4\sqrt{2+\sqrt{2-b}}}-\frac{\tan^{-1}\left(\frac{\sqrt{2+\sqrt{2-b}}-2x}{\sqrt{2-\sqrt{2-b}}}\right)}{4\sqrt{2-\sqrt{2-b}}}+\frac{\tan^{-1}\left(\frac{\sqrt{2-\sqrt{2-b}}+2x}{\sqrt{2+\sqrt{2-b}}}\right)}{4\sqrt{2+\sqrt{2-b}}}+\frac{\tan^{-1}\left(\frac{\sqrt{}}{}\right.}{4\sqrt{}}$$

```
[Out] -1/4*arctan((-2*x+(2+(2-b)^(1/2))^(1/2))/(2-(2-b)^(1/2))^(1/2))/(2-(2-b)^(1
/2))^(1/2)+1/4*arctan((2*x+(2+(2-b)^(1/2))^(1/2))/(2-(2-b)^(1/2))^(1/2))/(2
-(2-b)^(1/2))^(1/2)-1/8*ln(1+x^2-x*(2-(2-b)^(1/2))^(1/2))/(2-(2-b)^(1/2))^(
1/2)+1/8*ln(1+x^2+x*(2-(2-b)^(1/2))^(1/2))/(2-(2-b)^(1/2))^(1/2)-1/4*arctan
((-2*x+(2-(2-b)^(1/2))^(1/2))/(2+(2-b)^(1/2))^(1/2))/(2+(2-b)^(1/2))^(1/2)+
1/4*arctan((2*x+(2-(2-b)^(1/2))^(1/2))/(2+(2-b)^(1/2))^(1/2))/(2+(2-b)^(1/2
))^(1/2)-1/8*ln(1+x^2-x*(2+(2-b)^(1/2))^(1/2))/(2+(2-b)^(1/2))^(1/2)+1/8*ln
(1+x^2+x*(2+(2-b)^(1/2))^(1/2))/(2+(2-b)^(1/2))^(1/2)
```

Rubi [A]

time = 0.20, antiderivative size = 411, normalized size of antiderivative = 1.00, number of steps used = 19, number of rules used = 6, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {1433, 1108, 648, 632, 210, 642}

$$-\frac{\text{ArcTan}\left(\frac{\sqrt{2-\sqrt{2-b}}-2x}{\sqrt{\sqrt{2-b}+2}}\right)}{4\sqrt{\sqrt{2-b}+2}}-\frac{\text{ArcTan}\left(\frac{\sqrt{\sqrt{2-b}+2}-2x}{\sqrt{2-\sqrt{2-b}}}\right)}{4\sqrt{2-\sqrt{2-b}}}+\frac{\text{ArcTan}\left(\frac{\sqrt{2-\sqrt{2-b}}+2x}{\sqrt{\sqrt{2-b}+2}}\right)}{4\sqrt{\sqrt{2-b}+2}}+\frac{\text{ArcTan}\left(\frac{\sqrt{\sqrt{2-b}+2}+2x}{\sqrt{2-\sqrt{2-b}}}\right)}{4\sqrt{2-\sqrt{2-b}}}-\frac{\log\left(-\sqrt{2-\sqrt{2-b}}\,x+x^2+1\right)}{8\sqrt{2-\sqrt{2-b}}}+\frac{\log\left(\sqrt{2-\sqrt{2-b}}\,x+x^2+1\right)}{8\sqrt{2-\sqrt{2-b}}}-\frac{\log\left(-\sqrt{\sqrt{2-b}+2}\,x+x^2+1\right)}{8\sqrt{\sqrt{2-b}+2}}+\frac{\log\left(\sqrt{\sqrt{2-b}+2}\,x+x^2+1\right)}{8\sqrt{\sqrt{2-b}+2}}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + x^4)/(1 + b*x^4 + x^8),x]
```

```
[Out] -1/4*ArcTan[(Sqrt[2 - Sqrt[2 - b]] - 2*x)/Sqrt[2 + Sqrt[2 - b]]]/Sqrt[2 + S
qrt[2 - b]] - ArcTan[(Sqrt[2 + Sqrt[2 - b]] - 2*x)/Sqrt[2 - Sqrt[2 - b]]]/(
4*Sqrt[2 - Sqrt[2 - b]]) + ArcTan[(Sqrt[2 - Sqrt[2 - b]] + 2*x)/Sqrt[2 + Sq
rt[2 - b]]]/(4*Sqrt[2 + Sqrt[2 - b]]) + ArcTan[(Sqrt[2 + Sqrt[2 - b]] + 2*x
)/Sqrt[2 - Sqrt[2 - b]]]/(4*Sqrt[2 - Sqrt[2 - b]]) - Log[1 - Sqrt[2 - Sqrt[
2 - b]]*x + x^2]/(8*Sqrt[2 - Sqrt[2 - b]]) + Log[1 + Sqrt[2 - Sqrt[2 - b]]*
x + x^2]/(8*Sqrt[2 - Sqrt[2 - b]]) - Log[1 - Sqrt[2 + Sqrt[2 - b]]*x + x^2]
/(8*Sqrt[2 + Sqrt[2 - b]]) + Log[1 + Sqrt[2 + Sqrt[2 - b]]*x + x^2]/(8*Sqrt
[2 + Sqrt[2 - b]])
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1108

```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(-1), x_Symbol] :> With[{q = Rt[a/
c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Dist[1/(2*c*q*r), Int[(r - x)/(q - r*x
 + x^2), x], x] + Dist[1/(2*c*q*r), Int[(r + x)/(q + r*x + x^2), x], x]]] /
; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && NegQ[b^2 - 4*a*c]
```

Rule 1433

```
Int[((d_) + (e_.)*(x_)^(n_))/((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_)), x
_Symbol] :> With[{q = Rt[2*(d/e) - b/c, 2]}, Dist[e/(2*c), Int[1/Simp[d/e +
 q*x^(n/2) + x^n, x], x], x] + Dist[e/(2*c), Int[1/Simp[d/e - q*x^(n/2) + x
^n, x], x], x]] /; FreeQ[{a, b, c, d, e}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4
*a*c, 0] && EqQ[c*d^2 - a*e^2, 0] && IGtQ[n/2, 0] && (GtQ[2*(d/e) - b/c, 0]
 || ( !LtQ[2*(d/e) - b/c, 0] && EqQ[d, e*Rt[a/c, 2]]))
```

Rubi steps

$$\int \frac{1+x^4}{1+bx^4+x^8}\,dx = \frac{1}{2}\int \frac{1}{1-\sqrt{2-b}\,x^2+x^4}\,dx + \frac{1}{2}\int \frac{1}{1+\sqrt{2-b}\,x^2+x^4}\,dx$$

$$= \frac{\int \frac{\sqrt{2-\sqrt{2-b}}-x}{1-\sqrt{2-\sqrt{2-b}}\,x+x^2}\,dx}{4\sqrt{2-\sqrt{2-b}}} + \frac{\int \frac{\sqrt{2-\sqrt{2-b}}+x}{1+\sqrt{2-\sqrt{2-b}}\,x+x^2}\,dx}{4\sqrt{2-\sqrt{2-b}}} + \frac{\int \frac{\sqrt{2+\sqrt{2-b}}-x}{1-\sqrt{2+\sqrt{2-b}}\,x+x^2}}{4\sqrt{2+\sqrt{2-b}}}$$

$$= \frac{1}{8}\int \frac{1}{1-\sqrt{2-\sqrt{2-b}}\,x+x^2}\,dx + \frac{1}{8}\int \frac{1}{1+\sqrt{2-\sqrt{2-b}}\,x+x^2}\,dx + \frac{1}{8}\int \frac{}{1-}$$

$$= -\frac{\log\left(1-\sqrt{2-\sqrt{2-b}}\,x+x^2\right)}{8\sqrt{2-\sqrt{2-b}}} + \frac{\log\left(1+\sqrt{2-\sqrt{2-b}}\,x+x^2\right)}{8\sqrt{2-\sqrt{2-b}}} - \frac{\log\left(1-\right.}{8}$$

$$= -\frac{\tan^{-1}\left(\frac{\sqrt{2-\sqrt{2-b}}-2x}{\sqrt{2+\sqrt{2-b}}}\right)}{4\sqrt{2+\sqrt{2-b}}} - \frac{\tan^{-1}\left(\frac{\sqrt{2+\sqrt{2-b}}-2x}{\sqrt{2-\sqrt{2-b}}}\right)}{4\sqrt{2-\sqrt{2-b}}} + \frac{\tan^{-1}\left(\frac{\sqrt{2-\sqrt{2}}}{\sqrt{2+}}\right.}{4\sqrt{2+\sqrt{2}}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.
time = 0.02, size = 55, normalized size = 0.13

$$\frac{1}{4}\text{RootSum}\left[1+b\#1^4+\#1^8\&,\frac{\log(x-\#1)+\log(x-\#1)\#1^4}{b\#1^3+2\#1^7}\&\right]$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + x^4)/(1 + b*x^4 + x^8),x]
```

```
[Out] RootSum[1 + b*#1^4 + #1^8 & , (Log[x - #1] + Log[x - #1]*#1^4)/(b*#1^3 + 2*
#1^7) & ]/4
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 0.07, size = 42, normalized size = 0.10

method	result	size

default	$\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(\_Z^8+\_Z^4b+1\right)}\frac{\left(\_R^4+1\right)\ln\left(x-\_R\right)}{2\_R^7+\_R^3b}\right)}{4}$	42
risch	$\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(\_Z^8+\_Z^4b+1\right)}\frac{\left(\_R^4+1\right)\ln\left(x-\_R\right)}{2\_R^7+\_R^3b}\right)}{4}$	42

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4+1)/(x^8+b*x^4+1),x,method=_RETURNVERBOSE)
```

```
[Out] 1/4*sum((_R^4+1)/(2*_R^7+_R^3*b)*ln(x-_R),_R=RootOf(_Z^8+_Z^4*b+1))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4+1)/(x^8+b*x^4+1),x, algorithm="maxima")
```

```
[Out] integrate((x^4 + 1)/(x^8 + b*x^4 + 1), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 1443 vs. $2(321) = 642$.
time = 0.41, size = 1443, normalized size = 3.51

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4+1)/(x^8+b*x^4+1),x, algorithm="fricas")
```

```
[Out] sqrt(sqrt(1/2)*sqrt(((b^2 + 4*b + 4)*sqrt((b - 2)/(b^3 + 6*b^2 + 12*b + 8))
 - b)/(b^2 + 4*b + 4)))*arctan(1/2*sqrt(1/2)*(b^2 + (b^3 + 6*b^2 + 12*b + 8
)*sqrt((b - 2)/(b^3 + 6*b^2 + 12*b + 8)) + 4*b + 4)*sqrt(x^2 + 1/2*sqrt(1/2
)*(b^2 + (b^3 + 6*b^2 + 12*b + 8)*sqrt((b - 2)/(b^3 + 6*b^2 + 12*b + 8)) +
2*b)*sqrt(((b^2 + 4*b + 4)*sqrt((b - 2)/(b^3 + 6*b^2 + 12*b + 8)) - b)/(b^2
 + 4*b + 4)))*sqrt(sqrt(1/2)*sqrt(((b^2 + 4*b + 4)*sqrt((b - 2)/(b^3 + 6*b^
2 + 12*b + 8)) - b)/(b^2 + 4*b + 4)))*sqrt(((b^2 + 4*b + 4)*sqrt((b - 2)/(b
^3 + 6*b^2 + 12*b + 8)) - b)/(b^2 + 4*b + 4)) - 1/2*sqrt(1/2)*((b^3 + 6*b^2
 + 12*b + 8)*x*sqrt((b - 2)/(b^3 + 6*b^2 + 12*b + 8)) + (b^2 + 4*b + 4)*x)*
sqrt(sqrt(1/2)*sqrt(((b^2 + 4*b + 4)*sqrt((b - 2)/(b^3 + 6*b^2 + 12*b + 8))
 - b)/(b^2 + 4*b + 4)))*sqrt(((b^2 + 4*b + 4)*sqrt((b - 2)/(b^3 + 6*b^2 + 1
2*b + 8)) - b)/(b^2 + 4*b + 4))) - sqrt(sqrt(1/2)*sqrt(-((b^2 + 4*b + 4)*sq
```

```
rt((b - 2)/(b^3 + 6*b^2 + 12*b + 8)) + b)/(b^2 + 4*b + 4)))*arctan(-1/2*(sq
rt(1/2)*(b^2 - (b^3 + 6*b^2 + 12*b + 8)*sqrt((b - 2)/(b^3 + 6*b^2 + 12*b +
8)) + 4*b + 4)*sqrt(x^2 + 1/2*sqrt(1/2)*(b^2 - (b^3 + 6*b^2 + 12*b + 8)*sqr
t((b - 2)/(b^3 + 6*b^2 + 12*b + 8)) + 2*b)*sqrt(-((b^2 + 4*b + 4)*sqrt((b -
 2)/(b^3 + 6*b^2 + 12*b + 8)) + b)/(b^2 + 4*b + 4)))*sqrt(-((b^2 + 4*b + 4)
*sqrt((b - 2)/(b^3 + 6*b^2 + 12*b + 8)) + b)/(b^2 + 4*b + 4)) + sqrt(1/2)*(
(b^3 + 6*b^2 + 12*b + 8)*x*sqrt((b - 2)/(b^3 + 6*b^2 + 12*b + 8)) - (b^2 +
4*b + 4)*x)*sqrt(-((b^2 + 4*b + 4)*sqrt((b - 2)/(b^3 + 6*b^2 + 12*b + 8)) +
 b)/(b^2 + 4*b + 4)))*sqrt(sqrt(1/2)*sqrt(-((b^2 + 4*b + 4)*sqrt((b - 2)/(b
^3 + 6*b^2 + 12*b + 8)) + b)/(b^2 + 4*b + 4)))) - 1/4*sqrt(sqrt(1/2)*sqrt(-
((b^2 + 4*b + 4)*sqrt((b - 2)/(b^3 + 6*b^2 + 12*b + 8)) + b)/(b^2 + 4*b + 4
)))*log(1/2*((b^2 + 4*b + 4)*sqrt((b - 2)/(b^3 + 6*b^2 + 12*b + 8)) - b - 2
)*sqrt(sqrt(1/2)*sqrt(-((b^2 + 4*b + 4)*sqrt((b - 2)/(b^3 + 6*b^2 + 12*b +
8)) + b)/(b^2 + 4*b + 4))) + x) + 1/4*sqrt(sqrt(1/2)*sqrt(-((b^2 + 4*b + 4)
*sqrt((b - 2)/(b^3 + 6*b^2 + 12*b + 8)) + b)/(b^2 + 4*b + 4)))*log(-1/2*((b
^2 + 4*b + 4)*sqrt((b - 2)/(b^3 + 6*b^2 + 12*b + 8)) - b - 2)*sqrt(sqrt(1/2
)*sqrt(-((b^2 + 4*b + 4)*sqrt((b - 2)/(b^3 + 6*b^2 + 12*b + 8)) + b)/(b^2 +
 4*b + 4))) + x) + 1/4*sqrt(sqrt(1/2)*sqrt(((b^2 + 4*b + 4)*sqrt((b - 2)/(b
^3 + 6*b^2 + 12*b + 8)) - b)/(b^2 + 4*b + 4)))*log(1/2*((b^2 + 4*b + 4)*sqr
t((b - 2)/(b^3 + 6*b^2 + 12*b + 8)) + b + 2)*sqrt(sqrt(1/2)*sqrt(((b^2 + 4*
b + 4)*sqrt((b - 2)/(b^3 + 6*b^2 + 12*b + 8)) - b)/(b^2 + 4*b + 4))) + x) -
 1/4*sqrt(sqrt(1/2)*sqrt(((b^2 + 4*b + 4)*sqrt((b - 2)/(b^3 + 6*b^2 + 12*b
+ 8)) - b)/(b^2 + 4*b + 4)))*log(-1/2*((b^2 + 4*b + 4)*sqrt((b - 2)/(b^3 +
6*b^2 + 12*b + 8)) + b + 2)*sqrt(sqrt(1/2)*sqrt(((b^2 + 4*b + 4)*sqrt((b -
2)/(b^3 + 6*b^2 + 12*b + 8)) - b)/(b^2 + 4*b + 4))) + x)
```

Sympy [**A**]
time = 1.97, size = 75, normalized size = 0.18

$$\operatorname{RootSum}\left(t^8\cdot\left(65536b^4+524288b^3+1572864b^2+2097152b+1048576\right)+t^4\cdot\left(256b^3+1024b^2+1024b\right)+1,\left(t\mapsto t\log\left(1024t^5b^2+4096t^5b+4096t^5+4tb+4t+x\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**4+1)/(x**8+b*x**4+1),x)
```

```
[Out] RootSum(_t**8*(65536*b**4 + 524288*b**3 + 1572864*b**2 + 2097152*b + 104857
6) + _t**4*(256*b**3 + 1024*b**2 + 1024*b) + 1, Lambda(_t, _t*log(1024*_t**
5*b**2 + 4096*_t**5*b + 4096*_t**5 + 4*_t*b + 4*_t + x)))
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4+1)/(x^8+b*x^4+1),x, algorithm="giac")
```

```
[Out] integrate((x^4 + 1)/(x^8 + b*x^4 + 1), x)
```

Mupad [B]
time = 3.68, size = 2500, normalized size = 6.08

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4 + 1)/(b*x^4 + x^8 + 1),x)

[Out] - atan((((-(4*b + ((b - 2)*(b + 2)^5)^(1/2) + 4*b^2 + b^3)/(512*(32*b + 24*
b^2 + 8*b^3 + b^4 + 16)))^(1/4)*(((-(4*b + ((b - 2)*(b + 2)^5)^(1/2) + 4*b^
2 + b^3)/(512*(32*b + 24*b^2 + 8*b^3 + b^4 + 16)))^(1/4)*(262144*b + 196608
*b^2 - 196608*b^3 - 49152*b^4 + 49152*b^5 + 4096*b^6 - 4096*b^7 - 262144) +
 x*(32768*b + 65536*b^2 - 32768*b^3 - 20480*b^4 + 10240*b^5 + 2048*b^6 - 10
24*b^7 - 65536))*(-(4*b + ((b - 2)*(b + 2)^5)^(1/2) + 4*b^2 + b^3)/(512*(32
*b + 24*b^2 + 8*b^3 + b^4 + 16)))^(3/4) - 256*b + 64*b^3 - 16*b^4 + 256) +
x*(32*b - 48*b^2 + 24*b^3 - 4*b^4))*(-(4*b + ((b - 2)*(b + 2)^5)^(1/2) + 4*
b^2 + b^3)/(512*(32*b + 24*b^2 + 8*b^3 + b^4 + 16)))^(1/4)*1i - ((-(4*b + (
(b - 2)*(b + 2)^5)^(1/2) + 4*b^2 + b^3)/(512*(32*b + 24*b^2 + 8*b^3 + b^4 +
 16)))^(1/4)*(((-(4*b + ((b - 2)*(b + 2)^5)^(1/2) + 4*b^2 + b^3)/(512*(32*b
 + 24*b^2 + 8*b^3 + b^4 + 16)))^(1/4)*(262144*b + 196608*b^2 - 196608*b^3 -
 49152*b^4 + 49152*b^5 + 4096*b^6 - 4096*b^7 - 262144) - x*(32768*b + 65536
*b^2 - 32768*b^3 - 20480*b^4 + 10240*b^5 + 2048*b^6 - 1024*b^7 - 65536))*(-
(4*b + ((b - 2)*(b + 2)^5)^(1/2) + 4*b^2 + b^3)/(512*(32*b + 24*b^2 + 8*b^3
 + b^4 + 16)))^(3/4) - 256*b + 64*b^3 - 16*b^4 + 256) - x*(32*b - 48*b^2 +
24*b^3 - 4*b^4))*(-(4*b + ((b - 2)*(b + 2)^5)^(1/2) + 4*b^2 + b^3)/(512*(32
*b + 24*b^2 + 8*b^3 + b^4 + 16)))^(1/4)*1i)/(((-(4*b + ((b - 2)*(b + 2)^5)^
(1/2) + 4*b^2 + b^3)/(512*(32*b + 24*b^2 + 8*b^3 + b^4 + 16)))^(1/4)*(((-(4
*b + ((b - 2)*(b + 2)^5)^(1/2) + 4*b^2 + b^3)/(512*(32*b + 24*b^2 + 8*b^3 +
 b^4 + 16)))^(1/4)*(262144*b + 196608*b^2 - 196608*b^3 - 49152*b^4 + 49152*
b^5 + 4096*b^6 - 4096*b^7 - 262144) + x*(32768*b + 65536*b^2 - 32768*b^3 -
20480*b^4 + 10240*b^5 + 2048*b^6 - 1024*b^7 - 65536))*(-(4*b + ((b - 2)*(b
+ 2)^5)^(1/2) + 4*b^2 + b^3)/(512*(32*b + 24*b^2 + 8*b^3 + b^4 + 16)))^(3/4
) - 256*b + 64*b^3 - 16*b^4 + 256) + x*(32*b - 48*b^2 + 24*b^3 - 4*b^4))*(-
(4*b + ((b - 2)*(b + 2)^5)^(1/2) + 4*b^2 + b^3)/(512*(32*b + 24*b^2 + 8*b^3
 + b^4 + 16)))^(1/4) + ((-(4*b + ((b - 2)*(b + 2)^5)^(1/2) + 4*b^2 + b^3)/(
512*(32*b + 24*b^2 + 8*b^3 + b^4 + 16)))^(1/4)*(((-(4*b + ((b - 2)*(b + 2)^
5)^(1/2) + 4*b^2 + b^3)/(512*(32*b + 24*b^2 + 8*b^3 + b^4 + 16)))^(1/4)*(26
2144*b + 196608*b^2 - 196608*b^3 - 49152*b^4 + 49152*b^5 + 4096*b^6 - 4096*
b^7 - 262144) - x*(32768*b + 65536*b^2 - 32768*b^3 - 20480*b^4 + 10240*b^5
+ 2048*b^6 - 1024*b^7 - 65536))*(-(4*b + ((b - 2)*(b + 2)^5)^(1/2) + 4*b^2
+ b^3)/(512*(32*b + 24*b^2 + 8*b^3 + b^4 + 16)))^(3/4) - 256*b + 64*b^3 - 1
6*b^4 + 256) - x*(32*b - 48*b^2 + 24*b^3 - 4*b^4))*(-(4*b + ((b - 2)*(b + 2
)^5)^(1/2) + 4*b^2 + b^3)/(512*(32*b + 24*b^2 + 8*b^3 + b^4 + 16)))^(1/4)))
```

```
*(-(4*b + ((b - 2)*(b + 2)^5)^(1/2) + 4*b^2 + b^3)/(512*(32*b + 24*b^2 + 8*
b^3 + b^4 + 16)))^(1/4)*2i - 2*atan((((-(4*b + ((b - 2)*(b + 2)^5)^(1/2) +
4*b^2 + b^3)/(512*(32*b + 24*b^2 + 8*b^3 + b^4 + 16)))^(1/4)*(256*b + ((-(4
*b + ((b - 2)*(b + 2)^5)^(1/2) + 4*b^2 + b^3)/(512*(32*b + 24*b^2 + 8*b^3 +
 b^4 + 16)))^(1/4)*(262144*b + 196608*b^2 - 196608*b^3 - 49152*b^4 + 49152*
b^5 + 4096*b^6 - 4096*b^7 - 262144)*1i + x*(32768*b + 65536*b^2 - 32768*b^3
 - 20480*b^4 + 10240*b^5 + 2048*b^6 - 1024*b^7 - 65536))*(-(4*b + ((b - 2)*
(b + 2)^5)^(1/2) + 4*b^2 + b^3)/(512*(32*b + 24*b^2 + 8*b^3 + b^4 + 16)))^(
3/4)*1i - 64*b^3 + 16*b^4 - 256)*1i - x*(32*b - 48*b^2 + 24*b^3 - 4*b^4))*(
-(4*b + ((b - 2)*(b + 2)^5)^(1/2) + 4*b^2 + b^3)/(512*(32*b + 24*b^2 + 8*b^
3 + b^4 + 16)))^(1/4) - ((-(4*b + ((b - 2)*(b + 2)^5)^(1/2) + 4*b^2 + b^3)/
(512*(32*b + 24*b^2 + 8*b^3 + b^4 + 16)))^(1/4)*(256*b + ((-(4*b + ((b - 2)
*(b + 2)^5)^(1/2) + 4*b^2 + b^3)/(512*(32*b + 24*b^2 + 8*b^3 + b^4 + 16)))^
(1/4)*(262144*b + 196608*b^2 - 196608*b^3 - 49152*b^4 + 49152*b^5 + 4096*b^
6 - 4096*b^7 - 262144)*1i - x*(32768*b + 65536*b^2 - 32768*b^3 - 20480*b^4
+ 10240*b^5 + 2048*b^6 - 1024*b^7 - 65536))*(-(4*b + ((b - 2)*(b + 2)^5)^(1
/2) + 4*b^2 + b^3)/(512*(32*b + 24*b^2 + 8*b^3 + b^4 + 16)))^(3/4)*1i - 64*
b^3 + 16*b^4 - 256)*1i + x*(32*b - 48*b^2 + 24*b^3 - 4*b^4))*(-(4*b + ((b -
 2)*(b + 2)^5)^(1/2) + 4*b^2 + b^3)/(512*(32*b + 24*b^2 + 8*b^3 + b^4 + 16)
))^(1/4))/(((-(4*b + ((b - 2)*(b + 2)^5)^(1/2) + 4*b^2 + b^3)/(512*(32*b +
24*b^2 + 8*b^3 + b^4 + 16)))^(1/4)*(256*b + ((-(4*b + ((b - 2)*(b + 2)^5)^(
1/2) + 4*b^2 + b^3)/(512*(32*b + 24*b^2 + 8*b^3 + b^4 + 16)))^(1/4)*(262144
*b + 196608*b^2 - 196608*b^3 - 49152*b^4 + 49152*b^5 + 4096*b^6 - 4096*b^7
- 262144)*1i + x*(32768*b + 65536*b^2 - 32768*b^3 - 20480*b^4 + 10240*b^5 +
 2048*b^6 - 1024*b^7 - 65536))*(-(4*b + ((b - 2)*(b + 2)^5)^(1/2) + 4*b^2 +
 b^3)/(512*(32*b + 24*b^2 + 8*b^3 + b^4 + 16)))^(3/4)*1i - 64*b^3 + 16*b^4
- 256)*1i - x*(32*b - 48*b^2 + 24*b^3 - 4*b^4))*(-(4*b + ((b - 2)*(b + 2)^5
)^(1/2) + 4*b^2 + b^3)/(512*(32*b + 24*b^2 + 8*b^3 + b^4 + 16)))^(1/4)*1i +
 ((-(4*b + ((b - 2)*(b + 2)^5)^(1/2) + 4*b^2 + b^3)/(512*(32*b + 24*b^2 + 8
*b^3 + b^4 + 16)))^(1/4)*(256*b + ((-(4*b + ((b - 2)*(b + 2)^5)^(1/2) + 4*b
^2 + b^3)/(512*(32*b + 24*b^2 + 8*b^3 + b^4 + 16)))^(1/4)*(262144*b + 19660
8*b^2 - 196608*b^3 - 49152*b^4 + 49152*b^5 + 40...
```

3.10 $\int \frac{1+x^4}{1+3x^4+x^8}\,dx$

Optimal. Leaf size=451

$$-\frac{\sqrt[4]{3+\sqrt{5}}\,\tan^{-1}\left(1-\frac{2^{3/4}x}{\sqrt[4]{3-\sqrt{5}}}\right)}{2\,2^{3/4}\sqrt{5}}+\frac{\sqrt[4]{3+\sqrt{5}}\,\tan^{-1}\left(1+\frac{2^{3/4}x}{\sqrt[4]{3-\sqrt{5}}}\right)}{2\,2^{3/4}\sqrt{5}}-\frac{\sqrt[4]{3-\sqrt{5}}\,\tan^{-1}\left(1-\cdots\right.}{2\,2^{3/4}\sqrt{5}}$$

```
[Out] 1/20*arctan(-1+2^(3/4)*x/(3+5^(1/2))^(1/4))*(3-5^(1/2))^(1/4)*2^(1/4)*5^(1/
2)+1/20*arctan(1+2^(3/4)*x/(3+5^(1/2))^(1/4))*(3-5^(1/2))^(1/4)*2^(1/4)*5^(
1/2)-1/40*ln(2*x^2-2*2^(1/4)*x*(3+5^(1/2))^(1/4)+5^(1/2)+1)*(3-5^(1/2))^(1/
4)*2^(1/4)*5^(1/2)+1/40*ln(2*x^2+2*2^(1/4)*x*(3+5^(1/2))^(1/4)+5^(1/2)+1)*(
3-5^(1/2))^(1/4)*2^(1/4)*5^(1/2)+1/20*arctan(-1+2^(3/4)*x/(3-5^(1/2))^(1/4)
)*(3+5^(1/2))^(1/4)*2^(1/4)*5^(1/2)+1/20*arctan(1+2^(3/4)*x/(3-5^(1/2))^(1/
4))*(3+5^(1/2))^(1/4)*2^(1/4)*5^(1/2)-1/40*ln(2*x^2-2*2^(1/4)*x*(3-5^(1/2))
^(1/4)+5^(1/2)-1)*(3+5^(1/2))^(1/4)*2^(1/4)*5^(1/2)+1/40*ln(2*x^2+2*2^(1/4)
*x*(3-5^(1/2))^(1/4)+5^(1/2)-1)*(3+5^(1/2))^(1/4)*2^(1/4)*5^(1/2)
```

Rubi [A]

time = 0.27, antiderivative size = 451, normalized size of antiderivative = 1.00, number of steps used = 19, number of rules used = 7, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.389$, Rules used = {1434, 217, 1179, 642, 1176, 631, 210}

$$-\frac{\sqrt[4]{3+\sqrt{5}}\,\text{ArcTan}\left(1-\frac{2^{3/4}x}{\sqrt[4]{3-\sqrt{5}}}\right)}{2\,2^{3/4}\sqrt{5}}+\frac{\sqrt[4]{3+\sqrt{5}}\,\text{ArcTan}\left(\frac{2^{3/4}x}{\sqrt[4]{3-\sqrt{5}}}+1\right)}{2\,2^{3/4}\sqrt{5}}-\frac{\sqrt[4]{3-\sqrt{5}}\,\text{ArcTan}\left(1-\frac{2^{3/4}x}{\sqrt[4]{3+\sqrt{5}}}\right)}{2\,2^{3/4}\sqrt{5}}+\frac{\sqrt[4]{3-\sqrt{5}}\,\text{ArcTan}\left(\frac{2^{3/4}x}{\sqrt[4]{3+\sqrt{5}}}+1\right)}{2\,2^{3/4}\sqrt{5}}-\frac{\sqrt[4]{3+\sqrt{5}}\log\left(2x^2-2\sqrt[4]{2\left(3-\sqrt{5}\right)}x+\sqrt{2\left(3-\sqrt{5}\right)}\right)}{4\,2^{3/4}\sqrt{5}}+\frac{\sqrt[4]{3+\sqrt{5}}\log\left(2x^2+2\sqrt[4]{2\left(3-\sqrt{5}\right)}x+\sqrt{2\left(3-\sqrt{5}\right)}\right)}{4\,2^{3/4}\sqrt{5}}-\frac{\sqrt[4]{3-\sqrt{5}}\log\left(2x^2-2\sqrt[4]{2\left(3+\sqrt{5}\right)}x+\sqrt{2\left(3+\sqrt{5}\right)}\right)}{4\,2^{3/4}\sqrt{5}}+\frac{\sqrt[4]{3-\sqrt{5}}\log\left(2x^2+2\sqrt[4]{2\left(3+\sqrt{5}\right)}x+\sqrt{2\left(3+\sqrt{5}\right)}\right)}{4\,2^{3/4}\sqrt{5}}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + x^4)/(1 + 3*x^4 + x^8),x]
```

```
[Out] -1/2*((3 + Sqrt[5])^(1/4)*ArcTan[1 - (2^(3/4)*x)/(3 - Sqrt[5])^(1/4)])/(2^(
3/4)*Sqrt[5]) + ((3 + Sqrt[5])^(1/4)*ArcTan[1 + (2^(3/4)*x)/(3 - Sqrt[5])^(
1/4)])/(2*2^(3/4)*Sqrt[5]) - ((3 - Sqrt[5])^(1/4)*ArcTan[1 - (2^(3/4)*x)/(3
 + Sqrt[5])^(1/4)])/(2*2^(3/4)*Sqrt[5]) + ((3 - Sqrt[5])^(1/4)*ArcTan[1 + (
2^(3/4)*x)/(3 + Sqrt[5])^(1/4)])/(2*2^(3/4)*Sqrt[5]) - ((3 + Sqrt[5])^(1/4)
*Log[Sqrt[2*(3 - Sqrt[5])] - 2*(2*(3 - Sqrt[5]))^(1/4)*x + 2*x^2])/(4*2^(3/
4)*Sqrt[5]) + ((3 + Sqrt[5])^(1/4)*Log[Sqrt[2*(3 - Sqrt[5])] + 2*(2*(3 - Sq
rt[5]))^(1/4)*x + 2*x^2])/(4*2^(3/4)*Sqrt[5]) - ((3 - Sqrt[5])^(1/4)*Log[Sq
rt[2*(3 + Sqrt[5])] - 2*(2*(3 + Sqrt[5]))^(1/4)*x + 2*x^2])/(4*2^(3/4)*Sqrt
[5]) + ((3 - Sqrt[5])^(1/4)*Log[Sqrt[2*(3 + Sqrt[5])] + 2*(2*(3 + Sqrt[5]))
^(1/4)*x + 2*x^2])/(4*2^(3/4)*Sqrt[5])
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
```

```
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1434

```
Int[((d_) + (e_.)*(x_)^(n_))/((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_)), x
_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q),
Int[1/(b/2 - q/2 + c*x^n), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(
b/2 + q/2 + c*x^n), x], x]] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[n2, 2*n]
 && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - a*e^2, 0] && IGtQ[n/2, 0] && GtQ[b^2
 - 4*a*c, 0]
```

Rubi steps

$$\int \frac{1+x^4}{1+3x^4+x^8}\,dx = \frac{1}{10}\left(5-\sqrt{5}\right)\int \frac{1}{\frac{3}{2}-\frac{\sqrt{5}}{2}+x^4}\,dx+\frac{1}{10}\left(5+\sqrt{5}\right)\int \frac{1}{\frac{3}{2}+\frac{\sqrt{5}}{2}+x^4}\,dx$$

$$=\frac{\int \frac{\sqrt{3-\sqrt{5}}-\sqrt{2}x^2}{\frac{3}{2}-\frac{\sqrt{5}}{2}+x^4}\,dx}{2\sqrt{10}}+\frac{\int \frac{\sqrt{3-\sqrt{5}}+\sqrt{2}x^2}{\frac{3}{2}-\frac{\sqrt{5}}{2}+x^4}\,dx}{2\sqrt{10}}+\frac{\int \frac{\sqrt{3+\sqrt{5}}-\sqrt{2}x^2}{\frac{3}{2}+\frac{\sqrt{5}}{2}+x^4}\,dx}{2\sqrt{10}}+$$

$$=\frac{\int \frac{1}{\sqrt{\frac{1}{2}\left(3-\sqrt{5}\right)}-\sqrt[4]{2\left(3-\sqrt{5}\right)}x+x^2}\,dx}{4\sqrt{5}}+\frac{\int \frac{1}{\sqrt{\frac{1}{2}\left(3-\sqrt{5}\right)}+\sqrt[4]{2\left(3-\sqrt{5}\right)}x+x^2}}{4\sqrt{5}}$$

$$=-\frac{\sqrt[4]{3+\sqrt{5}}\log\left(\sqrt{2\left(3-\sqrt{5}\right)}-2\sqrt[4]{2\left(3-\sqrt{5}\right)}x+2x^2\right)}{4\,2^{3/4}\sqrt{5}}+\frac{\sqrt[4]{3+\sqrt{5}}\log\left(\right.}{}$$

$$=-\frac{\tan^{-1}\left(1-\frac{2^{3/4}x}{\sqrt[4]{3-\sqrt{5}}}\right)}{2\sqrt{5}\sqrt[4]{2\left(3-\sqrt{5}\right)}}+\frac{\tan^{-1}\left(1+\frac{2^{3/4}x}{\sqrt[4]{3-\sqrt{5}}}\right)}{2\sqrt{5}\sqrt[4]{2\left(3-\sqrt{5}\right)}}-\frac{\tan^{-1}\left(1-\frac{2^{3/4}x}{\sqrt[4]{3+\sqrt{5}}}\right)}{2\sqrt{5}\sqrt[4]{2\left(3+\sqrt{5}\right)}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.
time = 0.01, size = 55, normalized size = 0.12

$$\frac{1}{4}\text{RootSum}\left[1+3\#1^4+\#1^8\&,\frac{\log(x-\#1)+\log(x-\#1)\#1^4}{3\#1^3+2\#1^7}\&\right]$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + x^4)/(1 + 3*x^4 + x^8),x]
```

```
[Out] RootSum[1 + 3*#1^4 + #1^8 & , (Log[x - #1] + Log[x - #1]*#1^4)/(3*#1^3 + 2*
#1^7) & ]/4
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 0.02, size = 42, normalized size = 0.09

method	result	size
default	$\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(\_Z^{8}+3\_Z^{4}+1\right)}\frac{\left(\_R^{4}+1\right)\ln\left(x-\_R\right)}{2\_R^{7}+3\_R^{3}}\right)}{4}$	42
risch	$\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(\_Z^{8}+3\_Z^{4}+1\right)}\frac{\left(\_R^{4}+1\right)\ln\left(x-\_R\right)}{2\_R^{7}+3\_R^{3}}\right)}{4}$	42

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4+1)/(x^8+3*x^4+1),x,method=_RETURNVERBOSE)
```

```
[Out] 1/4*sum((_R^4+1)/(2*_R^7+3*_R^3)*ln(x-_R),_R=RootOf(_Z^8+3*_Z^4+1))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4+1)/(x^8+3*x^4+1),x, algorithm="maxima")
```

```
[Out] integrate((x^4 + 1)/(x^8 + 3*x^4 + 1), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 973 vs. 2(293) = 586.
time = 0.37, size = 973, normalized size = 2.16

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4+1)/(x^8+3*x^4+1),x, algorithm="fricas")
```

```
[Out] -1/80*sqrt(10)*(2*sqrt(5) + 6)^(3/4)*sqrt(sqrt(5) + 3)*(sqrt(5) - 3)*arctan
(1/400*sqrt(10)*sqrt(5)*sqrt(20*x^2 + sqrt(10)*(sqrt(5)*sqrt(2)*x - 5*sqrt(
2)*x)*(2*sqrt(5) + 6)^(1/4) - 5*sqrt(2*sqrt(5) + 6)*(sqrt(5) - 3))*(2*sqrt(
5) + 6)^(5/4)*(2*sqrt(5) - 5)*sqrt(sqrt(5) + 3) - 1/40*sqrt(10)*(2*sqrt(5)*
x - 5*x)*(2*sqrt(5) + 6)^(5/4)*sqrt(sqrt(5) + 3) + 1/8*(sqrt(5)*sqrt(2) - 3
*sqrt(2))*sqrt(2*sqrt(5) + 6)*sqrt(sqrt(5) + 3)) - 1/80*sqrt(10)*(2*sqrt(5)
 + 6)^(3/4)*sqrt(sqrt(5) + 3)*(sqrt(5) - 3)*arctan(1/400*sqrt(10)*sqrt(5)*s
qrt(20*x^2 - sqrt(10)*(sqrt(5)*sqrt(2)*x - 5*sqrt(2)*x)*(2*sqrt(5) + 6)^(1/
4) - 5*sqrt(2*sqrt(5) + 6)*(sqrt(5) - 3))*(2*sqrt(5) + 6)^(5/4)*(2*sqrt(5)
- 5)*sqrt(sqrt(5) + 3) - 1/40*sqrt(10)*(2*sqrt(5)*x - 5*x)*(2*sqrt(5) + 6)^
(5/4)*sqrt(sqrt(5) + 3) - 1/8*(sqrt(5)*sqrt(2) - 3*sqrt(2))*sqrt(2*sqrt(5)
```

```
+ 6)*sqrt(sqrt(5) + 3)) - 1/80*sqrt(10)*(sqrt(5) + 3)*sqrt(-sqrt(5) + 3)*(-
2*sqrt(5) + 6)^(3/4)*arctan(1/400*sqrt(10)*sqrt(5)*sqrt(20*x^2 + sqrt(10)*(
sqrt(5)*sqrt(2)*x + 5*sqrt(2)*x)*(-2*sqrt(5) + 6)^(1/4) + 5*(sqrt(5) + 3)*s
qrt(-2*sqrt(5) + 6))*(2*sqrt(5) + 5)*sqrt(-sqrt(5) + 3)*(-2*sqrt(5) + 6)^(5
/4) - 1/40*(sqrt(10)*(2*sqrt(5)*x + 5*x)*(-2*sqrt(5) + 6)^(5/4) + 5*(sqrt(5
)*sqrt(2) + 3*sqrt(2))*sqrt(-2*sqrt(5) + 6))*sqrt(-sqrt(5) + 3)) - 1/80*sqr
t(10)*(sqrt(5) + 3)*sqrt(-sqrt(5) + 3)*(-2*sqrt(5) + 6)^(3/4)*arctan(1/400*
sqrt(10)*sqrt(5)*sqrt(20*x^2 - sqrt(10)*(sqrt(5)*sqrt(2)*x + 5*sqrt(2)*x)*(
-2*sqrt(5) + 6)^(1/4) + 5*(sqrt(5) + 3)*sqrt(-2*sqrt(5) + 6))*(2*sqrt(5) +
5)*sqrt(-sqrt(5) + 3)*(-2*sqrt(5) + 6)^(5/4) - 1/40*(sqrt(10)*(2*sqrt(5)*x
+ 5*x)*(-2*sqrt(5) + 6)^(5/4) - 5*(sqrt(5)*sqrt(2) + 3*sqrt(2))*sqrt(-2*sqr
t(5) + 6))*sqrt(-sqrt(5) + 3)) - 1/80*sqrt(10)*sqrt(2)*(2*sqrt(5) + 6)^(1/4
)*log(400*x^2 + 20*sqrt(10)*(sqrt(5)*sqrt(2)*x - 5*sqrt(2)*x)*(2*sqrt(5) +
6)^(1/4) - 100*sqrt(2*sqrt(5) + 6)*(sqrt(5) - 3)) + 1/80*sqrt(10)*sqrt(2)*(
2*sqrt(5) + 6)^(1/4)*log(400*x^2 - 20*sqrt(10)*(sqrt(5)*sqrt(2)*x - 5*sqrt(
2)*x)*(2*sqrt(5) + 6)^(1/4) - 100*sqrt(2*sqrt(5) + 6)*(sqrt(5) - 3)) + 1/80
*sqrt(10)*sqrt(2)*(-2*sqrt(5) + 6)^(1/4)*log(400*x^2 + 20*sqrt(10)*(sqrt(5)
*sqrt(2)*x + 5*sqrt(2)*x)*(-2*sqrt(5) + 6)^(1/4) + 100*(sqrt(5) + 3)*sqrt(-
2*sqrt(5) + 6)) - 1/80*sqrt(10)*sqrt(2)*(-2*sqrt(5) + 6)^(1/4)*log(400*x^2
- 20*sqrt(10)*(sqrt(5)*sqrt(2)*x + 5*sqrt(2)*x)*(-2*sqrt(5) + 6)^(1/4) + 10
0*(sqrt(5) + 3)*sqrt(-2*sqrt(5) + 6))
```

<u>Sympy</u> **[A]**
time = 0.88, size = 24, normalized size = 0.05

$$\operatorname{RootSum}\left(40960000t^8 + 19200t^4 + 1, \left(t \mapsto t\log\left(25600t^5 + 16t + x\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**4+1)/(x**8+3*x**4+1),x)
```

```
[Out] RootSum(40960000*_t**8 + 19200*_t**4 + 1, Lambda(_t, _t*log(25600*_t**5 + 1
6*_t + x)))
```

<u>Giac</u> **[A]**
time = 4.57, size = 239, normalized size = 0.53

$$\frac{1}{80}\left(\pi + 4\arctan\left(x\sqrt{\sqrt{5}+1}+1\right)\right)\sqrt{5\sqrt{5}+5} - \frac{1}{80}\left(\pi + 4\arctan\left(-x\sqrt{\sqrt{5}+1}+1\right)\right)\sqrt{5\sqrt{5}+5} + \frac{1}{80}\left(\pi + 4\arctan\left(x\sqrt{\sqrt{5}-1}-1\right)\right)\sqrt{5\sqrt{5}-5} - \frac{1}{80}\left(\pi + 4\arctan\left(-x\sqrt{\sqrt{5}-1}-1\right)\right)\sqrt{5\sqrt{5}-5} + \frac{1}{40}\sqrt{5\sqrt{5}-5}\log\left(16900\left(x+\sqrt{\sqrt{5}+1}\right)^2 + 16900x^2\right) - \frac{1}{40}\sqrt{5\sqrt{5}-5}\log\left(16900\left(x-\sqrt{\sqrt{5}+1}\right)^2 + 16900x^2\right) + \frac{1}{40}\sqrt{5\sqrt{5}+5}\log\left(2500\left(x+\sqrt{\sqrt{5}-1}\right)^2 + 2500x^2\right) - \frac{1}{40}\sqrt{5\sqrt{5}+5}\log\left(2500\left(x-\sqrt{\sqrt{5}-1}\right)^2 + 2500x^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4+1)/(x^8+3*x^4+1),x, algorithm="giac")
```

```
[Out] 1/80*(pi + 4*arctan(x*sqrt(sqrt(5) + 1) + 1))*sqrt(5*sqrt(5) + 5) - 1/80*(p
i + 4*arctan(-x*sqrt(sqrt(5) + 1) + 1))*sqrt(5*sqrt(5) + 5) + 1/80*(pi + 4*
arctan(x*sqrt(sqrt(5) - 1) - 1))*sqrt(5*sqrt(5) - 5) - 1/80*(pi + 4*arctan(
-x*sqrt(sqrt(5) - 1) - 1))*sqrt(5*sqrt(5) - 5) + 1/40*sqrt(5*sqrt(5) - 5)*l
og(16900*(x + sqrt(sqrt(5) + 1))^2 + 16900*x^2) - 1/40*sqrt(5*sqrt(5) - 5)*
```

```
log(16900*(x - sqrt(sqrt(5) + 1))^2 + 16900*x^2) + 1/40*sqrt(5*sqrt(5) + 5)
*log(2500*(x + sqrt(sqrt(5) - 1))^2 + 2500*x^2) - 1/40*sqrt(5*sqrt(5) + 5)*
log(2500*(x - sqrt(sqrt(5) - 1))^2 + 2500*x^2)
```

Mupad [B]
time = 0.18, size = 459, normalized size = 1.02

$$\frac{2^{3/4}\,\sqrt{5}\,\mathrm{atan}\left(\frac{7\,2^{3/4}\,x\,{\left(-\sqrt{5}-3\right)}^{1/4}}{2\,\left(2\,\sqrt{2}\,\sqrt{-\sqrt{5}-3}+\sqrt{2}\,\sqrt{5}\,\sqrt{-\sqrt{5}-3}\right)}+\frac{3\,2^{3/4}\,\sqrt{5}\,x\,{\left(-\sqrt{5}-3\right)}^{1/4}}{2\,\left(2\,\sqrt{2}\,\sqrt{-\sqrt{5}-3}+\sqrt{2}\,\sqrt{5}\,\sqrt{-\sqrt{5}-3}\right)}\right)\,{\left(-\sqrt{5}-3\right)}^{1/4}}{20}-\frac{2^{3/4}\,\sqrt{5}\,\mathrm{atan}\left(\frac{2^{3/4}\,x\,{\left(-\sqrt{5}-3\right)}^{1/4}\,7{}\mathrm{i}}{2\,\left(2\,\sqrt{2}\,\sqrt{-\sqrt{5}-3}+\sqrt{2}\,\sqrt{5}\,\sqrt{-\sqrt{5}-3}\right)}+\frac{2^{3/4}\,\sqrt{5}\,x\,{\left(-\sqrt{5}-3\right)}^{1/4}\,3{}\mathrm{i}}{2\,\left(2\,\sqrt{2}\,\sqrt{-\sqrt{5}-3}+\sqrt{2}\,\sqrt{5}\,\sqrt{-\sqrt{5}-3}\right)}\right)\,{\left(-\sqrt{5}-3\right)}^{1/4}\,1{}\mathrm{i}}{20}-\frac{2^{3/4}\,\sqrt{5}\,\mathrm{atan}\left(\frac{7\,2^{3/4}\,x\,{\left(\sqrt{5}-3\right)}^{1/4}}{2\,\left(2\,\sqrt{2}\,\sqrt{\sqrt{5}-3}-\sqrt{2}\,\sqrt{5}\,\sqrt{\sqrt{5}-3}\right)}-\frac{3\,2^{3/4}\,\sqrt{5}\,x\,{\left(\sqrt{5}-3\right)}^{1/4}}{2\,\left(2\,\sqrt{2}\,\sqrt{\sqrt{5}-3}-\sqrt{2}\,\sqrt{5}\,\sqrt{\sqrt{5}-3}\right)}\right)\,{\left(\sqrt{5}-3\right)}^{1/4}}{20}+\frac{2^{3/4}\,\sqrt{5}\,\mathrm{atan}\left(\frac{2^{3/4}\,x\,{\left(\sqrt{5}-3\right)}^{1/4}\,7{}\mathrm{i}}{2\,\left(2\,\sqrt{2}\,\sqrt{\sqrt{5}-3}-\sqrt{2}\,\sqrt{5}\,\sqrt{\sqrt{5}-3}\right)}-\frac{2^{3/4}\,\sqrt{5}\,x\,{\left(\sqrt{5}-3\right)}^{1/4}\,3{}\mathrm{i}}{2\,\left(2\,\sqrt{2}\,\sqrt{\sqrt{5}-3}-\sqrt{2}\,\sqrt{5}\,\sqrt{\sqrt{5}-3}\right)}\right)\,{\left(\sqrt{5}-3\right)}^{1/4}\,1{}\mathrm{i}}{20}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4 + 1)/(3*x^4 + x^8 + 1),x)
```

```
[Out] (2^(3/4)*5^(1/2)*atan((7*2^(3/4)*x*(- 5^(1/2) - 3)^(1/4))/(2*(2*2^(1/2)*(-
5^(1/2) - 3)^(1/2) + 2^(1/2)*5^(1/2)*(- 5^(1/2) - 3)^(1/2))) + (3*2^(3/4)*5
^(1/2)*x*(- 5^(1/2) - 3)^(1/4))/(2*(2*2^(1/2)*(- 5^(1/2) - 3)^(1/2) + 2^(1/
2)*5^(1/2)*(- 5^(1/2) - 3)^(1/2))))*(- 5^(1/2) - 3)^(1/4))/20 - (2^(3/4)*5^
(1/2)*atan((2^(3/4)*x*(- 5^(1/2) - 3)^(1/4)*7i)/(2*(2*2^(1/2)*(- 5^(1/2) -
3)^(1/2) + 2^(1/2)*5^(1/2)*(- 5^(1/2) - 3)^(1/2))) + (2^(3/4)*5^(1/2)*x*(-
5^(1/2) - 3)^(1/4)*3i)/(2*(2*2^(1/2)*(- 5^(1/2) - 3)^(1/2) + 2^(1/2)*5^(1/2
)*(- 5^(1/2) - 3)^(1/2))))*(- 5^(1/2) - 3)^(1/4)*1i)/20 - (2^(3/4)*5^(1/2)*
atan((7*2^(3/4)*x*(5^(1/2) - 3)^(1/4))/(2*(2*2^(1/2)*(5^(1/2) - 3)^(1/2) -
2^(1/2)*5^(1/2)*(5^(1/2) - 3)^(1/2))) - (3*2^(3/4)*5^(1/2)*x*(5^(1/2) - 3)^
(1/4))/(2*(2*2^(1/2)*(5^(1/2) - 3)^(1/2) - 2^(1/2)*5^(1/2)*(5^(1/2) - 3)^(1
/2))))*(5^(1/2) - 3)^(1/4))/20 + (2^(3/4)*5^(1/2)*atan((2^(3/4)*x*(5^(1/2)
- 3)^(1/4)*7i)/(2*(2*2^(1/2)*(5^(1/2) - 3)^(1/2) - 2^(1/2)*5^(1/2)*(5^(1/2)
 - 3)^(1/2))) - (2^(3/4)*5^(1/2)*x*(5^(1/2) - 3)^(1/4)*3i)/(2*(2*2^(1/2)*(5
^(1/2) - 3)^(1/2) - 2^(1/2)*5^(1/2)*(5^(1/2) - 3)^(1/2))))*(5^(1/2) - 3)^(1
/4)*1i)/20
```

3.11 $\int \frac{1+x^4}{1+2x^4+x^8}\,dx$

Optimal. Leaf size=85

$$-\frac{\tan^{-1}\left(1-\sqrt{2}\,x\right)}{2\sqrt{2}}+\frac{\tan^{-1}\left(1+\sqrt{2}\,x\right)}{2\sqrt{2}}-\frac{\log\left(1-\sqrt{2}\,x+x^2\right)}{4\sqrt{2}}+\frac{\log\left(1+\sqrt{2}\,x+x^2\right)}{4\sqrt{2}}$$

```
[Out] 1/4*arctan(-1+x*2^(1/2))*2^(1/2)+1/4*arctan(1+x*2^(1/2))*2^(1/2)-1/8*ln(1+x
^2-x*2^(1/2))*2^(1/2)+1/8*ln(1+x^2+x*2^(1/2))*2^(1/2)
```

Rubi [A]
time = 0.03, antiderivative size = 85, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 7, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.389, Rules used = {28, 217, 1179, 642, 1176, 631, 210}

$$-\frac{\text{ArcTan}\left(1-\sqrt{2}\,x\right)}{2\sqrt{2}}+\frac{\text{ArcTan}\left(\sqrt{2}\,x+1\right)}{2\sqrt{2}}-\frac{\log\left(x^2-\sqrt{2}\,x+1\right)}{4\sqrt{2}}+\frac{\log\left(x^2+\sqrt{2}\,x+1\right)}{4\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + x^4)/(1 + 2*x^4 + x^8),x]
```

```
[Out] -1/2*ArcTan[1 - Sqrt[2]*x]/Sqrt[2] + ArcTan[1 + Sqrt[2]*x]/(2*Sqrt[2]) - Lo
g[1 - Sqrt[2]*x + x^2]/(4*Sqrt[2]) + Log[1 + Sqrt[2]*x + x^2]/(4*Sqrt[2])
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$
\begin{aligned}
\int \frac{1+x^4}{1+2x^4+x^8}\,dx &= \int \frac{1}{1+x^4}\,dx \\
&= \frac{1}{2}\int \frac{1-x^2}{1+x^4}\,dx + \frac{1}{2}\int \frac{1+x^2}{1+x^4}\,dx \\
&= \frac{1}{4}\int \frac{1}{1-\sqrt{2}\,x+x^2}\,dx + \frac{1}{4}\int \frac{1}{1+\sqrt{2}\,x+x^2}\,dx - \frac{\int \frac{\sqrt{2}+2x}{-1-\sqrt{2}\,x-x^2}\,dx}{4\sqrt{2}} - \frac{\int \frac{\sqrt{2}-2x}{-1+\sqrt{2}\,x-\dots}}{4\sqrt{2}} \\
&= -\frac{\log\left(1-\sqrt{2}\,x+x^2\right)}{4\sqrt{2}} + \frac{\log\left(1+\sqrt{2}\,x+x^2\right)}{4\sqrt{2}} + \frac{\mathrm{Subst}\left(\int \frac{1}{-1-x^2}\,dx, x, 1-\sqrt{2}\,x\right)}{2\sqrt{2}} \dots \\
&= -\frac{\tan^{-1}\left(1-\sqrt{2}\,x\right)}{2\sqrt{2}} + \frac{\tan^{-1}\left(1+\sqrt{2}\,x\right)}{2\sqrt{2}} - \frac{\log\left(1-\sqrt{2}\,x+x^2\right)}{4\sqrt{2}} + \frac{\log\left(1+\sqrt{2}\,x \dots\right.}{4\sqrt{2}}
\end{aligned}
$$

Mathematica [A]

time = 0.01, size = 64, normalized size = 0.75

$$\frac{-2\tan^{-1}\left(1-\sqrt{2}\,x\right)+2\tan^{-1}\left(1+\sqrt{2}\,x\right)-\log\left(1-\sqrt{2}\,x+x^2\right)+\log\left(1+\sqrt{2}\,x+x^2\right)}{4\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + x^4)/(1 + 2*x^4 + x^8),x]
```

```
[Out] (-2*ArcTan[1 - Sqrt[2]*x] + 2*ArcTan[1 + Sqrt[2]*x] - Log[1 - Sqrt[2]*x + x
^2] + Log[1 + Sqrt[2]*x + x^2])/(4*Sqrt[2])
```

Maple [A]
time = 0.03, size = 52, normalized size = 0.61

method	result	size
risch	$\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(\_Z^4+1\right)}\frac{\ln\left(x-\_R\right)}{\_R^3}\right)}{4}$	22
default	$\frac{\sqrt{2}\,\left(\ln\left(\frac{1+x^2+\sqrt{2}\,x}{1+x^2-\sqrt{2}\,x}\right)+2\arctan\left(\sqrt{2}\,x+1\right)+2\arctan\left(\sqrt{2}\,x-1\right)\right)}{8}$	52

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4+1)/(x^8+2*x^4+1),x,method=_RETURNVERBOSE)
```

```
[Out] 1/8*2^(1/2)*(ln((1+x^2+2^(1/2)*x)/(1+x^2-2^(1/2)*x))+2*arctan(2^(1/2)*x+1)+
2*arctan(2^(1/2)*x-1))
```

Maxima [A]
time = 0.50, size = 72, normalized size = 0.85

$$\frac{1}{4}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(2x+\sqrt{2}\right)\right)+\frac{1}{4}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(2x-\sqrt{2}\right)\right)+\frac{1}{8}\sqrt{2}\log\left(x^2+\sqrt{2}\,x+1\right)-\frac{1}{8}\sqrt{2}\log\left(x^2-\sqrt{2}\,x+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4+1)/(x^8+2*x^4+1),x, algorithm="maxima")
```

```
[Out] 1/4*sqrt(2)*arctan(1/2*sqrt(2)*(2*x + sqrt(2))) + 1/4*sqrt(2)*arctan(1/2*sq
rt(2)*(2*x - sqrt(2))) + 1/8*sqrt(2)*log(x^2 + sqrt(2)*x + 1) - 1/8*sqrt(2)
*log(x^2 - sqrt(2)*x + 1)
```

Fricas [A]
time = 0.36, size = 100, normalized size = 1.18

$$-\frac{1}{2}\sqrt{2}\arctan\left(-\sqrt{2}\,x+\sqrt{2}\sqrt{x^2+\sqrt{2}\,x+1}-1\right)-\frac{1}{2}\sqrt{2}\arctan\left(-\sqrt{2}\,x+\sqrt{2}\sqrt{x^2-\sqrt{2}\,x+1}+1\right)+\frac{1}{8}\sqrt{2}\log\left(4x^2+4\sqrt{2}\,x+4\right)-\frac{1}{8}\sqrt{2}\log\left(4x^2-4\sqrt{2}\,x+4\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4+1)/(x^8+2*x^4+1),x, algorithm="fricas")
```

```
[Out] -1/2*sqrt(2)*arctan(-sqrt(2)*x + sqrt(2)*sqrt(x^2 + sqrt(2)*x + 1) - 1) - 1
/2*sqrt(2)*arctan(-sqrt(2)*x + sqrt(2)*sqrt(x^2 - sqrt(2)*x + 1) + 1) + 1/8
*sqrt(2)*log(4*x^2 + 4*sqrt(2)*x + 4) - 1/8*sqrt(2)*log(4*x^2 - 4*sqrt(2)*x
 + 4)
```

Sympy [A]
time = 0.05, size = 73, normalized size = 0.86

$$-\frac{\sqrt{2}\log\left(x^2-\sqrt{2}\,x+1\right)}{8}+\frac{\sqrt{2}\log\left(x^2+\sqrt{2}\,x+1\right)}{8}+\frac{\sqrt{2}\operatorname{atan}\left(\sqrt{2}\,x-1\right)}{4}+\frac{\sqrt{2}\operatorname{atan}\left(\sqrt{2}\,x+1\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**4+1)/(x**8+2*x**4+1),x)
```

```
[Out] -sqrt(2)*log(x**2 - sqrt(2)*x + 1)/8 + sqrt(2)*log(x**2 + sqrt(2)*x + 1)/8
+ sqrt(2)*atan(sqrt(2)*x - 1)/4 + sqrt(2)*atan(sqrt(2)*x + 1)/4
```

Giac [A]
time = 3.59, size = 72, normalized size = 0.85

$$\frac{1}{4}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(2x+\sqrt{2}\right)\right)+\frac{1}{4}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(2x-\sqrt{2}\right)\right)+\frac{1}{8}\sqrt{2}\log\left(x^2+\sqrt{2}\,x+1\right)-\frac{1}{8}\sqrt{2}\log\left(x^2-\sqrt{2}\,x+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4+1)/(x^8+2*x^4+1),x, algorithm="giac")
```

```
[Out] 1/4*sqrt(2)*arctan(1/2*sqrt(2)*(2*x + sqrt(2))) + 1/4*sqrt(2)*arctan(1/2*sq
rt(2)*(2*x - sqrt(2))) + 1/8*sqrt(2)*log(x^2 + sqrt(2)*x + 1) - 1/8*sqrt(2)
*log(x^2 - sqrt(2)*x + 1)
```

Mupad [B]
time = 1.56, size = 33, normalized size = 0.39

$$\sqrt{2}\operatorname{atan}\left(\sqrt{2}\,x\left(\frac{1}{2}-\frac{1}{2}\mathrm{i}\right)\right)\left(\frac{1}{4}+\frac{1}{4}\mathrm{i}\right)+\sqrt{2}\operatorname{atan}\left(\sqrt{2}\,x\left(\frac{1}{2}+\frac{1}{2}\mathrm{i}\right)\right)\left(\frac{1}{4}-\frac{1}{4}\mathrm{i}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4 + 1)/(2*x^4 + x^8 + 1),x)
```

```
[Out] 2^(1/2)*atan(2^(1/2)*x*(1/2 - 1i/2))*(1/4 + 1i/4) + 2^(1/2)*atan(2^(1/2)*x*
(1/2 + 1i/2))*(1/4 - 1i/4)
```

3.12 $\int \frac{1+x^4}{1+x^4+x^8}\,dx$

Optimal. Leaf size=140

$$-\frac{\tan^{-1}\left(\frac{1-2x}{\sqrt{3}}\right)}{4\sqrt{3}}-\frac{1}{4}\tan^{-1}\left(\sqrt{3}-2x\right)+\frac{\tan^{-1}\left(\frac{1+2x}{\sqrt{3}}\right)}{4\sqrt{3}}+\frac{1}{4}\tan^{-1}\left(\sqrt{3}+2x\right)-\frac{1}{8}\log\left(1-x+x^2\right)+\frac{1}{8}\log\left(1\right.$$

```
[Out] 1/4*arctan(2*x-3^(1/2))+1/4*arctan(2*x+3^(1/2))-1/8*ln(x^2-x+1)+1/8*ln(x^2+
x+1)-1/12*arctan(1/3*(1-2*x)*3^(1/2))*3^(1/2)+1/12*arctan(1/3*(1+2*x)*3^(1/
2))*3^(1/2)-1/24*ln(1+x^2-x*3^(1/2))*3^(1/2)+1/24*ln(1+x^2+x*3^(1/2))*3^(1/
2)
```

Rubi [A]
time = 0.06, antiderivative size = 140, normalized size of antiderivative = 1.00, number of steps used = 19, number of rules used = 6, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$, Rules used = {1433, 1108, 648, 632, 210, 642}

$$-\frac{\text{ArcTan}\left(\frac{1-2x}{\sqrt{3}}\right)}{4\sqrt{3}}-\frac{1}{4}\text{ArcTan}\left(\sqrt{3}-2x\right)+\frac{\text{ArcTan}\left(\frac{2x+1}{\sqrt{3}}\right)}{4\sqrt{3}}+\frac{1}{4}\text{ArcTan}\left(2x+\sqrt{3}\right)-\frac{1}{8}\log\left(x^2-x+1\right)+\frac{1}{8}\log\left(x^2+x+1\right)-\frac{\log\left(x^2-\sqrt{3}x+1\right)}{8\sqrt{3}}+\frac{\log\left(x^2+\sqrt{3}x+1\right)}{8\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + x^4)/(1 + x^4 + x^8),x]
```

```
[Out] -1/4*ArcTan[(1 - 2*x)/Sqrt[3]]/Sqrt[3] - ArcTan[Sqrt[3] - 2*x]/4 + ArcTan[(
1 + 2*x)/Sqrt[3]]/(4*Sqrt[3]) + ArcTan[Sqrt[3] + 2*x]/4 - Log[1 - x + x^2]/
8 + Log[1 + x + x^2]/8 - Log[1 - Sqrt[3]*x + x^2]/(8*Sqrt[3]) + Log[1 + Sqr
t[3]*x + x^2]/(8*Sqrt[3])
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1108

```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(-1), x_Symbol] :> With[{q = Rt[a/
c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Dist[1/(2*c*q*r), Int[(r - x)/(q - r*x
 + x^2), x], x] + Dist[1/(2*c*q*r), Int[(r + x)/(q + r*x + x^2), x], x]]] /
; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && NegQ[b^2 - 4*a*c]
```

Rule 1433

```
Int[((d_) + (e_.)*(x_)^(n_))/((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_)), x
_Symbol] :> With[{q = Rt[2*(d/e) - b/c, 2]}, Dist[e/(2*c), Int[1/Simp[d/e +
 q*x^(n/2) + x^n, x], x], x] + Dist[e/(2*c), Int[1/Simp[d/e - q*x^(n/2) + x
^n, x], x], x]] /; FreeQ[{a, b, c, d, e}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4
*a*c, 0] && EqQ[c*d^2 - a*e^2, 0] && IGtQ[n/2, 0] && (GtQ[2*(d/e) - b/c, 0]
 || ( !LtQ[2*(d/e) - b/c, 0] && EqQ[d, e*Rt[a/c, 2]]))
```

Rubi steps

$$\begin{aligned}
\int \frac{1+x^4}{1+x^4+x^8}\,dx &= \frac{1}{2}\int \frac{1}{1-x^2+x^4}\,dx+\frac{1}{2}\int \frac{1}{1+x^2+x^4}\,dx\\
&= \frac{1}{4}\int \frac{1-x}{1-x+x^2}\,dx+\frac{1}{4}\int \frac{1+x}{1+x+x^2}\,dx+\frac{\int \frac{\sqrt{3}-x}{1-\sqrt{3}\,x+x^2}\,dx}{4\sqrt{3}}+\frac{\int \frac{\sqrt{3}+x}{1+\sqrt{3}\,x+x^2}\,dx}{4\sqrt{3}}\\
&= \frac{1}{8}\int \frac{1}{1-x+x^2}\,dx-\frac{1}{8}\int \frac{-1+2x}{1-x+x^2}\,dx+\frac{1}{8}\int \frac{1}{1+x+x^2}\,dx+\frac{1}{8}\int \frac{1+2x}{1+x+x^2}\,dx+\\
&= -\frac{1}{8}\log\left(1-x+x^2\right)+\frac{1}{8}\log\left(1+x+x^2\right)-\frac{\log\left(1-\sqrt{3}\,x+x^2\right)}{8\sqrt{3}}+\frac{\log\left(1+\sqrt{3}\,x+\right.}{8\sqrt{3}}\\
&= -\frac{\tan^{-1}\left(\frac{1-2x}{\sqrt{3}}\right)}{4\sqrt{3}}-\frac{1}{4}\tan^{-1}\left(\sqrt{3}-2x\right)+\frac{\tan^{-1}\left(\frac{1+2x}{\sqrt{3}}\right)}{4\sqrt{3}}+\frac{1}{4}\tan^{-1}\left(\sqrt{3}+2x\right)-\frac{1}{8}
\end{aligned}$$

Mathematica [C] Result contains complex when optimal does not.
time = 0.12, size = 135, normalized size = 0.96

$$\frac{1}{48}\left(4i\sqrt{-6-6i\sqrt{3}}\,\tan^{-1}\left(\frac{1}{2}\left(1-i\sqrt{3}\right)x\right)-4i\sqrt{-6+6i\sqrt{3}}\,\tan^{-1}\left(\frac{1}{2}\left(1+i\sqrt{3}\right)x\right)+4\sqrt{3}\,\tan^{-1}\left(\frac{-1+2x}{\sqrt{3}}\right)+4\sqrt{3}\,\tan^{-1}\left(\frac{1+2x}{\sqrt{3}}\right)-6\log\left(1-x+x^2\right)+6\log\left(1+x+x^2\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + x^4)/(1 + x^4 + x^8),x]
```

```
[Out] ((4*I)*Sqrt[-6 - (6*I)*Sqrt[3]]*ArcTan[((1 - I*Sqrt[3])*x)/2] - (4*I)*Sqrt[
-6 + (6*I)*Sqrt[3]]*ArcTan[((1 + I*Sqrt[3])*x)/2] + 4*Sqrt[3]*ArcTan[(-1 +
2*x)/Sqrt[3]] + 4*Sqrt[3]*ArcTan[(1 + 2*x)/Sqrt[3]] - 6*Log[1 - x + x^2] +
6*Log[1 + x + x^2])/48
```

Maple [A]
time = 0.05, size = 109, normalized size = 0.78

method	result
risch	$-\frac{\ln(4x^2-4x+4)}{8}+\frac{\sqrt{3}\,\arctan\left(\frac{(2x-1)\sqrt{3}}{3}\right)}{12}+\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(9\_Z^4+3\_Z^2+1\right)}\_R\ln\left(-3\_R^3+\_R+x\right)\right)}{4}+\frac{\ln(4x^2+4x}{8}$
default	$\frac{\ln(x^2+x+1)}{8}+\frac{\arctan\left(\frac{(2x+1)\sqrt{3}}{3}\right)\sqrt{3}}{12}+\frac{\arctan\left(2x-\sqrt{3}\right)}{4}+\frac{\arctan\left(2x+\sqrt{3}\right)}{4}-\frac{\ln\left(1+x^2-x\sqrt{3}\right)\sqrt{3}}{24}+\frac{\ln(}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4+1)/(x^8+x^4+1),x,method=_RETURNVERBOSE)
```

```
[Out] 1/8*ln(x^2+x+1)+1/12*arctan(1/3*(2*x+1)*3^(1/2))*3^(1/2)+1/4*arctan(2*x-3^(
1/2))+1/4*arctan(2*x+3^(1/2))-1/24*ln(1+x^2-x*3^(1/2))*3^(1/2)+1/24*ln(1+x^
2+x*3^(1/2))*3^(1/2)-1/8*ln(x^2-x+1)+1/12*3^(1/2)*arctan(1/3*(2*x-1)*3^(1/2
))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4+1)/(x^8+x^4+1),x, algorithm="maxima")
```

```
[Out] 1/12*sqrt(3)*arctan(1/3*sqrt(3)*(2*x + 1)) + 1/12*sqrt(3)*arctan(1/3*sqrt(3
)*(2*x - 1)) + 1/2*integrate(1/(x^4 - x^2 + 1), x) + 1/8*log(x^2 + x + 1) -
 1/8*log(x^2 - x + 1)
```

Fricas [A]
time = 0.36, size = 215, normalized size = 1.54

$$-\frac{1}{12}\sqrt{6}\sqrt{3}\sqrt{2}\arctan\left(-\frac{1}{3}\sqrt{6}\sqrt{3}\sqrt{2}x+\frac{1}{36}\sqrt{6}\sqrt{3}\sqrt{2}\sqrt{-72\sqrt{6}\sqrt{2}x+144x^2+144}+\sqrt{3}\right)-\frac{1}{12}\sqrt{6}\sqrt{3}\sqrt{2}\arctan\left(-\frac{1}{3}\sqrt{6}\sqrt{3}\sqrt{2}x+\frac{1}{3}\sqrt{6}\sqrt{3}\sqrt{\sqrt{6}\sqrt{2}x+2x^2+2}-\sqrt{3}\right)+\frac{1}{48}\sqrt{6}\sqrt{2}\log\left(72\sqrt{6}\sqrt{2}x+144x^2+144\right)-\frac{1}{48}\sqrt{6}\sqrt{2}\log\left(-72\sqrt{6}\sqrt{2}x+144x^2+144\right)+\frac{1}{12}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x+1)\right)+\frac{1}{12}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x-1)\right)+\frac{1}{8}\log\left(x^2+x+1\right)-\frac{1}{8}\log\left(x^2-x+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4+1)/(x^8+x^4+1),x, algorithm="fricas")
```

```
[Out] -1/12*sqrt(6)*sqrt(3)*sqrt(2)*arctan(-1/3*sqrt(6)*sqrt(3)*sqrt(2)*x + 1/36*
sqrt(6)*sqrt(3)*sqrt(2)*sqrt(-72*sqrt(6)*sqrt(2)*x + 144*x^2 + 144) + sqrt(
3)) - 1/12*sqrt(6)*sqrt(3)*sqrt(2)*arctan(-1/3*sqrt(6)*sqrt(3)*sqrt(2)*x +
1/3*sqrt(6)*sqrt(3)*sqrt(sqrt(6)*sqrt(2)*x + 2*x^2 + 2) - sqrt(3)) + 1/48*s
qrt(6)*sqrt(2)*log(72*sqrt(6)*sqrt(2)*x + 144*x^2 + 144) - 1/48*sqrt(6)*sqr
t(2)*log(-72*sqrt(6)*sqrt(2)*x + 144*x^2 + 144) + 1/12*sqrt(3)*arctan(1/3*s
qrt(3)*(2*x + 1)) + 1/12*sqrt(3)*arctan(1/3*sqrt(3)*(2*x - 1)) + 1/8*log(x^
2 + x + 1) - 1/8*log(x^2 - x + 1)
```

Sympy [C] Result contains complex when optimal does not.
time = 0.36, size = 190, normalized size = 1.36

$$\left(-\frac{1}{8}-\frac{\sqrt{3}\,i}{24}\right)\log\left(x-1-\frac{\sqrt{3}\,i}{3}+9216\left(-\frac{1}{8}-\frac{\sqrt{3}\,i}{24}\right)^{5}\right)+\left(-\frac{1}{8}+\frac{\sqrt{3}\,i}{24}\right)\log\left(x-1+9216\left(-\frac{1}{8}+\frac{\sqrt{3}\,i}{24}\right)^{5}+\frac{\sqrt{3}\,i}{3}\right)+\left(\frac{1}{8}-\frac{\sqrt{3}\,i}{24}\right)\log\left(x+1-\frac{\sqrt{3}\,i}{3}+9216\left(\frac{1}{8}-\frac{\sqrt{3}\,i}{24}\right)^{5}\right)+\left(\frac{1}{8}+\frac{\sqrt{3}\,i}{24}\right)\log\left(x+1+9216\left(\frac{1}{8}+\frac{\sqrt{3}\,i}{24}\right)^{5}+\frac{\sqrt{3}\,i}{3}\right)+\operatorname{RootSum}\left(2304t^{4}+48t^{2}+1,\left(t\mapsto t\log\left(9216t^{5}+8t+x\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**4+1)/(x**8+x**4+1),x)
```

```
[Out] (-1/8 - sqrt(3)*I/24)*log(x - 1 - sqrt(3)*I/3 + 9216*(-1/8 - sqrt(3)*I/24)*
*5) + (-1/8 + sqrt(3)*I/24)*log(x - 1 + 9216*(-1/8 + sqrt(3)*I/24)**5 + sqr
t(3)*I/3) + (1/8 - sqrt(3)*I/24)*log(x + 1 - sqrt(3)*I/3 + 9216*(1/8 - sqrt
(3)*I/24)**5) + (1/8 + sqrt(3)*I/24)*log(x + 1 + 9216*(1/8 + sqrt(3)*I/24)*
*5 + sqrt(3)*I/3) + RootSum(2304*_t**4 + 48*_t**2 + 1, Lambda(_t, _t*log(92
16*_t**5 + 8*_t + x)))
```

Giac [A]
time = 3.29, size = 108, normalized size = 0.77

$$\frac{1}{12}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\,(2x+1)\right)+\frac{1}{12}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\,(2x-1)\right)+\frac{1}{24}\sqrt{3}\log\left(x^{2}+\sqrt{3}\,x+1\right)-\frac{1}{24}\sqrt{3}\log\left(x^{2}-\sqrt{3}\,x+1\right)+\frac{1}{4}\arctan\left(2x+\sqrt{3}\right)+\frac{1}{4}\arctan\left(2x-\sqrt{3}\right)+\frac{1}{8}\log\left(x^{2}+x+1\right)-\frac{1}{8}\log\left(x^{2}-x+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4+1)/(x^8+x^4+1),x, algorithm="giac")
```

```
[Out] 1/12*sqrt(3)*arctan(1/3*sqrt(3)*(2*x + 1)) + 1/12*sqrt(3)*arctan(1/3*sqrt(3
)*(2*x - 1)) + 1/24*sqrt(3)*log(x^2 + sqrt(3)*x + 1) - 1/24*sqrt(3)*log(x^2
 - sqrt(3)*x + 1) + 1/4*arctan(2*x + sqrt(3)) + 1/4*arctan(2*x - sqrt(3)) +
 1/8*log(x^2 + x + 1) - 1/8*log(x^2 - x + 1)
```

Mupad [B]
time = 0.14, size = 95, normalized size = 0.68

$$\operatorname{atan}\left(\frac{2\,x}{-1+\sqrt{3}\,1\mathrm{i}}\right)\,\left(-\frac{1}{4}+\frac{\sqrt{3}\,1\mathrm{i}}{12}\right)+\operatorname{atan}\left(\frac{2\,x}{1+\sqrt{3}\,1\mathrm{i}}\right)\,\left(\frac{1}{4}+\frac{\sqrt{3}\,1\mathrm{i}}{12}\right)+\operatorname{atan}\left(\frac{x\,2\mathrm{i}}{-1+\sqrt{3}\,1\mathrm{i}}\right)\,\left(\frac{\sqrt{3}}{12}+\frac{1}{4}\mathrm{i}\right)+\operatorname{atan}\left(\frac{x\,2\mathrm{i}}{1+\sqrt{3}\,1\mathrm{i}}\right)\,\left(\frac{\sqrt{3}}{12}-\frac{1}{4}\mathrm{i}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4 + 1)/(x^4 + x^8 + 1),x)
```

```
[Out] atan((2*x)/(3^(1/2)*1i - 1))*((3^(1/2)*1i)/12 - 1/4) + atan((2*x)/(3^(1/2)*
1i + 1))*((3^(1/2)*1i)/12 + 1/4) + atan((x*2i)/(3^(1/2)*1i - 1))*(3^(1/2)/1
2 + 1i/4) + atan((x*2i)/(3^(1/2)*1i + 1))*(3^(1/2)/12 - 1i/4)
```

3.13 $\int \frac{1+x^4}{1+x^8}\,dx$

Optimal. Leaf size=347

$$-\frac{1}{4}\sqrt{\frac{1}{2}\left(2-\sqrt{2}\right)}\tan^{-1}\left(\frac{\sqrt{2-\sqrt{2}}-2x}{\sqrt{2+\sqrt{2}}}\right)-\frac{1}{4}\sqrt{\frac{1}{2}\left(2+\sqrt{2}\right)}\tan^{-1}\left(\frac{\sqrt{2+\sqrt{2}}-2x}{\sqrt{2-\sqrt{2}}}\right)+\frac{1}{4}\sqrt{\frac{1}{2}\left(2-\right.}$$

```
[Out] -1/8*arctan((-2*x+(2-2^(1/2))^(1/2))/(2+2^(1/2))^(1/2))*(4-2*2^(1/2))^(1/2)
+1/8*arctan((2*x+(2-2^(1/2))^(1/2))/(2+2^(1/2))^(1/2))*(4-2*2^(1/2))^(1/2)-
1/8*ln(1+x^2-x*(2-2^(1/2))^(1/2))/(2-2^(1/2))^(1/2)+1/8*ln(1+x^2+x*(2-2^(1/
2))^(1/2))/(2-2^(1/2))^(1/2)-1/8*arctan((-2*x+(2+2^(1/2))^(1/2))/(2-2^(1/2)
)^(1/2))*(4+2*2^(1/2))^(1/2)+1/8*arctan((2*x+(2+2^(1/2))^(1/2))/(2-2^(1/2))
^(1/2))*(4+2*2^(1/2))^(1/2)-1/8*ln(1+x^2-x*(2+2^(1/2))^(1/2))/(2+2^(1/2))^(
1/2)+1/8*ln(1+x^2+x*(2+2^(1/2))^(1/2))/(2+2^(1/2))^(1/2)
```

Rubi [A]
time = 0.17, antiderivative size = 347, normalized size of antiderivative = 1.00, number of steps used = 19, number of rules used = 6, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.462$, Rules used = {1427, 1108, 648, 632, 210, 642}

$$-\frac{1}{4}\sqrt{\frac{1}{2}\left(2-\sqrt{2}\right)}\text{ArcTan}\left(\frac{\sqrt{2-\sqrt{2}}-2x}{\sqrt{2+\sqrt{2}}}\right)-\frac{1}{4}\sqrt{\frac{1}{2}\left(2+\sqrt{2}\right)}\text{ArcTan}\left(\frac{\sqrt{2+\sqrt{2}}-2x}{\sqrt{2-\sqrt{2}}}\right)+\frac{1}{4}\sqrt{\frac{1}{2}\left(2-\sqrt{2}\right)}\text{ArcTan}\left(\frac{2x+\sqrt{2-\sqrt{2}}}{\sqrt{2+\sqrt{2}}}\right)+\frac{1}{4}\sqrt{\frac{1}{2}\left(2+\sqrt{2}\right)}\text{ArcTan}\left(\frac{2x+\sqrt{2+\sqrt{2}}}{\sqrt{2-\sqrt{2}}}\right)-\frac{\log\left(x^2-\sqrt{2-\sqrt{2}}\,x+1\right)}{8\sqrt{2-\sqrt{2}}}+\frac{\log\left(x^2+\sqrt{2-\sqrt{2}}\,x+1\right)}{8\sqrt{2-\sqrt{2}}}-\frac{\log\left(x^2-\sqrt{2+\sqrt{2}}\,x+1\right)}{8\sqrt{2+\sqrt{2}}}+\frac{\log\left(x^2+\sqrt{2+\sqrt{2}}\,x+1\right)}{8\sqrt{2+\sqrt{2}}}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + x^4)/(1 + x^8),x]
```

```
[Out] -1/4*(Sqrt[(2 - Sqrt[2])/2]*ArcTan[(Sqrt[2 - Sqrt[2]] - 2*x)/Sqrt[2 + Sqrt[
2]]]) - (Sqrt[(2 + Sqrt[2])/2]*ArcTan[(Sqrt[2 + Sqrt[2]] - 2*x)/Sqrt[2 - Sq
rt[2]]])/4 + (Sqrt[(2 - Sqrt[2])/2]*ArcTan[(Sqrt[2 - Sqrt[2]] + 2*x)/Sqrt[2
 + Sqrt[2]]])/4 + (Sqrt[(2 + Sqrt[2])/2]*ArcTan[(Sqrt[2 + Sqrt[2]] + 2*x)/S
qrt[2 - Sqrt[2]]])/4 - Log[1 - Sqrt[2 - Sqrt[2]]*x + x^2]/(8*Sqrt[2 - Sqrt[
2]]) + Log[1 + Sqrt[2 - Sqrt[2]]*x + x^2]/(8*Sqrt[2 - Sqrt[2]]) - Log[1 - S
qrt[2 + Sqrt[2]]*x + x^2]/(8*Sqrt[2 + Sqrt[2]]) + Log[1 + Sqrt[2 + Sqrt[2]]
*x + x^2]/(8*Sqrt[2 + Sqrt[2]])
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
```

```
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1108

```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(-1), x_Symbol] :> With[{q = Rt[a/
c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Dist[1/(2*c*q*r), Int[(r - x)/(q - r*x
 + x^2), x], x] + Dist[1/(2*c*q*r), Int[(r + x)/(q + r*x + x^2), x], x]]] /
; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && NegQ[b^2 - 4*a*c]
```

Rule 1427

```
Int[((d_) + (e_.)*(x_)^(n_))/((a_) + (c_.)*(x_)^(n2_)), x_Symbol] :> With[{
q = Rt[2*d*e, 2]}, Dist[e^2/(2*c), Int[1/(d + q*x^(n/2) + e*x^n), x], x] +
Dist[e^2/(2*c), Int[1/(d - q*x^(n/2) + e*x^n), x], x]] /; FreeQ[{a, c, d, e
}, x] && EqQ[n2, 2*n] && EqQ[c*d^2 - a*e^2, 0] && IGtQ[n/2, 0] && PosQ[d*e]
```

Rubi steps

$$\int \frac{1+x^4}{1+x^8}\,dx = \frac{1}{2}\int \frac{1}{1-\sqrt{2}\,x^2+x^4}\,dx + \frac{1}{2}\int \frac{1}{1+\sqrt{2}\,x^2+x^4}\,dx$$

$$= \frac{\int \frac{\sqrt{2-\sqrt{2}}-x}{1-\sqrt{2-\sqrt{2}}\,x+x^2}\,dx}{4\sqrt{2-\sqrt{2}}} + \frac{\int \frac{\sqrt{2-\sqrt{2}}+x}{1+\sqrt{2-\sqrt{2}}\,x+x^2}\,dx}{4\sqrt{2-\sqrt{2}}} + \frac{\int \frac{\sqrt{2+\sqrt{2}}-x}{1-\sqrt{2+\sqrt{2}}\,x+x^2}\,dx}{4\sqrt{2+\sqrt{2}}} + \frac{\int \frac{\sqrt{2+\sqrt{}}}{1+\sqrt{2+\sqrt{}}}}{4\sqrt{2+}}$$

$$= \frac{1}{8}\int \frac{1}{1-\sqrt{2-\sqrt{2}}\,x+x^2}\,dx + \frac{1}{8}\int \frac{1}{1+\sqrt{2-\sqrt{2}}\,x+x^2}\,dx + \frac{1}{8}\int \frac{1}{1-\sqrt{2+\sqrt{2}}\,x+x}$$

$$= -\frac{\log\left(1-\sqrt{2-\sqrt{2}}\,x+x^2\right)}{8\sqrt{2-\sqrt{2}}} + \frac{\log\left(1+\sqrt{2-\sqrt{2}}\,x+x^2\right)}{8\sqrt{2-\sqrt{2}}} - \frac{\log\left(1-\sqrt{2+\sqrt{2}}\,x+\right.}{8\sqrt{2+\sqrt{2}}}$$

$$= -\frac{\tan^{-1}\left(\frac{\sqrt{2-\sqrt{2}}-2x}{\sqrt{2+\sqrt{2}}}\right)}{4\sqrt{2+\sqrt{2}}} - \frac{\tan^{-1}\left(\frac{\sqrt{2+\sqrt{2}}-2x}{\sqrt{2-\sqrt{2}}}\right)}{4\sqrt{2-\sqrt{2}}} + \frac{\tan^{-1}\left(\frac{\sqrt{2-\sqrt{2}}+2x}{\sqrt{2+\sqrt{2}}}\right)}{4\sqrt{2+\sqrt{2}}} + \frac{\tan^{-}}{}$$

Mathematica [A]
time = 0.13, size = 258, normalized size = 0.74

$$\frac{1}{8}\left(2\tan^{-1}\left(\sec\left(\frac{\pi}{8}\right)\left(x+\sin\left(\frac{\pi}{8}\right)\right)\right)\left(\cos\left(\frac{\pi}{8}\right)-\sin\left(\frac{\pi}{8}\right)\right)+2\tan^{-1}\left(x\sec\left(\frac{\pi}{8}\right)-\tan\left(\frac{\pi}{8}\right)\right)\left(\cos\left(\frac{\pi}{8}\right)-\sin\left(\frac{\pi}{8}\right)\right)+\log\left(1+x^2+2x\cos\left(\frac{\pi}{8}\right)\right)\left(\cos\left(\frac{\pi}{8}\right)-\sin\left(\frac{\pi}{8}\right)\right)+\log\left(1+x^2-2x\cos\left(\frac{\pi}{8}\right)\right)\left(-\cos\left(\frac{\pi}{8}\right)+\sin\left(\frac{\pi}{8}\right)\right)+2\tan^{-1}\left(\left(x-\cos\left(\frac{\pi}{8}\right)\right)\csc\left(\frac{\pi}{8}\right)\right)\left(\cos\left(\frac{\pi}{8}\right)+\sin\left(\frac{\pi}{8}\right)\right)+2\tan^{-1}\left(\left(x+\cos\left(\frac{\pi}{8}\right)\right)\csc\left(\frac{\pi}{8}\right)\right)\left(\cos\left(\frac{\pi}{8}\right)+\sin\left(\frac{\pi}{8}\right)\right)-\log\left(1+x^2-2x\sin\left(\frac{\pi}{8}\right)\right)\left(\cos\left(\frac{\pi}{8}\right)+\sin\left(\frac{\pi}{8}\right)\right)+\log\left(1+x^2+2x\sin\left(\frac{\pi}{8}\right)\right)\left(\cos\left(\frac{\pi}{8}\right)+\sin\left(\frac{\pi}{8}\right)\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + x^4)/(1 + x^8),x]
```

```
[Out] (2*ArcTan[Sec[Pi/8]*(x + Sin[Pi/8])]*(Cos[Pi/8] - Sin[Pi/8]) + 2*ArcTan[x*S
ec[Pi/8] - Tan[Pi/8]]*(Cos[Pi/8] - Sin[Pi/8]) + Log[1 + x^2 + 2*x*Cos[Pi/8]
]*(Cos[Pi/8] - Sin[Pi/8]) + Log[1 + x^2 - 2*x*Cos[Pi/8]]*(-Cos[Pi/8] + Sin[
Pi/8]) + 2*ArcTan[(x - Cos[Pi/8])*Csc[Pi/8]]*(Cos[Pi/8] + Sin[Pi/8]) + 2*Ar
cTan[(x + Cos[Pi/8])*Csc[Pi/8]]*(Cos[Pi/8] + Sin[Pi/8]) - Log[1 + x^2 - 2*x
*Sin[Pi/8]]*(Cos[Pi/8] + Sin[Pi/8]) + Log[1 + x^2 + 2*x*Sin[Pi/8]]*(Cos[Pi/
8] + Sin[Pi/8]))/8
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 0.24, size = 27, normalized size = 0.08

method	result	size
default	$\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(\_Z^{8}+1\right)}\frac{\left(\_R^{4}+1\right)\ln\left(x-\_R\right)}{\_R^{7}}\right)}{8}$	27
risch	$\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(\_Z^{8}+1\right)}\frac{\left(\_R^{4}+1\right)\ln\left(x-\_R\right)}{\_R^{7}}\right)}{8}$	27
meijerg	Expression too large to display	566

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4+1)/(x^8+1),x,method=_RETURNVERBOSE)
```

```
[Out] 1/8*sum((_R^4+1)/_R^7*ln(x-_R),_R=RootOf(_Z^8+1))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4+1)/(x^8+1),x, algorithm="maxima")
```

```
[Out] integrate((x^4 + 1)/(x^8 + 1), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 1001 vs. $2(245) = 490$.
time = 0.39, size = 1001, normalized size = 2.88

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4+1)/(x^8+1),x, algorithm="fricas")
```

```
[Out] -1/8*(sqrt(sqrt(2) + 2) - sqrt(-sqrt(2) + 2))*arctan(-(2*x - 2*sqrt(x^2 + x
*sqrt(-sqrt(2) + 2) + 1) + sqrt(-sqrt(2) + 2))/sqrt(sqrt(2) + 2)) - 1/8*(sq
rt(sqrt(2) + 2) - sqrt(-sqrt(2) + 2))*arctan(-(2*x - 2*sqrt(x^2 - x*sqrt(-s
qrt(2) + 2) + 1) - sqrt(-sqrt(2) + 2))/sqrt(sqrt(2) + 2)) - 1/8*(sqrt(sqrt(
2) + 2) + sqrt(-sqrt(2) + 2))*arctan(-(2*x - 2*sqrt(x^2 + x*sqrt(sqrt(2) +
2) + 1) + sqrt(sqrt(2) + 2))/sqrt(-sqrt(2) + 2)) - 1/8*(sqrt(sqrt(2) + 2) +
 sqrt(-sqrt(2) + 2))*arctan(-(2*x - 2*sqrt(x^2 - x*sqrt(sqrt(2) + 2) + 1) -
 sqrt(sqrt(2) + 2))/sqrt(-sqrt(2) + 2)) - 1/8*sqrt(2)*sqrt(-sqrt(2) + 2)*ar
ctan(-(2*sqrt(2)*x - 2*sqrt(2*x^2 + sqrt(2)*x*sqrt(sqrt(2) + 2) - sqrt(2)*x
*sqrt(-sqrt(2) + 2) + 2) + sqrt(sqrt(2) + 2) - sqrt(-sqrt(2) + 2))/(sqrt(sq
```

```
rt(2) + 2) + sqrt(-sqrt(2) + 2))) - 1/8*sqrt(2)*sqrt(-sqrt(2) + 2)*arctan(-
(2*sqrt(2)*x - 2*sqrt(2*x^2 - sqrt(2)*x*sqrt(sqrt(2) + 2) + sqrt(2)*x*sqrt(
-sqrt(2) + 2) + 2) - sqrt(sqrt(2) + 2) + sqrt(-sqrt(2) + 2))/(sqrt(sqrt(2)
+ 2) + sqrt(-sqrt(2) + 2))) + 1/8*sqrt(2)*sqrt(sqrt(2) + 2)*arctan((2*sqrt(
2)*x - 2*sqrt(2*x^2 + sqrt(2)*x*sqrt(sqrt(2) + 2) + sqrt(2)*x*sqrt(-sqrt(2)
 + 2) + 2) + sqrt(sqrt(2) + 2) + sqrt(-sqrt(2) + 2))/(sqrt(sqrt(2) + 2) - s
qrt(-sqrt(2) + 2))) + 1/8*sqrt(2)*sqrt(sqrt(2) + 2)*arctan((2*sqrt(2)*x - 2
*sqrt(2*x^2 - sqrt(2)*x*sqrt(sqrt(2) + 2) - sqrt(2)*x*sqrt(-sqrt(2) + 2) +
2) - sqrt(sqrt(2) + 2) - sqrt(-sqrt(2) + 2))/(sqrt(sqrt(2) + 2) - sqrt(-sqr
t(2) + 2))) + 1/32*(sqrt(sqrt(2) + 2) - sqrt(-sqrt(2) + 2))*log(1024*x^2 +
1024*x*sqrt(sqrt(2) + 2) + 1024) - 1/32*(sqrt(sqrt(2) + 2) - sqrt(-sqrt(2)
+ 2))*log(1024*x^2 - 1024*x*sqrt(sqrt(2) + 2) + 1024) + 1/32*(sqrt(sqrt(2)
+ 2) + sqrt(-sqrt(2) + 2))*log(1024*x^2 + 1024*x*sqrt(-sqrt(2) + 2) + 1024)
 - 1/32*(sqrt(sqrt(2) + 2) + sqrt(-sqrt(2) + 2))*log(1024*x^2 - 1024*x*sqrt
(-sqrt(2) + 2) + 1024) + 1/32*sqrt(2)*sqrt(-sqrt(2) + 2)*log(256*x^2 + 128*
sqrt(2)*x*sqrt(sqrt(2) + 2) + 128*sqrt(2)*x*sqrt(-sqrt(2) + 2) + 256) + 1/3
2*sqrt(2)*sqrt(sqrt(2) + 2)*log(256*x^2 + 128*sqrt(2)*x*sqrt(sqrt(2) + 2) -
 128*sqrt(2)*x*sqrt(-sqrt(2) + 2) + 256) - 1/32*sqrt(2)*sqrt(sqrt(2) + 2)*l
og(256*x^2 - 128*sqrt(2)*x*sqrt(sqrt(2) + 2) + 128*sqrt(2)*x*sqrt(-sqrt(2)
+ 2) + 256) - 1/32*sqrt(2)*sqrt(-sqrt(2) + 2)*log(256*x^2 - 128*sqrt(2)*x*s
qrt(sqrt(2) + 2) - 128*sqrt(2)*x*sqrt(-sqrt(2) + 2) + 256)
```

Sympy [A]
time = 1.14, size = 19, normalized size = 0.05

$$\operatorname{RootSum}\left(1048576t^8 + 1, \left(t \mapsto t\log\left(4096t^5 + 4t + x\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**4+1)/(x**8+1),x)
```

```
[Out] RootSum(1048576*_t**8 + 1, Lambda(_t, _t*log(4096*_t**5 + 4*_t + x)))
```

Giac [A]
time = 3.01, size = 247, normalized size = 0.71

$$\frac{1}{8}\sqrt{-2\sqrt{2}+4}\arctan\left(\frac{2x+\sqrt{-\sqrt{2}+2}}{\sqrt{\sqrt{2}+2}}\right)+\frac{1}{8}\sqrt{-2\sqrt{2}+4}\arctan\left(\frac{2x-\sqrt{-\sqrt{2}+2}}{\sqrt{\sqrt{2}+2}}\right)+\frac{1}{8}\sqrt{2\sqrt{2}+4}\arctan\left(\frac{2x+\sqrt{\sqrt{2}+2}}{\sqrt{-\sqrt{2}+2}}\right)+\frac{1}{8}\sqrt{2\sqrt{2}+4}\arctan\left(\frac{2x-\sqrt{\sqrt{2}+2}}{\sqrt{-\sqrt{2}+2}}\right)+\frac{1}{16}\sqrt{-2\sqrt{2}+4}\log\left(x^2+x\sqrt{\sqrt{2}+2}+1\right)-\frac{1}{16}\sqrt{-2\sqrt{2}+4}\log\left(x^2-x\sqrt{\sqrt{2}+2}+1\right)+\frac{1}{16}\sqrt{2\sqrt{2}+4}\log\left(x^2+x\sqrt{-\sqrt{2}+2}+1\right)-\frac{1}{16}\sqrt{2\sqrt{2}+4}\log\left(x^2-x\sqrt{-\sqrt{2}+2}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4+1)/(x^8+1),x, algorithm="giac")
```

```
[Out] 1/8*sqrt(-2*sqrt(2) + 4)*arctan((2*x + sqrt(-sqrt(2) + 2))/sqrt(sqrt(2) + 2
)) + 1/8*sqrt(-2*sqrt(2) + 4)*arctan((2*x - sqrt(-sqrt(2) + 2))/sqrt(sqrt(2
) + 2)) + 1/8*sqrt(2*sqrt(2) + 4)*arctan((2*x + sqrt(sqrt(2) + 2))/sqrt(-sq
rt(2) + 2)) + 1/8*sqrt(2*sqrt(2) + 4)*arctan((2*x - sqrt(sqrt(2) + 2))/sqrt
(-sqrt(2) + 2)) + 1/16*sqrt(-2*sqrt(2) + 4)*log(x^2 + x*sqrt(sqrt(2) + 2) +
 1) - 1/16*sqrt(-2*sqrt(2) + 4)*log(x^2 - x*sqrt(sqrt(2) + 2) + 1) + 1/16*s
```

```
qrt(2*sqrt(2) + 4)*log(x^2 + x*sqrt(-sqrt(2) + 2) + 1) - 1/16*sqrt(2*sqrt(2
) + 4)*log(x^2 - x*sqrt(-sqrt(2) + 2) + 1)
```

Mupad [B]
time = 2.28, size = 311, normalized size = 0.90

$$-\ln\left(\left(\frac{\sqrt{-2\sqrt{2}-4}}{16}-\frac{\sqrt{4-2\sqrt{2}}}{16}\right)^3\left(65536\,x-16384\sqrt{-2\sqrt{2}-4}+16384\sqrt{4-2\sqrt{2}}\right)+256\right)\left(\frac{\sqrt{-2\sqrt{2}-4}}{16}-\frac{\sqrt{4-2\sqrt{2}}}{16}\right)+\mathrm{atan}\left(\frac{x\sqrt{\sqrt{2}-2}\,1\mathrm{i}}{2}+\frac{x\sqrt{\sqrt{2}+2}\,1\mathrm{i}}{2}+\frac{\sqrt{2}\,x\sqrt{\sqrt{2}-2}\,1\mathrm{i}}{2}-\frac{\sqrt{2}\,x\sqrt{\sqrt{2}+2}\,1\mathrm{i}}{2}\right)\left(\frac{\sqrt{2}\sqrt{\sqrt{2}-2}\,1\mathrm{i}}{8}+\frac{\sqrt{2}\sqrt{\sqrt{2}+2}\,1\mathrm{i}}{8}\right)-\frac{\mathrm{atan}\left(x\left(\sqrt{2}+2\right)^{3/2}\left(1-\frac{1}{2}\mathrm{i}\right)+\sqrt{2}\,x\left(\sqrt{2}+2\right)^{3/2}\left(-\frac{3}{4}+\frac{1}{4}\mathrm{i}\right)\right)\left(-2+\sqrt{2}\,(1-\mathrm{i})\right)\sqrt{\sqrt{2}+2}\,1\mathrm{i}}{8}+\frac{\mathrm{atan}\left(x\left(\sqrt{2}+2\right)^{3/2}\left(\frac{1}{2}+1\mathrm{i}\right)+\sqrt{2}\,x\left(\sqrt{2}+2\right)^{3/2}\left(-\frac{1}{4}-\frac{3}{4}\mathrm{i}\right)\right)\left(\sqrt{2}\,(1+1\mathrm{i})-2\mathrm{i}\right)\sqrt{\sqrt{2}+2}\,1\mathrm{i}}{8}+\sqrt{2}\,\ln\left(x+\left(\sqrt{2}+2\right)^{3/2}\left(-\frac{1}{2}-\mathrm{i}\right)+\sqrt{2}\left(\sqrt{2}+2\right)^{3/2}\left(\frac{1}{4}+\frac{3}{4}\mathrm{i}\right)\right)\left(\frac{\sqrt{\sqrt{2}-2}}{16}+\frac{\sqrt{\sqrt{2}+2}}{16}\right)1\mathrm{i}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4 + 1)/(x^8 + 1),x)
```

```
[Out] atan((x*(2^(1/2) - 2)^(1/2)*1i)/2 + (x*(2^(1/2) + 2)^(1/2)*1i)/2 + (2^(1/2)
*x*(2^(1/2) - 2)^(1/2)*1i)/2 - (2^(1/2)*x*(2^(1/2) + 2)^(1/2)*1i)/2)*((2^(1
/2)*(2^(1/2) - 2)^(1/2)*1i)/8 + (2^(1/2)*(2^(1/2) + 2)^(1/2)*1i)/8) - log((
(- 2*2^(1/2) - 4)^(1/2)/16 - (4 - 2*2^(1/2))^(1/2)/16)^3*(65536*x - 16384*(
- 2*2^(1/2) - 4)^(1/2) + 16384*(4 - 2*2^(1/2))^(1/2)) + 256)*((- 2*2^(1/2)
- 4)^(1/2)/16 - (4 - 2*2^(1/2))^(1/2)/16) - (atan(x*(2^(1/2) + 2)^(3/2)*(1
- 1i/2) - 2^(1/2)*x*(2^(1/2) + 2)^(3/2)*(3/4 - 1i/4))*(2^(1/2)*(1 - 1i) - 2
)*(2^(1/2) + 2)^(1/2)*1i)/8 + (atan(x*(2^(1/2) + 2)^(3/2)*(1/2 + 1i) - 2^(1
/2)*x*(2^(1/2) + 2)^(3/2)*(1/4 + 3i/4))*(2^(1/2)*(1 + 1i) - 2i)*(2^(1/2) +
2)^(1/2)*1i)/8 + 2^(1/2)*log(x - (2^(1/2) + 2)^(3/2)*(1/2 + 1i) + 2^(1/2)*(
2^(1/2) + 2)^(3/2)*(1/4 + 3i/4))*((2^(1/2) - 2)^(1/2)/16 + (2^(1/2) + 2)^(1
/2)/16)*1i
```

3.14 $\int \frac{1+x^4}{1-x^4+x^8}\,dx$

Optimal. Leaf size=331

$$-\frac{1}{4}\sqrt{2-\sqrt{3}}\,\tan^{-1}\left(\frac{\sqrt{2-\sqrt{3}}-2x}{\sqrt{2+\sqrt{3}}}\right)-\frac{1}{4}\sqrt{2+\sqrt{3}}\,\tan^{-1}\left(\frac{\sqrt{2+\sqrt{3}}-2x}{\sqrt{2-\sqrt{3}}}\right)+\frac{1}{4}\sqrt{2-\sqrt{3}}\,\tan^{-1}$$

```
[Out] -1/8*ln(1+x^2-x*(1/2*6^(1/2)-1/2*2^(1/2)))/(1/2*6^(1/2)-1/2*2^(1/2))+1/8*ln
(1+x^2+x*(1/2*6^(1/2)-1/2*2^(1/2)))/(1/2*6^(1/2)-1/2*2^(1/2))-1/4*arctan((-
2*x+1/2*6^(1/2)-1/2*2^(1/2))/(1/2*6^(1/2)+1/2*2^(1/2)))*(1/2*6^(1/2)-1/2*2^
(1/2))+1/4*arctan((2*x+1/2*6^(1/2)-1/2*2^(1/2))/(1/2*6^(1/2)+1/2*2^(1/2)))*
(1/2*6^(1/2)-1/2*2^(1/2))-1/8*ln(1+x^2-x*(1/2*6^(1/2)+1/2*2^(1/2)))/(1/2*6^
(1/2)+1/2*2^(1/2))+1/8*ln(1+x^2+x*(1/2*6^(1/2)+1/2*2^(1/2)))/(1/2*6^(1/2)+1
/2*2^(1/2))-1/4*arctan((-2*x+1/2*6^(1/2)+1/2*2^(1/2))/(1/2*6^(1/2)-1/2*2^(1
/2)))*(1/2*6^(1/2)+1/2*2^(1/2))+1/4*arctan((2*x+1/2*6^(1/2)+1/2*2^(1/2))/(1
/2*6^(1/2)-1/2*2^(1/2)))*(1/2*6^(1/2)+1/2*2^(1/2))
```

Rubi [A]

time = 0.17, antiderivative size = 331, normalized size of antiderivative = 1.00, number of steps used = 19, number of rules used = 6, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {1433, 1108, 648, 632, 210, 642}

$$-\frac{1}{4}\sqrt{2-\sqrt{3}}\,\text{ArcTan}\left(\frac{\sqrt{2-\sqrt{3}}-2x}{\sqrt{2+\sqrt{3}}}\right)-\frac{1}{4}\sqrt{2+\sqrt{3}}\,\text{ArcTan}\left(\frac{\sqrt{2+\sqrt{3}}-2x}{\sqrt{2-\sqrt{3}}}\right)+\frac{1}{4}\sqrt{2-\sqrt{3}}\,\text{ArcTan}\left(\frac{2x+\sqrt{2-\sqrt{3}}}{\sqrt{2+\sqrt{3}}}\right)+\frac{1}{4}\sqrt{2+\sqrt{3}}\,\text{ArcTan}\left(\frac{2x+\sqrt{2+\sqrt{3}}}{\sqrt{2-\sqrt{3}}}\right)-\frac{\log\left(x^2-\sqrt{2-\sqrt{3}}\,x+1\right)}{8\sqrt{2-\sqrt{3}}}+\frac{\log\left(x^2+\sqrt{2-\sqrt{3}}\,x+1\right)}{8\sqrt{2-\sqrt{3}}}-\frac{\log\left(x^2-\sqrt{2+\sqrt{3}}\,x+1\right)}{8\sqrt{2+\sqrt{3}}}+\frac{\log\left(x^2+\sqrt{2+\sqrt{3}}\,x+1\right)}{8\sqrt{2+\sqrt{3}}}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + x^4)/(1 - x^4 + x^8),x]
```

```
[Out] -1/4*(Sqrt[2 - Sqrt[3]]*ArcTan[(Sqrt[2 - Sqrt[3]] - 2*x)/Sqrt[2 + Sqrt[3]]]
) - (Sqrt[2 + Sqrt[3]]*ArcTan[(Sqrt[2 + Sqrt[3]] - 2*x)/Sqrt[2 - Sqrt[3]]])
/4 + (Sqrt[2 - Sqrt[3]]*ArcTan[(Sqrt[2 - Sqrt[3]] + 2*x)/Sqrt[2 + Sqrt[3]]]
)/4 + (Sqrt[2 + Sqrt[3]]*ArcTan[(Sqrt[2 + Sqrt[3]] + 2*x)/Sqrt[2 - Sqrt[3]]
])/4 - Log[1 - Sqrt[2 - Sqrt[3]]*x + x^2]/(8*Sqrt[2 - Sqrt[3]]) + Log[1 + S
qrt[2 - Sqrt[3]]*x + x^2]/(8*Sqrt[2 - Sqrt[3]]) - Log[1 - Sqrt[2 + Sqrt[3]]
*x + x^2]/(8*Sqrt[2 + Sqrt[3]]) + Log[1 + Sqrt[2 + Sqrt[3]]*x + x^2]/(8*Sqr
t[2 + Sqrt[3]])
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1108

```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(-1), x_Symbol] :> With[{q = Rt[a/
c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Dist[1/(2*c*q*r), Int[(r - x)/(q - r*x
 + x^2), x], x] + Dist[1/(2*c*q*r), Int[(r + x)/(q + r*x + x^2), x], x]]] /
; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && NegQ[b^2 - 4*a*c]
```

Rule 1433

```
Int[((d_) + (e_.)*(x_)^(n_))/((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_)), x
_Symbol] :> With[{q = Rt[2*(d/e) - b/c, 2]}, Dist[e/(2*c), Int[1/Simp[d/e +
 q*x^(n/2) + x^n, x], x], x] + Dist[e/(2*c), Int[1/Simp[d/e - q*x^(n/2) + x
^n, x], x], x]] /; FreeQ[{a, b, c, d, e}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4
*a*c, 0] && EqQ[c*d^2 - a*e^2, 0] && IGtQ[n/2, 0] && (GtQ[2*(d/e) - b/c, 0]
 || ( !LtQ[2*(d/e) - b/c, 0] && EqQ[d, e*Rt[a/c, 2]]))
```

Rubi steps

$$\int \frac{1+x^4}{1-x^4+x^8}\,dx = \frac{1}{2}\int \frac{1}{1-\sqrt{3}\,x^2+x^4}\,dx + \frac{1}{2}\int \frac{1}{1+\sqrt{3}\,x^2+x^4}\,dx$$

$$= \frac{\int \frac{\sqrt{2-\sqrt{3}}-x}{1-\sqrt{2-\sqrt{3}}\,x+x^2}\,dx}{4\sqrt{2-\sqrt{3}}} + \frac{\int \frac{\sqrt{2-\sqrt{3}}+x}{1+\sqrt{2-\sqrt{3}}\,x+x^2}\,dx}{4\sqrt{2-\sqrt{3}}} + \frac{\int \frac{\sqrt{2+\sqrt{3}}-x}{1-\sqrt{2+\sqrt{3}}\,x+x^2}\,dx}{4\sqrt{2+\sqrt{3}}} + \frac{\int \frac{\sqrt{2}}{1+\sqrt{2}}}{4}$$

$$= \frac{1}{8}\int \frac{1}{1-\sqrt{2-\sqrt{3}}\,x+x^2}\,dx + \frac{1}{8}\int \frac{1}{1+\sqrt{2-\sqrt{3}}\,x+x^2}\,dx + \frac{1}{8}\int \frac{1}{1-\sqrt{2+\sqrt{3}}}$$

$$= -\frac{\log\left(1-\sqrt{2-\sqrt{3}}\,x+x^2\right)}{8\sqrt{2-\sqrt{3}}} + \frac{\log\left(1+\sqrt{2-\sqrt{3}}\,x+x^2\right)}{8\sqrt{2-\sqrt{3}}} - \frac{\log\left(1-\sqrt{2+\sqrt{3}}\right.}{8\sqrt{2+\sqrt{3}}}$$

$$= -\frac{\tan^{-1}\left(\frac{\sqrt{2-\sqrt{3}}-2x}{\sqrt{2+\sqrt{3}}}\right)}{4\sqrt{2+\sqrt{3}}} - \frac{\tan^{-1}\left(\frac{\sqrt{2+\sqrt{3}}-2x}{\sqrt{2-\sqrt{3}}}\right)}{4\sqrt{2-\sqrt{3}}} + \frac{\tan^{-1}\left(\frac{\sqrt{2-\sqrt{3}}+2x}{\sqrt{2+\sqrt{3}}}\right)}{4\sqrt{2+\sqrt{3}}} +$$

Mathematica [C] Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.
time = 0.01, size = 55, normalized size = 0.17

$$\frac{1}{4}\text{RootSum}\left[1-\#1^4+\#1^8\&,\frac{\log(x-\#1)+\log(x-\#1)\#1^4}{-\#1^3+2\#1^7}\&\right]$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + x^4)/(1 - x^4 + x^8),x]
```

```
[Out] RootSum[1 - #1^4 + #1^8 & , (Log[x - #1] + Log[x - #1]*#1^4)/(-#1^3 + 2*#1^
7) & ]/4
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 0.02, size = 42, normalized size = 0.13

method	result	size

default	$\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(\_Z^8-\_Z^4+1\right)}\frac{\left(\_R^4+1\right)\ln\left(x-\_R\right)}{2\_R^7-\_R^3}\right)}{4}$	42
risch	$\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(\_Z^8-\_Z^4+1\right)}\frac{\left(\_R^4+1\right)\ln\left(x-\_R\right)}{2\_R^7-\_R^3}\right)}{4}$	42

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4+1)/(x^8-x^4+1),x,method=_RETURNVERBOSE)
```

```
[Out] 1/4*sum((_R^4+1)/(2*_R^7-_R^3)*ln(x-_R),_R=RootOf(_Z^8-_Z^4+1))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4+1)/(x^8-x^4+1),x, algorithm="maxima")
```

```
[Out] integrate((x^4 + 1)/(x^8 - x^4 + 1), x)
```

Fricas [A]
time = 0.39, size = 369, normalized size = 1.11

$$-\frac{1}{8}\sqrt{\sqrt{3}+2}\left(\sqrt{3}-2\right)\log\left(16x^2+16x\sqrt{\sqrt{3}+2}+16\right)+\frac{1}{8}\sqrt{\sqrt{3}+2}\left(\sqrt{3}-2\right)\log\left(16x^2-16x\sqrt{\sqrt{3}+2}+16\right)+\frac{1}{16}\left(\sqrt{3}+2\right)\sqrt{-4\sqrt{3}+8}\log\left(16x^2+8x\sqrt{-4\sqrt{3}+8}+16\right)-\frac{1}{16}\left(\sqrt{3}+2\right)\sqrt{-4\sqrt{3}+8}\log\left(16x^2-8x\sqrt{-4\sqrt{3}+8}+16\right)-\frac{1}{2}\sqrt{\sqrt{3}+2}\arctan\left(-2x\sqrt{\sqrt{3}+2}+2\sqrt{x^2+x\sqrt{\sqrt{3}+2}+1}\sqrt{\sqrt{3}+2}-\sqrt{3}-2\right)-\frac{1}{2}\sqrt{\sqrt{3}+2}\arctan\left(-2x\sqrt{\sqrt{3}+2}+2\sqrt{x^2-x\sqrt{\sqrt{3}+2}+1}\sqrt{\sqrt{3}+2}+\sqrt{3}+2\right)-\frac{1}{4}\sqrt{-4\sqrt{3}+8}\arctan\left(\frac{1}{2}\sqrt{2}\sqrt{2x^2+x\sqrt{-4\sqrt{3}+8}+2}\sqrt{-4\sqrt{3}+8}-x\sqrt{-4\sqrt{3}+8}+\sqrt{3}-2\right)-\frac{1}{4}\sqrt{-4\sqrt{3}+8}\arctan\left(\frac{1}{2}\sqrt{2}\sqrt{2x^2-x\sqrt{-4\sqrt{3}+8}+2}\sqrt{-4\sqrt{3}+8}-x\sqrt{-4\sqrt{3}+8}-\sqrt{3}+2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4+1)/(x^8-x^4+1),x, algorithm="fricas")
```

```
[Out] -1/8*sqrt(sqrt(3) + 2)*(sqrt(3) - 2)*log(16*x^2 + 16*x*sqrt(sqrt(3) + 2) +
16) + 1/8*sqrt(sqrt(3) + 2)*(sqrt(3) - 2)*log(16*x^2 - 16*x*sqrt(sqrt(3) +
2) + 16) + 1/16*(sqrt(3) + 2)*sqrt(-4*sqrt(3) + 8)*log(16*x^2 + 8*x*sqrt(-4
*sqrt(3) + 8) + 16) - 1/16*(sqrt(3) + 2)*sqrt(-4*sqrt(3) + 8)*log(16*x^2 -
8*x*sqrt(-4*sqrt(3) + 8) + 16) - 1/2*sqrt(sqrt(3) + 2)*arctan(-2*x*sqrt(sqr
t(3) + 2) + 2*sqrt(x^2 + x*sqrt(sqrt(3) + 2) + 1)*sqrt(sqrt(3) + 2) - sqrt(
3) - 2) - 1/2*sqrt(sqrt(3) + 2)*arctan(-2*x*sqrt(sqrt(3) + 2) + 2*sqrt(x^2
- x*sqrt(sqrt(3) + 2) + 1)*sqrt(sqrt(3) + 2) + sqrt(3) + 2) - 1/4*sqrt(-4*s
qrt(3) + 8)*arctan(1/2*sqrt(2)*sqrt(2*x^2 + x*sqrt(-4*sqrt(3) + 8) + 2)*sqr
t(-4*sqrt(3) + 8) - x*sqrt(-4*sqrt(3) + 8) + sqrt(3) - 2) - 1/4*sqrt(-4*sqr
t(3) + 8)*arctan(1/2*sqrt(2)*sqrt(2*x^2 - x*sqrt(-4*sqrt(3) + 8) + 2)*sqrt(
-4*sqrt(3) + 8) - x*sqrt(-4*sqrt(3) + 8) - sqrt(3) + 2)
```

Sympy [A]
time = 1.31, size = 20, normalized size = 0.06

$$\operatorname{RootSum}\left(65536t^{8}-256t^{4}+1,\left(t\mapsto t\log\left(1024t^{5}+x\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**4+1)/(x**8-x**4+1),x)
```

```
[Out] RootSum(65536*_t**8 - 256*_t**4 + 1, Lambda(_t, _t*log(1024*_t**5 + x)))
```

Giac [A]
time = 3.41, size = 245, normalized size = 0.74

$$\frac{1}{8}\left(\sqrt{6}-\sqrt{2}\right)\arctan\left(\frac{4x+\sqrt{6}-\sqrt{2}}{\sqrt{6}+\sqrt{2}}\right)+\frac{1}{8}\left(\sqrt{6}-\sqrt{2}\right)\arctan\left(\frac{4x-\sqrt{6}+\sqrt{2}}{\sqrt{6}+\sqrt{2}}\right)+\frac{1}{8}\left(\sqrt{6}+\sqrt{2}\right)\arctan\left(\frac{4x+\sqrt{6}+\sqrt{2}}{\sqrt{6}-\sqrt{2}}\right)+\frac{1}{8}\left(\sqrt{6}+\sqrt{2}\right)\arctan\left(\frac{4x-\sqrt{6}-\sqrt{2}}{\sqrt{6}-\sqrt{2}}\right)+\frac{1}{16}\left(\sqrt{6}-\sqrt{2}\right)\log\left(x^{2}+\frac{1}{2}x\left(\sqrt{6}+\sqrt{2}\right)+1\right)-\frac{1}{16}\left(\sqrt{6}-\sqrt{2}\right)\log\left(x^{2}-\frac{1}{2}x\left(\sqrt{6}+\sqrt{2}\right)+1\right)+\frac{1}{16}\left(\sqrt{6}+\sqrt{2}\right)\log\left(x^{2}+\frac{1}{2}x\left(\sqrt{6}-\sqrt{2}\right)+1\right)-\frac{1}{16}\left(\sqrt{6}+\sqrt{2}\right)\log\left(x^{2}-\frac{1}{2}x\left(\sqrt{6}-\sqrt{2}\right)+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4+1)/(x^8-x^4+1),x, algorithm="giac")
```

```
[Out] 1/8*(sqrt(6) - sqrt(2))*arctan((4*x + sqrt(6) - sqrt(2))/(sqrt(6) + sqrt(2)
)) + 1/8*(sqrt(6) - sqrt(2))*arctan((4*x - sqrt(6) + sqrt(2))/(sqrt(6) + sq
rt(2))) + 1/8*(sqrt(6) + sqrt(2))*arctan((4*x + sqrt(6) + sqrt(2))/(sqrt(6)
 - sqrt(2))) + 1/8*(sqrt(6) + sqrt(2))*arctan((4*x - sqrt(6) - sqrt(2))/(sq
rt(6) - sqrt(2))) + 1/16*(sqrt(6) - sqrt(2))*log(x^2 + 1/2*x*(sqrt(6) + sqr
t(2)) + 1) - 1/16*(sqrt(6) - sqrt(2))*log(x^2 - 1/2*x*(sqrt(6) + sqrt(2)) +
 1) + 1/16*(sqrt(6) + sqrt(2))*log(x^2 + 1/2*x*(sqrt(6) - sqrt(2)) + 1) - 1
/16*(sqrt(6) + sqrt(2))*log(x^2 - 1/2*x*(sqrt(6) - sqrt(2)) + 1)
```

Mupad [B]
time = 0.22, size = 145, normalized size = 0.44

$$-\operatorname{atan}\left(\frac{\sqrt{6}\,x\,\left(27-27\mathrm{i}\right)}{27\,\sqrt{3}-81\mathrm{i}}\right)\,\left(\sqrt{2}\,\left(\frac{1}{8}+\frac{1}{8}\mathrm{i}\right)+\sqrt{6}\,\left(-\frac{1}{8}+\frac{1}{8}\mathrm{i}\right)\right)-\operatorname{atan}\left(\frac{\sqrt{6}\,x\,\left(27+27\mathrm{i}\right)}{27\,\sqrt{3}-81\mathrm{i}}\right)\,\left(\sqrt{2}\,\left(\frac{1}{8}-\frac{1}{8}\mathrm{i}\right)+\sqrt{6}\,\left(\frac{1}{8}+\frac{1}{8}\mathrm{i}\right)\right)-\operatorname{atan}\left(\frac{\sqrt{6}\,x\,\left(27-27\mathrm{i}\right)}{27\,\sqrt{3}+81\mathrm{i}}\right)\,\left(\sqrt{2}\,\left(\frac{1}{8}+\frac{1}{8}\mathrm{i}\right)+\sqrt{6}\,\left(\frac{1}{8}-\frac{1}{8}\mathrm{i}\right)\right)-\operatorname{atan}\left(\frac{\sqrt{6}\,x\,\left(27+27\mathrm{i}\right)}{27\,\sqrt{3}+81\mathrm{i}}\right)\,\left(\sqrt{2}\,\left(\frac{1}{8}-\frac{1}{8}\mathrm{i}\right)+\sqrt{6}\,\left(-\frac{1}{8}-\frac{1}{8}\mathrm{i}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4 + 1)/(x^8 - x^4 + 1),x)
```

```
[Out] - atan((6^(1/2)*x*(27 - 27i))/(27*3^(1/2) - 81i))*(2^(1/2)*(1/8 + 1i/8) - 6
^(1/2)*(1/8 - 1i/8)) - atan((6^(1/2)*x*(27 + 27i))/(27*3^(1/2) - 81i))*(2^(
1/2)*(1/8 - 1i/8) + 6^(1/2)*(1/8 + 1i/8)) - atan((6^(1/2)*x*(27 - 27i))/(27
*3^(1/2) + 81i))*(2^(1/2)*(1/8 + 1i/8) + 6^(1/2)*(1/8 - 1i/8)) - atan((6^(1
/2)*x*(27 + 27i))/(27*3^(1/2) + 81i))*(2^(1/2)*(1/8 - 1i/8) - 6^(1/2)*(1/8
+ 1i/8))
```

3.15 $\int \frac{1+x^4}{1-2x^4+x^8}\,dx$

Optimal. Leaf size=27

$$\frac{x}{2(1-x^4)}+\frac{1}{4}\tan^{-1}(x)+\frac{1}{4}\tanh^{-1}(x)$$

```
[Out] 1/2*x/(-x^4+1)+1/4*arctan(x)+1/4*arctanh(x)
```

Rubi [A]
time = 0.01, antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.278, Rules used = {28, 393, 218, 212, 209}

$$\frac{\text{ArcTan}(x)}{4}+\frac{x}{2(1-x^4)}+\frac{1}{4}\tanh^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Int[(1 + x^4)/(1 - 2*x^4 + x^8),x]
```

```
[Out] x/(2*(1 - x^4)) + ArcTan[x]/4 + ArcTanh[x]/4
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 218

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x], x]
+ Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a/b
, 0]
```

Rule 393

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(-(b*c - a*d))*x*((a + b*x^n)^(p + 1)/(a*b*n*(p + 1))), x] - Dist[(a*d -
 b*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; F
reeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n
 + p, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{1+x^4}{1-2x^4+x^8}\,dx &= \int \frac{1+x^4}{(-1+x^4)^2}\,dx\\ &= \frac{x}{2(1-x^4)}-\frac{1}{2}\int \frac{1}{-1+x^4}\,dx\\ &= \frac{x}{2(1-x^4)}+\frac{1}{4}\int \frac{1}{1-x^2}\,dx+\frac{1}{4}\int \frac{1}{1+x^2}\,dx\\ &= \frac{x}{2(1-x^4)}+\frac{1}{4}\tan^{-1}(x)+\frac{1}{4}\tanh^{-1}(x)\end{aligned}$$

Mathematica [**A**]
time = 0.01, size = 31, normalized size = 1.15

$$\frac{1}{8}\left(-\frac{4x}{-1+x^4}+2\tan^{-1}(x)-\log(1-x)+\log(1+x)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + x^4)/(1 - 2*x^4 + x^8),x]
[Out] ((-4*x)/(-1 + x^4) + 2*ArcTan[x] - Log[1 - x] + Log[1 + x])/8
```

Maple [**A**]
time = 0.02, size = 42, normalized size = 1.56

method	result	size
risch	$-\frac{x}{2(x^4-1)}+\frac{\arctan(x)}{4}-\frac{\ln(-1+x)}{8}+\frac{\ln(1+x)}{8}$	28
default	$-\frac{1}{8(-1+x)}-\frac{\ln(-1+x)}{8}+\frac{x}{4x^2+4}+\frac{\arctan(x)}{4}-\frac{1}{8(1+x)}+\frac{\ln(1+x)}{8}$	42

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4+1)/(x^8-2*x^4+1),x,method=_RETURNVERBOSE)
[Out] -1/8/(-1+x)-1/8*ln(-1+x)+1/4*x/(x^2+1)+1/4*arctan(x)-1/8/(1+x)+1/8*ln(1+x)
```

Maxima [A]
time = 0.49, size = 27, normalized size = 1.00

$$-\frac{x}{2\,(x^4-1)}+\frac{1}{4}\,\arctan\left(x\right)+\frac{1}{8}\,\log\left(x+1\right)-\frac{1}{8}\,\log\left(x-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4+1)/(x^8-2*x^4+1),x, algorithm="maxima")
```

```
[Out] -1/2*x/(x^4 - 1) + 1/4*arctan(x) + 1/8*log(x + 1) - 1/8*log(x - 1)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 43 vs. $2(19) = 38$.
time = 0.36, size = 43, normalized size = 1.59

$$\frac{2\,(x^4-1)\arctan\left(x\right)+(x^4-1)\log\left(x+1\right)-(x^4-1)\log\left(x-1\right)-4\,x}{8\,(x^4-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4+1)/(x^8-2*x^4+1),x, algorithm="fricas")
```

```
[Out] 1/8*(2*(x^4 - 1)*arctan(x) + (x^4 - 1)*log(x + 1) - (x^4 - 1)*log(x - 1) -
4*x)/(x^4 - 1)
```

Sympy [A]
time = 0.06, size = 26, normalized size = 0.96

$$-\frac{x}{2x^4-2}-\frac{\log\left(x-1\right)}{8}+\frac{\log\left(x+1\right)}{8}+\frac{\operatorname{atan}\left(x\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**4+1)/(x**8-2*x**4+1),x)
```

```
[Out] -x/(2*x**4 - 2) - log(x - 1)/8 + log(x + 1)/8 + atan(x)/4
```

Giac [A]
time = 4.66, size = 29, normalized size = 1.07

$$-\frac{x}{2\,(x^4-1)}+\frac{1}{4}\,\arctan\left(x\right)+\frac{1}{8}\,\log\left(|x+1|\right)-\frac{1}{8}\,\log\left(|x-1|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4+1)/(x^8-2*x^4+1),x, algorithm="giac")
```

```
[Out] -1/2*x/(x^4 - 1) + 1/4*arctan(x) + 1/8*log(abs(x + 1)) - 1/8*log(abs(x - 1)
)
```

Mupad [B]
time = 0.05, size = 21, normalized size = 0.78

$$\frac{\mathrm{atan}(x)}{4}+\frac{\mathrm{atanh}(x)}{4}-\frac{x}{2\,\left(x^4-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4 + 1)/(x^8 - 2*x^4 + 1),x)
[Out] atan(x)/4 + atanh(x)/4 - x/(2*(x^4 - 1))
```

3.16 $\int \frac{1+x^4}{1-3x^4+x^8}\,dx$

Optimal. Leaf size=131

$$\frac{\tan^{-1}\left(\sqrt{\frac{2}{-1+\sqrt{5}}}\,x\right)}{\sqrt{2\left(-1+\sqrt{5}\right)}}-\frac{\tan^{-1}\left(\sqrt{\frac{2}{1+\sqrt{5}}}\,x\right)}{\sqrt{2\left(1+\sqrt{5}\right)}}+\frac{\tanh^{-1}\left(\sqrt{\frac{2}{-1+\sqrt{5}}}\,x\right)}{\sqrt{2\left(-1+\sqrt{5}\right)}}-\frac{\tanh^{-1}\left(\sqrt{\frac{2}{1+\sqrt{5}}}\,x\right)}{\sqrt{2\left(1+\sqrt{5}\right)}}$$

```
[Out] arctan(x*2^(1/2)/(5^(1/2)-1)^(1/2))/(-2+2*5^(1/2))^(1/2)+arctanh(x*2^(1/2)/
(5^(1/2)-1)^(1/2))/(-2+2*5^(1/2))^(1/2)-arctan(x*2^(1/2)/(5^(1/2)+1)^(1/2))
/(2+2*5^(1/2))^(1/2)-arctanh(x*2^(1/2)/(5^(1/2)+1)^(1/2))/(2+2*5^(1/2))^(1/
2)
```

Rubi [A]
time = 0.06, antiderivative size = 131, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 4, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {1433, 1107, 209, 213}

$$\frac{\text{ArcTan}\left(\sqrt{\frac{2}{\sqrt{5}-1}}\,x\right)}{\sqrt{2\left(\sqrt{5}-1\right)}}-\frac{\text{ArcTan}\left(\sqrt{\frac{2}{1+\sqrt{5}}}\,x\right)}{\sqrt{2\left(1+\sqrt{5}\right)}}+\frac{\tanh^{-1}\left(\sqrt{\frac{2}{\sqrt{5}-1}}\,x\right)}{\sqrt{2\left(\sqrt{5}-1\right)}}-\frac{\tanh^{-1}\left(\sqrt{\frac{2}{1+\sqrt{5}}}\,x\right)}{\sqrt{2\left(1+\sqrt{5}\right)}}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + x^4)/(1 - 3*x^4 + x^8),x]
```

```
[Out] ArcTan[Sqrt[2/(-1 + Sqrt[5])]*x]/Sqrt[2*(-1 + Sqrt[5])] - ArcTan[Sqrt[2/(1
+ Sqrt[5])]*x]/Sqrt[2*(1 + Sqrt[5])] + ArcTanh[Sqrt[2/(-1 + Sqrt[5])]*x]/Sq
rt[2*(-1 + Sqrt[5])] - ArcTanh[Sqrt[2/(1 + Sqrt[5])]*x]/Sqrt[2*(1 + Sqrt[5]
)]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 1107

```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(-1), x_Symbol] :> With[{q = Rt[b^
2 - 4*a*c, 2]}, Dist[c/q, Int[1/(b/2 - q/2 + c*x^2), x], x] - Dist[c/q, Int
[1/(b/2 + q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c,
0] && PosQ[b^2 - 4*a*c]
```

Rule 1433

```
Int[((d_) + (e_.)*(x_)^(n_))/((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_)), x
_Symbol] :> With[{q = Rt[2*(d/e) - b/c, 2]}, Dist[e/(2*c), Int[1/Simp[d/e +
 q*x^(n/2) + x^n, x], x], x] + Dist[e/(2*c), Int[1/Simp[d/e - q*x^(n/2) + x
^n, x], x], x]] /; FreeQ[{a, b, c, d, e}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4
*a*c, 0] && EqQ[c*d^2 - a*e^2, 0] && IGtQ[n/2, 0] && (GtQ[2*(d/e) - b/c, 0]
 || ( !LtQ[2*(d/e) - b/c, 0] && EqQ[d, e*Rt[a/c, 2]]))
```

Rubi steps

$$\begin{aligned}\int \frac{1+x^4}{1-3x^4+x^8}\,dx &= \frac{1}{2}\int \frac{1}{1-\sqrt{5}\,x^2+x^4}\,dx+\frac{1}{2}\int \frac{1}{1+\sqrt{5}\,x^2+x^4}\,dx\\ &= \frac{1}{2}\int \frac{1}{-\frac{1}{2}-\frac{\sqrt{5}}{2}+x^2}\,dx-\frac{1}{2}\int \frac{1}{\frac{1}{2}-\frac{\sqrt{5}}{2}+x^2}\,dx+\frac{1}{2}\int \frac{1}{-\frac{1}{2}+\frac{\sqrt{5}}{2}+x^2}\,dx-\frac{1}{2}\int \frac{1}{\frac{1}{2}\cdots}\\ &= \frac{\tan^{-1}\left(\sqrt{\frac{2}{-1+\sqrt{5}}}\,x\right)}{\sqrt{2\left(-1+\sqrt{5}\right)}}-\frac{\tan^{-1}\left(\sqrt{\frac{2}{1+\sqrt{5}}}\,x\right)}{\sqrt{2\left(1+\sqrt{5}\right)}}+\frac{\tanh^{-1}\left(\sqrt{\frac{2}{-1+\sqrt{5}}}\,x\right)}{\sqrt{2\left(-1+\sqrt{5}\right)}}-\cdots\end{aligned}$$

Mathematica [**A**]
time = 0.06, size = 131, normalized size = 1.00

$$\frac{\tan^{-1}\left(\sqrt{\frac{2}{-1+\sqrt{5}}}\,x\right)}{\sqrt{2\left(-1+\sqrt{5}\right)}}-\frac{\tan^{-1}\left(\sqrt{\frac{2}{1+\sqrt{5}}}\,x\right)}{\sqrt{2\left(1+\sqrt{5}\right)}}+\frac{\tanh^{-1}\left(\sqrt{\frac{2}{-1+\sqrt{5}}}\,x\right)}{\sqrt{2\left(-1+\sqrt{5}\right)}}-\frac{\tanh^{-1}\left(\sqrt{\frac{2}{1+\sqrt{5}}}\,x\right)}{\sqrt{2\left(1+\sqrt{5}\right)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + x^4)/(1 - 3*x^4 + x^8),x]
```

```
[Out] ArcTan[Sqrt[2/(-1 + Sqrt[5])]*x]/Sqrt[2*(-1 + Sqrt[5])] - ArcTan[Sqrt[2/(1
+ Sqrt[5])]*x]/Sqrt[2*(1 + Sqrt[5])] + ArcTanh[Sqrt[2/(-1 + Sqrt[5])]*x]/Sq
rt[2*(-1 + Sqrt[5])] - ArcTanh[Sqrt[2/(1 + Sqrt[5])]*x]/Sqrt[2*(1 + Sqrt[5]
)]
```

Maple [A]
time = 0.07, size = 96, normalized size = 0.73

method	result
risch	$\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(\_Z^4-\_Z^2-1\right)}\_R\ln\left(\_R^3-\_R+x\right)\right)}{4}+\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(\_Z^4+\_Z^2-1\right)}\_R\ln\left(-\_R^3-\_R+x\right)\right)}{4}$
default	$\frac{\operatorname{arctanh}\left(\frac{2x}{\sqrt{2\sqrt{5}-2}}\right)}{\sqrt{2\sqrt{5}-2}}-\frac{\arctan\left(\frac{2x}{\sqrt{2\sqrt{5}+2}}\right)}{\sqrt{2\sqrt{5}+2}}-\frac{\operatorname{arctanh}\left(\frac{2x}{\sqrt{2\sqrt{5}+2}}\right)}{\sqrt{2\sqrt{5}+2}}+\frac{\arctan\left(\frac{2x}{\sqrt{2\sqrt{5}-2}}\right)}{\sqrt{2\sqrt{5}-2}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4+1)/(x^8-3*x^4+1),x,method=_RETURNVERBOSE)
```

```
[Out] 1/(2*5^(1/2)-2)^(1/2)*arctanh(2*x/(2*5^(1/2)-2)^(1/2))-1/(2*5^(1/2)+2)^(1/2
)*arctan(2*x/(2*5^(1/2)+2)^(1/2))-1/(2*5^(1/2)+2)^(1/2)*arctanh(2*x/(2*5^(1
/2)+2)^(1/2))+1/(2*5^(1/2)-2)^(1/2)*arctan(2*x/(2*5^(1/2)-2)^(1/2))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4+1)/(x^8-3*x^4+1),x, algorithm="maxima")
```

```
[Out] integrate((x^4 + 1)/(x^8 - 3*x^4 + 1), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 247 vs. $2(95) = 190$.
time = 0.35, size = 247, normalized size = 1.89

$$-\frac{1}{2}\sqrt{2}\sqrt{\sqrt{5}+1}\arctan\left(-\frac{1}{2}\sqrt{2}x\sqrt{\sqrt{5}+1}+\frac{1}{2}\sqrt{2x^2+\sqrt{5}-1}\sqrt{\sqrt{5}+1}\right)+\frac{1}{2}\sqrt{2}\sqrt{\sqrt{5}-1}\arctan\left(-\frac{1}{2}\sqrt{2}x\sqrt{\sqrt{5}-1}+\frac{1}{2}\sqrt{2x^2+\sqrt{5}+1}\sqrt{\sqrt{5}-1}\right)+\frac{1}{8}\sqrt{2}\sqrt{\sqrt{5}+1}\log\left(\left(\sqrt{5}\sqrt{2}-\sqrt{2}\right)\sqrt{\sqrt{5}+1}+4x\right)-\frac{1}{8}\sqrt{2}\sqrt{\sqrt{5}+1}\log\left(-\left(\sqrt{5}\sqrt{2}-\sqrt{2}\right)\sqrt{\sqrt{5}+1}+4x\right)-\frac{1}{8}\sqrt{2}\sqrt{\sqrt{5}-1}\log\left(\left(\sqrt{5}\sqrt{2}+\sqrt{2}\right)\sqrt{\sqrt{5}-1}+4x\right)+\frac{1}{8}\sqrt{2}\sqrt{\sqrt{5}-1}\log\left(-\left(\sqrt{5}\sqrt{2}+\sqrt{2}\right)\sqrt{\sqrt{5}-1}+4x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4+1)/(x^8-3*x^4+1),x, algorithm="fricas")
```

```
[Out] -1/2*sqrt(2)*sqrt(sqrt(5) + 1)*arctan(-1/2*sqrt(2)*x*sqrt(sqrt(5) + 1) + 1/
2*sqrt(2*x^2 + sqrt(5) - 1)*sqrt(sqrt(5) + 1)) + 1/2*sqrt(2)*sqrt(sqrt(5) -
 1)*arctan(-1/2*sqrt(2)*x*sqrt(sqrt(5) - 1) + 1/2*sqrt(2*x^2 + sqrt(5) + 1)
*sqrt(sqrt(5) - 1)) + 1/8*sqrt(2)*sqrt(sqrt(5) + 1)*log((sqrt(5)*sqrt(2) -
sqrt(2))*sqrt(sqrt(5) + 1) + 4*x) - 1/8*sqrt(2)*sqrt(sqrt(5) + 1)*log(-(sqr
t(5)*sqrt(2) - sqrt(2))*sqrt(sqrt(5) + 1) + 4*x) - 1/8*sqrt(2)*sqrt(sqrt(5)
 - 1)*log((sqrt(5)*sqrt(2) + sqrt(2))*sqrt(sqrt(5) - 1) + 4*x) + 1/8*sqrt(2
```

```
)*sqrt(sqrt(5) - 1)*log(-(sqrt(5)*sqrt(2) + sqrt(2))*sqrt(sqrt(5) - 1) + 4*
x)
```

Sympy [A]
time = 0.68, size = 49, normalized size = 0.37

$$\operatorname{RootSum}\left(256t^4-16t^2-1,\left(t\mapsto t\log\left(1024t^5-8t+x\right)\right)\right)+\operatorname{RootSum}\left(256t^4+16t^2-1,\left(t\mapsto t\log\left(1024t^5-8t+x\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**4+1)/(x**8-3*x**4+1),x)
```

```
[Out] RootSum(256*_t**4 - 16*_t**2 - 1, Lambda(_t, _t*log(1024*_t**5 - 8*_t + x))
) + RootSum(256*_t**4 + 16*_t**2 - 1, Lambda(_t, _t*log(1024*_t**5 - 8*_t +
 x)))
```

Giac [A]
time = 4.58, size = 147, normalized size = 1.12

$$-\frac{1}{4}\sqrt{2\sqrt{5}-2}\arctan\left(\frac{x}{\sqrt{\frac{1}{2}\sqrt{5}+\frac{1}{2}}}\right)+\frac{1}{4}\sqrt{2\sqrt{5}+2}\arctan\left(\frac{x}{\sqrt{\frac{1}{2}\sqrt{5}-\frac{1}{2}}}\right)-\frac{1}{8}\sqrt{2\sqrt{5}-2}\log\left(\left|x+\sqrt{\frac{1}{2}\sqrt{5}+\frac{1}{2}}\right|\right)+\frac{1}{8}\sqrt{2\sqrt{5}-2}\log\left(\left|x-\sqrt{\frac{1}{2}\sqrt{5}+\frac{1}{2}}\right|\right)+\frac{1}{8}\sqrt{2\sqrt{5}+2}\log\left(\left|x+\sqrt{\frac{1}{2}\sqrt{5}-\frac{1}{2}}\right|\right)-\frac{1}{8}\sqrt{2\sqrt{5}+2}\log\left(\left|x-\sqrt{\frac{1}{2}\sqrt{5}-\frac{1}{2}}\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4+1)/(x^8-3*x^4+1),x, algorithm="giac")
```

```
[Out] -1/4*sqrt(2*sqrt(5) - 2)*arctan(x/sqrt(1/2*sqrt(5) + 1/2)) + 1/4*sqrt(2*sqr
t(5) + 2)*arctan(x/sqrt(1/2*sqrt(5) - 1/2)) - 1/8*sqrt(2*sqrt(5) - 2)*log(a
bs(x + sqrt(1/2*sqrt(5) + 1/2))) + 1/8*sqrt(2*sqrt(5) - 2)*log(abs(x - sqrt
(1/2*sqrt(5) + 1/2))) + 1/8*sqrt(2*sqrt(5) + 2)*log(abs(x + sqrt(1/2*sqrt(5
) - 1/2))) - 1/8*sqrt(2*sqrt(5) + 2)*log(abs(x - sqrt(1/2*sqrt(5) - 1/2)))
```

Mupad [B]
time = 0.20, size = 269, normalized size = 2.05

$$-\frac{\sqrt{2}\,\mathrm{atan}\left(\frac{\sqrt{2}\,x\,\sqrt{\sqrt{5}-1}\,1875{}\mathrm{i}}{2\,\left(875\,\sqrt{5}-1875\right)}-\frac{\sqrt{2}\,\sqrt{5}\,x\,\sqrt{\sqrt{5}-1}\,875{}\mathrm{i}}{2\,\left(875\,\sqrt{5}-1875\right)}\right)\,\sqrt{\sqrt{5}-1}\,1{}\mathrm{i}}{4}-\frac{\sqrt{2}\,\mathrm{atan}\left(\frac{\sqrt{2}\,x\,\sqrt{\sqrt{5}+1}\,1875{}\mathrm{i}}{2\,\left(875\,\sqrt{5}+1875\right)}+\frac{\sqrt{2}\,\sqrt{5}\,x\,\sqrt{\sqrt{5}+1}\,875{}\mathrm{i}}{2\,\left(875\,\sqrt{5}+1875\right)}\right)\,\sqrt{\sqrt{5}+1}\,1{}\mathrm{i}}{4}+\frac{\sqrt{2}\,\mathrm{atan}\left(\frac{\sqrt{2}\,x\,\sqrt{1-\sqrt{5}}\,1875{}\mathrm{i}}{2\,\left(875\,\sqrt{5}-1875\right)}-\frac{\sqrt{2}\,\sqrt{5}\,x\,\sqrt{1-\sqrt{5}}\,875{}\mathrm{i}}{2\,\left(875\,\sqrt{5}-1875\right)}\right)\,\sqrt{1-\sqrt{5}}\,1{}\mathrm{i}}{4}+\frac{\sqrt{2}\,\mathrm{atan}\left(\frac{\sqrt{2}\,x\,\sqrt{-\sqrt{5}-1}\,1875{}\mathrm{i}}{2\,\left(875\,\sqrt{5}+1875\right)}+\frac{\sqrt{2}\,\sqrt{5}\,x\,\sqrt{-\sqrt{5}-1}\,875{}\mathrm{i}}{2\,\left(875\,\sqrt{5}+1875\right)}\right)\,\sqrt{-\sqrt{5}-1}\,1{}\mathrm{i}}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4 + 1)/(x^8 - 3*x^4 + 1),x)
```

```
[Out] (2^(1/2)*atan((2^(1/2)*x*(1 - 5^(1/2))^(1/2)*1875i)/(2*(875*5^(1/2) - 1875)
) - (2^(1/2)*5^(1/2)*x*(1 - 5^(1/2))^(1/2)*875i)/(2*(875*5^(1/2) - 1875)))*
(1 - 5^(1/2))^(1/2)*1i)/4 - (2^(1/2)*atan((2^(1/2)*x*(5^(1/2) + 1)^(1/2)*18
75i)/(2*(875*5^(1/2) + 1875)) + (2^(1/2)*5^(1/2)*x*(5^(1/2) + 1)^(1/2)*875i
)/(2*(875*5^(1/2) + 1875)))*(5^(1/2) + 1)^(1/2)*1i)/4 - (2^(1/2)*atan((2^(1
/2)*x*(5^(1/2) - 1)^(1/2)*1875i)/(2*(875*5^(1/2) - 1875)) - (2^(1/2)*5^(1/2
)*x*(5^(1/2) - 1)^(1/2)*875i)/(2*(875*5^(1/2) - 1875)))*(5^(1/2) - 1)^(1/2)
*1i)/4 + (2^(1/2)*atan((2^(1/2)*x*(- 5^(1/2) - 1)^(1/2)*1875i)/(2*(875*5^(1
/2) + 1875)) + (2^(1/2)*5^(1/2)*x*(- 5^(1/2) - 1)^(1/2)*875i)/(2*(875*5^(1/
2) + 1875)))*(- 5^(1/2) - 1)^(1/2)*1i)/4
```

3.17 $\int \frac{1+x^4}{1-4x^4+x^8}\,dx$

Optimal. Leaf size=157

$$\frac{\tan^{-1}\left(\frac{\sqrt[4]{2}\,x}{\sqrt{-1+\sqrt{3}}}\right)}{2\sqrt[4]{2}\,\sqrt{-1+\sqrt{3}}}-\frac{\tan^{-1}\left(\frac{\sqrt[4]{2}\,x}{\sqrt{1+\sqrt{3}}}\right)}{2\sqrt[4]{2}\,\sqrt{1+\sqrt{3}}}+\frac{\tanh^{-1}\left(\frac{\sqrt[4]{2}\,x}{\sqrt{-1+\sqrt{3}}}\right)}{2\sqrt[4]{2}\,\sqrt{-1+\sqrt{3}}}-\frac{\tanh^{-1}\left(\frac{\sqrt[4]{2}\,x}{\sqrt{1+\sqrt{3}}}\right)}{2\sqrt[4]{2}\,\sqrt{1+\sqrt{3}}}$$

```
[Out] 1/4*arctan(2^(1/4)*x/(3^(1/2)-1)^(1/2))*2^(3/4)/(3^(1/2)-1)^(1/2)+1/4*arcta
nh(2^(1/4)*x/(3^(1/2)-1)^(1/2))*2^(3/4)/(3^(1/2)-1)^(1/2)-1/4*arctan(2^(1/4
)*x/(1+3^(1/2))^(1/2))*2^(3/4)/(1+3^(1/2))^(1/2)-1/4*arctanh(2^(1/4)*x/(1+3
^(1/2))^(1/2))*2^(3/4)/(1+3^(1/2))^(1/2)
```

Rubi [A]

time = 0.06, antiderivative size = 157, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 4, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {1433, 1107, 209, 213}

$$\frac{\text{ArcTan}\left(\frac{\sqrt[4]{2}\,x}{\sqrt{\sqrt{3}-1}}\right)}{2\sqrt[4]{2}\,\sqrt{\sqrt{3}-1}}-\frac{\text{ArcTan}\left(\frac{\sqrt[4]{2}\,x}{\sqrt{1+\sqrt{3}}}\right)}{2\sqrt[4]{2}\,\sqrt{1+\sqrt{3}}}+\frac{\tanh^{-1}\left(\frac{\sqrt[4]{2}\,x}{\sqrt{\sqrt{3}-1}}\right)}{2\sqrt[4]{2}\,\sqrt{\sqrt{3}-1}}-\frac{\tanh^{-1}\left(\frac{\sqrt[4]{2}\,x}{\sqrt{1+\sqrt{3}}}\right)}{2\sqrt[4]{2}\,\sqrt{1+\sqrt{3}}}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + x^4)/(1 - 4*x^4 + x^8),x]
```

```
[Out] ArcTan[(2^(1/4)*x)/Sqrt[-1 + Sqrt[3]]]/(2*2^(1/4)*Sqrt[-1 + Sqrt[3]]) - Arc
Tan[(2^(1/4)*x)/Sqrt[1 + Sqrt[3]]]/(2*2^(1/4)*Sqrt[1 + Sqrt[3]]) + ArcTanh[
(2^(1/4)*x)/Sqrt[-1 + Sqrt[3]]]/(2*2^(1/4)*Sqrt[-1 + Sqrt[3]]) - ArcTanh[(2
^(1/4)*x)/Sqrt[1 + Sqrt[3]]]/(2*2^(1/4)*Sqrt[1 + Sqrt[3]])
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 1107

```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(-1), x_Symbol] :> With[{q = Rt[b^
2 - 4*a*c, 2]}, Dist[c/q, Int[1/(b/2 - q/2 + c*x^2), x], x] - Dist[c/q, Int
[1/(b/2 + q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c,
0] && PosQ[b^2 - 4*a*c]
```

Rule 1433

```
Int[((d_) + (e_.)*(x_)^(n_))/((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_)), x
_Symbol] :> With[{q = Rt[2*(d/e) - b/c, 2]}, Dist[e/(2*c), Int[1/Simp[d/e +
 q*x^(n/2) + x^n, x], x], x] + Dist[e/(2*c), Int[1/Simp[d/e - q*x^(n/2) + x
^n, x], x], x]] /; FreeQ[{a, b, c, d, e}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4
*a*c, 0] && EqQ[c*d^2 - a*e^2, 0] && IGtQ[n/2, 0] && (GtQ[2*(d/e) - b/c, 0]
 || ( !LtQ[2*(d/e) - b/c, 0] && EqQ[d, e*Rt[a/c, 2]]))
```

Rubi steps

$$\int \frac{1+x^4}{1-4x^4+x^8}\,dx = \frac{1}{2}\int \frac{1}{1-\sqrt{6}\,x^2+x^4}\,dx + \frac{1}{2}\int \frac{1}{1+\sqrt{6}\,x^2+x^4}\,dx$$

$$= \frac{\int \frac{1}{-\sqrt{\frac{3}{2}}-\frac{1}{\sqrt{2}}+x^2}\,dx}{2\sqrt{2}} + \frac{\int \frac{1}{\sqrt{\frac{3}{2}}-\frac{1}{\sqrt{2}}+x^2}\,dx}{2\sqrt{2}} - \frac{\int \frac{1}{-\sqrt{\frac{3}{2}}+\frac{1}{\sqrt{2}}+x^2}\,dx}{2\sqrt{2}} - \frac{\int \frac{1}{\sqrt{\frac{3}{2}}+\frac{1}{\sqrt{2}}+x}}{2\sqrt{2}}$$

$$= \frac{\tan^{-1}\left(\frac{\sqrt[4]{2}\,x}{\sqrt{-1+\sqrt{3}}}\right)}{2\sqrt[4]{2}\,\sqrt{-1+\sqrt{3}}} - \frac{\tan^{-1}\left(\frac{\sqrt[4]{2}\,x}{\sqrt{1+\sqrt{3}}}\right)}{2\sqrt[4]{2}\,\sqrt{1+\sqrt{3}}} + \frac{\tanh^{-1}\left(\frac{\sqrt[4]{2}\,x}{\sqrt{-1+\sqrt{3}}}\right)}{2\sqrt[4]{2}\,\sqrt{-1+\sqrt{3}}} - \frac{\tanh}{2}$$

Mathematica [C] Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.
time = 0.01, size = 53, normalized size = 0.34

$$\frac{1}{8}\text{RootSum}\left[1-4\#1^4+\#1^8\&,\frac{\log(x-\#1)+\log(x-\#1)\#1^4}{-2\#1^3+\#1^7}\&\right]$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + x^4)/(1 - 4*x^4 + x^8),x]
```

```
[Out] RootSum[1 - 4*#1^4 + #1^8 & , (Log[x - #1] + Log[x - #1]*#1^4)/(-2*#1^3 + #
1^7) & ]/8
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 0.03, size = 40, normalized size = 0.25

method	result	size
default	$\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(\_Z^{8}-4\_Z^{4}+1\right)}\frac{\left(\_R^{4}+1\right)\ln\left(x-\_R\right)}{\_R^{7}-2\_R^{3}}\right)}{8}$	40
risch	$\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(\_Z^{8}-4\_Z^{4}+1\right)}\frac{\left(\_R^{4}+1\right)\ln\left(x-\_R\right)}{\_R^{7}-2\_R^{3}}\right)}{8}$	40

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4+1)/(x^8-4*x^4+1),x,method=_RETURNVERBOSE)
```

```
[Out] 1/8*sum((_R^4+1)/(_R^7-2*_R^3)*ln(x-_R),_R=RootOf(_Z^8-4*_Z^4+1))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4+1)/(x^8-4*x^4+1),x, algorithm="maxima")
```

```
[Out] integrate((x^4 + 1)/(x^8 - 4*x^4 + 1), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 331 vs. $2(101) = 202$.
time = 0.36, size = 331, normalized size = 2.11

$$\frac{1}{2}\sqrt{2}\left(-\sqrt{3}+2\right)^{\frac{1}{4}}\arctan\left(\frac{1}{2}\sqrt{x^{2}+\left(\sqrt{3}+2\right)\sqrt{-\sqrt{3}+2}}\left(\sqrt{3}\sqrt{2}+\sqrt{2}\right)\left(-\sqrt{3}+2\right)^{\frac{3}{4}}-\frac{1}{2}\left(\sqrt{3}\sqrt{2}x+\sqrt{2}x\right)\left(-\sqrt{3}+2\right)^{\frac{3}{4}}\right)-\frac{1}{2}\sqrt{2}\left(\sqrt{3}+2\right)^{\frac{1}{4}}\arctan\left(\frac{1}{2}\left(\sqrt{x^{2}-\sqrt{\sqrt{3}+2}\left(\sqrt{3}-2\right)}\left(\sqrt{3}\sqrt{2}-\sqrt{2}\right)\sqrt{\sqrt{3}+2}-\left(\sqrt{3}\sqrt{2}x-\sqrt{2}x\right)\sqrt{\sqrt{3}+2}\right)\left(\sqrt{3}+2\right)^{\frac{1}{4}}\right)+\frac{1}{8}\sqrt{2}\left(\sqrt{3}+2\right)^{\frac{1}{4}}\log\left(\left(\sqrt{3}\sqrt{2}-\sqrt{2}\right)\left(\sqrt{3}+2\right)^{\frac{1}{4}}+2x\right)-\frac{1}{8}\sqrt{2}\left(\sqrt{3}+2\right)^{\frac{1}{4}}\log\left(-\left(\sqrt{3}\sqrt{2}-\sqrt{2}\right)\left(\sqrt{3}+2\right)^{\frac{1}{4}}+2x\right)-\frac{1}{8}\sqrt{2}\left(-\sqrt{3}+2\right)^{\frac{1}{4}}\log\left(\left(\sqrt{3}\sqrt{2}+\sqrt{2}\right)\left(-\sqrt{3}+2\right)^{\frac{1}{4}}+2x\right)+\frac{1}{8}\sqrt{2}\left(-\sqrt{3}+2\right)^{\frac{1}{4}}\log\left(-\left(\sqrt{3}\sqrt{2}+\sqrt{2}\right)\left(-\sqrt{3}+2\right)^{\frac{1}{4}}+2x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4+1)/(x^8-4*x^4+1),x, algorithm="fricas")
```

```
[Out] 1/2*sqrt(2)*(-sqrt(3) + 2)^(1/4)*arctan(1/2*sqrt(x^2 + (sqrt(3) + 2)*sqrt(-
sqrt(3) + 2))*(sqrt(3)*sqrt(2) + sqrt(2))*(-sqrt(3) + 2)^(3/4) - 1/2*(sqrt(
3)*sqrt(2)*x + sqrt(2)*x)*(-sqrt(3) + 2)^(3/4)) - 1/2*sqrt(2)*(sqrt(3) + 2)
^(1/4)*arctan(1/2*(sqrt(x^2 - sqrt(sqrt(3) + 2)*(sqrt(3) - 2))*(sqrt(3)*sqr
t(2) - sqrt(2))*sqrt(sqrt(3) + 2) - (sqrt(3)*sqrt(2)*x - sqrt(2)*x)*sqrt(sq
rt(3) + 2))*(sqrt(3) + 2)^(1/4)) + 1/8*sqrt(2)*(sqrt(3) + 2)^(1/4)*log((sqr
t(3)*sqrt(2) - sqrt(2))*(sqrt(3) + 2)^(1/4) + 2*x) - 1/8*sqrt(2)*(sqrt(3) +
 2)^(1/4)*log(-(sqrt(3)*sqrt(2) - sqrt(2))*(sqrt(3) + 2)^(1/4) + 2*x) - 1/8
*sqrt(2)*(-sqrt(3) + 2)^(1/4)*log((sqrt(3)*sqrt(2) + sqrt(2))*(-sqrt(3) + 2
)^(1/4) + 2*x) + 1/8*sqrt(2)*(-sqrt(3) + 2)^(1/4)*log(-(sqrt(3)*sqrt(2) + s
qrt(2))*(-sqrt(3) + 2)^(1/4) + 2*x)
```

Sympy [A]
time = 0.08, size = 24, normalized size = 0.15

$$\operatorname{RootSum}\left(1048576t^8 - 4096t^4 + 1, \left(t \mapsto t \log\left(4096t^5 - 12t + x\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**4+1)/(x**8-4*x**4+1),x)
```

```
[Out] RootSum(1048576*_t**8 - 4096*_t**4 + 1, Lambda(_t, _t*log(4096*_t**5 - 12*_
t + x)))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4+1)/(x^8-4*x^4+1),x, algorithm="giac")
```

```
[Out] integrate((x^4 + 1)/(x^8 - 4*x^4 + 1), x)
```

Mupad [B]
time = 1.72, size = 399, normalized size = 2.54

$$\frac{\sqrt{2}\,\mathrm{atan}\left(\frac{5184\sqrt{2}\,x\left(\sqrt{3}+2\right)^{1/4}}{3888\sqrt{\sqrt{3}+2}+2160\sqrt{3}\sqrt{\sqrt{3}+2}}+\frac{3024\sqrt{2}\sqrt{3}\,x\left(\sqrt{3}+2\right)^{1/4}}{3888\sqrt{\sqrt{3}+2}+2160\sqrt{3}\sqrt{\sqrt{3}+2}}\right)\left(\sqrt{3}+2\right)^{1/4}}{4}+\frac{\sqrt{2}\,\mathrm{atan}\left(\frac{\sqrt{2}\,x\left(2-\sqrt{3}\right)^{1/4}5184\mathrm{i}}{2160\sqrt{3}\sqrt{2-\sqrt{3}}-3888\sqrt{2-\sqrt{3}}}-\frac{\sqrt{2}\sqrt{3}\,x\left(2-\sqrt{3}\right)^{1/4}3024\mathrm{i}}{2160\sqrt{3}\sqrt{2-\sqrt{3}}-3888\sqrt{2-\sqrt{3}}}\right)\left(2-\sqrt{3}\right)^{1/4}1\mathrm{i}}{4}-\frac{\sqrt{2}\,\mathrm{atan}\left(\frac{5184\sqrt{2}\,x\left(2-\sqrt{3}\right)^{1/4}}{2160\sqrt{3}\sqrt{2-\sqrt{3}}-3888\sqrt{2-\sqrt{3}}}-\frac{3024\sqrt{2}\sqrt{3}\,x\left(2-\sqrt{3}\right)^{1/4}}{2160\sqrt{3}\sqrt{2-\sqrt{3}}-3888\sqrt{2-\sqrt{3}}}\right)\left(2-\sqrt{3}\right)^{1/4}}{4}-\frac{\sqrt{2}\,\mathrm{atan}\left(\frac{\sqrt{2}\,x\left(\sqrt{3}+2\right)^{1/4}5184\mathrm{i}}{3888\sqrt{\sqrt{3}+2}+2160\sqrt{3}\sqrt{\sqrt{3}+2}}+\frac{\sqrt{2}\sqrt{3}\,x\left(\sqrt{3}+2\right)^{1/4}3024\mathrm{i}}{3888\sqrt{\sqrt{3}+2}+2160\sqrt{3}\sqrt{\sqrt{3}+2}}\right)\left(\sqrt{3}+2\right)^{1/4}1\mathrm{i}}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4 + 1)/(x^8 - 4*x^4 + 1),x)
```

```
[Out] (2^(1/2)*atan((2^(1/2)*x*(2 - 3^(1/2))^(1/4)*5184i)/(2160*3^(1/2)*(2 - 3^(1
/2))^(1/2) - 3888*(2 - 3^(1/2))^(1/2)) - (2^(1/2)*3^(1/2)*x*(2 - 3^(1/2))^(
1/4)*3024i)/(2160*3^(1/2)*(2 - 3^(1/2))^(1/2) - 3888*(2 - 3^(1/2))^(1/2)))*
(2 - 3^(1/2))^(1/4)*1i)/4 - (2^(1/2)*atan((5184*2^(1/2)*x*(2 - 3^(1/2))^(1/
4))/(2160*3^(1/2)*(2 - 3^(1/2))^(1/2) - 3888*(2 - 3^(1/2))^(1/2)) - (3024*2
^(1/2)*3^(1/2)*x*(2 - 3^(1/2))^(1/4))/(2160*3^(1/2)*(2 - 3^(1/2))^(1/2) - 3
888*(2 - 3^(1/2))^(1/2)))*(2 - 3^(1/2))^(1/4))/4 + (2^(1/2)*atan((5184*2^(1
/2)*x*(3^(1/2) + 2)^(1/4))/(3888*(3^(1/2) + 2)^(1/2) + 2160*3^(1/2)*(3^(1/2
) + 2)^(1/2)) + (3024*2^(1/2)*3^(1/2)*x*(3^(1/2) + 2)^(1/4))/(3888*(3^(1/2)
 + 2)^(1/2) + 2160*3^(1/2)*(3^(1/2) + 2)^(1/2)))*(3^(1/2) + 2)^(1/4))/4 - (
2^(1/2)*atan((2^(1/2)*x*(3^(1/2) + 2)^(1/4)*5184i)/(3888*(3^(1/2) + 2)^(1/2
) + 2160*3^(1/2)*(3^(1/2) + 2)^(1/2)) + (2^(1/2)*3^(1/2)*x*(3^(1/2) + 2)^(1
/4)*3024i)/(3888*(3^(1/2) + 2)^(1/2) + 2160*3^(1/2)*(3^(1/2) + 2)^(1/2)))*(
3^(1/2) + 2)^(1/4)*1i)/4
```

3.18 $\int \frac{1+x^4}{1-5x^4+x^8}\,dx$

Optimal. Leaf size=171

$$\frac{\tan^{-1}\left(\sqrt{\frac{2}{-\sqrt{3}+\sqrt{7}}}\,x\right)}{\sqrt{6\left(-\sqrt{3}+\sqrt{7}\right)}}-\frac{\tan^{-1}\left(\sqrt{\frac{2}{\sqrt{3}+\sqrt{7}}}\,x\right)}{\sqrt{6\left(\sqrt{3}+\sqrt{7}\right)}}+\frac{\tanh^{-1}\left(\sqrt{\frac{2}{-\sqrt{3}+\sqrt{7}}}\,x\right)}{\sqrt{6\left(-\sqrt{3}+\sqrt{7}\right)}}-\frac{\tanh^{-1}\left(\sqrt{\frac{2}{\sqrt{3}+\ldots}}\right.}{\sqrt{6\left(\sqrt{3}+\ldots\right.}}$$

```
[Out] arctan(x*2^(1/2)/(7^(1/2)-3^(1/2))^(1/2))/(-6*3^(1/2)+6*7^(1/2))^(1/2)+arct
anh(x*2^(1/2)/(7^(1/2)-3^(1/2))^(1/2))/(-6*3^(1/2)+6*7^(1/2))^(1/2)-arctan(
x*2^(1/2)/(7^(1/2)+3^(1/2))^(1/2))/(6*3^(1/2)+6*7^(1/2))^(1/2)-arctanh(x*2^
(1/2)/(7^(1/2)+3^(1/2))^(1/2))/(6*3^(1/2)+6*7^(1/2))^(1/2)
```

Rubi [A]
time = 0.11, antiderivative size = 171, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 4, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {1433, 1107, 209, 213}

$$\frac{\text{ArcTan}\left(\sqrt{\frac{2}{\sqrt{7}-\sqrt{3}}}\,x\right)}{\sqrt{6\left(\sqrt{7}-\sqrt{3}\right)}}-\frac{\text{ArcTan}\left(\sqrt{\frac{2}{\sqrt{3}+\sqrt{7}}}\,x\right)}{\sqrt{6\left(\sqrt{3}+\sqrt{7}\right)}}+\frac{\tanh^{-1}\left(\sqrt{\frac{2}{\sqrt{7}-\sqrt{3}}}\,x\right)}{\sqrt{6\left(\sqrt{7}-\sqrt{3}\right)}}-\frac{\tanh^{-1}\left(\sqrt{\frac{2}{\sqrt{3}+\sqrt{7}}}\,x\right)}{\sqrt{6\left(\sqrt{3}+\sqrt{7}\right)}}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + x^4)/(1 - 5*x^4 + x^8),x]
```

```
[Out] ArcTan[Sqrt[2/(-Sqrt[3] + Sqrt[7])]*x]/Sqrt[6*(-Sqrt[3] + Sqrt[7])] - ArcTa
n[Sqrt[2/(Sqrt[3] + Sqrt[7])]*x]/Sqrt[6*(Sqrt[3] + Sqrt[7])] + ArcTanh[Sqrt
[2/(-Sqrt[3] + Sqrt[7])]*x]/Sqrt[6*(-Sqrt[3] + Sqrt[7])] - ArcTanh[Sqrt[2/(
Sqrt[3] + Sqrt[7])]*x]/Sqrt[6*(Sqrt[3] + Sqrt[7])]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 1107

```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(-1), x_Symbol] :> With[{q = Rt[b^
2 - 4*a*c, 2]}, Dist[c/q, Int[1/(b/2 - q/2 + c*x^2), x], x] - Dist[c/q, Int
[1/(b/2 + q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c,
0] && PosQ[b^2 - 4*a*c]
```

Rule 1433

```
Int[((d_) + (e_.)*(x_)^(n_))/((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_)), x
_Symbol] :> With[{q = Rt[2*(d/e) - b/c, 2]}, Dist[e/(2*c), Int[1/Simp[d/e +
 q*x^(n/2) + x^n, x], x], x] + Dist[e/(2*c), Int[1/Simp[d/e - q*x^(n/2) + x
^n, x], x], x]] /; FreeQ[{a, b, c, d, e}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4
*a*c, 0] && EqQ[c*d^2 - a*e^2, 0] && IGtQ[n/2, 0] && (GtQ[2*(d/e) - b/c, 0]
 || ( !LtQ[2*(d/e) - b/c, 0] && EqQ[d, e*Rt[a/c, 2]]))
```

Rubi steps

$$
\begin{aligned}
\int \frac{1+x^4}{1-5x^4+x^8}\,dx &= \frac{1}{2}\int \frac{1}{1-\sqrt{7}\,x^2+x^4}\,dx+\frac{1}{2}\int \frac{1}{1+\sqrt{7}\,x^2+x^4}\,dx\\
&= \frac{\int \frac{1}{-\frac{\sqrt{3}}{2}-\frac{\sqrt{7}}{2}+x^2}\,dx}{2\sqrt{3}}-\frac{\int \frac{1}{\frac{\sqrt{3}}{2}-\frac{\sqrt{7}}{2}+x^2}\,dx}{2\sqrt{3}}+\frac{\int \frac{1}{-\frac{\sqrt{3}}{2}+\frac{\sqrt{7}}{2}+x^2}\,dx}{2\sqrt{3}}-\frac{\int \frac{1}{\frac{\sqrt{3}}{2}+\frac{\sqrt{7}}{2}+x^2}
\\
&= \frac{\tan^{-1}\left(\sqrt{\frac{2}{-\sqrt{3}+\sqrt{7}}}\,x\right)}{\sqrt{6\left(-\sqrt{3}+\sqrt{7}\right)}}-\frac{\tan^{-1}\left(\sqrt{\frac{2}{\sqrt{3}+\sqrt{7}}}\,x\right)}{\sqrt{6\left(\sqrt{3}+\sqrt{7}\right)}}+\frac{\tanh^{-1}\left(\sqrt{\frac{2}{-\sqrt{3}+\sqrt{7}}}\right.}{\sqrt{6\left(-\sqrt{3}+\sqrt{7}\right)}}
\end{aligned}
$$

Mathematica [C] Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.
time = 0.01, size = 55, normalized size = 0.32

$$\frac{1}{4}\text{RootSum}\left[1-5\#1^4+\#1^8\&,\frac{\log(x-\#1)+\log(x-\#1)\#1^4}{-5\#1^3+2\#1^7}\&\right]$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + x^4)/(1 - 5*x^4 + x^8),x]
```

```
[Out] RootSum[1 - 5*#1^4 + #1^8 & , (Log[x - #1] + Log[x - #1]*#1^4)/(-5*#1^3 + 2
*#1^7) & ]/4
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 0.02, size = 42, normalized size = 0.25

method	result	size
default	$\frac{\sum_{\_R=\mathrm{RootOf}\left(\_Z^8-5\_Z^4+1\right)}\frac{\left(\_R^4+1\right)\ln\left(x-\_R\right)}{2\_R^7-5\_R^3}}{4}$	42
risch	$\frac{\sum_{\_R=\mathrm{RootOf}\left(\_Z^8-5\_Z^4+1\right)}\frac{\left(\_R^4+1\right)\ln\left(x-\_R\right)}{2\_R^7-5\_R^3}}{4}$	42

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4+1)/(x^8-5*x^4+1),x,method=_RETURNVERBOSE)
```

```
[Out] 1/4*sum((_R^4+1)/(2*_R^7-5*_R^3)*ln(x-_R),_R=RootOf(_Z^8-5*_Z^4+1))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4+1)/(x^8-5*x^4+1),x, algorithm="maxima")
```

```
[Out] integrate((x^4 + 1)/(x^8 - 5*x^4 + 1), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 574 vs. $2(123) = 246$.
time = 0.36, size = 574, normalized size = 3.36

$$\frac{1}{6}\sqrt{6}\sqrt{\sqrt{2}\sqrt{-\sqrt{7}\sqrt{3}+5}}\arctan\left(\frac{1}{48}\left(\sqrt{7}\sqrt{6}\sqrt{3}\sqrt{2}+3\sqrt{6}\sqrt{2}\right)\sqrt{4x^2+\left(\sqrt{7}\sqrt{3}\sqrt{2}+5\sqrt{2}\right)\sqrt{-\sqrt{7}\sqrt{3}+5}}\sqrt{-\sqrt{7}\sqrt{3}+5}\sqrt{\sqrt{2}\sqrt{-\sqrt{7}\sqrt{3}+5}}-\frac{1}{24}\left(\sqrt{7}\sqrt{6}\sqrt{3}\sqrt{2}x+3\sqrt{6}\sqrt{2}x\right)\sqrt{-\sqrt{7}\sqrt{3}+5}\sqrt{\sqrt{2}\sqrt{-\sqrt{7}\sqrt{3}+5}}\right)-\frac{1}{6}\sqrt{6}\sqrt{\sqrt{2}\sqrt{\sqrt{7}\sqrt{3}+5}}\arctan\left(\frac{1}{48}\left(\left(\sqrt{7}\sqrt{6}\sqrt{3}\sqrt{2}-3\sqrt{6}\sqrt{2}\right)\sqrt{4x^2-\left(\sqrt{7}\sqrt{3}\sqrt{2}-5\sqrt{2}\right)\sqrt{\sqrt{7}\sqrt{3}+5}}\sqrt{\sqrt{7}\sqrt{3}+5}-2\left(\sqrt{7}\sqrt{6}\sqrt{3}\sqrt{2}x-3\sqrt{6}\sqrt{2}x\right)\sqrt{\sqrt{7}\sqrt{3}+5}\right)\sqrt{\sqrt{2}\sqrt{\sqrt{7}\sqrt{3}+5}}\right)+\frac{1}{24}\sqrt{6}\sqrt{\sqrt{2}\sqrt{\sqrt{7}\sqrt{3}+5}}\log\left(\left(\sqrt{7}\sqrt{6}\sqrt{3}-3\sqrt{6}\right)\sqrt{\sqrt{2}\sqrt{\sqrt{7}\sqrt{3}+5}}+12x\right)-\frac{1}{24}\sqrt{6}\sqrt{\sqrt{2}\sqrt{\sqrt{7}\sqrt{3}+5}}\log\left(-\left(\sqrt{7}\sqrt{6}\sqrt{3}-3\sqrt{6}\right)\sqrt{\sqrt{2}\sqrt{\sqrt{7}\sqrt{3}+5}}+12x\right)-\frac{1}{24}\sqrt{6}\sqrt{\sqrt{2}\sqrt{-\sqrt{7}\sqrt{3}+5}}\log\left(\left(\sqrt{7}\sqrt{6}\sqrt{3}+3\sqrt{6}\right)\sqrt{\sqrt{2}\sqrt{-\sqrt{7}\sqrt{3}+5}}+12x\right)+\frac{1}{24}\sqrt{6}\sqrt{\sqrt{2}\sqrt{-\sqrt{7}\sqrt{3}+5}}\log\left(-\left(\sqrt{7}\sqrt{6}\sqrt{3}+3\sqrt{6}\right)\sqrt{\sqrt{2}\sqrt{-\sqrt{7}\sqrt{3}+5}}+12x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4+1)/(x^8-5*x^4+1),x, algorithm="fricas")
```

```
[Out] 1/6*sqrt(6)*sqrt(sqrt(2)*sqrt(-sqrt(7)*sqrt(3) + 5))*arctan(1/48*(sqrt(7)*s
qrt(6)*sqrt(3)*sqrt(2) + 3*sqrt(6)*sqrt(2))*sqrt(4*x^2 + (sqrt(7)*sqrt(3)*s
qrt(2) + 5*sqrt(2))*sqrt(-sqrt(7)*sqrt(3) + 5))*sqrt(-sqrt(7)*sqrt(3) + 5)*
sqrt(sqrt(2)*sqrt(-sqrt(7)*sqrt(3) + 5)) - 1/24*(sqrt(7)*sqrt(6)*sqrt(3)*sq
rt(2)*x + 3*sqrt(6)*sqrt(2)*x)*sqrt(-sqrt(7)*sqrt(3) + 5)*sqrt(sqrt(2)*sqrt
(-sqrt(7)*sqrt(3) + 5))) - 1/6*sqrt(6)*sqrt(sqrt(2)*sqrt(sqrt(7)*sqrt(3) +
5))*arctan(1/48*((sqrt(7)*sqrt(6)*sqrt(3)*sqrt(2) - 3*sqrt(6)*sqrt(2))*sqrt
(4*x^2 - (sqrt(7)*sqrt(3)*sqrt(2) - 5*sqrt(2))*sqrt(sqrt(7)*sqrt(3) + 5))*s
qrt(sqrt(7)*sqrt(3) + 5) - 2*(sqrt(7)*sqrt(6)*sqrt(3)*sqrt(2)*x - 3*sqrt(6)
*sqrt(2)*x)*sqrt(sqrt(7)*sqrt(3) + 5))*sqrt(sqrt(2)*sqrt(sqrt(7)*sqrt(3) +
5))) + 1/24*sqrt(6)*sqrt(sqrt(2)*sqrt(sqrt(7)*sqrt(3) + 5))*log((sqrt(7)*sq
```

```
rt(6)*sqrt(3) - 3*sqrt(6))*sqrt(sqrt(2)*sqrt(sqrt(7)*sqrt(3) + 5)) + 12*x)
- 1/24*sqrt(6)*sqrt(sqrt(2)*sqrt(sqrt(7)*sqrt(3) + 5))*log(-(sqrt(7)*sqrt(6
)*sqrt(3) - 3*sqrt(6))*sqrt(sqrt(2)*sqrt(sqrt(7)*sqrt(3) + 5)) + 12*x) - 1/
24*sqrt(6)*sqrt(sqrt(2)*sqrt(-sqrt(7)*sqrt(3) + 5))*log((sqrt(7)*sqrt(6)*sq
rt(3) + 3*sqrt(6))*sqrt(sqrt(2)*sqrt(-sqrt(7)*sqrt(3) + 5)) + 12*x) + 1/24*
sqrt(6)*sqrt(sqrt(2)*sqrt(-sqrt(7)*sqrt(3) + 5))*log(-(sqrt(7)*sqrt(6)*sqrt
(3) + 3*sqrt(6))*sqrt(sqrt(2)*sqrt(-sqrt(7)*sqrt(3) + 5)) + 12*x)
```

Sympy [**A**]
time = 0.08, size = 24, normalized size = 0.14

$$\operatorname{RootSum}\left(5308416t^8-11520t^4+1,\left(t\mapsto t\log\left(9216t^5-16t+x\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**4+1)/(x**8-5*x**4+1),x)
```

```
[Out] RootSum(5308416*_t**8 - 11520*_t**4 + 1, Lambda(_t, _t*log(9216*_t**5 - 16*
_t + x)))
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4+1)/(x^8-5*x^4+1),x, algorithm="giac")
```

```
[Out] integrate((x^4 + 1)/(x^8 - 5*x^4 + 1), x)
```

Mupad [**B**]
time = 1.76, size = 483, normalized size = 2.82

$$\frac{2^{3/4}\sqrt{3}\,\mathrm{atan}\left(\frac{12005\,2^{3/4}\sqrt{3}\,x\left(5-\sqrt{21}\right)^{1/4}}{2\left(4802\sqrt{2}\sqrt{5-\sqrt{21}}-1029\sqrt{2}\sqrt{21}\sqrt{5-\sqrt{21}}\right)}-\frac{7889\,2^{3/4}\sqrt{3}\sqrt{21}\,x\left(5-\sqrt{21}\right)^{1/4}}{6\left(4802\sqrt{2}\sqrt{5-\sqrt{21}}-1029\sqrt{2}\sqrt{21}\sqrt{5-\sqrt{21}}\right)}\right)\left(5-\sqrt{21}\right)^{1/4}}{12}-\frac{2^{3/4}\sqrt{3}\,\mathrm{atan}\left(\frac{2^{3/4}\sqrt{3}\,x\left(5-\sqrt{21}\right)^{1/4}12005i}{2\left(4802\sqrt{2}\sqrt{5-\sqrt{21}}-1029\sqrt{2}\sqrt{21}\sqrt{5-\sqrt{21}}\right)}-\frac{2^{3/4}\sqrt{3}\sqrt{21}\,x\left(5-\sqrt{21}\right)^{1/4}7889i}{6\left(4802\sqrt{2}\sqrt{5-\sqrt{21}}-1029\sqrt{2}\sqrt{21}\sqrt{5-\sqrt{21}}\right)}\right)\left(5-\sqrt{21}\right)^{1/4}1i}{12}+\frac{2^{3/4}\sqrt{3}\,\mathrm{atan}\left(\frac{12005\,2^{3/4}\sqrt{3}\,x\left(\sqrt{21}+5\right)^{1/4}}{2\left(4802\sqrt{2}\sqrt{\sqrt{21}+5}+1029\sqrt{2}\sqrt{21}\sqrt{\sqrt{21}+5}\right)}+\frac{7889\,2^{3/4}\sqrt{3}\sqrt{21}\,x\left(\sqrt{21}+5\right)^{1/4}}{6\left(4802\sqrt{2}\sqrt{\sqrt{21}+5}+1029\sqrt{2}\sqrt{21}\sqrt{\sqrt{21}+5}\right)}\right)\left(\sqrt{21}+5\right)^{1/4}}{12}-\frac{2^{3/4}\sqrt{3}\,\mathrm{atan}\left(\frac{2^{3/4}\sqrt{3}\,x\left(\sqrt{21}+5\right)^{1/4}12005i}{2\left(4802\sqrt{2}\sqrt{\sqrt{21}+5}+1029\sqrt{2}\sqrt{21}\sqrt{\sqrt{21}+5}\right)}+\frac{2^{3/4}\sqrt{3}\sqrt{21}\,x\left(\sqrt{21}+5\right)^{1/4}7889i}{6\left(4802\sqrt{2}\sqrt{\sqrt{21}+5}+1029\sqrt{2}\sqrt{21}\sqrt{\sqrt{21}+5}\right)}\right)\left(\sqrt{21}+5\right)^{1/4}1i}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4 + 1)/(x^8 - 5*x^4 + 1),x)
```

```
[Out] (2^(3/4)*3^(1/2)*atan((12005*2^(3/4)*3^(1/2)*x*(5 - 21^(1/2))^(1/4))/(2*(48
02*2^(1/2)*(5 - 21^(1/2))^(1/2) - 1029*2^(1/2)*21^(1/2)*(5 - 21^(1/2))^(1/2
))) - (7889*2^(3/4)*3^(1/2)*21^(1/2)*x*(5 - 21^(1/2))^(1/4))/(6*(4802*2^(1/
2)*(5 - 21^(1/2))^(1/2) - 1029*2^(1/2)*21^(1/2)*(5 - 21^(1/2))^(1/2))))*(5
- 21^(1/2))^(1/4))/12 - (2^(3/4)*3^(1/2)*atan((2^(3/4)*3^(1/2)*x*(5 - 21^(1
/2))^(1/4)*12005i)/(2*(4802*2^(1/2)*(5 - 21^(1/2))^(1/2) - 1029*2^(1/2)*21^
(1/2)*(5 - 21^(1/2))^(1/2))) - (2^(3/4)*3^(1/2)*21^(1/2)*x*(5 - 21^(1/2))^(
1/4)*7889i)/(6*(4802*2^(1/2)*(5 - 21^(1/2))^(1/2) - 1029*2^(1/2)*21^(1/2)*(
5 - 21^(1/2))^(1/2))))*(5 - 21^(1/2))^(1/4)*1i)/12 + (2^(3/4)*3^(1/2)*atan(
```

```
(12005*2^(3/4)*3^(1/2)*x*(21^(1/2) + 5)^(1/4))/(2*(4802*2^(1/2)*(21^(1/2) +
 5)^(1/2) + 1029*2^(1/2)*21^(1/2)*(21^(1/2) + 5)^(1/2))) + (7889*2^(3/4)*3^
(1/2)*21^(1/2)*x*(21^(1/2) + 5)^(1/4))/(6*(4802*2^(1/2)*(21^(1/2) + 5)^(1/2
) + 1029*2^(1/2)*21^(1/2)*(21^(1/2) + 5)^(1/2))))*(21^(1/2) + 5)^(1/4))/12
- (2^(3/4)*3^(1/2)*atan((2^(3/4)*3^(1/2)*x*(21^(1/2) + 5)^(1/4)*12005i)/(2*
(4802*2^(1/2)*(21^(1/2) + 5)^(1/2) + 1029*2^(1/2)*21^(1/2)*(21^(1/2) + 5)^(
1/2))) + (2^(3/4)*3^(1/2)*21^(1/2)*x*(21^(1/2) + 5)^(1/4)*7889i)/(6*(4802*2
^(1/2)*(21^(1/2) + 5)^(1/2) + 1029*2^(1/2)*21^(1/2)*(21^(1/2) + 5)^(1/2))))
*(21^(1/2) + 5)^(1/4)*1i)/12
```

3.19 $\int \frac{1+x^4}{1-6x^4+x^8}\,dx$

Optimal. Leaf size=117

$$\frac{\tan^{-1}\left(\frac{x}{\sqrt{-1+\sqrt{2}}}\right)}{4\sqrt{-1+\sqrt{2}}}-\frac{\tan^{-1}\left(\frac{x}{\sqrt{1+\sqrt{2}}}\right)}{4\sqrt{1+\sqrt{2}}}+\frac{\tanh^{-1}\left(\frac{x}{\sqrt{-1+\sqrt{2}}}\right)}{4\sqrt{-1+\sqrt{2}}}-\frac{\tanh^{-1}\left(\frac{x}{\sqrt{1+\sqrt{2}}}\right)}{4\sqrt{1+\sqrt{2}}}$$

```
[Out] 1/4*arctan(x/(2^(1/2)-1)^(1/2))/(2^(1/2)-1)^(1/2)+1/4*arctanh(x/(2^(1/2)-1)
^(1/2))/(2^(1/2)-1)^(1/2)-1/4*arctan(x/(1+2^(1/2))^(1/2))/(1+2^(1/2))^(1/2)
-1/4*arctanh(x/(1+2^(1/2))^(1/2))/(1+2^(1/2))^(1/2)
```

Rubi [A]
time = 0.04, antiderivative size = 117, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 4, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {1433, 1107, 209, 213}

$$\frac{\text{ArcTan}\left(\frac{x}{\sqrt{\sqrt{2}-1}}\right)}{4\sqrt{\sqrt{2}-1}}-\frac{\text{ArcTan}\left(\frac{x}{\sqrt{1+\sqrt{2}}}\right)}{4\sqrt{1+\sqrt{2}}}+\frac{\tanh^{-1}\left(\frac{x}{\sqrt{\sqrt{2}-1}}\right)}{4\sqrt{\sqrt{2}-1}}-\frac{\tanh^{-1}\left(\frac{x}{\sqrt{1+\sqrt{2}}}\right)}{4\sqrt{1+\sqrt{2}}}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + x^4)/(1 - 6*x^4 + x^8),x]
```

```
[Out] ArcTan[x/Sqrt[-1 + Sqrt[2]]]/(4*Sqrt[-1 + Sqrt[2]]) - ArcTan[x/Sqrt[1 + Sqr
t[2]]]/(4*Sqrt[1 + Sqrt[2]]) + ArcTanh[x/Sqrt[-1 + Sqrt[2]]]/(4*Sqrt[-1 + S
qrt[2]]) - ArcTanh[x/Sqrt[1 + Sqrt[2]]]/(4*Sqrt[1 + Sqrt[2]])
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 1107

```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(-1), x_Symbol] :> With[{q = Rt[b^
2 - 4*a*c, 2]}, Dist[c/q, Int[1/(b/2 - q/2 + c*x^2), x], x] - Dist[c/q, Int
```

```
[1/(b/2 + q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c,
0] && PosQ[b^2 - 4*a*c]
```

Rule 1433

```
Int[((d_) + (e_.)*(x_)^(n_))/((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_)), x
_Symbol] :> With[{q = Rt[2*(d/e) - b/c, 2]}, Dist[e/(2*c), Int[1/Simp[d/e +
 q*x^(n/2) + x^n, x], x], x] + Dist[e/(2*c), Int[1/Simp[d/e - q*x^(n/2) + x
^n, x], x], x]] /; FreeQ[{a, b, c, d, e}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4
*a*c, 0] && EqQ[c*d^2 - a*e^2, 0] && IGtQ[n/2, 0] && (GtQ[2*(d/e) - b/c, 0]
 || ( !LtQ[2*(d/e) - b/c, 0] && EqQ[d, e*Rt[a/c, 2]]))
```

Rubi steps

$$
\begin{aligned}
\int \frac{1+x^4}{1-6x^4+x^8}\,dx &= \frac{1}{2}\int \frac{1}{1-2\sqrt{2}\,x^2+x^4}\,dx+\frac{1}{2}\int \frac{1}{1+2\sqrt{2}\,x^2+x^4}\,dx \\
&= \frac{1}{4}\int \frac{1}{-1-\sqrt{2}+x^2}\,dx-\frac{1}{4}\int \frac{1}{1-\sqrt{2}+x^2}\,dx+\frac{1}{4}\int \frac{1}{-1+\sqrt{2}+x^2}\,dx-\frac{1}{4}\int \frac{}{1 \cdots} \\
&= \frac{\tan^{-1}\left(\frac{x}{\sqrt{-1+\sqrt{2}}}\right)}{4\sqrt{-1+\sqrt{2}}}-\frac{\tan^{-1}\left(\frac{x}{\sqrt{1+\sqrt{2}}}\right)}{4\sqrt{1+\sqrt{2}}}+\frac{\tanh^{-1}\left(\frac{x}{\sqrt{-1+\sqrt{2}}}\right)}{4\sqrt{-1+\sqrt{2}}}-\frac{\tan\cdots}{\cdots}
\end{aligned}
$$

Mathematica [A]
time = 0.03, size = 111, normalized size = 0.95

$$\frac{1}{4}\left(\sqrt{1+\sqrt{2}}\,\tan^{-1}\left(\frac{x}{\sqrt{-1+\sqrt{2}}}\right)-\sqrt{-1+\sqrt{2}}\,\tan^{-1}\left(\frac{x}{\sqrt{1+\sqrt{2}}}\right)+\sqrt{1+\sqrt{2}}\,\tanh^{-1}\left(\frac{x}{\sqrt{-1+\sqrt{2}}}\right)-\sqrt{-1+\sqrt{2}}\,\tanh^{-1}\left(\frac{x}{\sqrt{1+\sqrt{2}}}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + x^4)/(1 - 6*x^4 + x^8),x]
```

```
[Out] (Sqrt[1 + Sqrt[2]]*ArcTan[x/Sqrt[-1 + Sqrt[2]]] - Sqrt[-1 + Sqrt[2]]*ArcTan
[x/Sqrt[1 + Sqrt[2]]] + Sqrt[1 + Sqrt[2]]*ArcTanh[x/Sqrt[-1 + Sqrt[2]]] - S
qrt[-1 + Sqrt[2]]*ArcTanh[x/Sqrt[1 + Sqrt[2]]])/4
```

Maple [A]
time = 0.07, size = 78, normalized size = 0.67

method	result	si

risch	$\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(\_Z^4+2\_Z^2-1\right)}\_R\ln\left(-\_R^3-2\_R+x\right)\right)}{8}+\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(\_Z^4-2\_Z^2-1\right)}\_R\ln\left(\_R^3-2\_R+x\right)\right)}{8}$	58
default	$\frac{\arctan\left(\frac{x}{\sqrt{\sqrt{2}-1}}\right)}{4\sqrt{\sqrt{2}-1}}+\frac{\operatorname{arctanh}\left(\frac{x}{\sqrt{\sqrt{2}-1}}\right)}{4\sqrt{\sqrt{2}-1}}-\frac{\arctan\left(\frac{x}{\sqrt{1+\sqrt{2}}}\right)}{4\sqrt{1+\sqrt{2}}}-\frac{\operatorname{arctanh}\left(\frac{x}{\sqrt{1+\sqrt{2}}}\right)}{4\sqrt{1+\sqrt{2}}}$	78

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4+1)/(x^8-6*x^4+1),x,method=_RETURNVERBOSE)
```

```
[Out] 1/4*arctan(x/(2^(1/2)-1)^(1/2))/(2^(1/2)-1)^(1/2)+1/4*arctanh(x/(2^(1/2)-1)
^(1/2))/(2^(1/2)-1)^(1/2)-1/4*arctan(x/(1+2^(1/2))^(1/2))/(1+2^(1/2))^(1/2)
-1/4*arctanh(x/(1+2^(1/2))^(1/2))/(1+2^(1/2))^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4+1)/(x^8-6*x^4+1),x, algorithm="maxima")
```

```
[Out] integrate((x^4 + 1)/(x^8 - 6*x^4 + 1), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 181 vs. $2(77) = 154$.
time = 0.37, size = 181, normalized size = 1.55

$$-\frac{1}{2}\sqrt{\sqrt{2}+1}\arctan\left(-x\sqrt{\sqrt{2}+1}+\sqrt{x^2+\sqrt{2}-1}\sqrt{\sqrt{2}+1}\right)+\frac{1}{2}\sqrt{\sqrt{2}-1}\arctan\left(-x\sqrt{\sqrt{2}-1}+\sqrt{x^2+\sqrt{2}+1}\sqrt{\sqrt{2}-1}\right)-\frac{1}{8}\sqrt{\sqrt{2}-1}\log\left(\left(\sqrt{2}+1\right)\sqrt{\sqrt{2}-1}+x\right)+\frac{1}{8}\sqrt{\sqrt{2}-1}\log\left(-\left(\sqrt{2}+1\right)\sqrt{\sqrt{2}-1}+x\right)+\frac{1}{8}\sqrt{\sqrt{2}+1}\log\left(\sqrt{\sqrt{2}+1}\left(\sqrt{2}-1\right)+x\right)-\frac{1}{8}\sqrt{\sqrt{2}+1}\log\left(-\sqrt{\sqrt{2}+1}\left(\sqrt{2}-1\right)+x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4+1)/(x^8-6*x^4+1),x, algorithm="fricas")
```

```
[Out] -1/2*sqrt(sqrt(2) + 1)*arctan(-x*sqrt(sqrt(2) + 1) + sqrt(x^2 + sqrt(2) - 1
)*sqrt(sqrt(2) + 1)) + 1/2*sqrt(sqrt(2) - 1)*arctan(-x*sqrt(sqrt(2) - 1) +
sqrt(x^2 + sqrt(2) + 1)*sqrt(sqrt(2) - 1)) - 1/8*sqrt(sqrt(2) - 1)*log((sqr
t(2) + 1)*sqrt(sqrt(2) - 1) + x) + 1/8*sqrt(sqrt(2) - 1)*log(-(sqrt(2) + 1)
*sqrt(sqrt(2) - 1) + x) + 1/8*sqrt(sqrt(2) + 1)*log(sqrt(sqrt(2) + 1)*(sqrt
(2) - 1) + x) - 1/8*sqrt(sqrt(2) + 1)*log(-sqrt(sqrt(2) + 1)*(sqrt(2) - 1)
+ x)
```

Sympy [A]
time = 0.68, size = 49, normalized size = 0.42

$$\mathrm{RootSum}\left(4096t^4-128t^2-1,\left(t\mapsto t\log\left(16384t^5-20t+x\right)\right)\right)+\mathrm{RootSum}\left(4096t^4+128t^2-1,\left(t\mapsto t\log\left(16384t^5-20t+x\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**4+1)/(x**8-6*x**4+1),x)
```

```
[Out] RootSum(4096*_t**4 - 128*_t**2 - 1, Lambda(_t, _t*log(16384*_t**5 - 20*_t +
 x))) + RootSum(4096*_t**4 + 128*_t**2 - 1, Lambda(_t, _t*log(16384*_t**5 -
 20*_t + x)))
```

Giac [A]
time = 3.86, size = 123, normalized size = 1.05

$$-\frac{1}{4}\sqrt{\sqrt{2}-1}\arctan\left(\frac{x}{\sqrt{\sqrt{2}+1}}\right)+\frac{1}{4}\sqrt{\sqrt{2}+1}\arctan\left(\frac{x}{\sqrt{\sqrt{2}-1}}\right)-\frac{1}{8}\sqrt{\sqrt{2}-1}\log\left(\left|x+\sqrt{\sqrt{2}+1}\right|\right)+\frac{1}{8}\sqrt{\sqrt{2}-1}\log\left(\left|x-\sqrt{\sqrt{2}+1}\right|\right)+\frac{1}{8}\sqrt{\sqrt{2}+1}\log\left(\left|x+\sqrt{\sqrt{2}-1}\right|\right)-\frac{1}{8}\sqrt{\sqrt{2}+1}\log\left(\left|x-\sqrt{\sqrt{2}-1}\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4+1)/(x^8-6*x^4+1),x, algorithm="giac")
```

```
[Out] -1/4*sqrt(sqrt(2) - 1)*arctan(x/sqrt(sqrt(2) + 1)) + 1/4*sqrt(sqrt(2) + 1)*
arctan(x/sqrt(sqrt(2) - 1)) - 1/8*sqrt(sqrt(2) - 1)*log(abs(x + sqrt(sqrt(2
) + 1))) + 1/8*sqrt(sqrt(2) - 1)*log(abs(x - sqrt(sqrt(2) + 1))) + 1/8*sqrt
(sqrt(2) + 1)*log(abs(x + sqrt(sqrt(2) - 1))) - 1/8*sqrt(sqrt(2) + 1)*log(a
bs(x - sqrt(sqrt(2) - 1)))
```

Mupad [B]
time = 0.19, size = 233, normalized size = 1.99

$$-\frac{\mathrm{atan}\left(\frac{x\sqrt{\sqrt{2}-1}\,49152\mathrm{i}}{34816\sqrt{2}-49152}-\frac{\sqrt{2}\,x\sqrt{\sqrt{2}-1}\,34816\mathrm{i}}{34816\sqrt{2}-49152}\right)\sqrt{\sqrt{2}-1}\,1\mathrm{i}}{4}-\frac{\mathrm{atan}\left(\frac{x\sqrt{\sqrt{2}+1}\,49152\mathrm{i}}{34816\sqrt{2}+49152}+\frac{\sqrt{2}\,x\sqrt{\sqrt{2}+1}\,34816\mathrm{i}}{34816\sqrt{2}+49152}\right)\sqrt{\sqrt{2}+1}\,1\mathrm{i}}{4}+\frac{\mathrm{atan}\left(\frac{x\sqrt{1-\sqrt{2}}\,49152\mathrm{i}}{34816\sqrt{2}-49152}-\frac{\sqrt{2}\,x\sqrt{1-\sqrt{2}}\,34816\mathrm{i}}{34816\sqrt{2}-49152}\right)\sqrt{1-\sqrt{2}}\,1\mathrm{i}}{4}+\frac{\mathrm{atan}\left(\frac{x\sqrt{-\sqrt{2}-1}\,49152\mathrm{i}}{34816\sqrt{2}+49152}+\frac{\sqrt{2}\,x\sqrt{-\sqrt{2}-1}\,34816\mathrm{i}}{34816\sqrt{2}+49152}\right)\sqrt{-\sqrt{2}-1}\,1\mathrm{i}}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4 + 1)/(x^8 - 6*x^4 + 1),x)
```

```
[Out] (atan((x*(1 - 2^(1/2))^(1/2)*49152i)/(34816*2^(1/2) - 49152) - (2^(1/2)*x*(
1 - 2^(1/2))^(1/2)*34816i)/(34816*2^(1/2) - 49152))*(1 - 2^(1/2))^(1/2)*1i)
/4 - (atan((x*(2^(1/2) + 1)^(1/2)*49152i)/(34816*2^(1/2) + 49152) + (2^(1/2
)*x*(2^(1/2) + 1)^(1/2)*34816i)/(34816*2^(1/2) + 49152))*(2^(1/2) + 1)^(1/2
)*1i)/4 - (atan((x*(2^(1/2) - 1)^(1/2)*49152i)/(34816*2^(1/2) - 49152) - (2
^(1/2)*x*(2^(1/2) - 1)^(1/2)*34816i)/(34816*2^(1/2) - 49152))*(2^(1/2) - 1)
^(1/2)*1i)/4 + (atan((x*(- 2^(1/2) - 1)^(1/2)*49152i)/(34816*2^(1/2) + 4915
2) + (2^(1/2)*x*(- 2^(1/2) - 1)^(1/2)*34816i)/(34816*2^(1/2) + 49152))*(- 2
^(1/2) - 1)^(1/2)*1i)/4
```

3.20 $\int \frac{1-x^4}{1+bx^4+x^8}\,dx$

<u>**Optimal**</u>. Leaf size=511

$$-\frac{\sqrt{2+b}\,\tan^{-1}\left(\frac{\sqrt{2-\sqrt{2-b}}-2x}{\sqrt{2+\sqrt{2-b}}}\right)}{4\sqrt{2-\sqrt{2-b}}\,\sqrt{2-b}}+\frac{\sqrt{2+b}\,\tan^{-1}\left(\frac{\sqrt{2+\sqrt{2-b}}-2x}{\sqrt{2-\sqrt{2-b}}}\right)}{4\sqrt{2+\sqrt{2-b}}\,\sqrt{2-b}}+\frac{\sqrt{2+b}\,\tan^{-1}\left(\frac{\sqrt{2-\sqrt{2}}}{\sqrt{2+}}\right.}{4\sqrt{2-\sqrt{2-b}}\,\sqrt{2}}$$

```
[Out] -1/4*arctan((-2*x+(2-(2-b)^(1/2))^(1/2))/(2+(2-b)^(1/2))^(1/2))*(2+b)^(1/2)
/(2-b)^(1/2)/(2-(2-b)^(1/2))^(1/2)+1/4*arctan((2*x+(2-(2-b)^(1/2))^(1/2))/(
2+(2-b)^(1/2))^(1/2))*(2+b)^(1/2)/(2-b)^(1/2)/(2-(2-b)^(1/2))^(1/2)+1/8*ln(
1+x^2-x*(2-(2-b)^(1/2))^(1/2))*(2-(2-b)^(1/2))^(1/2)/(2-b)^(1/2)-1/8*ln(1+x
^2+x*(2-(2-b)^(1/2))^(1/2))*(2-(2-b)^(1/2))^(1/2)/(2-b)^(1/2)+1/4*arctan((-
2*x+(2+(2-b)^(1/2))^(1/2))/(2-(2-b)^(1/2))^(1/2))*(2+b)^(1/2)/(2-b)^(1/2)/(
2+(2-b)^(1/2))^(1/2)-1/4*arctan((2*x+(2+(2-b)^(1/2))^(1/2))/(2-(2-b)^(1/2))
^(1/2))*(2+b)^(1/2)/(2-b)^(1/2)/(2+(2-b)^(1/2))^(1/2)-1/8*ln(1+x^2-x*(2+(2-
b)^(1/2))^(1/2))*(2+(2-b)^(1/2))^(1/2)/(2-b)^(1/2)+1/8*ln(1+x^2+x*(2+(2-b)^
(1/2))^(1/2))*(2+(2-b)^(1/2))^(1/2)/(2-b)^(1/2)
```

<u>**Rubi**</u> [A]
time = 0.26, antiderivative size = 511, normalized size of antiderivative = 1.00, number of steps used = 19, number of rules used = 6, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.300$, Rules used = {1435, 1183, 648, 632, 210, 642}

$$-\frac{\sqrt{b+2}\,\text{ArcTan}\left(\frac{\sqrt{2-\sqrt{2-b}}-2x}{\sqrt{\sqrt{2-b}+2}}\right)}{4\sqrt{2-\sqrt{2-b}}\,\sqrt{2-b}}+\frac{\sqrt{b+2}\,\text{ArcTan}\left(\frac{\sqrt{\sqrt{2-b}+2}-2x}{\sqrt{2-\sqrt{2-b}}}\right)}{4\sqrt{\sqrt{2-b}+2}\,\sqrt{2-b}}+\frac{\sqrt{b+2}\,\text{ArcTan}\left(\frac{\sqrt{2-\sqrt{2-b}}+2x}{\sqrt{\sqrt{2-b}+2}}\right)}{4\sqrt{2-\sqrt{2-b}}\,\sqrt{2-b}}-\frac{\sqrt{b+2}\,\text{ArcTan}\left(\frac{\sqrt{\sqrt{2-b}+2}+2x}{\sqrt{2-\sqrt{2-b}}}\right)}{4\sqrt{\sqrt{2-b}+2}\,\sqrt{2-b}}+\frac{\sqrt{2-\sqrt{2-b}}\,\log\left(-\sqrt{2-\sqrt{2-b}}\,x+x^2+1\right)}{8\sqrt{2-b}}-\frac{\sqrt{2-\sqrt{2-b}}\,\log\left(\sqrt{2-\sqrt{2-b}}\,x+x^2+1\right)}{8\sqrt{2-b}}-\frac{\sqrt{\sqrt{2-b}+2}\,\log\left(-\sqrt{\sqrt{2-b}+2}\,x+x^2+1\right)}{8\sqrt{2-b}}+\frac{\sqrt{\sqrt{2-b}+2}\,\log\left(\sqrt{\sqrt{2-b}+2}\,x+x^2+1\right)}{8\sqrt{2-b}}$$

Antiderivative was successfully verified.

```
[In] Int[(1 - x^4)/(1 + b*x^4 + x^8),x]
```

```
[Out] -1/4*(Sqrt[2 + b]*ArcTan[(Sqrt[2 - Sqrt[2 - b]] - 2*x)/Sqrt[2 + Sqrt[2 - b]
]])/(Sqrt[2 - Sqrt[2 - b]]*Sqrt[2 - b]) + (Sqrt[2 + b]*ArcTan[(Sqrt[2 + Sqr
t[2 - b]] - 2*x)/Sqrt[2 - Sqrt[2 - b]]])/(4*Sqrt[2 + Sqrt[2 - b]]*Sqrt[2 -
b]) + (Sqrt[2 + b]*ArcTan[(Sqrt[2 - Sqrt[2 - b]] + 2*x)/Sqrt[2 + Sqrt[2 - b
]]])/(4*Sqrt[2 - Sqrt[2 - b]]*Sqrt[2 - b]) - (Sqrt[2 + b]*ArcTan[(Sqrt[2 +
Sqrt[2 - b]] + 2*x)/Sqrt[2 - Sqrt[2 - b]]])/(4*Sqrt[2 + Sqrt[2 - b]]*Sqrt[2
 - b]) + (Sqrt[2 - Sqrt[2 - b]]*Log[1 - Sqrt[2 - Sqrt[2 - b]]*x + x^2])/(8*
Sqrt[2 - b]) - (Sqrt[2 - Sqrt[2 - b]]*Log[1 + Sqrt[2 - Sqrt[2 - b]]*x + x^2
])/(8*Sqrt[2 - b]) - (Sqrt[2 + Sqrt[2 - b]]*Log[1 - Sqrt[2 + Sqrt[2 - b]]*x
 + x^2])/(8*Sqrt[2 - b]) + (Sqrt[2 + Sqrt[2 - b]]*Log[1 + Sqrt[2 + Sqrt[2 -
 b]]*x + x^2])/(8*Sqrt[2 - b])
```

<u>Rule 210</u>

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
```

```
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1183

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[a/c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Dist[1/(2*c*q*r), Int
[(d*r - (d - e*q)*x)/(q - r*x + x^2), x], x] + Dist[1/(2*c*q*r), Int[(d*r +
 (d - e*q)*x)/(q + r*x + x^2), x], x]]] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NegQ[b^2 - 4*a*c]
```

Rule 1435

```
Int[((d_) + (e_.)*(x_)^(n_))/((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_)), x
_Symbol] :> With[{q = Rt[-2*(d/e) - b/c, 2]}, Dist[e/(2*c*q), Int[(q - 2*x^
(n/2))/Simp[d/e + q*x^(n/2) - x^n, x], x], x] + Dist[e/(2*c*q), Int[(q + 2*
x^(n/2))/Simp[d/e - q*x^(n/2) - x^n, x], x], x]] /; FreeQ[{a, b, c, d, e},
x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - a*e^2, 0] && IGtQ[
n/2, 0] &&  !GtQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\int \frac{1-x^4}{1+bx^4+x^8}\,dx = -\frac{\int \frac{\sqrt{2-b}+2x^2}{-1-\sqrt{2-b}\,x^2-x^4}\,dx}{2\sqrt{2-b}} - \frac{\int \frac{\sqrt{2-b}-2x^2}{-1+\sqrt{2-b}\,x^2-x^4}\,dx}{2\sqrt{2-b}}$$

$$= \frac{\int \frac{\sqrt{2-\sqrt{2-b}}\,\sqrt{2-b}-\left(-2+\sqrt{2-b}\right)x}{1-\sqrt{2-\sqrt{2-b}}\,x+x^2}\,dx}{4\sqrt{2-\sqrt{2-b}}\,\sqrt{2-b}} + \frac{\int \frac{\sqrt{2-\sqrt{2-b}}\,\sqrt{2-b}+\left(-2+\sqrt{2-\cdots\right.}}{1+\sqrt{2-\sqrt{2-b}}\,x+x^2}}{4\sqrt{2-\sqrt{2-b}}\,\sqrt{2-b}}$$

$$= -\left(\frac{1}{8}\left(-1+\frac{2}{\sqrt{2-b}}\right)\int \frac{1}{1-\sqrt{2+\sqrt{2-b}}\,x+x^2}\,dx\right) - \frac{1}{8}\left(-1+\frac{2}{\sqrt{2-b}}\right)\int \frac{\cdots}{1\cdots}$$

$$= \frac{\sqrt{2-\sqrt{2-b}}\,\log\left(1-\sqrt{2-\sqrt{2-b}}\,x+x^2\right)}{8\sqrt{2-b}} - \frac{\sqrt{2-\sqrt{2-b}}\,\log\left(1+\sqrt{2-\sqrt{\cdots}}\right.}{8\sqrt{2-b}}$$

$$= -\frac{\sqrt{2+\sqrt{2-b}}\,\tan^{-1}\left(\frac{\sqrt{2-\sqrt{2-b}}-2x}{\sqrt{2+\sqrt{2-b}}}\right)}{4\sqrt{2-b}} + \frac{\sqrt{2-\sqrt{2-b}}\,\tan^{-1}\left(\frac{\sqrt{2+\sqrt{2-\cdots}}}{\sqrt{2-\sqrt{2\cdots}}}\right.}{4\sqrt{2-b}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.
time = 0.02, size = 57, normalized size = 0.11

$$-\frac{1}{4}\text{RootSum}\left[1+b\#1^4+\#1^8\&,\frac{-\log(x-\#1)+\log(x-\#1)\#1^4}{b\#1^3+2\#1^7}\&\right]$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 - x^4)/(1 + b*x^4 + x^8),x]
```

```
[Out] -1/4*RootSum[1 + b*#1^4 + #1^8 & , (-Log[x - #1] + Log[x - #1]*#1^4)/(b*#1^
3 + 2*#1^7) & ]
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 0.03, size = 44, normalized size = 0.09

method	result	size

default	$\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(\_Z^8+\_Z^4 b+1\right)} \frac{\left(-\_R^4+1\right)\ln\left(x-\_R\right)}{2\_R^7+\_R^3 b}\right)}{4}$	44
risch	$\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(\_Z^8+\_Z^4 b+1\right)} \frac{\left(-\_R^4+1\right)\ln\left(x-\_R\right)}{2\_R^7+\_R^3 b}\right)}{4}$	44

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-x^4+1)/(x^8+b*x^4+1),x,method=_RETURNVERBOSE)
```

```
[Out] 1/4*sum((-_R^4+1)/(2*_R^7+_R^3*b)*ln(x-_R),_R=RootOf(_Z^8+_Z^4*b+1))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^4+1)/(x^8+b*x^4+1),x, algorithm="maxima")
```

```
[Out] -integrate((x^4 - 1)/(x^8 + b*x^4 + 1), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 1443 vs. $2(397) = 794$.
time = 0.37, size = 1443, normalized size = 2.82

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^4+1)/(x^8+b*x^4+1),x, algorithm="fricas")
```

```
[Out] -sqrt(sqrt(1/2)*sqrt(((b^2 - 4*b + 4)*sqrt((b + 2)/(b^3 - 6*b^2 + 12*b - 8)
) - b)/(b^2 - 4*b + 4)))*arctan(1/2*sqrt(1/2)*(b^2 + (b^3 - 6*b^2 + 12*b -
8)*sqrt((b + 2)/(b^3 - 6*b^2 + 12*b - 8)) - 4*b + 4)*sqrt(x^2 + 1/2*sqrt(1/
2)*(b^2 + (b^3 - 6*b^2 + 12*b - 8)*sqrt((b + 2)/(b^3 - 6*b^2 + 12*b - 8)) -
 2*b)*sqrt(((b^2 - 4*b + 4)*sqrt((b + 2)/(b^3 - 6*b^2 + 12*b - 8)) - b)/(b^
2 - 4*b + 4)))*sqrt(sqrt(1/2)*sqrt(((b^2 - 4*b + 4)*sqrt((b + 2)/(b^3 - 6*b
^2 + 12*b - 8)) - b)/(b^2 - 4*b + 4)))*sqrt(((b^2 - 4*b + 4)*sqrt((b + 2)/(
b^3 - 6*b^2 + 12*b - 8)) - b)/(b^2 - 4*b + 4)) - 1/2*sqrt(1/2)*((b^3 - 6*b^
2 + 12*b - 8)*x*sqrt((b + 2)/(b^3 - 6*b^2 + 12*b - 8)) + (b^2 - 4*b + 4)*x)
*sqrt(sqrt(1/2)*sqrt(((b^2 - 4*b + 4)*sqrt((b + 2)/(b^3 - 6*b^2 + 12*b - 8)
) - b)/(b^2 - 4*b + 4)))*sqrt(((b^2 - 4*b + 4)*sqrt((b + 2)/(b^3 - 6*b^2 +
12*b - 8)) - b)/(b^2 - 4*b + 4))) + sqrt(sqrt(1/2)*sqrt(-((b^2 - 4*b + 4)*s
```

```
qrt((b + 2)/(b^3 - 6*b^2 + 12*b - 8)) + b)/(b^2 - 4*b + 4)))*arctan(-1/2*(s
qrt(1/2)*(b^2 - (b^3 - 6*b^2 + 12*b - 8)*sqrt((b + 2)/(b^3 - 6*b^2 + 12*b -
 8)) - 4*b + 4)*sqrt(x^2 + 1/2*sqrt(1/2)*(b^2 - (b^3 - 6*b^2 + 12*b - 8)*sq
rt((b + 2)/(b^3 - 6*b^2 + 12*b - 8)) - 2*b)*sqrt(-((b^2 - 4*b + 4)*sqrt((b
+ 2)/(b^3 - 6*b^2 + 12*b - 8)) + b)/(b^2 - 4*b + 4)))*sqrt(-((b^2 - 4*b + 4
)*sqrt((b + 2)/(b^3 - 6*b^2 + 12*b - 8)) + b)/(b^2 - 4*b + 4)) + sqrt(1/2)*
((b^3 - 6*b^2 + 12*b - 8)*x*sqrt((b + 2)/(b^3 - 6*b^2 + 12*b - 8)) - (b^2 -
 4*b + 4)*x)*sqrt(-((b^2 - 4*b + 4)*sqrt((b + 2)/(b^3 - 6*b^2 + 12*b - 8))
+ b)/(b^2 - 4*b + 4)))*sqrt(sqrt(1/2)*sqrt(-((b^2 - 4*b + 4)*sqrt((b + 2)/(
b^3 - 6*b^2 + 12*b - 8)) + b)/(b^2 - 4*b + 4)))) + 1/4*sqrt(sqrt(1/2)*sqrt(
-((b^2 - 4*b + 4)*sqrt((b + 2)/(b^3 - 6*b^2 + 12*b - 8)) + b)/(b^2 - 4*b +
4)))*log(1/2*((b^2 - 4*b + 4)*sqrt((b + 2)/(b^3 - 6*b^2 + 12*b - 8)) - b +
2)*sqrt(sqrt(1/2)*sqrt(-((b^2 - 4*b + 4)*sqrt((b + 2)/(b^3 - 6*b^2 + 12*b -
 8)) + b)/(b^2 - 4*b + 4))) + x) - 1/4*sqrt(sqrt(1/2)*sqrt(-((b^2 - 4*b + 4
)*sqrt((b + 2)/(b^3 - 6*b^2 + 12*b - 8)) + b)/(b^2 - 4*b + 4)))*log(-1/2*((
b^2 - 4*b + 4)*sqrt((b + 2)/(b^3 - 6*b^2 + 12*b - 8)) - b + 2)*sqrt(sqrt(1/
2)*sqrt(-((b^2 - 4*b + 4)*sqrt((b + 2)/(b^3 - 6*b^2 + 12*b - 8)) + b)/(b^2
- 4*b + 4))) + x) - 1/4*sqrt(sqrt(1/2)*sqrt(((b^2 - 4*b + 4)*sqrt((b + 2)/(
b^3 - 6*b^2 + 12*b - 8)) - b)/(b^2 - 4*b + 4)))*log(1/2*((b^2 - 4*b + 4)*sq
rt((b + 2)/(b^3 - 6*b^2 + 12*b - 8)) + b - 2)*sqrt(sqrt(1/2)*sqrt(((b^2 - 4
*b + 4)*sqrt((b + 2)/(b^3 - 6*b^2 + 12*b - 8)) - b)/(b^2 - 4*b + 4))) + x)
+ 1/4*sqrt(sqrt(1/2)*sqrt(((b^2 - 4*b + 4)*sqrt((b + 2)/(b^3 - 6*b^2 + 12*b
 - 8)) - b)/(b^2 - 4*b + 4)))*log(-1/2*((b^2 - 4*b + 4)*sqrt((b + 2)/(b^3 -
 6*b^2 + 12*b - 8)) + b - 2)*sqrt(sqrt(1/2)*sqrt(((b^2 - 4*b + 4)*sqrt((b +
 2)/(b^3 - 6*b^2 + 12*b - 8)) - b)/(b^2 - 4*b + 4))) + x)
```

Sympy [A]
time = 2.10, size = 76, normalized size = 0.15

$$-\operatorname{RootSum}\left(t^{8}\cdot\left(65536b^{4}-524288b^{3}+1572864b^{2}-2097152b+1048576\right)+t^{4}\cdot\left(256b^{3}-1024b^{2}+1024b\right)+1,\left(t\mapsto t\log\left(1024t^{5}b^{2}-4096t^{5}b+4096t^{5}+4tb-4t+x\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x**4+1)/(x**8+b*x**4+1),x)
```

```
[Out] -RootSum(_t**8*(65536*b**4 - 524288*b**3 + 1572864*b**2 - 2097152*b + 10485
76) + _t**4*(256*b**3 - 1024*b**2 + 1024*b) + 1, Lambda(_t, _t*log(1024*_t*
*5*b**2 - 4096*_t**5*b + 4096*_t**5 + 4*_t*b - 4*_t + x)))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^4+1)/(x^8+b*x^4+1),x, algorithm="giac")
```

```
[Out] integrate(-(x^4 - 1)/(x^8 + b*x^4 + 1), x)
```

Mupad [B]
time = 3.74, size = 2500, normalized size = 4.89

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(x^4 - 1)/(b*x^4 + x^8 + 1),x)
[Out] - atan((((-(4*b + ((b - 2)^5*(b + 2))^(1/2) - 4*b^2 + b^3)/(512*(24*b^2 - 3
2*b - 8*b^3 + b^4 + 16)))^(1/4)*(256*b + ((-(4*b + ((b - 2)^5*(b + 2))^(1/2
) - 4*b^2 + b^3)/(512*(24*b^2 - 32*b - 8*b^3 + b^4 + 16)))^(1/4)*(262144*b
- 196608*b^2 - 196608*b^3 + 49152*b^4 + 49152*b^5 - 4096*b^6 - 4096*b^7 + 2
62144) + x*(32768*b - 65536*b^2 - 32768*b^3 + 20480*b^4 + 10240*b^5 - 2048*
b^6 - 1024*b^7 + 65536))*(-(4*b + ((b - 2)^5*(b + 2))^(1/2) - 4*b^2 + b^3)/
(512*(24*b^2 - 32*b - 8*b^3 + b^4 + 16)))^(3/4) - 64*b^3 - 16*b^4 + 256) -
x*(32*b + 48*b^2 + 24*b^3 + 4*b^4))*(-(4*b + ((b - 2)^5*(b + 2))^(1/2) - 4*
b^2 + b^3)/(512*(24*b^2 - 32*b - 8*b^3 + b^4 + 16)))^(1/4)*1i - ((-(4*b + (
(b - 2)^5*(b + 2))^(1/2) - 4*b^2 + b^3)/(512*(24*b^2 - 32*b - 8*b^3 + b^4 +
 16)))^(1/4)*(256*b + ((-(4*b + ((b - 2)^5*(b + 2))^(1/2) - 4*b^2 + b^3)/(5
12*(24*b^2 - 32*b - 8*b^3 + b^4 + 16)))^(1/4)*(262144*b - 196608*b^2 - 1966
08*b^3 + 49152*b^4 + 49152*b^5 - 4096*b^6 - 4096*b^7 + 262144) - x*(32768*b
 - 65536*b^2 - 32768*b^3 + 20480*b^4 + 10240*b^5 - 2048*b^6 - 1024*b^7 + 65
536))*(-(4*b + ((b - 2)^5*(b + 2))^(1/2) - 4*b^2 + b^3)/(512*(24*b^2 - 32*b
 - 8*b^3 + b^4 + 16)))^(3/4) - 64*b^3 - 16*b^4 + 256) + x*(32*b + 48*b^2 +
24*b^3 + 4*b^4))*(-(4*b + ((b - 2)^5*(b + 2))^(1/2) - 4*b^2 + b^3)/(512*(24
*b^2 - 32*b - 8*b^3 + b^4 + 16)))^(1/4)*1i)/(((-(4*b + ((b - 2)^5*(b + 2))^
(1/2) - 4*b^2 + b^3)/(512*(24*b^2 - 32*b - 8*b^3 + b^4 + 16)))^(1/4)*(256*b
 + ((-(4*b + ((b - 2)^5*(b + 2))^(1/2) - 4*b^2 + b^3)/(512*(24*b^2 - 32*b -
 8*b^3 + b^4 + 16)))^(1/4)*(262144*b - 196608*b^2 - 196608*b^3 + 49152*b^4
+ 49152*b^5 - 4096*b^6 - 4096*b^7 + 262144) + x*(32768*b - 65536*b^2 - 3276
8*b^3 + 20480*b^4 + 10240*b^5 - 2048*b^6 - 1024*b^7 + 65536))*(-(4*b + ((b
- 2)^5*(b + 2))^(1/2) - 4*b^2 + b^3)/(512*(24*b^2 - 32*b - 8*b^3 + b^4 + 16
)))^(3/4) - 64*b^3 - 16*b^4 + 256) - x*(32*b + 48*b^2 + 24*b^3 + 4*b^4))*(-
(4*b + ((b - 2)^5*(b + 2))^(1/2) - 4*b^2 + b^3)/(512*(24*b^2 - 32*b - 8*b^3
 + b^4 + 16)))^(1/4) + ((-(4*b + ((b - 2)^5*(b + 2))^(1/2) - 4*b^2 + b^3)/(
512*(24*b^2 - 32*b - 8*b^3 + b^4 + 16)))^(1/4)*(256*b + ((-(4*b + ((b - 2)^
5*(b + 2))^(1/2) - 4*b^2 + b^3)/(512*(24*b^2 - 32*b - 8*b^3 + b^4 + 16)))^(
1/4)*(262144*b - 196608*b^2 - 196608*b^3 + 49152*b^4 + 49152*b^5 - 4096*b^6
 - 4096*b^7 + 262144) - x*(32768*b - 65536*b^2 - 32768*b^3 + 20480*b^4 + 10
240*b^5 - 2048*b^6 - 1024*b^7 + 65536))*(-(4*b + ((b - 2)^5*(b + 2))^(1/2)
- 4*b^2 + b^3)/(512*(24*b^2 - 32*b - 8*b^3 + b^4 + 16)))^(3/4) - 64*b^3 - 1
6*b^4 + 256) + x*(32*b + 48*b^2 + 24*b^3 + 4*b^4))*(-(4*b + ((b - 2)^5*(b +
 2))^(1/2) - 4*b^2 + b^3)/(512*(24*b^2 - 32*b - 8*b^3 + b^4 + 16)))^(1/4)))
```

```
*(-(4*b + ((b - 2)^5*(b + 2))^(1/2) - 4*b^2 + b^3)/(512*(24*b^2 - 32*b - 8*
b^3 + b^4 + 16)))^(1/4)*2i - 2*atan((((-(4*b + ((b - 2)^5*(b + 2))^(1/2) -
4*b^2 + b^3)/(512*(24*b^2 - 32*b - 8*b^3 + b^4 + 16)))^(1/4)*(((-(4*b + ((b
 - 2)^5*(b + 2))^(1/2) - 4*b^2 + b^3)/(512*(24*b^2 - 32*b - 8*b^3 + b^4 + 1
6)))^(1/4)*(262144*b - 196608*b^2 - 196608*b^3 + 49152*b^4 + 49152*b^5 - 40
96*b^6 - 4096*b^7 + 262144)*1i + x*(32768*b - 65536*b^2 - 32768*b^3 + 20480
*b^4 + 10240*b^5 - 2048*b^6 - 1024*b^7 + 65536))*(-(4*b + ((b - 2)^5*(b + 2
))^(1/2) - 4*b^2 + b^3)/(512*(24*b^2 - 32*b - 8*b^3 + b^4 + 16)))^(3/4)*1i
- 256*b + 64*b^3 + 16*b^4 - 256)*1i + x*(32*b + 48*b^2 + 24*b^3 + 4*b^4))*(
-(4*b + ((b - 2)^5*(b + 2))^(1/2) - 4*b^2 + b^3)/(512*(24*b^2 - 32*b - 8*b^
3 + b^4 + 16)))^(1/4) - ((-(4*b + ((b - 2)^5*(b + 2))^(1/2) - 4*b^2 + b^3)/
(512*(24*b^2 - 32*b - 8*b^3 + b^4 + 16)))^(1/4)*(((-(4*b + ((b - 2)^5*(b +
2))^(1/2) - 4*b^2 + b^3)/(512*(24*b^2 - 32*b - 8*b^3 + b^4 + 16)))^(1/4)*(2
62144*b - 196608*b^2 - 196608*b^3 + 49152*b^4 + 49152*b^5 - 4096*b^6 - 4096
*b^7 + 262144)*1i - x*(32768*b - 65536*b^2 - 32768*b^3 + 20480*b^4 + 10240*
b^5 - 2048*b^6 - 1024*b^7 + 65536))*(-(4*b + ((b - 2)^5*(b + 2))^(1/2) - 4*
b^2 + b^3)/(512*(24*b^2 - 32*b - 8*b^3 + b^4 + 16)))^(3/4)*1i - 256*b + 64*
b^3 + 16*b^4 - 256)*1i - x*(32*b + 48*b^2 + 24*b^3 + 4*b^4))*(-(4*b + ((b -
 2)^5*(b + 2))^(1/2) - 4*b^2 + b^3)/(512*(24*b^2 - 32*b - 8*b^3 + b^4 + 16)
))^(1/4))/(((-(4*b + ((b - 2)^5*(b + 2))^(1/2) - 4*b^2 + b^3)/(512*(24*b^2
- 32*b - 8*b^3 + b^4 + 16)))^(1/4)*(((-(4*b + ((b - 2)^5*(b + 2))^(1/2) - 4
*b^2 + b^3)/(512*(24*b^2 - 32*b - 8*b^3 + b^4 + 16)))^(1/4)*(262144*b - 196
608*b^2 - 196608*b^3 + 49152*b^4 + 49152*b^5 - 4096*b^6 - 4096*b^7 + 262144
)*1i + x*(32768*b - 65536*b^2 - 32768*b^3 + 20480*b^4 + 10240*b^5 - 2048*b^
6 - 1024*b^7 + 65536))*(-(4*b + ((b - 2)^5*(b + 2))^(1/2) - 4*b^2 + b^3)/(5
12*(24*b^2 - 32*b - 8*b^3 + b^4 + 16)))^(3/4)*1i - 256*b + 64*b^3 + 16*b^4
- 256)*1i + x*(32*b + 48*b^2 + 24*b^3 + 4*b^4))*(-(4*b + ((b - 2)^5*(b + 2)
)^(1/2) - 4*b^2 + b^3)/(512*(24*b^2 - 32*b - 8*b^3 + b^4 + 16)))^(1/4)*1i +
 ((-(4*b + ((b - 2)^5*(b + 2))^(1/2) - 4*b^2 + b^3)/(512*(24*b^2 - 32*b - 8
*b^3 + b^4 + 16)))^(1/4)*(((-(4*b + ((b - 2)^5*(b + 2))^(1/2) - 4*b^2 + b^3
)/(512*(24*b^2 - 32*b - 8*b^3 + b^4 + 16)))^(1/4)*(262144*b - 196608*b^2 -
196608*b^3 + 49152*b^4 + 49152*b^5 - 4096*b^6 -...
```

3.21 $\int \frac{1-x^4}{1+3x^4+x^8}\,dx$

Optimal. Leaf size=411

$$-\frac{\sqrt[4]{3+\sqrt{5}}\,\tan^{-1}\left(1-\frac{2^{3/4}x}{\sqrt[4]{3-\sqrt{5}}}\right)}{2\ 2^{3/4}}+\frac{\sqrt[4]{3+\sqrt{5}}\,\tan^{-1}\left(1+\frac{2^{3/4}x}{\sqrt[4]{3-\sqrt{5}}}\right)}{2\ 2^{3/4}}+\frac{\sqrt[4]{3-\sqrt{5}}\,\tan^{-1}\left(1-\right.}{2\ 2^{3/4}}$$

```
[Out] -1/4*arctan(-1+2^(3/4)*x/(3+5^(1/2))^(1/4))*(3-5^(1/2))^(1/4)*2^(1/4)-1/4*a
rctan(1+2^(3/4)*x/(3+5^(1/2))^(1/4))*(3-5^(1/2))^(1/4)*2^(1/4)+1/8*ln(2*x^2
-2*2^(1/4)*x*(3+5^(1/2))^(1/4)+5^(1/2)+1)*(3-5^(1/2))^(1/4)*2^(1/4)-1/8*ln(
2*x^2+2*2^(1/4)*x*(3+5^(1/2))^(1/4)+5^(1/2)+1)*(3-5^(1/2))^(1/4)*2^(1/4)+1/
4*arctan(-1+2^(3/4)*x/(3-5^(1/2))^(1/4))*(3+5^(1/2))^(1/4)*2^(1/4)+1/4*arct
an(1+2^(3/4)*x/(3-5^(1/2))^(1/4))*(3+5^(1/2))^(1/4)*2^(1/4)-1/8*ln(2*x^2-2*
2^(1/4)*x*(3-5^(1/2))^(1/4)+5^(1/2)-1)*(3+5^(1/2))^(1/4)*2^(1/4)+1/8*ln(2*x
^2+2*2^(1/4)*x*(3-5^(1/2))^(1/4)+5^(1/2)-1)*(3+5^(1/2))^(1/4)*2^(1/4)
```

Rubi [A]
time = 0.22, antiderivative size = 411, normalized size of antiderivative = 1.00, number of steps used = 19, number of rules used = 7, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.350$, Rules used = {1434, 217, 1179, 642, 1176, 631, 210}

$$-\frac{\sqrt[4]{3+\sqrt{5}}\,\text{ArcTan}\left(1-\frac{2^{3/4}x}{\sqrt[4]{3-\sqrt{5}}}\right)}{2\ 2^{3/4}}+\frac{\sqrt[4]{3+\sqrt{5}}\,\text{ArcTan}\left(\frac{2^{3/4}x}{\sqrt[4]{3-\sqrt{5}}}+1\right)}{2\ 2^{3/4}}+\frac{\sqrt[4]{3-\sqrt{5}}\,\text{ArcTan}\left(1-\frac{2^{3/4}x}{\sqrt[4]{3+\sqrt{5}}}\right)}{2\ 2^{3/4}}-\frac{\sqrt[4]{3-\sqrt{5}}\,\text{ArcTan}\left(\frac{2^{3/4}x}{\sqrt[4]{3+\sqrt{5}}}+1\right)}{2\ 2^{3/4}}-\frac{\sqrt[4]{3+\sqrt{5}}\,\log\left(2x^2-2\sqrt[4]{2\left(3-\sqrt{5}\right)}x+\sqrt{2\left(3-\sqrt{5}\right)}\right)}{4\ 2^{3/4}}+\frac{\sqrt[4]{3+\sqrt{5}}\,\log\left(2x^2+2\sqrt[4]{2\left(3-\sqrt{5}\right)}x+\sqrt{2\left(3-\sqrt{5}\right)}\right)}{4\ 2^{3/4}}+\frac{\sqrt[4]{3-\sqrt{5}}\,\log\left(2x^2-2\sqrt[4]{2\left(3+\sqrt{5}\right)}x+\sqrt{2\left(3+\sqrt{5}\right)}\right)}{4\ 2^{3/4}}-\frac{\sqrt[4]{3-\sqrt{5}}\,\log\left(2x^2+2\sqrt[4]{2\left(3+\sqrt{5}\right)}x+\sqrt{2\left(3+\sqrt{5}\right)}\right)}{4\ 2^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Int[(1 - x^4)/(1 + 3*x^4 + x^8),x]
```

```
[Out] -1/2*((3 + Sqrt[5])^(1/4)*ArcTan[1 - (2^(3/4)*x)/(3 - Sqrt[5])^(1/4)])/2^(3
/4) + ((3 + Sqrt[5])^(1/4)*ArcTan[1 + (2^(3/4)*x)/(3 - Sqrt[5])^(1/4)])/(2*
2^(3/4)) + ((3 - Sqrt[5])^(1/4)*ArcTan[1 - (2^(3/4)*x)/(3 + Sqrt[5])^(1/4)]
)/(2*2^(3/4)) - ((3 - Sqrt[5])^(1/4)*ArcTan[1 + (2^(3/4)*x)/(3 + Sqrt[5])^(
1/4)])/(2*2^(3/4)) - ((3 + Sqrt[5])^(1/4)*Log[Sqrt[2*(3 - Sqrt[5])] - 2*(2*
(3 - Sqrt[5]))^(1/4)*x + 2*x^2])/(4*2^(3/4)) + ((3 + Sqrt[5])^(1/4)*Log[Sqr
t[2*(3 - Sqrt[5])] + 2*(2*(3 - Sqrt[5]))^(1/4)*x + 2*x^2])/(4*2^(3/4)) + ((
3 - Sqrt[5])^(1/4)*Log[Sqrt[2*(3 + Sqrt[5])] - 2*(2*(3 + Sqrt[5]))^(1/4)*x
+ 2*x^2])/(4*2^(3/4)) - ((3 - Sqrt[5])^(1/4)*Log[Sqrt[2*(3 + Sqrt[5])] + 2*
(2*(3 + Sqrt[5]))^(1/4)*x + 2*x^2])/(4*2^(3/4))
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1434

```
Int[((d_) + (e_.)*(x_)^(n_))/((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_)), x
_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q),
Int[1/(b/2 - q/2 + c*x^n), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(
b/2 + q/2 + c*x^n), x], x]] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[n2, 2*n]
 && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - a*e^2, 0] && IGtQ[n/2, 0] && GtQ[b^2
 - 4*a*c, 0]
```

Rubi steps

$$\int \frac{1-x^4}{1+3x^4+x^8}\,dx = \frac{1}{2}\left(-1-\sqrt{5}\right)\int \frac{1}{\frac{3}{2}+\frac{\sqrt{5}}{2}+x^4}\,dx + \frac{1}{2}\left(-1+\sqrt{5}\right)\int \frac{1}{\frac{3}{2}-\frac{\sqrt{5}}{2}+x^4}\,dx$$

$$= \frac{\int \frac{\sqrt{3-\sqrt{5}}-\sqrt{2}\,x^2}{\frac{3}{2}-\frac{\sqrt{5}}{2}+x^4}\,dx}{2\sqrt{2}} + \frac{\int \frac{\sqrt{3-\sqrt{5}}+\sqrt{2}\,x^2}{\frac{3}{2}-\frac{\sqrt{5}}{2}+x^4}\,dx}{2\sqrt{2}} - \frac{\int \frac{\sqrt{3+\sqrt{5}}-\sqrt{2}\,x^2}{\frac{3}{2}+\frac{\sqrt{5}}{2}+x^4}\,dx}{2\sqrt{2}} - \cdots$$

$$= \frac{1}{4}\int \frac{1}{\sqrt{\frac{1}{2}\left(3-\sqrt{5}\right)} - \sqrt[4]{2\left(3-\sqrt{5}\right)}\,x + x^2}\,dx + \frac{1}{4}\int \frac{1}{\sqrt{\frac{1}{2}\left(3-\sqrt{5}\right)} + \sqrt[4]{2\left(3 \cdots\right.}}$$

$$= -\frac{\sqrt[4]{3+\sqrt{5}}\,\log\left(\sqrt{2\left(3-\sqrt{5}\right)} - 2\sqrt[4]{2\left(3-\sqrt{5}\right)}\,x + 2x^2\right)}{4\ 2^{3/4}} + \frac{\sqrt[4]{3+\sqrt{5}}\,\log\left(\cdots\right.}{\cdots}$$

$$= -\frac{\tan^{-1}\left(1-\frac{2^{3/4}x}{\sqrt[4]{3-\sqrt{5}}}\right)}{2\sqrt[4]{2\left(3-\sqrt{5}\right)}} + \frac{\tan^{-1}\left(1+\frac{2^{3/4}x}{\sqrt[4]{3-\sqrt{5}}}\right)}{2\sqrt[4]{2\left(3-\sqrt{5}\right)}} + \frac{\tan^{-1}\left(1-\frac{2^{3/4}x}{\sqrt[4]{3+\sqrt{5}}}\cdots\right.}{2\sqrt[4]{2\left(3+\sqrt{5}\right)}}\cdots$$

Mathematica [C] Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.
time = 0.02, size = 57, normalized size = 0.14

$$-\frac{1}{4}\text{RootSum}\left[1+3\#1^4+\#1^8\&,\frac{-\log(x-\#1)+\log(x-\#1)\#1^4}{3\#1^3+2\#1^7}\&\right]$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 - x^4)/(1 + 3*x^4 + x^8),x]
```

```
[Out] -1/4*RootSum[1 + 3*#1^4 + #1^8 & , (-Log[x - #1] + Log[x - #1]*#1^4)/(3*#1^
3 + 2*#1^7) & ]
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 0.03, size = 44, normalized size = 0.11

method	result	size
default	$\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(\_Z^{8}+3\_Z^{4}+1\right)}\frac{\left(-\_R^{4}+1\right)\ln\left(x-\_R\right)}{2\_R^{7}+3\_R^{3}}\right)}{4}$	44
risch	$\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(\_Z^{8}+3\_Z^{4}+1\right)}\frac{\left(-\_R^{4}+1\right)\ln\left(x-\_R\right)}{2\_R^{7}+3\_R^{3}}\right)}{4}$	44

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-x^4+1)/(x^8+3*x^4+1),x,method=_RETURNVERBOSE)
[Out] 1/4*sum((-_R^4+1)/(2*_R^7+3*_R^3)*ln(x-_R),_R=RootOf(_Z^8+3*_Z^4+1))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^4+1)/(x^8+3*x^4+1),x, algorithm="maxima")
[Out] -integrate((x^4 - 1)/(x^8 + 3*x^4 + 1), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 896 vs. 2(269) = 538.
time = 0.39, size = 896, normalized size = 2.18

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^4+1)/(x^8+3*x^4+1),x, algorithm="fricas")
[Out] 1/16*(sqrt(5)*sqrt(2) - 3*sqrt(2))*(2*sqrt(5) + 6)^(3/4)*sqrt(sqrt(5) + 3)*
arctan(1/16*sqrt(4*x^2 - sqrt(2*sqrt(5) + 6)*(sqrt(5) - 3) + 2*(sqrt(5)*x -
 x)*(2*sqrt(5) + 6)^(1/4))*(sqrt(5)*sqrt(2) - 2*sqrt(2))*(2*sqrt(5) + 6)^(5
/4)*sqrt(sqrt(5) + 3) - 1/8*(sqrt(5)*sqrt(2)*x - 2*sqrt(2)*x)*(2*sqrt(5) +
6)^(5/4)*sqrt(sqrt(5) + 3) + 1/8*(sqrt(5)*sqrt(2) - 3*sqrt(2))*sqrt(2*sqrt(
5) + 6)*sqrt(sqrt(5) + 3)) + 1/16*(sqrt(5)*sqrt(2) - 3*sqrt(2))*(2*sqrt(5)
+ 6)^(3/4)*sqrt(sqrt(5) + 3)*arctan(1/16*sqrt(4*x^2 - sqrt(2*sqrt(5) + 6)*(
sqrt(5) - 3) - 2*(sqrt(5)*x - x)*(2*sqrt(5) + 6)^(1/4))*(sqrt(5)*sqrt(2) -
2*sqrt(2))*(2*sqrt(5) + 6)^(5/4)*sqrt(sqrt(5) + 3) - 1/8*(sqrt(5)*sqrt(2)*x
 - 2*sqrt(2)*x)*(2*sqrt(5) + 6)^(5/4)*sqrt(sqrt(5) + 3) - 1/8*(sqrt(5)*sqrt
(2) - 3*sqrt(2))*sqrt(2*sqrt(5) + 6)*sqrt(sqrt(5) + 3)) + 1/16*(sqrt(5)*sqr
```

```
t(2) + 3*sqrt(2))*sqrt(-sqrt(5) + 3)*(-2*sqrt(5) + 6)^(3/4)*arctan(1/16*sqr
t(4*x^2 + (sqrt(5) + 3)*sqrt(-2*sqrt(5) + 6) + 2*(sqrt(5)*x + x)*(-2*sqrt(5
) + 6)^(1/4))*(sqrt(5)*sqrt(2) + 2*sqrt(2))*sqrt(-sqrt(5) + 3)*(-2*sqrt(5)
+ 6)^(5/4) - 1/8*((sqrt(5)*sqrt(2)*x + 2*sqrt(2)*x)*(-2*sqrt(5) + 6)^(5/4)
+ (sqrt(5)*sqrt(2) + 3*sqrt(2))*sqrt(-2*sqrt(5) + 6))*sqrt(-sqrt(5) + 3)) +
 1/16*(sqrt(5)*sqrt(2) + 3*sqrt(2))*sqrt(-sqrt(5) + 3)*(-2*sqrt(5) + 6)^(3/
4)*arctan(1/16*sqrt(4*x^2 + (sqrt(5) + 3)*sqrt(-2*sqrt(5) + 6) - 2*(sqrt(5)
*x + x)*(-2*sqrt(5) + 6)^(1/4))*(sqrt(5)*sqrt(2) + 2*sqrt(2))*sqrt(-sqrt(5)
 + 3)*(-2*sqrt(5) + 6)^(5/4) - 1/8*((sqrt(5)*sqrt(2)*x + 2*sqrt(2)*x)*(-2*s
qrt(5) + 6)^(5/4) - (sqrt(5)*sqrt(2) + 3*sqrt(2))*sqrt(-2*sqrt(5) + 6))*sqr
t(-sqrt(5) + 3)) + 1/8*(2*sqrt(5) + 6)^(1/4)*log(16*x^2 - 4*sqrt(2*sqrt(5)
+ 6)*(sqrt(5) - 3) + 8*(sqrt(5)*x - x)*(2*sqrt(5) + 6)^(1/4)) - 1/8*(2*sqrt
(5) + 6)^(1/4)*log(16*x^2 - 4*sqrt(2*sqrt(5) + 6)*(sqrt(5) - 3) - 8*(sqrt(5
)*x - x)*(2*sqrt(5) + 6)^(1/4)) - 1/8*(-2*sqrt(5) + 6)^(1/4)*log(16*x^2 + 4
*(sqrt(5) + 3)*sqrt(-2*sqrt(5) + 6) + 8*(sqrt(5)*x + x)*(-2*sqrt(5) + 6)^(1
/4)) + 1/8*(-2*sqrt(5) + 6)^(1/4)*log(16*x^2 + 4*(sqrt(5) + 3)*sqrt(-2*sqrt
(5) + 6) - 8*(sqrt(5)*x + x)*(-2*sqrt(5) + 6)^(1/4))
```

Sympy [**A**]
time = 0.91, size = 26, normalized size = 0.06

$$-\operatorname{RootSum}\left(65536t^8 + 768t^4 + 1, \left(t \mapsto t\log\left(1024t^5 + 8t + x\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x**4+1)/(x**8+3*x**4+1),x)
```

```
[Out] -RootSum(65536*_t**8 + 768*_t**4 + 1, Lambda(_t, _t*log(1024*_t**5 + 8*_t +
 x)))
```

Giac [**A**]
time = 3.70, size = 223, normalized size = 0.54

$$\frac{1}{16}\left(\pi + 4\arctan\left(x\sqrt{\sqrt{5}+1} + 1\right)\right)\sqrt{\sqrt{5}+1} - \frac{1}{16}\left(\pi + 4\arctan\left(-x\sqrt{\sqrt{5}+1} + 1\right)\right)\sqrt{\sqrt{5}+1} - \frac{1}{16}\left(\pi + 4\arctan\left(x\sqrt{\sqrt{5}-1} - 1\right)\right)\sqrt{\sqrt{5}-1} + \frac{1}{16}\left(\pi + 4\arctan\left(-x\sqrt{\sqrt{5}-1} - 1\right)\right)\sqrt{\sqrt{5}-1} - \frac{1}{8}\sqrt{\sqrt{5}-1}\log\left(2500\left(x + \sqrt{\sqrt{5}+1}\right)^2 + 2500x^2\right) + \frac{1}{8}\sqrt{\sqrt{5}-1}\log\left(2500\left(x - \sqrt{\sqrt{5}+1}\right)^2 + 2500x^2\right) + \frac{1}{8}\sqrt{\sqrt{5}+1}\log\left(1156\left(x + \sqrt{\sqrt{5}-1}\right)^2 + 1156x^2\right) - \frac{1}{8}\sqrt{\sqrt{5}+1}\log\left(1156\left(x - \sqrt{\sqrt{5}-1}\right)^2 + 1156x^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^4+1)/(x^8+3*x^4+1),x, algorithm="giac")
```

```
[Out] 1/16*(pi + 4*arctan(x*sqrt(sqrt(5) + 1) + 1))*sqrt(sqrt(5) + 1) - 1/16*(pi
+ 4*arctan(-x*sqrt(sqrt(5) + 1) + 1))*sqrt(sqrt(5) + 1) - 1/16*(pi + 4*arct
an(x*sqrt(sqrt(5) - 1) - 1))*sqrt(sqrt(5) - 1) + 1/16*(pi + 4*arctan(-x*sqr
t(sqrt(5) - 1) - 1))*sqrt(sqrt(5) - 1) - 1/8*sqrt(sqrt(5) - 1)*log(2500*(x
+ sqrt(sqrt(5) + 1))^2 + 2500*x^2) + 1/8*sqrt(sqrt(5) - 1)*log(2500*(x - sq
rt(sqrt(5) + 1))^2 + 2500*x^2) + 1/8*sqrt(sqrt(5) + 1)*log(1156*(x + sqrt(s
qrt(5) - 1))^2 + 1156*x^2) - 1/8*sqrt(sqrt(5) + 1)*log(1156*(x - sqrt(sqrt(
5) - 1))^2 + 1156*x^2)
```

Mupad [B]

time = 1.68, size = 447, normalized size = 1.09

$$\frac{2^{3/4}\,\mathrm{atan}\left(\frac{1875\,2^{3/4}\,x\,{\left(\sqrt{5}-3\right)}^{1/4}}{2\,\left(625\,\sqrt{2}\,\sqrt{\sqrt{5}-3}-250\,\sqrt{2}\,\sqrt{5}\,\sqrt{\sqrt{5}-3}\right)}-\frac{875\,2^{3/4}\,\sqrt{5}\,x\,{\left(\sqrt{5}-3\right)}^{1/4}}{2\,\left(625\,\sqrt{2}\,\sqrt{\sqrt{5}-3}-250\,\sqrt{2}\,\sqrt{5}\,\sqrt{\sqrt{5}-3}\right)}\right)\,{\left(\sqrt{5}-3\right)}^{1/4}}{4}-\frac{2^{3/4}\,\mathrm{atan}\left(\frac{2^{3/4}\,x\,{\left(\sqrt{5}-3\right)}^{1/4}\,1875{}\mathrm{i}}{2\,\left(625\,\sqrt{2}\,\sqrt{\sqrt{5}-3}-250\,\sqrt{2}\,\sqrt{5}\,\sqrt{\sqrt{5}-3}\right)}-\frac{2^{3/4}\,\sqrt{5}\,x\,{\left(\sqrt{5}-3\right)}^{1/4}\,875{}\mathrm{i}}{2\,\left(625\,\sqrt{2}\,\sqrt{\sqrt{5}-3}-250\,\sqrt{2}\,\sqrt{5}\,\sqrt{\sqrt{5}-3}\right)}\right)\,{\left(\sqrt{5}-3\right)}^{1/4}\,1{}\mathrm{i}}{4}+\frac{2^{3/4}\,\mathrm{atan}\left(\frac{1875\,2^{3/4}\,x\,{\left(-\sqrt{5}-3\right)}^{1/4}}{2\,\left(625\,\sqrt{2}\,\sqrt{-\sqrt{5}-3}+250\,\sqrt{2}\,\sqrt{5}\,\sqrt{-\sqrt{5}-3}\right)}+\frac{875\,2^{3/4}\,\sqrt{5}\,x\,{\left(-\sqrt{5}-3\right)}^{1/4}}{2\,\left(625\,\sqrt{2}\,\sqrt{-\sqrt{5}-3}+250\,\sqrt{2}\,\sqrt{5}\,\sqrt{-\sqrt{5}-3}\right)}\right)\,{\left(-\sqrt{5}-3\right)}^{1/4}}{4}-\frac{2^{3/4}\,\mathrm{atan}\left(\frac{2^{3/4}\,x\,{\left(-\sqrt{5}-3\right)}^{1/4}\,1875{}\mathrm{i}}{2\,\left(625\,\sqrt{2}\,\sqrt{-\sqrt{5}-3}+250\,\sqrt{2}\,\sqrt{5}\,\sqrt{-\sqrt{5}-3}\right)}+\frac{2^{3/4}\,\sqrt{5}\,x\,{\left(-\sqrt{5}-3\right)}^{1/4}\,875{}\mathrm{i}}{2\,\left(625\,\sqrt{2}\,\sqrt{-\sqrt{5}-3}+250\,\sqrt{2}\,\sqrt{5}\,\sqrt{-\sqrt{5}-3}\right)}\right)\,{\left(-\sqrt{5}-3\right)}^{1/4}\,1{}\mathrm{i}}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(x^4 - 1)/(3*x^4 + x^8 + 1),x)
```

```
[Out] (2^(3/4)*atan((1875*2^(3/4)*x*(5^(1/2) - 3)^(1/4))/(2*(625*2^(1/2)*(5^(1/2)
 - 3)^(1/2) - 250*2^(1/2)*5^(1/2)*(5^(1/2) - 3)^(1/2))) - (875*2^(3/4)*5^(1
/2)*x*(5^(1/2) - 3)^(1/4))/(2*(625*2^(1/2)*(5^(1/2) - 3)^(1/2) - 250*2^(1/2
)*5^(1/2)*(5^(1/2) - 3)^(1/2))))*(5^(1/2) - 3)^(1/4))/4 - (2^(3/4)*atan((2^
(3/4)*x*(5^(1/2) - 3)^(1/4)*1875i)/(2*(625*2^(1/2)*(5^(1/2) - 3)^(1/2) - 25
0*2^(1/2)*5^(1/2)*(5^(1/2) - 3)^(1/2))) - (2^(3/4)*5^(1/2)*x*(5^(1/2) - 3)^
(1/4)*875i)/(2*(625*2^(1/2)*(5^(1/2) - 3)^(1/2) - 250*2^(1/2)*5^(1/2)*(5^(1
/2) - 3)^(1/2))))*(5^(1/2) - 3)^(1/4)*1i)/4 + (2^(3/4)*atan((1875*2^(3/4)*x
*(- 5^(1/2) - 3)^(1/4))/(2*(625*2^(1/2)*(- 5^(1/2) - 3)^(1/2) + 250*2^(1/2)
*5^(1/2)*(- 5^(1/2) - 3)^(1/2))) + (875*2^(3/4)*5^(1/2)*x*(- 5^(1/2) - 3)^(
1/4))/(2*(625*2^(1/2)*(- 5^(1/2) - 3)^(1/2) + 250*2^(1/2)*5^(1/2)*(- 5^(1/2
) - 3)^(1/2))))*(- 5^(1/2) - 3)^(1/4))/4 - (2^(3/4)*atan((2^(3/4)*x*(- 5^(1
/2) - 3)^(1/4)*1875i)/(2*(625*2^(1/2)*(- 5^(1/2) - 3)^(1/2) + 250*2^(1/2)*5
^(1/2)*(- 5^(1/2) - 3)^(1/2))) + (2^(3/4)*5^(1/2)*x*(- 5^(1/2) - 3)^(1/4)*8
75i)/(2*(625*2^(1/2)*(- 5^(1/2) - 3)^(1/2) + 250*2^(1/2)*5^(1/2)*(- 5^(1/2)
 - 3)^(1/2))))*(- 5^(1/2) - 3)^(1/4)*1i)/4
```

3.22 $\int \frac{1-x^4}{1+2x^4+x^8}\,dx$

Optimal. Leaf size=97

$$\frac{x}{2(1+x^4)}-\frac{\tan^{-1}\left(1-\sqrt{2}\,x\right)}{4\sqrt{2}}+\frac{\tan^{-1}\left(1+\sqrt{2}\,x\right)}{4\sqrt{2}}-\frac{\log\left(1-\sqrt{2}\,x+x^2\right)}{8\sqrt{2}}+\frac{\log\left(1+\sqrt{2}\,x+x^2\right)}{8\sqrt{2}}$$

```
[Out] 1/2*x/(x^4+1)+1/8*arctan(-1+x*2^(1/2))*2^(1/2)+1/8*arctan(1+x*2^(1/2))*2^(1
/2)-1/16*ln(1+x^2-x*2^(1/2))*2^(1/2)+1/16*ln(1+x^2+x*2^(1/2))*2^(1/2)
```

Rubi [A]
time = 0.04, antiderivative size = 97, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 8, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {28, 393, 217, 1179, 642, 1176, 631, 210}

$$-\frac{\text{ArcTan}\left(1-\sqrt{2}\,x\right)}{4\sqrt{2}}+\frac{\text{ArcTan}\left(\sqrt{2}\,x+1\right)}{4\sqrt{2}}+\frac{x}{2(x^4+1)}-\frac{\log\left(x^2-\sqrt{2}\,x+1\right)}{8\sqrt{2}}+\frac{\log\left(x^2+\sqrt{2}\,x+1\right)}{8\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[(1 - x^4)/(1 + 2*x^4 + x^8),x]
```

```
[Out] x/(2*(1 + x^4)) - ArcTan[1 - Sqrt[2]*x]/(4*Sqrt[2]) + ArcTan[1 + Sqrt[2]*x]
/(4*Sqrt[2]) - Log[1 - Sqrt[2]*x + x^2]/(8*Sqrt[2]) + Log[1 + Sqrt[2]*x + x
^2]/(8*Sqrt[2])
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 393

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(-(b*c - a*d))*x*((a + b*x^n)^(p + 1)/(a*b*n*(p + 1))), x] - Dist[(a*d -
 b*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; F
reeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n
 + p, 0])
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$
\begin{aligned}
\int \frac{1-x^4}{1+2x^4+x^8}\,dx &= \int \frac{1-x^4}{(1+x^4)^2}\,dx \\
&= \frac{x}{2(1+x^4)}+\frac{1}{2}\int \frac{1}{1+x^4}\,dx \\
&= \frac{x}{2(1+x^4)}+\frac{1}{4}\int \frac{1-x^2}{1+x^4}\,dx+\frac{1}{4}\int \frac{1+x^2}{1+x^4}\,dx \\
&= \frac{x}{2(1+x^4)}+\frac{1}{8}\int \frac{1}{1-\sqrt{2}\,x+x^2}\,dx+\frac{1}{8}\int \frac{1}{1+\sqrt{2}\,x+x^2}\,dx-\frac{\int \frac{\sqrt{2}+2x}{-1-\sqrt{2}\,x-x^2}\,dx}{8\sqrt{2}}- \\
&= \frac{x}{2(1+x^4)}-\frac{\log\left(1-\sqrt{2}\,x+x^2\right)}{8\sqrt{2}}+\frac{\log\left(1+\sqrt{2}\,x+x^2\right)}{8\sqrt{2}}+\frac{\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx,x,1\right.}{4\sqrt{2}} \\
&= \frac{x}{2(1+x^4)}-\frac{\tan^{-1}\left(1-\sqrt{2}\,x\right)}{4\sqrt{2}}+\frac{\tan^{-1}\left(1+\sqrt{2}\,x\right)}{4\sqrt{2}}-\frac{\log\left(1-\sqrt{2}\,x+x^2\right)}{8\sqrt{2}}+\frac{\log}{}
\end{aligned}
$$

Mathematica [A]
time = 0.04, size = 90, normalized size = 0.93

$$\frac{1}{16}\left(\frac{8x}{1+x^4}-2\sqrt{2}\tan^{-1}\left(1-\sqrt{2}\,x\right)+2\sqrt{2}\tan^{-1}\left(1+\sqrt{2}\,x\right)-\sqrt{2}\log\left(1-\sqrt{2}\,x+x^2\right)+\sqrt{2}\log\left(1+\sqrt{2}\,x+x^2\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 - x^4)/(1 + 2*x^4 + x^8),x]
```

```
[Out] ((8*x)/(1 + x^4) - 2*Sqrt[2]*ArcTan[1 - Sqrt[2]*x] + 2*Sqrt[2]*ArcTan[1 + S
qrt[2]*x] - Sqrt[2]*Log[1 - Sqrt[2]*x + x^2] + Sqrt[2]*Log[1 + Sqrt[2]*x +
x^2])/16
```

Maple [A]
time = 0.02, size = 63, normalized size = 0.65

method	result	size
risch	$\frac{x}{2x^4+2}+\frac{\left(\sum_{\_R=\text{RootOf}(\_Z^4+1)}\frac{\ln(x-\_R)}{\_R^3}\right)}{8}$	33
default	$\frac{x}{2x^4+2}+\frac{\sqrt{2}\left(\ln\left(\frac{1+x^2+\sqrt{2}\,x}{1+x^2-\sqrt{2}\,x}\right)+2\arctan\left(\sqrt{2}\,x+1\right)+2\arctan\left(\sqrt{2}\,x-1\right)\right)}{16}$	63

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-x^4+1)/(x^8+2*x^4+1),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*x/(x^4+1)+1/16*2^(1/2)*(ln((1+x^2+2^(1/2)*x)/(1+x^2-2^(1/2)*x))+2*arcta
n(2^(1/2)*x+1)+2*arctan(2^(1/2)*x-1))
```

Maxima [A]
time = 0.53, size = 82, normalized size = 0.85

$$\frac{1}{8}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(2x+\sqrt{2}\right)\right)+\frac{1}{8}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(2x-\sqrt{2}\right)\right)+\frac{1}{16}\sqrt{2}\log\left(x^2+\sqrt{2}x+1\right)-\frac{1}{16}\sqrt{2}\log\left(x^2-\sqrt{2}x+1\right)+\frac{x}{2(x^4+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^4+1)/(x^8+2*x^4+1),x, algorithm="maxima")
```

```
[Out] 1/8*sqrt(2)*arctan(1/2*sqrt(2)*(2*x + sqrt(2))) + 1/8*sqrt(2)*arctan(1/2*sq
rt(2)*(2*x - sqrt(2))) + 1/16*sqrt(2)*log(x^2 + sqrt(2)*x + 1) - 1/16*sqrt(
2)*log(x^2 - sqrt(2)*x + 1) + 1/2*x/(x^4 + 1)
```

Fricas [A]
time = 0.33, size = 131, normalized size = 1.35

$$-\frac{4\sqrt{2}(x^4+1)\arctan\left(-\sqrt{2}x+\sqrt{2}\sqrt{x^2+\sqrt{2}x+1}-1\right)+4\sqrt{2}(x^4+1)\arctan\left(-\sqrt{2}x+\sqrt{2}\sqrt{x^2-\sqrt{2}x+1}+1\right)-\sqrt{2}(x^4+1)\log\left(4x^2+4\sqrt{2}x+4\right)+\sqrt{2}(x^4+1)\log\left(4x^2-4\sqrt{2}x+4\right)-8x}{16(x^4+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^4+1)/(x^8+2*x^4+1),x, algorithm="fricas")
```

```
[Out] -1/16*(4*sqrt(2)*(x^4 + 1)*arctan(-sqrt(2)*x + sqrt(2)*sqrt(x^2 + sqrt(2)*x
 + 1) - 1) + 4*sqrt(2)*(x^4 + 1)*arctan(-sqrt(2)*x + sqrt(2)*sqrt(x^2 - sqr
t(2)*x + 1) + 1) - sqrt(2)*(x^4 + 1)*log(4*x^2 + 4*sqrt(2)*x + 4) + sqrt(2)
*(x^4 + 1)*log(4*x^2 - 4*sqrt(2)*x + 4) - 8*x)/(x^4 + 1)
```

Sympy [A]
time = 0.06, size = 82, normalized size = 0.85

$$\frac{x}{2x^4+2}-\frac{\sqrt{2}\log\left(x^2-\sqrt{2}x+1\right)}{16}+\frac{\sqrt{2}\log\left(x^2+\sqrt{2}x+1\right)}{16}+\frac{\sqrt{2}\operatorname{atan}\left(\sqrt{2}x-1\right)}{8}+\frac{\sqrt{2}\operatorname{atan}\left(\sqrt{2}x+1\right)}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x**4+1)/(x**8+2*x**4+1),x)
```

```
[Out] x/(2*x**4 + 2) - sqrt(2)*log(x**2 - sqrt(2)*x + 1)/16 + sqrt(2)*log(x**2 +
sqrt(2)*x + 1)/16 + sqrt(2)*atan(sqrt(2)*x - 1)/8 + sqrt(2)*atan(sqrt(2)*x
+ 1)/8
```

Giac [A]
time = 4.00, size = 82, normalized size = 0.85

$$\frac{1}{8}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(2x+\sqrt{2}\right)\right)+\frac{1}{8}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(2x-\sqrt{2}\right)\right)+\frac{1}{16}\sqrt{2}\log\left(x^2+\sqrt{2}x+1\right)-\frac{1}{16}\sqrt{2}\log\left(x^2-\sqrt{2}x+1\right)+\frac{x}{2(x^4+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^4+1)/(x^8+2*x^4+1),x, algorithm="giac")
```

```
[Out] 1/8*sqrt(2)*arctan(1/2*sqrt(2)*(2*x + sqrt(2))) + 1/8*sqrt(2)*arctan(1/2*sq
rt(2)*(2*x - sqrt(2))) + 1/16*sqrt(2)*log(x^2 + sqrt(2)*x + 1) - 1/16*sqrt(
2)*log(x^2 - sqrt(2)*x + 1) + 1/2*x/(x^4 + 1)
```

Mupad [B]
time = 1.62, size = 44, normalized size = 0.45

$$\frac{x}{2\,\left(x^4+1\right)}+\sqrt{2}\,\mathrm{atan}\left(\sqrt{2}\,x\,\left(\frac{1}{2}-\frac{1}{2}\mathrm{i}\right)\right)\,\left(\frac{1}{8}+\frac{1}{8}\mathrm{i}\right)+\sqrt{2}\,\mathrm{atan}\left(\sqrt{2}\,x\,\left(\frac{1}{2}+\frac{1}{2}\mathrm{i}\right)\right)\,\left(\frac{1}{8}-\frac{1}{8}\mathrm{i}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(x^4 - 1)/(2*x^4 + x^8 + 1),x)
```

```
[Out] 2^(1/2)*atan(2^(1/2)*x*(1/2 - 1i/2))*(1/8 + 1i/8) + 2^(1/2)*atan(2^(1/2)*x*
(1/2 + 1i/2))*(1/8 - 1i/8) + x/(2*(x^4 + 1))
```

3.23 $\int \frac{1-x^4}{1+x^4+x^8} \, dx$

Optimal. Leaf size=140

$$-\frac{1}{4}\sqrt{3}\tan^{-1}\left(\frac{1-2x}{\sqrt{3}}\right)+\frac{1}{4}\tan^{-1}\left(\sqrt{3}-2x\right)+\frac{1}{4}\sqrt{3}\tan^{-1}\left(\frac{1+2x}{\sqrt{3}}\right)-\frac{1}{4}\tan^{-1}\left(\sqrt{3}+2x\right)+\frac{1}{8}\log\left(1-x\right.$$

```
[Out] -1/4*arctan(2*x-3^(1/2))-1/4*arctan(2*x+3^(1/2))+1/8*ln(x^2-x+1)-1/8*ln(x^2
+x+1)-1/4*arctan(1/3*(1-2*x)*3^(1/2))*3^(1/2)+1/4*arctan(1/3*(1+2*x)*3^(1/2
))*3^(1/2)-1/8*ln(1+x^2-x*3^(1/2))*3^(1/2)+1/8*ln(1+x^2+x*3^(1/2))*3^(1/2)
```

Rubi [A]
time = 0.07, antiderivative size = 140, normalized size of antiderivative = 1.00, number of steps used = 19, number of rules used = 6, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {1435, 1183, 648, 632, 210, 642}

$$-\frac{1}{4}\sqrt{3}\,\text{ArcTan}\left(\frac{1-2x}{\sqrt{3}}\right)+\frac{1}{4}\text{ArcTan}\left(\sqrt{3}-2x\right)+\frac{1}{4}\sqrt{3}\,\text{ArcTan}\left(\frac{2x+1}{\sqrt{3}}\right)-\frac{1}{4}\text{ArcTan}\left(2x+\sqrt{3}\right)+\frac{1}{8}\log\left(x^2-x+1\right)-\frac{1}{8}\log\left(x^2+x+1\right)-\frac{1}{8}\sqrt{3}\log\left(x^2-\sqrt{3}x+1\right)+\frac{1}{8}\sqrt{3}\log\left(x^2+\sqrt{3}x+1\right)$$

Antiderivative was successfully verified.

```
[In] Int[(1 - x^4)/(1 + x^4 + x^8),x]
```

```
[Out] -1/4*(Sqrt[3]*ArcTan[(1 - 2*x)/Sqrt[3]]) + ArcTan[Sqrt[3] - 2*x]/4 + (Sqrt[
3]*ArcTan[(1 + 2*x)/Sqrt[3]])/4 - ArcTan[Sqrt[3] + 2*x]/4 + Log[1 - x + x^2
]/8 - Log[1 + x + x^2]/8 - (Sqrt[3]*Log[1 - Sqrt[3]*x + x^2])/8 + (Sqrt[3]*
Log[1 + Sqrt[3]*x + x^2])/8
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1183

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[a/c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Dist[1/(2*c*q*r), Int
[(d*r - (d - e*q)*x)/(q - r*x + x^2), x], x] + Dist[1/(2*c*q*r), Int[(d*r +
 (d - e*q)*x)/(q + r*x + x^2), x], x]]] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NegQ[b^2 - 4*a*c]
```

Rule 1435

```
Int[((d_) + (e_.)*(x_)^(n_))/((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_)), x
_Symbol] :> With[{q = Rt[-2*(d/e) - b/c, 2]}, Dist[e/(2*c*q), Int[(q - 2*x^
(n/2))/Simp[d/e + q*x^(n/2) - x^n, x], x], x] + Dist[e/(2*c*q), Int[(q + 2*
x^(n/2))/Simp[d/e - q*x^(n/2) - x^n, x], x], x]] /; FreeQ[{a, b, c, d, e},
x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - a*e^2, 0] && IGtQ[
n/2, 0] &&  !GtQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{1-x^4}{1+x^4+x^8}\,dx &= -\left(\frac{1}{2}\int \frac{1+2x^2}{-1-x^2-x^4}\,dx\right)-\frac{1}{2}\int \frac{1-2x^2}{-1+x^2-x^4}\,dx\\
&= \frac{1}{4}\int \frac{1+x}{1-x+x^2}\,dx+\frac{1}{4}\int \frac{1-x}{1+x+x^2}\,dx+\frac{\int \frac{\sqrt{3}-3x}{1-\sqrt{3}\,x+x^2}\,dx}{4\sqrt{3}}+\frac{\int \frac{\sqrt{3}+3x}{1+\sqrt{3}\,x+x^2}\,dx}{4\sqrt{3}}\\
&= \frac{1}{8}\int \frac{-1+2x}{1-x+x^2}\,dx-\frac{1}{8}\int \frac{1+2x}{1+x+x^2}\,dx-\frac{1}{8}\int \frac{1}{1-\sqrt{3}\,x+x^2}\,dx-\frac{1}{8}\int \frac{1}{1+\sqrt{3}\,x+\cdots}\\
&= \frac{1}{8}\log\left(1-x+x^2\right)-\frac{1}{8}\log\left(1+x+x^2\right)-\frac{1}{8}\sqrt{3}\,\log\left(1-\sqrt{3}\,x+x^2\right)+\frac{1}{8}\sqrt{3}\,\log\left(1-\cdots\right.\\
&= -\frac{1}{4}\sqrt{3}\,\tan^{-1}\left(\frac{1-2x}{\sqrt{3}}\right)+\frac{1}{4}\tan^{-1}\left(\sqrt{3}-2x\right)+\frac{1}{4}\sqrt{3}\,\tan^{-1}\left(\frac{1+2x}{\sqrt{3}}\right)-\frac{1}{4}\tan^{-1}\left(\cdots\right.
\end{aligned}$$

Mathematica [C] Result contains complex when optimal does not.
time = 0.10, size = 129, normalized size = 0.92

$$\frac{1}{8}\left(-2\sqrt{-2-2i\sqrt{3}}\,\tan^{-1}\left(\frac{1}{2}\left(1-i\sqrt{3}\right)x\right)-2\sqrt{-2+2i\sqrt{3}}\,\tan^{-1}\left(\frac{1}{2}\left(1+i\sqrt{3}\right)x\right)+2\sqrt{3}\,\tan^{-1}\left(\frac{-1+2x}{\sqrt{3}}\right)+2\sqrt{3}\,\tan^{-1}\left(\frac{1+2x}{\sqrt{3}}\right)+\log\left(1-x+x^2\right)-\log\left(1+x+x^2\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 - x^4)/(1 + x^4 + x^8),x]
```

```
[Out] (-2*Sqrt[-2 - (2*I)*Sqrt[3]]*ArcTan[((1 - I*Sqrt[3])*x)/2] - 2*Sqrt[-2 + (2
*I)*Sqrt[3]]*ArcTan[((1 + I*Sqrt[3])*x)/2] + 2*Sqrt[3]*ArcTan[(-1 + 2*x)/Sq
rt[3]] + 2*Sqrt[3]*ArcTan[(1 + 2*x)/Sqrt[3]] + Log[1 - x + x^2] - Log[1 + x
 + x^2])/8
```

Maple [A]
time = 0.04, size = 109, normalized size = 0.78

method	result
risch	$\frac{\ln(4x^2-4x+4)}{8}+\frac{\sqrt{3}\,\arctan\left(\frac{(2x-1)\sqrt{3}}{3}\right)}{4}-\frac{\ln(4x^2+4x+4)}{8}+\frac{\arctan\left(\frac{(2x+1)\sqrt{3}}{3}\right)\sqrt{3}}{4}+\frac{\left(\sum_{\_R=\mathrm{RootOf}(\_Z^4-\_Z^2+}\right.}{}$
default	$-\frac{\ln(x^2+x+1)}{8}+\frac{\arctan\left(\frac{(2x+1)\sqrt{3}}{3}\right)\sqrt{3}}{4}-\frac{\ln\left(1+x^2-x\sqrt{3}\right)\sqrt{3}}{8}-\frac{\arctan\left(2x-\sqrt{3}\right)}{4}+\frac{\ln\left(1+x^2+x\sqrt{3}\right)\sqrt{3}}{8}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-x^4+1)/(x^8+x^4+1),x,method=_RETURNVERBOSE)
```

```
[Out] -1/8*ln(x^2+x+1)+1/4*arctan(1/3*(2*x+1)*3^(1/2))*3^(1/2)-1/8*ln(1+x^2-x*3^(
1/2))*3^(1/2)-1/4*arctan(2*x-3^(1/2))+1/8*ln(1+x^2+x*3^(1/2))*3^(1/2)-1/4*a
rctan(2*x+3^(1/2))+1/8*ln(x^2-x+1)+1/4*3^(1/2)*arctan(1/3*(2*x-1)*3^(1/2))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^4+1)/(x^8+x^4+1),x, algorithm="maxima")
```

```
[Out] 1/4*sqrt(3)*arctan(1/3*sqrt(3)*(2*x + 1)) + 1/4*sqrt(3)*arctan(1/3*sqrt(3)*
(2*x - 1)) - 1/2*integrate((2*x^2 - 1)/(x^4 - x^2 + 1), x) - 1/8*log(x^2 +
x + 1) + 1/8*log(x^2 - x + 1)
```

Fricas [A]
time = 0.36, size = 142, normalized size = 1.01

$$\frac{1}{4}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x+1)\right)+\frac{1}{4}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x-1)\right)+\frac{1}{8}\sqrt{3}\log\left(16x^2+16\sqrt{3}x+16\right)-\frac{1}{8}\sqrt{3}\log\left(16x^2-16\sqrt{3}x+16\right)+\frac{1}{2}\arctan\left(-2x+\sqrt{3}+2\sqrt{x^2-\sqrt{3}x+1}\right)+\frac{1}{2}\arctan\left(-2x-\sqrt{3}+2\sqrt{x^2+\sqrt{3}x+1}\right)-\frac{1}{8}\log\left(x^2+x+1\right)+\frac{1}{8}\log\left(x^2-x+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^4+1)/(x^8+x^4+1),x, algorithm="fricas")
```

```
[Out] 1/4*sqrt(3)*arctan(1/3*sqrt(3)*(2*x + 1)) + 1/4*sqrt(3)*arctan(1/3*sqrt(3)*
(2*x - 1)) + 1/8*sqrt(3)*log(16*x^2 + 16*sqrt(3)*x + 16) - 1/8*sqrt(3)*log(
```

```
16*x^2 - 16*sqrt(3)*x + 16) + 1/2*arctan(-2*x + sqrt(3) + 2*sqrt(x^2 - sqrt
(3)*x + 1)) + 1/2*arctan(-2*x - sqrt(3) + 2*sqrt(x^2 + sqrt(3)*x + 1)) - 1/
8*log(x^2 + x + 1) + 1/8*log(x^2 - x + 1)
```

Sympy [C] Result contains complex when optimal does not.
time = 0.32, size = 148, normalized size = 1.06

$$-\left(-\frac{1}{8}-\frac{\sqrt{3}\,i}{8}\right)\log\left(x+1024\left(-\frac{1}{8}-\frac{\sqrt{3}\,i}{8}\right)^5\right)-\left(-\frac{1}{8}+\frac{\sqrt{3}\,i}{8}\right)\log\left(x+1024\left(-\frac{1}{8}+\frac{\sqrt{3}\,i}{8}\right)^5\right)-\left(\frac{1}{8}-\frac{\sqrt{3}\,i}{8}\right)\log\left(x+1024\left(\frac{1}{8}-\frac{\sqrt{3}\,i}{8}\right)^5\right)-\left(\frac{1}{8}+\frac{\sqrt{3}\,i}{8}\right)\log\left(x+1024\left(\frac{1}{8}+\frac{\sqrt{3}\,i}{8}\right)^5\right)-\operatorname{RootSum}\left(256t^4-16t^2+1,\ \left(t\mapsto t\log\left(1024t^5+x\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x**4+1)/(x**8+x**4+1),x)
```

```
[Out] -(-1/8 - sqrt(3)*I/8)*log(x + 1024*(-1/8 - sqrt(3)*I/8)**5) - (-1/8 + sqrt(
3)*I/8)*log(x + 1024*(-1/8 + sqrt(3)*I/8)**5) - (1/8 - sqrt(3)*I/8)*log(x +
 1024*(1/8 - sqrt(3)*I/8)**5) - (1/8 + sqrt(3)*I/8)*log(x + 1024*(1/8 + sqr
t(3)*I/8)**5) - RootSum(256*_t**4 - 16*_t**2 + 1, Lambda(_t, _t*log(1024*_t
**5 + x)))
```

Giac [A]
time = 3.21, size = 108, normalized size = 0.77

$$\frac{1}{4}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2x+1\right)\right)+\frac{1}{4}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2x-1\right)\right)+\frac{1}{8}\sqrt{3}\log\left(x^2+\sqrt{3}x+1\right)-\frac{1}{8}\sqrt{3}\log\left(x^2-\sqrt{3}x+1\right)-\frac{1}{4}\arctan\left(2x+\sqrt{3}\right)-\frac{1}{4}\arctan\left(2x-\sqrt{3}\right)-\frac{1}{8}\log\left(x^2+x+1\right)+\frac{1}{8}\log\left(x^2-x+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^4+1)/(x^8+x^4+1),x, algorithm="giac")
```

```
[Out] 1/4*sqrt(3)*arctan(1/3*sqrt(3)*(2*x + 1)) + 1/4*sqrt(3)*arctan(1/3*sqrt(3)*
(2*x - 1)) + 1/8*sqrt(3)*log(x^2 + sqrt(3)*x + 1) - 1/8*sqrt(3)*log(x^2 - s
qrt(3)*x + 1) - 1/4*arctan(2*x + sqrt(3)) - 1/4*arctan(2*x - sqrt(3)) - 1/8
*log(x^2 + x + 1) + 1/8*log(x^2 - x + 1)
```

Mupad [B]
time = 0.19, size = 109, normalized size = 0.78

$$-\operatorname{atan}\left(\frac{54\sqrt{3}\,x}{-81+\sqrt{3}\,27\mathrm{i}}\right)\left(\frac{\sqrt{3}}{4}+\frac{1}{4}\mathrm{i}\right)+\operatorname{atan}\left(\frac{54\sqrt{3}\,x}{81+\sqrt{3}\,27\mathrm{i}}\right)\left(\frac{\sqrt{3}}{4}-\frac{1}{4}\mathrm{i}\right)+\operatorname{atan}\left(\frac{\sqrt{3}\,x\,54\mathrm{i}}{-81+\sqrt{3}\,27\mathrm{i}}\right)\left(-\frac{1}{4}+\frac{\sqrt{3}\,1\mathrm{i}}{4}\right)-\operatorname{atan}\left(\frac{\sqrt{3}\,x\,54\mathrm{i}}{81+\sqrt{3}\,27\mathrm{i}}\right)\left(\frac{1}{4}+\frac{\sqrt{3}\,1\mathrm{i}}{4}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(x^4 - 1)/(x^4 + x^8 + 1),x)
```

```
[Out] atan((54*3^(1/2)*x)/(3^(1/2)*27i + 81))*(3^(1/2)/4 - 1i/4) - atan((54*3^(1/
2)*x)/(3^(1/2)*27i - 81))*(3^(1/2)/4 + 1i/4) + atan((3^(1/2)*x*54i)/(3^(1/2
)*27i - 81))*((3^(1/2)*1i)/4 - 1/4) - atan((3^(1/2)*x*54i)/(3^(1/2)*27i + 8
1))*((3^(1/2)*1i)/4 + 1/4)
```

3.24 $\int \frac{1-x^4}{1+x^8}\,dx$

Optimal. Leaf size=347

$$-\frac{\tan^{-1}\left(\frac{\sqrt{2-\sqrt{2}}-2x}{\sqrt{2+\sqrt{2}}}\right)}{4\sqrt{2-\sqrt{2}}}+\frac{\tan^{-1}\left(\frac{\sqrt{2+\sqrt{2}}-2x}{\sqrt{2-\sqrt{2}}}\right)}{4\sqrt{2+\sqrt{2}}}+\frac{\tan^{-1}\left(\frac{\sqrt{2-\sqrt{2}}+2x}{\sqrt{2+\sqrt{2}}}\right)}{4\sqrt{2-\sqrt{2}}}-\frac{\tan^{-1}\left(\frac{\sqrt{2+\sqrt{2}}+2x}{\sqrt{2-\sqrt{2}}}\right)}{4\sqrt{2+\sqrt{2}}}$$

```
[Out] 1/16*ln(1+x^2-x*(2-2^(1/2))^(1/2))*(4-2*2^(1/2))^(1/2)-1/16*ln(1+x^2+x*(2-2
^(1/2))^(1/2))*(4-2*2^(1/2))^(1/2)-1/4*arctan((-2*x+(2-2^(1/2))^(1/2))/(2+2
^(1/2))^(1/2))/(2-2^(1/2))^(1/2)+1/4*arctan((2*x+(2-2^(1/2))^(1/2))/(2+2^(1
/2))^(1/2))/(2-2^(1/2))^(1/2)-1/16*ln(1+x^2-x*(2+2^(1/2))^(1/2))*(4+2*2^(1/
2))^(1/2)+1/16*ln(1+x^2+x*(2+2^(1/2))^(1/2))*(4+2*2^(1/2))^(1/2)+1/4*arctan
((-2*x+(2+2^(1/2))^(1/2))/(2-2^(1/2))^(1/2))/(2+2^(1/2))^(1/2)-1/4*arctan((
2*x+(2+2^(1/2))^(1/2))/(2-2^(1/2))^(1/2))/(2+2^(1/2))^(1/2)
```

Rubi [A]

time = 0.18, antiderivative size = 347, normalized size of antiderivative = 1.00, number of steps used = 19, number of rules used = 6, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {1428, 1183, 648, 632, 210, 642}

$$-\frac{\text{ArcTan}\left(\frac{\sqrt{2-\sqrt{2}}-2x}{\sqrt{2+\sqrt{2}}}\right)}{4\sqrt{2-\sqrt{2}}}+\frac{\text{ArcTan}\left(\frac{\sqrt{2+\sqrt{2}}-2x}{\sqrt{2-\sqrt{2}}}\right)}{4\sqrt{2+\sqrt{2}}}+\frac{\text{ArcTan}\left(\frac{2x+\sqrt{2-\sqrt{2}}}{\sqrt{2+\sqrt{2}}}\right)}{4\sqrt{2-\sqrt{2}}}-\frac{\text{ArcTan}\left(\frac{2x+\sqrt{2+\sqrt{2}}}{\sqrt{2-\sqrt{2}}}\right)}{4\sqrt{2+\sqrt{2}}}+\frac{1}{8}\sqrt{\frac{1}{2}\left(2-\sqrt{2}\right)}\log\left(x^2-\sqrt{2-\sqrt{2}}\,x+1\right)-\frac{1}{8}\sqrt{\frac{1}{2}\left(2-\sqrt{2}\right)}\log\left(x^2+\sqrt{2-\sqrt{2}}\,x+1\right)-\frac{1}{8}\sqrt{\frac{1}{2}\left(2+\sqrt{2}\right)}\log\left(x^2-\sqrt{2+\sqrt{2}}\,x+1\right)+\frac{1}{8}\sqrt{\frac{1}{2}\left(2+\sqrt{2}\right)}\log\left(x^2+\sqrt{2+\sqrt{2}}\,x+1\right)$$

Antiderivative was successfully verified.

```
[In] Int[(1 - x^4)/(1 + x^8),x]
```

```
[Out] -1/4*ArcTan[(Sqrt[2 - Sqrt[2]] - 2*x)/Sqrt[2 + Sqrt[2]]]/Sqrt[2 - Sqrt[2]]
+ ArcTan[(Sqrt[2 + Sqrt[2]] - 2*x)/Sqrt[2 - Sqrt[2]]]/(4*Sqrt[2 + Sqrt[2]])
 + ArcTan[(Sqrt[2 - Sqrt[2]] + 2*x)/Sqrt[2 + Sqrt[2]]]/(4*Sqrt[2 - Sqrt[2]]
) - ArcTan[(Sqrt[2 + Sqrt[2]] + 2*x)/Sqrt[2 - Sqrt[2]]]/(4*Sqrt[2 + Sqrt[2]
]) + (Sqrt[(2 - Sqrt[2])/2]*Log[1 - Sqrt[2 - Sqrt[2]]*x + x^2])/8 - (Sqrt[(
2 - Sqrt[2])/2]*Log[1 + Sqrt[2 - Sqrt[2]]*x + x^2])/8 - (Sqrt[(2 + Sqrt[2])
/2]*Log[1 - Sqrt[2 + Sqrt[2]]*x + x^2])/8 + (Sqrt[(2 + Sqrt[2])/2]*Log[1 +
Sqrt[2 + Sqrt[2]]*x + x^2])/8
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
```

```
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1183

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[a/c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Dist[1/(2*c*q*r), Int
[(d*r - (d - e*q)*x)/(q - r*x + x^2), x], x] + Dist[1/(2*c*q*r), Int[(d*r +
 (d - e*q)*x)/(q + r*x + x^2), x], x]]] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NegQ[b^2 - 4*a*c]
```

Rule 1428

```
Int[((d_) + (e_.)*(x_)^(n_))/((a_) + (c_.)*(x_)^(n2_)), x_Symbol] :> With[{
q = Rt[-2*d*e, 2]}, Dist[d/(2*a), Int[(d - q*x^(n/2))/(d - q*x^(n/2) - e*x^
n), x], x] + Dist[d/(2*a), Int[(d + q*x^(n/2))/(d + q*x^(n/2) - e*x^n), x],
 x]] /; FreeQ[{a, c, d, e}, x] && EqQ[n2, 2*n] && EqQ[c*d^2 - a*e^2, 0] &&
IGtQ[n/2, 0] && NegQ[d*e]
```

Rubi steps

$$\int \frac{1-x^4}{1+x^8}\,dx = \frac{1}{2}\int \frac{1-\sqrt{2}\,x^2}{1-\sqrt{2}\,x^2+x^4}\,dx + \frac{1}{2}\int \frac{1+\sqrt{2}\,x^2}{1+\sqrt{2}\,x^2+x^4}\,dx$$

$$= \frac{\int \frac{\sqrt{2-\sqrt{2}}-\left(1-\sqrt{2}\right)x}{1-\sqrt{2-\sqrt{2}}\,x+x^2}\,dx}{4\sqrt{2-\sqrt{2}}} + \frac{\int \frac{\sqrt{2-\sqrt{2}}+\left(1-\sqrt{2}\right)x}{1+\sqrt{2-\sqrt{2}}\,x+x^2}\,dx}{4\sqrt{2-\sqrt{2}}} + \frac{\int \frac{\sqrt{2+\sqrt{2}}-\left(1+\sqrt{2}\right)x}{1-\sqrt{2+\sqrt{2}}\,x+x^2}\,dx}{4\sqrt{2+\sqrt{2}}} +$$

$$= -\left(\frac{1}{8}\sqrt{3-2\sqrt{2}}\int \frac{1}{1-\sqrt{2+\sqrt{2}}\,x+x^2}\,dx\right) - \frac{1}{8}\sqrt{3-2\sqrt{2}}\int \frac{1}{1+\sqrt{2+\sqrt{2}}\,x+x^2}$$

$$= \frac{1}{8}\sqrt{1-\frac{1}{\sqrt{2}}}\,\log\left(1-\sqrt{2-\sqrt{2}}\,x+x^2\right) - \frac{1}{8}\sqrt{1-\frac{1}{\sqrt{2}}}\,\log\left(1+\sqrt{2-\sqrt{2}}\,x+x^2\right) -$$

$$= -\frac{1}{4}\sqrt{\frac{1}{2}\left(2+\sqrt{2}\right)}\,\tan^{-1}\left(\frac{\sqrt{2-\sqrt{2}}-2x}{\sqrt{2+\sqrt{2}}}\right) + \frac{1}{4}\sqrt{\frac{1}{2}\left(2-\sqrt{2}\right)}\,\tan^{-1}\left(\frac{\sqrt{2+\sqrt{2}}-}{\sqrt{2-\sqrt{2}}}\right.$$

Mathematica [**A**]
time = 0.10, size = 257, normalized size = 0.74

$$\frac{1}{8}\left(2\tan^{-1}\left(\cot\left(\frac{\pi}{8}\right)-x\csc\left(\frac{\pi}{8}\right)\right)\left(\cos\left(\frac{\pi}{8}\right)-\sin\left(\frac{\pi}{8}\right)\right)+\log\left(1+x^2-2x\sin\left(\frac{\pi}{8}\right)\right)\left(\cos\left(\frac{\pi}{8}\right)-\sin\left(\frac{\pi}{8}\right)\right)+2\tan^{-1}\left(\left(x+\cos\left(\frac{\pi}{8}\right)\right)\csc\left(\frac{\pi}{8}\right)\right)\left(-\cos\left(\frac{\pi}{8}\right)+\sin\left(\frac{\pi}{8}\right)\right)+\log\left(1+x^2+2x\sin\left(\frac{\pi}{8}\right)\right)\left(-\cos\left(\frac{\pi}{8}\right)+\sin\left(\frac{\pi}{8}\right)\right)+2\tan^{-1}\left(\sec\left(\frac{\pi}{8}\right)\left(x+\sin\left(\frac{\pi}{8}\right)\right)\right)\left(\cos\left(\frac{\pi}{8}\right)+\sin\left(\frac{\pi}{8}\right)\right)+2\tan^{-1}\left(x\sec\left(\frac{\pi}{8}\right)-\tan\left(\frac{\pi}{8}\right)\right)\left(\cos\left(\frac{\pi}{8}\right)+\sin\left(\frac{\pi}{8}\right)\right)-\log\left(1+x^2-2x\cos\left(\frac{\pi}{8}\right)\right)\left(\cos\left(\frac{\pi}{8}\right)+\sin\left(\frac{\pi}{8}\right)\right)+\log\left(1+x^2+2x\cos\left(\frac{\pi}{8}\right)\right)\left(\cos\left(\frac{\pi}{8}\right)+\sin\left(\frac{\pi}{8}\right)\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 - x^4)/(1 + x^8),x]
```

```
[Out] (2*ArcTan[Cot[Pi/8] - x*Csc[Pi/8]]*(Cos[Pi/8] - Sin[Pi/8]) + Log[1 + x^2 -
2*x*Sin[Pi/8]]*(Cos[Pi/8] - Sin[Pi/8]) + 2*ArcTan[(x + Cos[Pi/8])*Csc[Pi/8]
]*(-Cos[Pi/8] + Sin[Pi/8]) + Log[1 + x^2 + 2*x*Sin[Pi/8]]*(-Cos[Pi/8] + Sin
[Pi/8]) + 2*ArcTan[Sec[Pi/8]*(x + Sin[Pi/8])]*(Cos[Pi/8] + Sin[Pi/8]) + 2*A
rcTan[x*Sec[Pi/8] - Tan[Pi/8]]*(Cos[Pi/8] + Sin[Pi/8]) - Log[1 + x^2 - 2*x*
Cos[Pi/8]]*(Cos[Pi/8] + Sin[Pi/8]) + Log[1 + x^2 + 2*x*Cos[Pi/8]]*(Cos[Pi/8
] + Sin[Pi/8]))/8
```

Maple [**C**] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 0.18, size = 29, normalized size = 0.08

method	result	size

default	$\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(\_Z^8+1\right)}\frac{\left(-\_R^4+1\right)\ln\left(x-\_R\right)}{\_R^7}\right)}{8}$	29
risch	$\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(\_Z^8+1\right)}\frac{\left(-\_R^4+1\right)\ln\left(x-\_R\right)}{\_R^7}\right)}{8}$	29
meijerg	Expression too large to display	566

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-x^4+1)/(x^8+1),x,method=_RETURNVERBOSE)
```

```
[Out] 1/8*sum((-_R^4+1)/_R^7*ln(x-_R),_R=RootOf(_Z^8+1))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^4+1)/(x^8+1),x, algorithm="maxima")
```

```
[Out] -integrate((x^4 - 1)/(x^8 + 1), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 1001 vs. $2(245) = 490$.
time = 0.42, size = 1001, normalized size = 2.88

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^4+1)/(x^8+1),x, algorithm="fricas")
```

```
[Out] -1/8*(sqrt(sqrt(2) + 2) + sqrt(-sqrt(2) + 2))*arctan(-(2*x - 2*sqrt(x^2 + x
*sqrt(-sqrt(2) + 2) + 1) + sqrt(-sqrt(2) + 2))/sqrt(sqrt(2) + 2)) - 1/8*(sq
rt(sqrt(2) + 2) + sqrt(-sqrt(2) + 2))*arctan(-(2*x - 2*sqrt(x^2 - x*sqrt(-s
qrt(2) + 2) + 1) - sqrt(-sqrt(2) + 2))/sqrt(sqrt(2) + 2)) + 1/8*(sqrt(sqrt(
2) + 2) - sqrt(-sqrt(2) + 2))*arctan(-(2*x - 2*sqrt(x^2 + x*sqrt(sqrt(2) +
2) + 1) + sqrt(sqrt(2) + 2))/sqrt(-sqrt(2) + 2)) + 1/8*(sqrt(sqrt(2) + 2) -
 sqrt(-sqrt(2) + 2))*arctan(-(2*x - 2*sqrt(x^2 - x*sqrt(sqrt(2) + 2) + 1) -
 sqrt(sqrt(2) + 2))/sqrt(-sqrt(2) + 2)) - 1/8*sqrt(2)*sqrt(sqrt(2) + 2)*arc
tan(-(2*sqrt(2)*x - 2*sqrt(2*x^2 + sqrt(2)*x*sqrt(sqrt(2) + 2) - sqrt(2)*x*
sqrt(-sqrt(2) + 2) + 2) + sqrt(sqrt(2) + 2) - sqrt(-sqrt(2) + 2))/(sqrt(sqr
t(2) + 2) + sqrt(-sqrt(2) + 2))) - 1/8*sqrt(2)*sqrt(sqrt(2) + 2)*arctan(-(2
```

```
*sqrt(2)*x - 2*sqrt(2*x^2 - sqrt(2)*x*sqrt(sqrt(2) + 2) + sqrt(2)*x*sqrt(-s
qrt(2) + 2) + 2) - sqrt(sqrt(2) + 2) + sqrt(-sqrt(2) + 2))/(sqrt(sqrt(2) +
2) + sqrt(-sqrt(2) + 2))) - 1/8*sqrt(2)*sqrt(-sqrt(2) + 2)*arctan((2*sqrt(2
)*x - 2*sqrt(2*x^2 + sqrt(2)*x*sqrt(sqrt(2) + 2) + sqrt(2)*x*sqrt(-sqrt(2)
+ 2) + 2) + sqrt(sqrt(2) + 2) + sqrt(-sqrt(2) + 2))/(sqrt(sqrt(2) + 2) - sq
rt(-sqrt(2) + 2))) - 1/8*sqrt(2)*sqrt(-sqrt(2) + 2)*arctan((2*sqrt(2)*x - 2
*sqrt(2*x^2 - sqrt(2)*x*sqrt(sqrt(2) + 2) - sqrt(2)*x*sqrt(-sqrt(2) + 2) +
2) - sqrt(sqrt(2) + 2) - sqrt(-sqrt(2) + 2))/(sqrt(sqrt(2) + 2) - sqrt(-sqr
t(2) + 2))) + 1/32*(sqrt(sqrt(2) + 2) + sqrt(-sqrt(2) + 2))*log(1024*x^2 +
1024*x*sqrt(sqrt(2) + 2) + 1024) - 1/32*(sqrt(sqrt(2) + 2) + sqrt(-sqrt(2)
+ 2))*log(1024*x^2 - 1024*x*sqrt(sqrt(2) + 2) + 1024) - 1/32*(sqrt(sqrt(2)
+ 2) - sqrt(-sqrt(2) + 2))*log(1024*x^2 + 1024*x*sqrt(-sqrt(2) + 2) + 1024)
 + 1/32*(sqrt(sqrt(2) + 2) - sqrt(-sqrt(2) + 2))*log(1024*x^2 - 1024*x*sqrt
(-sqrt(2) + 2) + 1024) + 1/32*sqrt(2)*sqrt(sqrt(2) + 2)*log(256*x^2 + 128*s
qrt(2)*x*sqrt(sqrt(2) + 2) + 128*sqrt(2)*x*sqrt(-sqrt(2) + 2) + 256) - 1/32
*sqrt(2)*sqrt(-sqrt(2) + 2)*log(256*x^2 + 128*sqrt(2)*x*sqrt(sqrt(2) + 2) -
 128*sqrt(2)*x*sqrt(-sqrt(2) + 2) + 256) + 1/32*sqrt(2)*sqrt(-sqrt(2) + 2)*
log(256*x^2 - 128*sqrt(2)*x*sqrt(sqrt(2) + 2) + 128*sqrt(2)*x*sqrt(-sqrt(2)
 + 2) + 256) - 1/32*sqrt(2)*sqrt(sqrt(2) + 2)*log(256*x^2 - 128*sqrt(2)*x*s
qrt(sqrt(2) + 2) - 128*sqrt(2)*x*sqrt(-sqrt(2) + 2) + 256)
```

Sympy [**A**]
time = 1.11, size = 20, normalized size = 0.06

$$-\operatorname{RootSum}\left(1048576t^{8}+1,\left(t\mapsto t\log\left(4096t^{5}-4t+x\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x**4+1)/(x**8+1),x)
```

```
[Out] -RootSum(1048576*_t**8 + 1, Lambda(_t, _t*log(4096*_t**5 - 4*_t + x)))
```

Giac [**A**]
time = 4.77, size = 247, normalized size = 0.71

$$\frac{1}{8}\sqrt{2\sqrt{2}+4}\arctan\left(\frac{2x+\sqrt{-\sqrt{2}+2}}{\sqrt{\sqrt{2}+2}}\right)+\frac{1}{8}\sqrt{2\sqrt{2}+4}\arctan\left(\frac{2x-\sqrt{-\sqrt{2}+2}}{\sqrt{\sqrt{2}+2}}\right)-\frac{1}{8}\sqrt{-2\sqrt{2}+4}\arctan\left(\frac{2x+\sqrt{\sqrt{2}+2}}{\sqrt{-\sqrt{2}+2}}\right)-\frac{1}{8}\sqrt{-2\sqrt{2}+4}\arctan\left(\frac{2x-\sqrt{\sqrt{2}+2}}{\sqrt{-\sqrt{2}+2}}\right)+\frac{1}{16}\sqrt{2\sqrt{2}+4}\log\left(x^{2}+x\sqrt{\sqrt{2}+2}+1\right)-\frac{1}{16}\sqrt{2\sqrt{2}+4}\log\left(x^{2}-x\sqrt{\sqrt{2}+2}+1\right)-\frac{1}{16}\sqrt{-2\sqrt{2}+4}\log\left(x^{2}+x\sqrt{-\sqrt{2}+2}+1\right)+\frac{1}{16}\sqrt{-2\sqrt{2}+4}\log\left(x^{2}-x\sqrt{-\sqrt{2}+2}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^4+1)/(x^8+1),x, algorithm="giac")
```

```
[Out] 1/8*sqrt(2*sqrt(2) + 4)*arctan((2*x + sqrt(-sqrt(2) + 2))/sqrt(sqrt(2) + 2)
) + 1/8*sqrt(2*sqrt(2) + 4)*arctan((2*x - sqrt(-sqrt(2) + 2))/sqrt(sqrt(2)
+ 2)) - 1/8*sqrt(-2*sqrt(2) + 4)*arctan((2*x + sqrt(sqrt(2) + 2))/sqrt(-sqr
t(2) + 2)) - 1/8*sqrt(-2*sqrt(2) + 4)*arctan((2*x - sqrt(sqrt(2) + 2))/sqrt
(-sqrt(2) + 2)) + 1/16*sqrt(2*sqrt(2) + 4)*log(x^2 + x*sqrt(sqrt(2) + 2) +
1) - 1/16*sqrt(2*sqrt(2) + 4)*log(x^2 - x*sqrt(sqrt(2) + 2) + 1) - 1/16*sqr
```

```
t(-2*sqrt(2) + 4)*log(x^2 + x*sqrt(-sqrt(2) + 2) + 1) + 1/16*sqrt(-2*sqrt(2
) + 4)*log(x^2 - x*sqrt(-sqrt(2) + 2) + 1)
```

Mupad [B]
time = 1.96, size = 312, normalized size = 0.90

$$-\ln\left(\left(\frac{\sqrt{-2\sqrt{2}-4}}{16}-\frac{\sqrt{4-2\sqrt{2}}}{16}\right)^3\left(65536\,x-16384\sqrt{-2\sqrt{2}-4}+16384\sqrt{4-2\sqrt{2}}\right)-256\right)\left(\frac{\sqrt{-2\sqrt{2}-4}}{16}-\frac{\sqrt{4-2\sqrt{2}}}{16}\right)-\mathrm{atan}\left(-\frac{x\,1\mathrm{i}}{\sqrt{\sqrt{2}-2}}+\frac{x\,1\mathrm{i}}{\sqrt{\sqrt{2}+2}}+\frac{\sqrt{2}\,x\,1\mathrm{i}}{2\sqrt{\sqrt{2}-2}}+\frac{\sqrt{2}\,x\,1\mathrm{i}}{2\sqrt{\sqrt{2}+2}}\right)\left(\frac{\sqrt{2}\,\sqrt{\sqrt{2}-2}\,1\mathrm{i}}{8}+\frac{\sqrt{2}\,\sqrt{\sqrt{2}+2}\,1\mathrm{i}}{8}\right)+\frac{\mathrm{atan}\left(x\left(\sqrt{2}+2\right)^{3/2}\left(\frac{1}{2}+1\mathrm{i}\right)+\sqrt{2}\,x\left(\sqrt{2}+2\right)^{3/2}\left(-\frac{1}{4}-\frac{3}{4}\mathrm{i}\right)\right)\left(-2+\sqrt{2}\,(1-\mathrm{i})\right)\sqrt{\sqrt{2}+2}\,1\mathrm{i}}{8}+\frac{\mathrm{atan}\left(x\left(\sqrt{2}+2\right)^{3/2}\left(1-\frac{1}{2}\mathrm{i}\right)+\sqrt{2}\,x\left(\sqrt{2}+2\right)^{3/2}\left(-\frac{3}{4}+\frac{1}{4}\mathrm{i}\right)\right)\left(\sqrt{2}\,(1+1\mathrm{i})-2\mathrm{i}\right)\sqrt{\sqrt{2}+2}\,1\mathrm{i}}{8}+\sqrt{2}\,\ln\left(x+\left(\sqrt{2}+2\right)^{3/2}\left(-1+\frac{1}{2}\mathrm{i}\right)+\sqrt{2}\left(\sqrt{2}+2\right)^{3/2}\left(\frac{3}{4}-\frac{1}{4}\mathrm{i}\right)\right)\left(\frac{\sqrt{\sqrt{2}-2}}{16}+\frac{\sqrt{\sqrt{2}+2}}{16}\right)1\mathrm{i}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(x^4 - 1)/(x^8 + 1),x)
```

```
[Out] (atan(x*(2^(1/2) + 2)^(3/2)*(1/2 + 1i) - 2^(1/2)*x*(2^(1/2) + 2)^(3/2)*(1/4
 + 3i/4))*(2^(1/2)*(1 - 1i) - 2)*(2^(1/2) + 2)^(1/2)*1i)/8 - atan((x*1i)/(2
^(1/2) + 2)^(1/2) - (x*1i)/(2^(1/2) - 2)^(1/2) + (2^(1/2)*x*1i)/(2*(2^(1/2)
 - 2)^(1/2)) + (2^(1/2)*x*1i)/(2*(2^(1/2) + 2)^(1/2)))*((2^(1/2)*(2^(1/2) -
 2)^(1/2)*1i)/8 + (2^(1/2)*(2^(1/2) + 2)^(1/2)*1i)/8) - log(((- 2*2^(1/2) -
 4)^(1/2)/16 - (4 - 2*2^(1/2))^(1/2)/16)^3*(65536*x - 16384*(- 2*2^(1/2) -
4)^(1/2) + 16384*(4 - 2*2^(1/2))^(1/2)) - 256)*((- 2*2^(1/2) - 4)^(1/2)/16
- (4 - 2*2^(1/2))^(1/2)/16) + (atan(x*(2^(1/2) + 2)^(3/2)*(1 - 1i/2) - 2^(1
/2)*x*(2^(1/2) + 2)^(3/2)*(3/4 - 1i/4))*(2^(1/2)*(1 + 1i) - 2i)*(2^(1/2) +
2)^(1/2)*1i)/8 + 2^(1/2)*log(x - (2^(1/2) + 2)^(3/2)*(1 - 1i/2) + 2^(1/2)*(
2^(1/2) + 2)^(3/2)*(3/4 - 1i/4))*((2^(1/2) - 2)^(1/2)/16 + (2^(1/2) + 2)^(1
/2)/16)*1i
```

3.25 $\int \frac{1-x^4}{1-x^4+x^8}\,dx$

Optimal. Leaf size=355

$$-\frac{\tan^{-1}\left(\frac{\sqrt{2-\sqrt{3}}-2x}{\sqrt{2+\sqrt{3}}}\right)}{4\sqrt{3\left(2-\sqrt{3}\right)}}+\frac{\tan^{-1}\left(\frac{\sqrt{2+\sqrt{3}}-2x}{\sqrt{2-\sqrt{3}}}\right)}{4\sqrt{3\left(2+\sqrt{3}\right)}}+\frac{\tan^{-1}\left(\frac{\sqrt{2-\sqrt{3}}+2x}{\sqrt{2+\sqrt{3}}}\right)}{4\sqrt{3\left(2-\sqrt{3}\right)}}-\frac{\tan^{-1}\left(\frac{\sqrt{2+\sqrt{3}}+2x}{\sqrt{2-\sqrt{3}}}\right)}{4\sqrt{3\left(2+\sqrt{3}\right)}}$$

```
[Out] 1/8*ln(1+x^2-x*(1/2*6^(1/2)-1/2*2^(1/2)))*(1/2*2^(1/2)-1/6*6^(1/2))-1/8*ln(
1+x^2+x*(1/2*6^(1/2)-1/2*2^(1/2)))*(1/2*2^(1/2)-1/6*6^(1/2))-1/4*arctan((-2
*x+1/2*6^(1/2)-1/2*2^(1/2))/(1/2*6^(1/2)+1/2*2^(1/2)))/(3/2*2^(1/2)-1/2*6^(
1/2))+1/4*arctan((2*x+1/2*6^(1/2)-1/2*2^(1/2))/(1/2*6^(1/2)+1/2*2^(1/2)))/(
3/2*2^(1/2)-1/2*6^(1/2))-1/8*ln(1+x^2-x*(1/2*6^(1/2)+1/2*2^(1/2)))*(1/2*2^(
1/2)+1/6*6^(1/2))+1/8*ln(1+x^2+x*(1/2*6^(1/2)+1/2*2^(1/2)))*(1/2*2^(1/2)+1/
6*6^(1/2))+1/4*arctan((-2*x+1/2*6^(1/2)+1/2*2^(1/2))/(1/2*6^(1/2)-1/2*2^(1/
2)))/(3/2*2^(1/2)+1/2*6^(1/2))-1/4*arctan((2*x+1/2*6^(1/2)+1/2*2^(1/2))/(1/
2*6^(1/2)-1/2*2^(1/2)))/(3/2*2^(1/2)+1/2*6^(1/2))
```

Rubi [A]

time = 0.19, antiderivative size = 355, normalized size of antiderivative = 1.00, number of steps used = 19, number of rules used = 6, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.300$, Rules used = {1435, 1183, 648, 632, 210, 642}

$$-\frac{\text{ArcTan}\left(\frac{\sqrt{2-\sqrt{3}}-2x}{\sqrt{2+\sqrt{3}}}\right)}{4\sqrt{3\left(2-\sqrt{3}\right)}}+\frac{\text{ArcTan}\left(\frac{\sqrt{2+\sqrt{3}}-2x}{\sqrt{2-\sqrt{3}}}\right)}{4\sqrt{3\left(2+\sqrt{3}\right)}}+\frac{\text{ArcTan}\left(\frac{2x+\sqrt{2-\sqrt{3}}}{\sqrt{2+\sqrt{3}}}\right)}{4\sqrt{3\left(2-\sqrt{3}\right)}}-\frac{\text{ArcTan}\left(\frac{2x+\sqrt{2+\sqrt{3}}}{\sqrt{2-\sqrt{3}}}\right)}{4\sqrt{3\left(2+\sqrt{3}\right)}}+\frac{1}{8}\sqrt{\frac{1}{3}\left(2-\sqrt{3}\right)}\log\left(x^2-\sqrt{2-\sqrt{3}}\,x+1\right)-\frac{1}{8}\sqrt{\frac{1}{3}\left(2-\sqrt{3}\right)}\log\left(x^2+\sqrt{2-\sqrt{3}}\,x+1\right)-\frac{1}{8}\sqrt{\frac{1}{3}\left(2+\sqrt{3}\right)}\log\left(x^2-\sqrt{2+\sqrt{3}}\,x+1\right)+\frac{1}{8}\sqrt{\frac{1}{3}\left(2+\sqrt{3}\right)}\log\left(x^2+\sqrt{2+\sqrt{3}}\,x+1\right)$$

Antiderivative was successfully verified.

```
[In] Int[(1 - x^4)/(1 - x^4 + x^8),x]
```

```
[Out] -1/4*ArcTan[(Sqrt[2 - Sqrt[3]] - 2*x)/Sqrt[2 + Sqrt[3]]]/Sqrt[3*(2 - Sqrt[3
])] + ArcTan[(Sqrt[2 + Sqrt[3]] - 2*x)/Sqrt[2 - Sqrt[3]]]/(4*Sqrt[3*(2 + Sq
rt[3])]) + ArcTan[(Sqrt[2 - Sqrt[3]] + 2*x)/Sqrt[2 + Sqrt[3]]]/(4*Sqrt[3*(2
 - Sqrt[3])]) - ArcTan[(Sqrt[2 + Sqrt[3]] + 2*x)/Sqrt[2 - Sqrt[3]]]/(4*Sqrt
[3*(2 + Sqrt[3])]) + (Sqrt[(2 - Sqrt[3])/3]*Log[1 - Sqrt[2 - Sqrt[3]]*x + x
^2])/8 - (Sqrt[(2 - Sqrt[3])/3]*Log[1 + Sqrt[2 - Sqrt[3]]*x + x^2])/8 - (Sq
rt[(2 + Sqrt[3])/3]*Log[1 - Sqrt[2 + Sqrt[3]]*x + x^2])/8 + (Sqrt[(2 + Sqrt
[3])/3]*Log[1 + Sqrt[2 + Sqrt[3]]*x + x^2])/8
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1183

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[a/c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Dist[1/(2*c*q*r), Int
[(d*r - (d - e*q)*x)/(q - r*x + x^2), x], x] + Dist[1/(2*c*q*r), Int[(d*r +
 (d - e*q)*x)/(q + r*x + x^2), x], x]]] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NegQ[b^2 - 4*a*c]
```

Rule 1435

```
Int[((d_) + (e_.)*(x_)^(n_))/((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_)), x
_Symbol] :> With[{q = Rt[-2*(d/e) - b/c, 2]}, Dist[e/(2*c*q), Int[(q - 2*x^
(n/2))/Simp[d/e + q*x^(n/2) - x^n, x], x], x] + Dist[e/(2*c*q), Int[(q + 2*
x^(n/2))/Simp[d/e - q*x^(n/2) - x^n, x], x], x]] /; FreeQ[{a, b, c, d, e},
x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - a*e^2, 0] && IGtQ[
n/2, 0] &&  !GtQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\int \frac{1-x^4}{1-x^4+x^8}\,dx = -\frac{\int \frac{\sqrt{3}+2x^2}{-1-\sqrt{3}\,x^2-x^4}\,dx}{2\sqrt{3}} - \frac{\int \frac{\sqrt{3}-2x^2}{-1+\sqrt{3}\,x^2-x^4}\,dx}{2\sqrt{3}}$$

$$= \frac{\int \frac{\sqrt{3\left(2-\sqrt{3}\right)}-\left(-2+\sqrt{3}\right)x}{1-\sqrt{2-\sqrt{3}}\,x+x^2}\,dx}{4\sqrt{3\left(2-\sqrt{3}\right)}} + \frac{\int \frac{\sqrt{3\left(2-\sqrt{3}\right)}+\left(-2+\sqrt{3}\right)x}{1+\sqrt{2-\sqrt{3}}\,x+x^2}\,dx}{4\sqrt{3\left(2-\sqrt{3}\right)}} + \frac{\int \frac{\sqrt{3\left(2+\right.}}{1-\sqrt{2}}}{4\sqrt{3}}$$

$$= -\left(\frac{1}{8}\sqrt{\frac{1}{3}\left(7-4\sqrt{3}\right)}\int \frac{1}{1-\sqrt{2+\sqrt{3}}\,x+x^2}\,dx\right) - \frac{1}{8}\sqrt{\frac{1}{3}\left(7-4\sqrt{3}\right)}\int \frac{}{1+\sqrt{2}}$$

$$= \frac{1}{8}\sqrt{\frac{1}{3}\left(2-\sqrt{3}\right)}\log\left(1-\sqrt{2-\sqrt{3}}\,x+x^2\right) - \frac{1}{8}\sqrt{\frac{2}{3}-\frac{1}{\sqrt{3}}}\log\left(1+\sqrt{2-\sqrt{3}}\,x\right.$$

$$= -\frac{1}{4}\sqrt{\frac{1}{3}\left(2+\sqrt{3}\right)}\tan^{-1}\left(\frac{\sqrt{2-\sqrt{3}}-2x}{\sqrt{2+\sqrt{3}}}\right) + \frac{1}{4}\sqrt{\frac{1}{3}\left(2-\sqrt{3}\right)}\tan^{-1}\left(\frac{\sqrt{2+\sqrt{}}}{\sqrt{2-}}\right.$$

Mathematica [C] Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.
time = 0.01, size = 57, normalized size = 0.16

$$-\frac{1}{4}\text{RootSum}\left[1-\#1^4+\#1^8\&,\frac{-\log(x-\#1)+\log(x-\#1)\#1^4}{-\#1^3+2\#1^7}\&\right]$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 - x^4)/(1 - x^4 + x^8),x]
```

```
[Out] -1/4*RootSum[1 - #1^4 + #1^8 & , (-Log[x - #1] + Log[x - #1]*#1^4)/(-#1^3 +
 2*#1^7) & ]
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 0.03, size = 44, normalized size = 0.12

method	result	size

default	$\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(\_Z^{8}-\_Z^{4}+1\right)}\frac{\left(-\_R^{4}+1\right)\ln\left(x-\_R\right)}{2\_R^{7}-\_R^{3}}\right)}{4}$	44
risch	$\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(\_Z^{8}-\_Z^{4}+1\right)}\frac{\left(-\_R^{4}+1\right)\ln\left(x-\_R\right)}{2\_R^{7}-\_R^{3}}\right)}{4}$	44

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-x^4+1)/(x^8-x^4+1),x,method=_RETURNVERBOSE)
[Out] 1/4*sum((-_R^4+1)/(2*_R^7-_R^3)*ln(x-_R),_R=RootOf(_Z^8-_Z^4+1))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^4+1)/(x^8-x^4+1),x, algorithm="maxima")
[Out] -integrate((x^4 - 1)/(x^8 - x^4 + 1), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 719 vs. $2(263) = 526$.
time = 0.38, size = 719, normalized size = 2.03

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^4+1)/(x^8-x^4+1),x, algorithm="fricas")
[Out] 1/48*sqrt(6)*(sqrt(3)*sqrt(2) - 2*sqrt(2))*sqrt(sqrt(3) + 2)*log(144*x^2 +
24*sqrt(6)*(2*sqrt(3)*sqrt(2)*x - 3*sqrt(2)*x)*sqrt(sqrt(3) + 2) + 144) - 1
/48*sqrt(6)*(sqrt(3)*sqrt(2) - 2*sqrt(2))*sqrt(sqrt(3) + 2)*log(144*x^2 - 2
4*sqrt(6)*(2*sqrt(3)*sqrt(2)*x - 3*sqrt(2)*x)*sqrt(sqrt(3) + 2) + 144) + 1/
96*sqrt(6)*(sqrt(3)*sqrt(2) + 2*sqrt(2))*sqrt(-4*sqrt(3) + 8)*log(144*x^2 +
 12*sqrt(6)*(2*sqrt(3)*sqrt(2)*x + 3*sqrt(2)*x)*sqrt(-4*sqrt(3) + 8) + 144)
 - 1/96*sqrt(6)*(sqrt(3)*sqrt(2) + 2*sqrt(2))*sqrt(-4*sqrt(3) + 8)*log(144*
x^2 - 12*sqrt(6)*(2*sqrt(3)*sqrt(2)*x + 3*sqrt(2)*x)*sqrt(-4*sqrt(3) + 8) +
 144) + 1/24*sqrt(6)*sqrt(2)*sqrt(-4*sqrt(3) + 8)*arctan(1/36*sqrt(6)*sqrt(
3)*sqrt(12*x^2 + sqrt(6)*(2*sqrt(3)*sqrt(2)*x + 3*sqrt(2)*x)*sqrt(-4*sqrt(3
) + 8) + 12)*(2*sqrt(3)*sqrt(2) + 3*sqrt(2))*sqrt(-4*sqrt(3) + 8) - 1/6*sqr
t(6)*(2*sqrt(3)*sqrt(2)*x + 3*sqrt(2)*x)*sqrt(-4*sqrt(3) + 8) - sqrt(3) - 2
```

```
) + 1/24*sqrt(6)*sqrt(2)*sqrt(-4*sqrt(3) + 8)*arctan(1/36*sqrt(6)*sqrt(3)*s
qrt(12*x^2 - sqrt(6)*(2*sqrt(3)*sqrt(2)*x + 3*sqrt(2)*x)*sqrt(-4*sqrt(3) +
8) + 12)*(2*sqrt(3)*sqrt(2) + 3*sqrt(2))*sqrt(-4*sqrt(3) + 8) - 1/6*sqrt(6)
*(2*sqrt(3)*sqrt(2)*x + 3*sqrt(2)*x)*sqrt(-4*sqrt(3) + 8) + sqrt(3) + 2) -
1/12*sqrt(6)*sqrt(2)*sqrt(sqrt(3) + 2)*arctan(-1/3*sqrt(6)*(2*sqrt(3)*sqrt(
2)*x - 3*sqrt(2)*x)*sqrt(sqrt(3) + 2) + 1/3*sqrt(6*x^2 + sqrt(6)*(2*sqrt(3)
*sqrt(2)*x - 3*sqrt(2)*x)*sqrt(sqrt(3) + 2) + 6)*(2*sqrt(3)*sqrt(2) - 3*sqr
t(2))*sqrt(sqrt(3) + 2) + sqrt(3) - 2) - 1/12*sqrt(6)*sqrt(2)*sqrt(sqrt(3)
+ 2)*arctan(-1/3*sqrt(6)*(2*sqrt(3)*sqrt(2)*x - 3*sqrt(2)*x)*sqrt(sqrt(3) +
 2) + 1/3*sqrt(6*x^2 - sqrt(6)*(2*sqrt(3)*sqrt(2)*x - 3*sqrt(2)*x)*sqrt(sqr
t(3) + 2) + 6)*(2*sqrt(3)*sqrt(2) - 3*sqrt(2))*sqrt(sqrt(3) + 2) - sqrt(3)
+ 2)
```

Sympy [**A**]
time = 1.36, size = 26, normalized size = 0.07

$$-\operatorname{RootSum}\left(5308416t^8 - 2304t^4 + 1, \left(t \mapsto t\log\left(9216t^5 - 8t + x\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x**4+1)/(x**8-x**4+1),x)
```

```
[Out] -RootSum(5308416*_t**8 - 2304*_t**4 + 1, Lambda(_t, _t*log(9216*_t**5 - 8*_
t + x)))
```

Giac [**A**]
time = 4.41, size = 253, normalized size = 0.71

$$\frac{1}{24}\left(\sqrt{6}+3\sqrt{2}\right)\arctan\left(\frac{4x+\sqrt{6}-\sqrt{2}}{\sqrt{6}+\sqrt{2}}\right)+\frac{1}{24}\left(\sqrt{6}+3\sqrt{2}\right)\arctan\left(\frac{4x-\sqrt{6}+\sqrt{2}}{\sqrt{6}+\sqrt{2}}\right)+\frac{1}{24}\left(\sqrt{6}-3\sqrt{2}\right)\arctan\left(\frac{4x+\sqrt{6}+\sqrt{2}}{\sqrt{6}-\sqrt{2}}\right)+\frac{1}{24}\left(\sqrt{6}-3\sqrt{2}\right)\arctan\left(\frac{4x-\sqrt{6}-\sqrt{2}}{\sqrt{6}-\sqrt{2}}\right)+\frac{1}{48}\left(\sqrt{6}+3\sqrt{2}\right)\log\left(x^2+\frac{1}{2}x\left(\sqrt{6}+\sqrt{2}\right)+1\right)-\frac{1}{48}\left(\sqrt{6}+3\sqrt{2}\right)\log\left(x^2-\frac{1}{2}x\left(\sqrt{6}+\sqrt{2}\right)+1\right)+\frac{1}{48}\left(\sqrt{6}-3\sqrt{2}\right)\log\left(x^2+\frac{1}{2}x\left(\sqrt{6}-\sqrt{2}\right)+1\right)-\frac{1}{48}\left(\sqrt{6}-3\sqrt{2}\right)\log\left(x^2-\frac{1}{2}x\left(\sqrt{6}-\sqrt{2}\right)+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^4+1)/(x^8-x^4+1),x, algorithm="giac")
```

```
[Out] 1/24*(sqrt(6) + 3*sqrt(2))*arctan((4*x + sqrt(6) - sqrt(2))/(sqrt(6) + sqrt
(2))) + 1/24*(sqrt(6) + 3*sqrt(2))*arctan((4*x - sqrt(6) + sqrt(2))/(sqrt(6
) + sqrt(2))) + 1/24*(sqrt(6) - 3*sqrt(2))*arctan((4*x + sqrt(6) + sqrt(2))
/(sqrt(6) - sqrt(2))) + 1/24*(sqrt(6) - 3*sqrt(2))*arctan((4*x - sqrt(6) -
sqrt(2))/(sqrt(6) - sqrt(2))) + 1/48*(sqrt(6) + 3*sqrt(2))*log(x^2 + 1/2*x*
(sqrt(6) + sqrt(2)) + 1) - 1/48*(sqrt(6) + 3*sqrt(2))*log(x^2 - 1/2*x*(sqrt
(6) + sqrt(2)) + 1) + 1/48*(sqrt(6) - 3*sqrt(2))*log(x^2 + 1/2*x*(sqrt(6) -
 sqrt(2)) + 1) - 1/48*(sqrt(6) - 3*sqrt(2))*log(x^2 - 1/2*x*(sqrt(6) - sqrt
(2)) + 1)
```

Mupad [**B**]
time = 1.67, size = 208, normalized size = 0.59

$$-\frac{\sqrt{3}\,\mathrm{atan}\left(\frac{x}{{\left(8-\sqrt{3}\,8\mathrm{i}\right)}^{1/4}}+\frac{\sqrt{3}\,x\,1\mathrm{i}}{{\left(8-\sqrt{3}\,8\mathrm{i}\right)}^{1/4}}\right)\,{\left(8-\sqrt{3}\,8\mathrm{i}\right)}^{1/4}\,1\mathrm{i}}{12}-\frac{\sqrt{3}\,\mathrm{atan}\left(\frac{x\,1\mathrm{i}}{{\left(8-\sqrt{3}\,8\mathrm{i}\right)}^{1/4}}-\frac{\sqrt{3}\,x}{{\left(8-\sqrt{3}\,8\mathrm{i}\right)}^{1/4}}\right)\,{\left(8-\sqrt{3}\,8\mathrm{i}\right)}^{1/4}}{12}+\frac{2^{3/4}\,\sqrt{3}\,\mathrm{atan}\left(\frac{2^{1/4}\,x}{2\,{\left(1+\sqrt{3}\,1\mathrm{i}\right)}^{1/4}}-\frac{2^{1/4}\,\sqrt{3}\,x\,1\mathrm{i}}{2\,{\left(1+\sqrt{3}\,1\mathrm{i}\right)}^{1/4}}\right)\,{\left(1+\sqrt{3}\,1\mathrm{i}\right)}^{1/4}\,1\mathrm{i}}{12}+\frac{2^{3/4}\,\sqrt{3}\,\mathrm{atan}\left(\frac{2^{1/4}\,x\,1\mathrm{i}}{2\,{\left(1+\sqrt{3}\,1\mathrm{i}\right)}^{1/4}}+\frac{2^{1/4}\,\sqrt{3}\,x}{2\,{\left(1+\sqrt{3}\,1\mathrm{i}\right)}^{1/4}}\right)\,{\left(1+\sqrt{3}\,1\mathrm{i}\right)}^{1/4}}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(x^4 - 1)/(x^8 - x^4 + 1),x)
```

```
[Out] (2^(3/4)*3^(1/2)*atan((2^(1/4)*x)/(2*(3^(1/2)*1i + 1)^(1/4)) - (2^(1/4)*3^(
1/2)*x*1i)/(2*(3^(1/2)*1i + 1)^(1/4)))*(3^(1/2)*1i + 1)^(1/4)*1i)/12 - (3^(
1/2)*atan((x*1i)/(8 - 3^(1/2)*8i)^(1/4) - (3^(1/2)*x)/(8 - 3^(1/2)*8i)^(1/4
))*(8 - 3^(1/2)*8i)^(1/4))/12 - (3^(1/2)*atan(x/(8 - 3^(1/2)*8i)^(1/4) + (3
^(1/2)*x*1i)/(8 - 3^(1/2)*8i)^(1/4))*(8 - 3^(1/2)*8i)^(1/4)*1i)/12 + (2^(3/
4)*3^(1/2)*atan((2^(1/4)*x*1i)/(2*(3^(1/2)*1i + 1)^(1/4)) + (2^(1/4)*3^(1/2
)*x)/(2*(3^(1/2)*1i + 1)^(1/4)))*(3^(1/2)*1i + 1)^(1/4))/12
```

3.26 $\int \frac{1-x^4}{1-2x^4+x^8} \, dx$

Optimal. Leaf size=13

$$\frac{1}{2}\tan^{-1}(x)+\frac{1}{2}\tanh^{-1}(x)$$

```
[Out] 1/2*arctan(x)+1/2*arctanh(x)
```

Rubi [A]
time = 0.00, antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {28, 21, 218, 212, 209}

$$\frac{\text{ArcTan}(x)}{2}+\frac{1}{2}\tanh^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Int[(1 - x^4)/(1 - 2*x^4 + x^8),x]
```

```
[Out] ArcTan[x]/2 + ArcTanh[x]/2
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 218

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x], x]
+ Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a/b
, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{1-x^4}{1-2x^4+x^8}\,dx &= \int \frac{1-x^4}{(-1+x^4)^2}\,dx\\
&= -\int \frac{1}{-1+x^4}\,dx\\
&= \frac{1}{2}\int \frac{1}{1-x^2}\,dx + \frac{1}{2}\int \frac{1}{1+x^2}\,dx\\
&= \frac{1}{2}\tan^{-1}(x) + \frac{1}{2}\tanh^{-1}(x)
\end{aligned}$$

Mathematica [A]
time = 0.00, size = 25, normalized size = 1.92

$$\frac{1}{2}\tan^{-1}(x) - \frac{1}{4}\log(1-x) + \frac{1}{4}\log(1+x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 - x^4)/(1 - 2*x^4 + x^8),x]
[Out] ArcTan[x]/2 - Log[1 - x]/4 + Log[1 + x]/4
```

Maple [A]
time = 0.02, size = 10, normalized size = 0.77

method	result	size
default	$\frac{\arctan(x)}{2}+\frac{\operatorname{arctanh}(x)}{2}$	10
risch	$\frac{\arctan(x)}{2}-\frac{\ln(-1+x)}{4}+\frac{\ln(1+x)}{4}$	18

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-x^4+1)/(x^8-2*x^4+1),x,method=_RETURNVERBOSE)
[Out] 1/2*arctan(x)+1/2*arctanh(x)
```

Maxima [A]
time = 0.48, size = 17, normalized size = 1.31

$$\frac{1}{2}\arctan\left(x\right) + \frac{1}{4}\log\left(x+1\right) - \frac{1}{4}\log\left(x-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^4+1)/(x^8-2*x^4+1),x, algorithm="maxima")
```

```
[Out] 1/2*arctan(x) + 1/4*log(x + 1) - 1/4*log(x - 1)
```

Fricas [A]
time = 0.33, size = 17, normalized size = 1.31

$$\frac{1}{2}\arctan\left(x\right)+\frac{1}{4}\log\left(x+1\right)-\frac{1}{4}\log\left(x-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^4+1)/(x^8-2*x^4+1),x, algorithm="fricas")
```

```
[Out] 1/2*arctan(x) + 1/4*log(x + 1) - 1/4*log(x - 1)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 17 vs. $2(8) = 16$.
time = 0.05, size = 17, normalized size = 1.31

$$-\frac{\log\left(x-1\right)}{4}+\frac{\log\left(x+1\right)}{4}+\frac{\operatorname{atan}\left(x\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x**4+1)/(x**8-2*x**4+1),x)
```

```
[Out] -log(x - 1)/4 + log(x + 1)/4 + atan(x)/2
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 19 vs. $2(9) = 18$.
time = 5.84, size = 19, normalized size = 1.46

$$\frac{1}{2}\arctan\left(x\right)+\frac{1}{4}\log\left(|x+1|\right)-\frac{1}{4}\log\left(|x-1|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^4+1)/(x^8-2*x^4+1),x, algorithm="giac")
```

```
[Out] 1/2*arctan(x) + 1/4*log(abs(x + 1)) - 1/4*log(abs(x - 1))
```

Mupad [B]
time = 0.02, size = 9, normalized size = 0.69

$$\frac{\operatorname{atan}(x)}{2}+\frac{\operatorname{atanh}(x)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(x^4 - 1)/(x^8 - 2*x^4 + 1),x)
```

```
[Out] atan(x)/2 + atanh(x)/2
```

3.27 $\int \frac{1-x^4}{1-3x^4+x^8}\,dx$

Optimal. Leaf size=129

$$\frac{\tan^{-1}\left(\sqrt{\frac{2}{-1+\sqrt{5}}}\,x\right)}{\sqrt{10\left(-1+\sqrt{5}\right)}}+\frac{\tan^{-1}\left(\sqrt{\frac{2}{1+\sqrt{5}}}\,x\right)}{\sqrt{10\left(1+\sqrt{5}\right)}}+\frac{\tanh^{-1}\left(\sqrt{\frac{2}{-1+\sqrt{5}}}\,x\right)}{\sqrt{10\left(-1+\sqrt{5}\right)}}+\frac{\tanh^{-1}\left(\sqrt{\frac{2}{1+\sqrt{5}}}\,x\right)}{\sqrt{10\left(1+\sqrt{5}\right)}}$$

```
[Out] arctan(x*2^(1/2)/(5^(1/2)-1)^(1/2))/(-10+10*5^(1/2))^(1/2)+arctanh(x*2^(1/2
)/(5^(1/2)-1)^(1/2))/(-10+10*5^(1/2))^(1/2)+arctan(x*2^(1/2)/(5^(1/2)+1)^(1
/2))/(10+10*5^(1/2))^(1/2)+arctanh(x*2^(1/2)/(5^(1/2)+1)^(1/2))/(10+10*5^(1
/2))^(1/2)
```

Rubi [A]
time = 0.08, antiderivative size = 129, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 4, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {1433, 1107, 213, 209}

$$\frac{\text{ArcTan}\left(\sqrt{\frac{2}{\sqrt{5}-1}}\,x\right)}{\sqrt{10\left(\sqrt{5}-1\right)}}+\frac{\text{ArcTan}\left(\sqrt{\frac{2}{1+\sqrt{5}}}\,x\right)}{\sqrt{10\left(1+\sqrt{5}\right)}}+\frac{\tanh^{-1}\left(\sqrt{\frac{2}{\sqrt{5}-1}}\,x\right)}{\sqrt{10\left(\sqrt{5}-1\right)}}+\frac{\tanh^{-1}\left(\sqrt{\frac{2}{1+\sqrt{5}}}\,x\right)}{\sqrt{10\left(1+\sqrt{5}\right)}}$$

Antiderivative was successfully verified.

```
[In] Int[(1 - x^4)/(1 - 3*x^4 + x^8),x]
```

```
[Out] ArcTan[Sqrt[2/(-1 + Sqrt[5])]*x]/Sqrt[10*(-1 + Sqrt[5])] + ArcTan[Sqrt[2/(1
 + Sqrt[5])]*x]/Sqrt[10*(1 + Sqrt[5])] + ArcTanh[Sqrt[2/(-1 + Sqrt[5])]*x]/
Sqrt[10*(-1 + Sqrt[5])] + ArcTanh[Sqrt[2/(1 + Sqrt[5])]*x]/Sqrt[10*(1 + Sqr
t[5])]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 1107

```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(-1), x_Symbol] :> With[{q = Rt[b^
2 - 4*a*c, 2]}, Dist[c/q, Int[1/(b/2 - q/2 + c*x^2), x], x] - Dist[c/q, Int
[1/(b/2 + q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c,
0] && PosQ[b^2 - 4*a*c]
```

Rule 1433

```
Int[((d_) + (e_.)*(x_)^(n_))/((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_)), x
_Symbol] :> With[{q = Rt[2*(d/e) - b/c, 2]}, Dist[e/(2*c), Int[1/Simp[d/e +
 q*x^(n/2) + x^n, x], x], x] + Dist[e/(2*c), Int[1/Simp[d/e - q*x^(n/2) + x
^n, x], x], x]] /; FreeQ[{a, b, c, d, e}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4
*a*c, 0] && EqQ[c*d^2 - a*e^2, 0] && IGtQ[n/2, 0] && (GtQ[2*(d/e) - b/c, 0]
 || ( !LtQ[2*(d/e) - b/c, 0] && EqQ[d, e*Rt[a/c, 2]]))
```

Rubi steps

$$\begin{aligned}
\int \frac{1-x^4}{1-3x^4+x^8}\,dx &= -\left(\frac{1}{2}\int \frac{1}{-1-x^2+x^4}\,dx\right)-\frac{1}{2}\int \frac{1}{-1+x^2+x^4}\,dx \\
&= -\frac{\int \frac{1}{-\frac{1}{2}-\frac{\sqrt{5}}{2}+x^2}\,dx}{2\sqrt{5}}-\frac{\int \frac{1}{\frac{1}{2}-\frac{\sqrt{5}}{2}+x^2}\,dx}{2\sqrt{5}}+\frac{\int \frac{1}{-\frac{1}{2}+\frac{\sqrt{5}}{2}+x^2}\,dx}{2\sqrt{5}}+\frac{\int \frac{1}{\frac{1}{2}+\frac{\sqrt{5}}{2}+x^2}\,dx}{2\sqrt{5}} \\
&= \frac{\tan^{-1}\left(\sqrt{\frac{2}{-1+\sqrt{5}}}\,x\right)}{\sqrt{10\left(-1+\sqrt{5}\right)}}+\frac{\tan^{-1}\left(\sqrt{\frac{2}{1+\sqrt{5}}}\,x\right)}{\sqrt{10\left(1+\sqrt{5}\right)}}+\frac{\tanh^{-1}\left(\sqrt{\frac{2}{-1+\sqrt{5}}}\,x\right)}{\sqrt{10\left(-1+\sqrt{5}\right)}}+ \cdots
\end{aligned}$$

Mathematica [A]
time = 0.05, size = 129, normalized size = 1.00

$$\frac{\tan^{-1}\left(\sqrt{\frac{2}{-1+\sqrt{5}}}\,x\right)}{\sqrt{10\left(-1+\sqrt{5}\right)}}+\frac{\tan^{-1}\left(\sqrt{\frac{2}{1+\sqrt{5}}}\,x\right)}{\sqrt{10\left(1+\sqrt{5}\right)}}+\frac{\tanh^{-1}\left(\sqrt{\frac{2}{-1+\sqrt{5}}}\,x\right)}{\sqrt{10\left(-1+\sqrt{5}\right)}}+\frac{\tanh^{-1}\left(\sqrt{\frac{2}{1+\sqrt{5}}}\,x\right)}{\sqrt{10\left(1+\sqrt{5}\right)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 - x^4)/(1 - 3*x^4 + x^8),x]
```

```
[Out] ArcTan[Sqrt[2/(-1 + Sqrt[5])]*x]/Sqrt[10*(-1 + Sqrt[5])] + ArcTan[Sqrt[2/(1
 + Sqrt[5])]*x]/Sqrt[10*(1 + Sqrt[5])] + ArcTanh[Sqrt[2/(-1 + Sqrt[5])]*x]/
Sqrt[10*(-1 + Sqrt[5])] + ArcTanh[Sqrt[2/(1 + Sqrt[5])]*x]/Sqrt[10*(1 + Sqr
t[5])]
```

Maple [A]
time = 0.05, size = 110, normalized size = 0.85

method	result
risch	$\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(25\_Z^4-5\_Z^2-1\right)}\_R\ln\left(-5\_R^3+3\_R+x\right)\right)}{4}+\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(25\_Z^4+5\_Z^2-1\right)}\_R\ln\left(5\_R^3+3\_R+x\right)\right)}{4}$
default	$\frac{\sqrt{5}\,\mathrm{arctanh}\left(\frac{2x}{\sqrt{2\sqrt{5}-2}}\right)}{5\sqrt{2\sqrt{5}-2}}+\frac{\sqrt{5}\,\arctan\left(\frac{2x}{\sqrt{2\sqrt{5}+2}}\right)}{5\sqrt{2\sqrt{5}+2}}+\frac{\sqrt{5}\,\mathrm{arctanh}\left(\frac{2x}{\sqrt{2\sqrt{5}+2}}\right)}{5\sqrt{2\sqrt{5}+2}}+\frac{\sqrt{5}\,\mathrm{arc}}{5}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-x^4+1)/(x^8-3*x^4+1),x,method=_RETURNVERBOSE)
```

```
[Out] 1/5*5^(1/2)/(2*5^(1/2)-2)^(1/2)*arctanh(2*x/(2*5^(1/2)-2)^(1/2))+1/5*5^(1/2
)/(2*5^(1/2)+2)^(1/2)*arctan(2*x/(2*5^(1/2)+2)^(1/2))+1/5*5^(1/2)/(2*5^(1/2
)+2)^(1/2)*arctanh(2*x/(2*5^(1/2)+2)^(1/2))+1/5*5^(1/2)/(2*5^(1/2)-2)^(1/2)
*arctan(2*x/(2*5^(1/2)-2)^(1/2))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^4+1)/(x^8-3*x^4+1),x, algorithm="maxima")
```

```
[Out] -integrate((x^4 - 1)/(x^8 - 3*x^4 + 1), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 255 vs. 2(93) = 186.
time = 0.39, size = 255, normalized size = 1.98

$$-\frac{1}{10}\sqrt{10}\sqrt{\sqrt{5}+1}\arctan\left(\frac{1}{20}\sqrt{10}\sqrt{5}\sqrt{2}\sqrt{2x^2+\sqrt{5}-1}\sqrt{\sqrt{5}+1}-\frac{1}{10}\sqrt{10}\sqrt{5}x\sqrt{\sqrt{5}+1}\right)-\frac{1}{10}\sqrt{10}\sqrt{\sqrt{5}-1}\arctan\left(\frac{1}{20}\sqrt{10}\sqrt{5}\sqrt{2}\sqrt{2x^2+\sqrt{5}+1}\sqrt{\sqrt{5}-1}-\frac{1}{10}\sqrt{10}\sqrt{5}x\sqrt{\sqrt{5}-1}\right)+\frac{1}{40}\sqrt{10}\sqrt{\sqrt{5}-1}\log\left(\sqrt{10}\left(\sqrt{5}+5\right)\sqrt{\sqrt{5}-1}+20x\right)-\frac{1}{40}\sqrt{10}\sqrt{\sqrt{5}-1}\log\left(-\sqrt{10}\left(\sqrt{5}+5\right)\sqrt{\sqrt{5}-1}+20x\right)-\frac{1}{40}\sqrt{10}\sqrt{\sqrt{5}+1}\log\left(\sqrt{10}\sqrt{\sqrt{5}+1}\left(\sqrt{5}-5\right)+20x\right)+\frac{1}{40}\sqrt{10}\sqrt{\sqrt{5}+1}\log\left(-\sqrt{10}\sqrt{\sqrt{5}+1}\left(\sqrt{5}-5\right)+20x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^4+1)/(x^8-3*x^4+1),x, algorithm="fricas")
```

```
[Out] -1/10*sqrt(10)*sqrt(sqrt(5) + 1)*arctan(1/20*sqrt(10)*sqrt(5)*sqrt(2)*sqrt(
2*x^2 + sqrt(5) - 1)*sqrt(sqrt(5) + 1) - 1/10*sqrt(10)*sqrt(5)*x*sqrt(sqrt(
5) + 1)) - 1/10*sqrt(10)*sqrt(sqrt(5) - 1)*arctan(1/20*sqrt(10)*sqrt(5)*sqr
t(2)*sqrt(2*x^2 + sqrt(5) + 1)*sqrt(sqrt(5) - 1) - 1/10*sqrt(10)*sqrt(5)*x*
sqrt(sqrt(5) - 1)) + 1/40*sqrt(10)*sqrt(sqrt(5) - 1)*log(sqrt(10)*(sqrt(5)
+ 5)*sqrt(sqrt(5) - 1) + 20*x) - 1/40*sqrt(10)*sqrt(sqrt(5) - 1)*log(-sqrt(
```

```
10)*(sqrt(5) + 5)*sqrt(sqrt(5) - 1) + 20*x) - 1/40*sqrt(10)*sqrt(sqrt(5) +
1)*log(sqrt(10)*sqrt(sqrt(5) + 1)*(sqrt(5) - 5) + 20*x) + 1/40*sqrt(10)*sqr
t(sqrt(5) + 1)*log(-sqrt(10)*sqrt(sqrt(5) + 1)*(sqrt(5) - 5) + 20*x)
```

Sympy [A]
time = 0.70, size = 51, normalized size = 0.40

$$-\operatorname{RootSum}\left(6400t^4-80t^2-1,\left(t\mapsto t\log\left(25600t^5-16t+x\right)\right)\right)-\operatorname{RootSum}\left(6400t^4+80t^2-1,\left(t\mapsto t\log\left(25600t^5-16t+x\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x**4+1)/(x**8-3*x**4+1),x)
```

```
[Out] -RootSum(6400*_t**4 - 80*_t**2 - 1, Lambda(_t, _t*log(25600*_t**5 - 16*_t +
 x))) - RootSum(6400*_t**4 + 80*_t**2 - 1, Lambda(_t, _t*log(25600*_t**5 -
16*_t + x)))
```

Giac [A]
time = 3.44, size = 147, normalized size = 1.14

$$\frac{1}{20}\sqrt{10\sqrt{5}-10}\arctan\left(\frac{x}{\sqrt{\frac{1}{2}\sqrt{5}+\frac{1}{2}}}\right)+\frac{1}{20}\sqrt{10\sqrt{5}+10}\arctan\left(\frac{x}{\sqrt{\frac{1}{2}\sqrt{5}-\frac{1}{2}}}\right)+\frac{1}{40}\sqrt{10\sqrt{5}-10}\log\left(\left|x+\sqrt{\frac{1}{2}\sqrt{5}+\frac{1}{2}}\right|\right)-\frac{1}{40}\sqrt{10\sqrt{5}-10}\log\left(\left|x-\sqrt{\frac{1}{2}\sqrt{5}+\frac{1}{2}}\right|\right)+\frac{1}{40}\sqrt{10\sqrt{5}+10}\log\left(\left|x+\sqrt{\frac{1}{2}\sqrt{5}-\frac{1}{2}}\right|\right)-\frac{1}{40}\sqrt{10\sqrt{5}+10}\log\left(\left|x-\sqrt{\frac{1}{2}\sqrt{5}-\frac{1}{2}}\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^4+1)/(x^8-3*x^4+1),x, algorithm="giac")
```

```
[Out] 1/20*sqrt(10*sqrt(5) - 10)*arctan(x/sqrt(1/2*sqrt(5) + 1/2)) + 1/20*sqrt(10
*sqrt(5) + 10)*arctan(x/sqrt(1/2*sqrt(5) - 1/2)) + 1/40*sqrt(10*sqrt(5) - 1
0)*log(abs(x + sqrt(1/2*sqrt(5) + 1/2))) - 1/40*sqrt(10*sqrt(5) - 10)*log(a
bs(x - sqrt(1/2*sqrt(5) + 1/2))) + 1/40*sqrt(10*sqrt(5) + 10)*log(abs(x + s
qrt(1/2*sqrt(5) - 1/2))) - 1/40*sqrt(10*sqrt(5) + 10)*log(abs(x - sqrt(1/2*
sqrt(5) - 1/2)))
```

Mupad [B]
time = 1.71, size = 269, normalized size = 2.09

$$-\frac{\sqrt{10}\,\mathrm{atan}\left(\frac{\sqrt{10}\,x\,\sqrt{\sqrt{5}-1}\,3{}\mathrm{i}}{2\,\left(3\,\sqrt{5}-7\right)}-\frac{\sqrt{5}\,\sqrt{10}\,x\,\sqrt{\sqrt{5}-1}\,7{}\mathrm{i}}{10\,\left(3\,\sqrt{5}-7\right)}\right)\,\sqrt{\sqrt{5}-1}\,1{}\mathrm{i}}{20}-\frac{\sqrt{10}\,\mathrm{atan}\left(\frac{\sqrt{10}\,x\,\sqrt{\sqrt{5}+1}\,3{}\mathrm{i}}{2\,\left(3\,\sqrt{5}+7\right)}+\frac{\sqrt{5}\,\sqrt{10}\,x\,\sqrt{\sqrt{5}+1}\,7{}\mathrm{i}}{10\,\left(3\,\sqrt{5}+7\right)}\right)\,\sqrt{\sqrt{5}+1}\,1{}\mathrm{i}}{20}+\frac{\sqrt{10}\,\mathrm{atan}\left(\frac{\sqrt{10}\,x\,\sqrt{1-\sqrt{5}}\,3{}\mathrm{i}}{2\,\left(3\,\sqrt{5}-7\right)}-\frac{\sqrt{5}\,\sqrt{10}\,x\,\sqrt{1-\sqrt{5}}\,7{}\mathrm{i}}{10\,\left(3\,\sqrt{5}-7\right)}\right)\,\sqrt{1-\sqrt{5}}\,1{}\mathrm{i}}{20}+\frac{\sqrt{10}\,\mathrm{atan}\left(\frac{\sqrt{10}\,x\,\sqrt{-\sqrt{5}-1}\,3{}\mathrm{i}}{2\,\left(3\,\sqrt{5}+7\right)}+\frac{\sqrt{5}\,\sqrt{10}\,x\,\sqrt{-\sqrt{5}-1}\,7{}\mathrm{i}}{10\,\left(3\,\sqrt{5}+7\right)}\right)\,\sqrt{-\sqrt{5}-1}\,1{}\mathrm{i}}{20}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(x^4 - 1)/(x^8 - 3*x^4 + 1),x)
```

```
[Out] (10^(1/2)*atan((10^(1/2)*x*(1 - 5^(1/2))^(1/2)*3i)/(2*(3*5^(1/2) - 7)) - (5
^(1/2)*10^(1/2)*x*(1 - 5^(1/2))^(1/2)*7i)/(10*(3*5^(1/2) - 7)))*(1 - 5^(1/2
))^(1/2)*1i)/20 - (10^(1/2)*atan((10^(1/2)*x*(5^(1/2) + 1)^(1/2)*3i)/(2*(3*
5^(1/2) + 7)) + (5^(1/2)*10^(1/2)*x*(5^(1/2) + 1)^(1/2)*7i)/(10*(3*5^(1/2)
+ 7)))*(5^(1/2) + 1)^(1/2)*1i)/20 - (10^(1/2)*atan((10^(1/2)*x*(5^(1/2) - 1
)^(1/2)*3i)/(2*(3*5^(1/2) - 7)) - (5^(1/2)*10^(1/2)*x*(5^(1/2) - 1)^(1/2)*7
i)/(10*(3*5^(1/2) - 7)))*(5^(1/2) - 1)^(1/2)*1i)/20 + (10^(1/2)*atan((10^(1
/2)*x*(- 5^(1/2) - 1)^(1/2)*3i)/(2*(3*5^(1/2) + 7)) + (5^(1/2)*10^(1/2)*x*(
- 5^(1/2) - 1)^(1/2)*7i)/(10*(3*5^(1/2) + 7)))*(- 5^(1/2) - 1)^(1/2)*1i)/20
```

3.28 $\int \frac{1-x^4}{1-4x^4+x^8}\,dx$

Optimal. Leaf size=165

$$\frac{\tan^{-1}\left(\frac{\sqrt[4]{2}\,x}{\sqrt{-1+\sqrt{3}}}\right)}{2\sqrt[4]{2}\sqrt{3\left(-1+\sqrt{3}\right)}}+\frac{\tan^{-1}\left(\frac{\sqrt[4]{2}\,x}{\sqrt{1+\sqrt{3}}}\right)}{2\sqrt[4]{2}\sqrt{3\left(1+\sqrt{3}\right)}}+\frac{\tanh^{-1}\left(\frac{\sqrt[4]{2}\,x}{\sqrt{-1+\sqrt{3}}}\right)}{2\sqrt[4]{2}\sqrt{3\left(-1+\sqrt{3}\right)}}+\frac{\tanh^{-1}\left(\frac{\sqrt[4]{2}\,x}{\sqrt{1+\sqrt{3}}}\right)}{2\sqrt[4]{2}\sqrt{3\left(1+\sqrt{3}\right)}}$$

```
[Out] 1/4*arctan(2^(1/4)*x/(3^(1/2)-1)^(1/2))*2^(3/4)/(-3+3*3^(1/2))^(1/2)+1/4*ar
ctanh(2^(1/4)*x/(3^(1/2)-1)^(1/2))*2^(3/4)/(-3+3*3^(1/2))^(1/2)+1/4*arctan(
2^(1/4)*x/(1+3^(1/2))^(1/2))*2^(3/4)/(3+3*3^(1/2))^(1/2)+1/4*arctanh(2^(1/4
)*x/(1+3^(1/2))^(1/2))*2^(3/4)/(3+3*3^(1/2))^(1/2)
```

Rubi [A]
time = 0.07, antiderivative size = 165, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 4, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {1433, 1107, 213, 209}

$$\frac{\text{ArcTan}\left(\frac{\sqrt[4]{2}\,x}{\sqrt{\sqrt{3}-1}}\right)}{2\sqrt[4]{2}\sqrt{3\left(\sqrt{3}-1\right)}}+\frac{\text{ArcTan}\left(\frac{\sqrt[4]{2}\,x}{\sqrt{1+\sqrt{3}}}\right)}{2\sqrt[4]{2}\sqrt{3\left(1+\sqrt{3}\right)}}+\frac{\tanh^{-1}\left(\frac{\sqrt[4]{2}\,x}{\sqrt{\sqrt{3}-1}}\right)}{2\sqrt[4]{2}\sqrt{3\left(\sqrt{3}-1\right)}}+\frac{\tanh^{-1}\left(\frac{\sqrt[4]{2}\,x}{\sqrt{1+\sqrt{3}}}\right)}{2\sqrt[4]{2}\sqrt{3\left(1+\sqrt{3}\right)}}$$

Antiderivative was successfully verified.

```
[In] Int[(1 - x^4)/(1 - 4*x^4 + x^8),x]
```

```
[Out] ArcTan[(2^(1/4)*x)/Sqrt[-1 + Sqrt[3]]]/(2*2^(1/4)*Sqrt[3*(-1 + Sqrt[3])]) +
 ArcTan[(2^(1/4)*x)/Sqrt[1 + Sqrt[3]]]/(2*2^(1/4)*Sqrt[3*(1 + Sqrt[3])]) +
ArcTanh[(2^(1/4)*x)/Sqrt[-1 + Sqrt[3]]]/(2*2^(1/4)*Sqrt[3*(-1 + Sqrt[3])])
+ ArcTanh[(2^(1/4)*x)/Sqrt[1 + Sqrt[3]]]/(2*2^(1/4)*Sqrt[3*(1 + Sqrt[3])])
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 1107

```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(-1), x_Symbol] :> With[{q = Rt[b^
2 - 4*a*c, 2]}, Dist[c/q, Int[1/(b/2 - q/2 + c*x^2), x], x] - Dist[c/q, Int
[1/(b/2 + q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c,
0] && PosQ[b^2 - 4*a*c]
```

Rule 1433

```
Int[((d_) + (e_.)*(x_)^(n_))/((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_)), x
_Symbol] :> With[{q = Rt[2*(d/e) - b/c, 2]}, Dist[e/(2*c), Int[1/Simp[d/e +
 q*x^(n/2) + x^n, x], x], x] + Dist[e/(2*c), Int[1/Simp[d/e - q*x^(n/2) + x
^n, x], x], x]] /; FreeQ[{a, b, c, d, e}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4
*a*c, 0] && EqQ[c*d^2 - a*e^2, 0] && IGtQ[n/2, 0] && (GtQ[2*(d/e) - b/c, 0]
 || ( !LtQ[2*(d/e) - b/c, 0] && EqQ[d, e*Rt[a/c, 2]]))
```

Rubi steps

$$\int \frac{1-x^4}{1-4x^4+x^8}\,dx = -\left(\frac{1}{2}\int \frac{1}{-1-\sqrt{2}\,x^2+x^4}\,dx\right)-\frac{1}{2}\int \frac{1}{-1+\sqrt{2}\,x^2+x^4}\,dx$$

$$= -\frac{\int \frac{1}{-\sqrt{\frac{3}{2}-\frac{1}{\sqrt{2}}}+x^2}\,dx}{2\sqrt{6}}+\frac{\int \frac{1}{\sqrt{\frac{3}{2}-\frac{1}{\sqrt{2}}}+x^2}\,dx}{2\sqrt{6}}-\frac{\int \frac{1}{-\sqrt{\frac{3}{2}+\frac{1}{\sqrt{2}}}+x^2}\,dx}{2\sqrt{6}}+\frac{\int \frac{1}{\sqrt{\frac{3}{2}+\frac{1}{\sqrt{2}}}+x^2}\,dx}{2\sqrt{6}}$$

$$= \frac{\tan^{-1}\left(\frac{\sqrt[4]{2}\,x}{\sqrt{-1+\sqrt{3}}}\right)}{2\sqrt[4]{2}\sqrt{3\left(-1+\sqrt{3}\right)}}+\frac{\tan^{-1}\left(\frac{\sqrt[4]{2}\,x}{\sqrt{1+\sqrt{3}}}\right)}{2\sqrt[4]{2}\sqrt{3\left(1+\sqrt{3}\right)}}+\frac{\tanh^{-1}\left(\frac{\sqrt[4]{2}\,x}{\sqrt{-1+\sqrt{3}}}\right)}{2\sqrt[4]{2}\sqrt{3\left(-1+\sqrt{3}\right)}}+\dots$$

Mathematica [C] Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.
time = 0.01, size = 55, normalized size = 0.33

$$-\frac{1}{8}\text{RootSum}\left[1-4\#1^4+\#1^8\&,\frac{-\log(x-\#1)+\log(x-\#1)\#1^4}{-2\#1^3+\#1^7}\&\right]$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 - x^4)/(1 - 4*x^4 + x^8),x]
```

```
[Out] -1/8*RootSum[1 - 4*#1^4 + #1^8 & , (-Log[x - #1] + Log[x - #1]*#1^4)/(-2*#1
^3 + #1^7) & ]
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 0.02, size = 42, normalized size = 0.25

method	result	size
default	$\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(\_Z^8-4\_Z^4+1\right)}\frac{\left(-\_R^4+1\right)\ln\left(x-\_R\right)}{\_R^7-2\_R^3}\right)}{8}$	42
risch	$\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(\_Z^8-4\_Z^4+1\right)}\frac{\left(-\_R^4+1\right)\ln\left(x-\_R\right)}{\_R^7-2\_R^3}\right)}{8}$	42

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-x^4+1)/(x^8-4*x^4+1),x,method=_RETURNVERBOSE)
```

```
[Out] 1/8*sum((-_R^4+1)/(_R^7-2*_R^3)*ln(x-_R),_R=RootOf(_Z^8-4*_Z^4+1))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^4+1)/(x^8-4*x^4+1),x, algorithm="maxima")
```

```
[Out] -integrate((x^4 - 1)/(x^8 - 4*x^4 + 1), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 302 vs. 2(109) = 218.
time = 0.36, size = 302, normalized size = 1.83

$$-\frac{1}{6}\sqrt{6}\left(-\sqrt{3}+2\right)^{\frac{1}{4}}\arctan\left(\frac{1}{6}\sqrt{6}\sqrt{x^2+\left(\sqrt{3}+2\right)\sqrt{-\sqrt{3}+2}}\left(\sqrt{3}+3\right)\left(-\sqrt{3}+2\right)^{\frac{3}{4}}-\frac{1}{6}\sqrt{6}\left(\sqrt{3}x+3x\right)\left(-\sqrt{3}+2\right)^{\frac{3}{4}}\right)+\frac{1}{6}\sqrt{6}\left(\sqrt{3}+2\right)^{\frac{1}{4}}\arctan\left(\frac{1}{6}\left(\sqrt{6}\sqrt{x^2-\sqrt{\sqrt{3}+2}\left(\sqrt{3}-2\right)}\sqrt{\sqrt{3}+2}\left(\sqrt{3}-3\right)-\sqrt{6}\left(\sqrt{3}x-3x\right)\sqrt{\sqrt{3}+2}\right)\left(\sqrt{3}+2\right)^{\frac{1}{4}}\right)-\frac{1}{24}\sqrt{6}\left(\sqrt{3}+2\right)^{\frac{1}{4}}\log\left(\sqrt{6}\left(\sqrt{3}+2\right)^{\frac{1}{4}}\left(\sqrt{3}-3\right)+6x\right)+\frac{1}{24}\sqrt{6}\left(\sqrt{3}+2\right)^{\frac{1}{4}}\log\left(-\sqrt{6}\left(\sqrt{3}+2\right)^{\frac{1}{4}}\left(\sqrt{3}-3\right)+6x\right)+\frac{1}{24}\sqrt{6}\left(-\sqrt{3}+2\right)^{\frac{1}{4}}\log\left(\sqrt{6}\left(\sqrt{3}+3\right)\left(-\sqrt{3}+2\right)^{\frac{1}{4}}+6x\right)-\frac{1}{24}\sqrt{6}\left(-\sqrt{3}+2\right)^{\frac{1}{4}}\log\left(-\sqrt{6}\left(\sqrt{3}+3\right)\left(-\sqrt{3}+2\right)^{\frac{1}{4}}+6x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^4+1)/(x^8-4*x^4+1),x, algorithm="fricas")
```

```
[Out] -1/6*sqrt(6)*(-sqrt(3) + 2)^(1/4)*arctan(1/6*sqrt(6)*sqrt(x^2 + (sqrt(3) +
2)*sqrt(-sqrt(3) + 2))*(sqrt(3) + 3)*(-sqrt(3) + 2)^(3/4) - 1/6*sqrt(6)*(sq
rt(3)*x + 3*x)*(-sqrt(3) + 2)^(3/4)) + 1/6*sqrt(6)*(sqrt(3) + 2)^(1/4)*arct
an(1/6*(sqrt(6)*sqrt(x^2 - sqrt(sqrt(3) + 2)*(sqrt(3) - 2))*sqrt(sqrt(3) +
2)*(sqrt(3) - 3) - sqrt(6)*(sqrt(3)*x - 3*x)*sqrt(sqrt(3) + 2))*(sqrt(3) +
2)^(1/4)) - 1/24*sqrt(6)*(sqrt(3) + 2)^(1/4)*log(sqrt(6)*(sqrt(3) + 2)^(1/4
)*(sqrt(3) - 3) + 6*x) + 1/24*sqrt(6)*(sqrt(3) + 2)^(1/4)*log(-sqrt(6)*(sqr
t(3) + 2)^(1/4)*(sqrt(3) - 3) + 6*x) + 1/24*sqrt(6)*(-sqrt(3) + 2)^(1/4)*lo
g(sqrt(6)*(sqrt(3) + 3)*(-sqrt(3) + 2)^(1/4) + 6*x) - 1/24*sqrt(6)*(-sqrt(3
) + 2)^(1/4)*log(-sqrt(6)*(sqrt(3) + 3)*(-sqrt(3) + 2)^(1/4) + 6*x)
```

Sympy [A]
time = 0.08, size = 26, normalized size = 0.16

$$-\operatorname{RootSum}\left(84934656t^{8}-36864t^{4}+1,\left(t\mapsto t\log\left(36864t^{5}-20t+x\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x**4+1)/(x**8-4*x**4+1),x)
```

```
[Out] -RootSum(84934656*_t**8 - 36864*_t**4 + 1, Lambda(_t, _t*log(36864*_t**5 -
20*_t + x)))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^4+1)/(x^8-4*x^4+1),x, algorithm="giac")
```

```
[Out] integrate(-(x^4 - 1)/(x^8 - 4*x^4 + 1), x)
```

Mupad [B]
time = 0.18, size = 399, normalized size = 2.42

$$\frac{\sqrt{6}\,\mathrm{atan}\left(\frac{64\sqrt{6}\,x\left(\sqrt{3}+2\right)^{1/4}}{80\sqrt{\sqrt{3}+2}+48\sqrt{3}\sqrt{\sqrt{3}+2}}+\frac{112\sqrt{3}\sqrt{6}\,x\left(\sqrt{3}+2\right)^{1/4}}{3\left(80\sqrt{\sqrt{3}+2}+48\sqrt{3}\sqrt{\sqrt{3}+2}\right)}\right)\left(\sqrt{3}+2\right)^{1/4}}{12}+\frac{\sqrt{6}\,\mathrm{atan}\left(\frac{\sqrt{6}\,x\left(2-\sqrt{3}\right)^{1/4}64\mathrm{i}}{48\sqrt{3}\sqrt{2-\sqrt{3}}-80\sqrt{2-\sqrt{3}}}-\frac{\sqrt{3}\sqrt{6}\,x\left(2-\sqrt{3}\right)^{1/4}112\mathrm{i}}{3\left(48\sqrt{3}\sqrt{2-\sqrt{3}}-80\sqrt{2-\sqrt{3}}\right)}\right)\left(2-\sqrt{3}\right)^{1/4}1\mathrm{i}}{12}-\frac{\sqrt{6}\,\mathrm{atan}\left(\frac{64\sqrt{6}\,x\left(2-\sqrt{3}\right)^{1/4}}{48\sqrt{3}\sqrt{2-\sqrt{3}}-80\sqrt{2-\sqrt{3}}}-\frac{112\sqrt{3}\sqrt{6}\,x\left(2-\sqrt{3}\right)^{1/4}}{3\left(48\sqrt{3}\sqrt{2-\sqrt{3}}-80\sqrt{2-\sqrt{3}}\right)}\right)\left(2-\sqrt{3}\right)^{1/4}}{12}-\frac{\sqrt{6}\,\mathrm{atan}\left(\frac{\sqrt{6}\,x\left(\sqrt{3}+2\right)^{1/4}64\mathrm{i}}{80\sqrt{\sqrt{3}+2}+48\sqrt{3}\sqrt{\sqrt{3}+2}}+\frac{\sqrt{3}\sqrt{6}\,x\left(\sqrt{3}+2\right)^{1/4}112\mathrm{i}}{3\left(80\sqrt{\sqrt{3}+2}+48\sqrt{3}\sqrt{\sqrt{3}+2}\right)}\right)\left(\sqrt{3}+2\right)^{1/4}1\mathrm{i}}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(x^4 - 1)/(x^8 - 4*x^4 + 1),x)
```

```
[Out] (6^(1/2)*atan((6^(1/2)*x*(2 - 3^(1/2))^(1/4)*64i)/(48*3^(1/2)*(2 - 3^(1/2))
^(1/2) - 80*(2 - 3^(1/2))^(1/2)) - (3^(1/2)*6^(1/2)*x*(2 - 3^(1/2))^(1/4)*1
12i)/(3*(48*3^(1/2)*(2 - 3^(1/2))^(1/2) - 80*(2 - 3^(1/2))^(1/2))))*(2 - 3^
(1/2))^(1/4)*1i)/12 - (6^(1/2)*atan((64*6^(1/2)*x*(2 - 3^(1/2))^(1/4))/(48*
3^(1/2)*(2 - 3^(1/2))^(1/2) - 80*(2 - 3^(1/2))^(1/2)) - (112*3^(1/2)*6^(1/2
)*x*(2 - 3^(1/2))^(1/4))/(3*(48*3^(1/2)*(2 - 3^(1/2))^(1/2) - 80*(2 - 3^(1/
2))^(1/2))))*(2 - 3^(1/2))^(1/4))/12 + (6^(1/2)*atan((64*6^(1/2)*x*(3^(1/2)
 + 2)^(1/4))/(80*(3^(1/2) + 2)^(1/2) + 48*3^(1/2)*(3^(1/2) + 2)^(1/2)) + (1
12*3^(1/2)*6^(1/2)*x*(3^(1/2) + 2)^(1/4))/(3*(80*(3^(1/2) + 2)^(1/2) + 48*3
^(1/2)*(3^(1/2) + 2)^(1/2))))*(3^(1/2) + 2)^(1/4))/12 - (6^(1/2)*atan((6^(1
/2)*x*(3^(1/2) + 2)^(1/4)*64i)/(80*(3^(1/2) + 2)^(1/2) + 48*3^(1/2)*(3^(1/2
) + 2)^(1/2)) + (3^(1/2)*6^(1/2)*x*(3^(1/2) + 2)^(1/4)*112i)/(3*(80*(3^(1/2
) + 2)^(1/2) + 48*3^(1/2)*(3^(1/2) + 2)^(1/2))))*(3^(1/2) + 2)^(1/4)*1i)/12
```

3.29 $\int \frac{1-x^4}{1-5x^4+x^8}\,dx$

Optimal. Leaf size=169

$$\frac{\tan^{-1}\left(\sqrt{\frac{2}{-\sqrt{3}+\sqrt{7}}}\,x\right)}{\sqrt{14\left(-\sqrt{3}+\sqrt{7}\right)}}+\frac{\tan^{-1}\left(\sqrt{\frac{2}{\sqrt{3}+\sqrt{7}}}\,x\right)}{\sqrt{14\left(\sqrt{3}+\sqrt{7}\right)}}+\frac{\tanh^{-1}\left(\sqrt{\frac{2}{-\sqrt{3}+\sqrt{7}}}\,x\right)}{\sqrt{14\left(-\sqrt{3}+\sqrt{7}\right)}}+\frac{\tanh^{-1}\left(\sqrt{\frac{2}{\sqrt{3}+\sqrt{7}}}\,x\right)}{\sqrt{14\left(\sqrt{3}+\sqrt{7}\right)}}$$

```
[Out] arctan(x*2^(1/2)/(7^(1/2)-3^(1/2))^(1/2))/(-14*3^(1/2)+14*7^(1/2))^(1/2)+ar
ctanh(x*2^(1/2)/(7^(1/2)-3^(1/2))^(1/2))/(-14*3^(1/2)+14*7^(1/2))^(1/2)+arc
tan(x*2^(1/2)/(7^(1/2)+3^(1/2))^(1/2))/(14*3^(1/2)+14*7^(1/2))^(1/2)+arctan
h(x*2^(1/2)/(7^(1/2)+3^(1/2))^(1/2))/(14*3^(1/2)+14*7^(1/2))^(1/2)
```

Rubi [A]
time = 0.10, antiderivative size = 169, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 4, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {1433, 1107, 213, 209}

$$\frac{\text{ArcTan}\left(\sqrt{\frac{2}{\sqrt{7}-\sqrt{3}}}\,x\right)}{\sqrt{14\left(\sqrt{7}-\sqrt{3}\right)}}+\frac{\text{ArcTan}\left(\sqrt{\frac{2}{\sqrt{3}+\sqrt{7}}}\,x\right)}{\sqrt{14\left(\sqrt{3}+\sqrt{7}\right)}}+\frac{\tanh^{-1}\left(\sqrt{\frac{2}{\sqrt{7}-\sqrt{3}}}\,x\right)}{\sqrt{14\left(\sqrt{7}-\sqrt{3}\right)}}+\frac{\tanh^{-1}\left(\sqrt{\frac{2}{\sqrt{3}+\sqrt{7}}}\,x\right)}{\sqrt{14\left(\sqrt{3}+\sqrt{7}\right)}}$$

Antiderivative was successfully verified.

```
[In] Int[(1 - x^4)/(1 - 5*x^4 + x^8),x]
```

```
[Out] ArcTan[Sqrt[2/(-Sqrt[3] + Sqrt[7])]*x]/Sqrt[14*(-Sqrt[3] + Sqrt[7])] + ArcT
an[Sqrt[2/(Sqrt[3] + Sqrt[7])]*x]/Sqrt[14*(Sqrt[3] + Sqrt[7])] + ArcTanh[Sq
rt[2/(-Sqrt[3] + Sqrt[7])]*x]/Sqrt[14*(-Sqrt[3] + Sqrt[7])] + ArcTanh[Sqrt[
2/(Sqrt[3] + Sqrt[7])]*x]/Sqrt[14*(Sqrt[3] + Sqrt[7])]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 1107

```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(-1), x_Symbol] :> With[{q = Rt[b^
2 - 4*a*c, 2]}, Dist[c/q, Int[1/(b/2 - q/2 + c*x^2), x], x] - Dist[c/q, Int
[1/(b/2 + q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c,
0] && PosQ[b^2 - 4*a*c]
```

Rule 1433

```
Int[((d_) + (e_.)*(x_)^(n_))/((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_)), x
_Symbol] :> With[{q = Rt[2*(d/e) - b/c, 2]}, Dist[e/(2*c), Int[1/Simp[d/e +
 q*x^(n/2) + x^n, x], x], x] + Dist[e/(2*c), Int[1/Simp[d/e - q*x^(n/2) + x
^n, x], x], x]] /; FreeQ[{a, b, c, d, e}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4
*a*c, 0] && EqQ[c*d^2 - a*e^2, 0] && IGtQ[n/2, 0] && (GtQ[2*(d/e) - b/c, 0]
 || ( !LtQ[2*(d/e) - b/c, 0] && EqQ[d, e*Rt[a/c, 2]]))
```

Rubi steps

$$\int \frac{1-x^4}{1-5x^4+x^8}\,dx = -\left(\frac{1}{2}\int \frac{1}{-1-\sqrt{3}\,x^2+x^4}\,dx\right)-\frac{1}{2}\int \frac{1}{-1+\sqrt{3}\,x^2+x^4}\,dx$$

$$= -\frac{\int \frac{1}{-\frac{\sqrt{3}}{2}-\frac{\sqrt{7}}{2}+x^2}\,dx}{2\sqrt{7}} - \frac{\int \frac{1}{\frac{\sqrt{3}}{2}-\frac{\sqrt{7}}{2}+x^2}\,dx}{2\sqrt{7}} + \frac{\int \frac{1}{-\frac{\sqrt{3}}{2}+\frac{\sqrt{7}}{2}+x^2}\,dx}{2\sqrt{7}} + \frac{\int \frac{1}{\frac{\sqrt{3}}{2}+\frac{\sqrt{7}}{2}+x^2}\,dx}{2\sqrt{7}}$$

$$= \frac{\tan^{-1}\left(\sqrt{\frac{2}{-\sqrt{3}+\sqrt{7}}}\,x\right)}{\sqrt{14\left(-\sqrt{3}+\sqrt{7}\right)}} + \frac{\tan^{-1}\left(\sqrt{\frac{2}{\sqrt{3}+\sqrt{7}}}\,x\right)}{\sqrt{14\left(\sqrt{3}+\sqrt{7}\right)}} + \frac{\tanh^{-1}\left(\sqrt{\frac{2}{-\sqrt{3}+\sqrt{7}}}\right.}{\sqrt{14\left(-\sqrt{3}+\sqrt{7}\right)}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.
time = 0.01, size = 57, normalized size = 0.34

$$-\frac{1}{4}\text{RootSum}\left[1-5\#1^4+\#1^8\&,\frac{-\log(x-\#1)+\log(x-\#1)\#1^4}{-5\#1^3+2\#1^7}\&\right]$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 - x^4)/(1 - 5*x^4 + x^8),x]
```

```
[Out] -1/4*RootSum[1 - 5*#1^4 + #1^8 & , (-Log[x - #1] + Log[x - #1]*#1^4)/(-5*#1
^3 + 2*#1^7) & ]
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 0.02, size = 44, normalized size = 0.26

method	result	size
default	$\frac{\left(\sum_{\_R=\text{RootOf}\left(\_Z^8-5\_Z^4+1\right)}\frac{\left(-\_R^4+1\right)\ln\left(x-\_R\right)}{2\_R^7-5\_R^3}\right)}{4}$	44
risch	$\frac{\left(\sum_{\_R=\text{RootOf}\left(\_Z^8-5\_Z^4+1\right)}\frac{\left(-\_R^4+1\right)\ln\left(x-\_R\right)}{2\_R^7-5\_R^3}\right)}{4}$	44

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-x^4+1)/(x^8-5*x^4+1),x,method=_RETURNVERBOSE)
```

```
[Out] 1/4*sum((-_R^4+1)/(2*_R^7-5*_R^3)*ln(x-_R),_R=RootOf(_Z^8-5*_Z^4+1))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^4+1)/(x^8-5*x^4+1),x, algorithm="maxima")
```

```
[Out] -integrate((x^4 - 1)/(x^8 - 5*x^4 + 1), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 546 vs. $2(121) = 242$.
time = 0.38, size = 546, normalized size = 3.23

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^4+1)/(x^8-5*x^4+1),x, algorithm="fricas")
```

```
[Out] -1/14*sqrt(14)*sqrt(sqrt(2)*sqrt(-sqrt(7)*sqrt(3) + 5))*arctan(1/112*sqrt(1
4)*sqrt(4*x^2 + (sqrt(7)*sqrt(3)*sqrt(2) + 5*sqrt(2))*sqrt(-sqrt(7)*sqrt(3)
 + 5))*(sqrt(7)*sqrt(3)*sqrt(2) + 7*sqrt(2))*sqrt(-sqrt(7)*sqrt(3) + 5)*sqr
t(sqrt(2)*sqrt(-sqrt(7)*sqrt(3) + 5)) - 1/56*sqrt(14)*(sqrt(7)*sqrt(3)*sqrt
(2)*x + 7*sqrt(2)*x)*sqrt(-sqrt(7)*sqrt(3) + 5)*sqrt(sqrt(2)*sqrt(-sqrt(7)*
sqrt(3) + 5))) + 1/14*sqrt(14)*sqrt(sqrt(2)*sqrt(sqrt(7)*sqrt(3) + 5))*arct
an(1/112*(sqrt(14)*sqrt(4*x^2 - (sqrt(7)*sqrt(3)*sqrt(2) - 5*sqrt(2))*sqrt(
sqrt(7)*sqrt(3) + 5))*(sqrt(7)*sqrt(3)*sqrt(2) - 7*sqrt(2))*sqrt(sqrt(7)*sq
rt(3) + 5) - 2*sqrt(14)*(sqrt(7)*sqrt(3)*sqrt(2)*x - 7*sqrt(2)*x)*sqrt(sqrt
(7)*sqrt(3) + 5))*sqrt(sqrt(2)*sqrt(sqrt(7)*sqrt(3) + 5))) - 1/56*sqrt(14)*
sqrt(sqrt(2)*sqrt(sqrt(7)*sqrt(3) + 5))*log(sqrt(14)*(sqrt(7)*sqrt(3) - 7)*
```

```
sqrt(sqrt(2)*sqrt(sqrt(7)*sqrt(3) + 5)) + 28*x) + 1/56*sqrt(14)*sqrt(sqrt(2
)*sqrt(sqrt(7)*sqrt(3) + 5))*log(-sqrt(14)*(sqrt(7)*sqrt(3) - 7)*sqrt(sqrt(
2)*sqrt(sqrt(7)*sqrt(3) + 5)) + 28*x) + 1/56*sqrt(14)*sqrt(sqrt(2)*sqrt(-sq
rt(7)*sqrt(3) + 5))*log(sqrt(14)*(sqrt(7)*sqrt(3) + 7)*sqrt(sqrt(2)*sqrt(-s
qrt(7)*sqrt(3) + 5)) + 28*x) - 1/56*sqrt(14)*sqrt(sqrt(2)*sqrt(-sqrt(7)*sqr
t(3) + 5))*log(-sqrt(14)*(sqrt(7)*sqrt(3) + 7)*sqrt(sqrt(2)*sqrt(-sqrt(7)*s
qrt(3) + 5)) + 28*x)
```

Sympy [A]
time = 0.08, size = 26, normalized size = 0.15

$$-\operatorname{RootSum}\left(157351936t^8-62720t^4+1,\left(t\mapsto t\log\left(50176t^5-24t+x\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x**4+1)/(x**8-5*x**4+1),x)
```

```
[Out] -RootSum(157351936*_t**8 - 62720*_t**4 + 1, Lambda(_t, _t*log(50176*_t**5 -
 24*_t + x)))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^4+1)/(x^8-5*x^4+1),x, algorithm="giac")
```

```
[Out] integrate(-(x^4 - 1)/(x^8 - 5*x^4 + 1), x)
```

Mupad [B]
time = 1.79, size = 483, normalized size = 2.86

$$\frac{2^{3/4}\sqrt{7}\,\mathrm{atan}\left(\frac{405\,2^{3/4}\sqrt{7}\,x\,\left(5-\sqrt{21}\right)^{1/4}}{2\left(243\sqrt{2}\sqrt{5-\sqrt{21}}-54\sqrt{2}\sqrt{21}\sqrt{5-\sqrt{21}}\right)}-\frac{621\,2^{3/4}\sqrt{7}\sqrt{21}\,x\,\left(5-\sqrt{21}\right)^{1/4}}{14\left(243\sqrt{2}\sqrt{5-\sqrt{21}}-54\sqrt{2}\sqrt{21}\sqrt{5-\sqrt{21}}\right)}\right)\left(5-\sqrt{21}\right)^{1/4}}{28}-\frac{2^{3/4}\sqrt{7}\,\mathrm{atan}\left(\frac{2^{3/4}\sqrt{7}\,x\,\left(5-\sqrt{21}\right)^{1/4}405\mathrm{i}}{2\left(243\sqrt{2}\sqrt{5-\sqrt{21}}-54\sqrt{2}\sqrt{21}\sqrt{5-\sqrt{21}}\right)}-\frac{2^{3/4}\sqrt{7}\sqrt{21}\,x\,\left(5-\sqrt{21}\right)^{1/4}621\mathrm{i}}{14\left(243\sqrt{2}\sqrt{5-\sqrt{21}}-54\sqrt{2}\sqrt{21}\sqrt{5-\sqrt{21}}\right)}\right)\left(5-\sqrt{21}\right)^{1/4}1\mathrm{i}}{28}+\frac{2^{3/4}\sqrt{7}\,\mathrm{atan}\left(\frac{405\,2^{3/4}\sqrt{7}\,x\,\left(\sqrt{21}+5\right)^{1/4}}{2\left(243\sqrt{2}\sqrt{\sqrt{21}+5}+54\sqrt{2}\sqrt{21}\sqrt{\sqrt{21}+5}\right)}+\frac{621\,2^{3/4}\sqrt{7}\sqrt{21}\,x\,\left(\sqrt{21}+5\right)^{1/4}}{14\left(243\sqrt{2}\sqrt{\sqrt{21}+5}+54\sqrt{2}\sqrt{21}\sqrt{\sqrt{21}+5}\right)}\right)\left(\sqrt{21}+5\right)^{1/4}}{28}-\frac{2^{3/4}\sqrt{7}\,\mathrm{atan}\left(\frac{2^{3/4}\sqrt{7}\,x\,\left(\sqrt{21}+5\right)^{1/4}405\mathrm{i}}{2\left(243\sqrt{2}\sqrt{\sqrt{21}+5}+54\sqrt{2}\sqrt{21}\sqrt{\sqrt{21}+5}\right)}+\frac{2^{3/4}\sqrt{7}\sqrt{21}\,x\,\left(\sqrt{21}+5\right)^{1/4}621\mathrm{i}}{14\left(243\sqrt{2}\sqrt{\sqrt{21}+5}+54\sqrt{2}\sqrt{21}\sqrt{\sqrt{21}+5}\right)}\right)\left(\sqrt{21}+5\right)^{1/4}1\mathrm{i}}{28}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(x^4 - 1)/(x^8 - 5*x^4 + 1),x)
```

```
[Out] (2^(3/4)*7^(1/2)*atan((405*2^(3/4)*7^(1/2)*x*(5 - 21^(1/2))^(1/4))/(2*(243*
2^(1/2)*(5 - 21^(1/2))^(1/2) - 54*2^(1/2)*21^(1/2)*(5 - 21^(1/2))^(1/2))) -
 (621*2^(3/4)*7^(1/2)*21^(1/2)*x*(5 - 21^(1/2))^(1/4))/(14*(243*2^(1/2)*(5
- 21^(1/2))^(1/2) - 54*2^(1/2)*21^(1/2)*(5 - 21^(1/2))^(1/2))))*(5 - 21^(1/
2))^(1/4))/28 - (2^(3/4)*7^(1/2)*atan((2^(3/4)*7^(1/2)*x*(5 - 21^(1/2))^(1/
4)*405i)/(2*(243*2^(1/2)*(5 - 21^(1/2))^(1/2) - 54*2^(1/2)*21^(1/2)*(5 - 21
^(1/2))^(1/2))) - (2^(3/4)*7^(1/2)*21^(1/2)*x*(5 - 21^(1/2))^(1/4)*621i)/(1
4*(243*2^(1/2)*(5 - 21^(1/2))^(1/2) - 54*2^(1/2)*21^(1/2)*(5 - 21^(1/2))^(1
/2))))*(5 - 21^(1/2))^(1/4)*1i)/28 + (2^(3/4)*7^(1/2)*atan((405*2^(3/4)*7^(
```

```
1/2)*x*(21^(1/2) + 5)^(1/4))/(2*(243*2^(1/2)*(21^(1/2) + 5)^(1/2) + 54*2^(1
/2)*21^(1/2)*(21^(1/2) + 5)^(1/2))) + (621*2^(3/4)*7^(1/2)*21^(1/2)*x*(21^(
1/2) + 5)^(1/4))/(14*(243*2^(1/2)*(21^(1/2) + 5)^(1/2) + 54*2^(1/2)*21^(1/2
)*(21^(1/2) + 5)^(1/2))))*(21^(1/2) + 5)^(1/4))/28 - (2^(3/4)*7^(1/2)*atan(
(2^(3/4)*7^(1/2)*x*(21^(1/2) + 5)^(1/4)*405i)/(2*(243*2^(1/2)*(21^(1/2) + 5
)^(1/2) + 54*2^(1/2)*21^(1/2)*(21^(1/2) + 5)^(1/2))) + (2^(3/4)*7^(1/2)*21^
(1/2)*x*(21^(1/2) + 5)^(1/4)*621i)/(14*(243*2^(1/2)*(21^(1/2) + 5)^(1/2) +
54*2^(1/2)*21^(1/2)*(21^(1/2) + 5)^(1/2))))*(21^(1/2) + 5)^(1/4)*1i)/28
```

3.30 $\int \frac{1-x^4}{1-6x^4+x^8}\,dx$

Optimal. Leaf size=125

$$\frac{\tan^{-1}\left(\frac{x}{\sqrt{-1+\sqrt{2}}}\right)}{4\sqrt{2\left(-1+\sqrt{2}\right)}}+\frac{\tan^{-1}\left(\frac{x}{\sqrt{1+\sqrt{2}}}\right)}{4\sqrt{2\left(1+\sqrt{2}\right)}}+\frac{\tanh^{-1}\left(\frac{x}{\sqrt{-1+\sqrt{2}}}\right)}{4\sqrt{2\left(-1+\sqrt{2}\right)}}+\frac{\tanh^{-1}\left(\frac{x}{\sqrt{1+\sqrt{2}}}\right)}{4\sqrt{2\left(1+\sqrt{2}\right)}}$$

```
[Out] 1/4*arctan(x/(2^(1/2)-1)^(1/2))/(-2+2*2^(1/2))^(1/2)+1/4*arctanh(x/(2^(1/2)
-1)^(1/2))/(-2+2*2^(1/2))^(1/2)+1/4*arctan(x/(1+2^(1/2))^(1/2))/(2+2*2^(1/2
))^(1/2)+1/4*arctanh(x/(1+2^(1/2))^(1/2))/(2+2*2^(1/2))^(1/2)
```

Rubi [A]
time = 0.05, antiderivative size = 125, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 4, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {1433, 1107, 213, 209}

$$\frac{\text{ArcTan}\left(\frac{x}{\sqrt{\sqrt{2}-1}}\right)}{4\sqrt{2\left(\sqrt{2}-1\right)}}+\frac{\text{ArcTan}\left(\frac{x}{\sqrt{1+\sqrt{2}}}\right)}{4\sqrt{2\left(1+\sqrt{2}\right)}}+\frac{\tanh^{-1}\left(\frac{x}{\sqrt{\sqrt{2}-1}}\right)}{4\sqrt{2\left(\sqrt{2}-1\right)}}+\frac{\tanh^{-1}\left(\frac{x}{\sqrt{1+\sqrt{2}}}\right)}{4\sqrt{2\left(1+\sqrt{2}\right)}}$$

Antiderivative was successfully verified.

```
[In] Int[(1 - x^4)/(1 - 6*x^4 + x^8),x]
```

```
[Out] ArcTan[x/Sqrt[-1 + Sqrt[2]]]/(4*Sqrt[2*(-1 + Sqrt[2])]) + ArcTan[x/Sqrt[1 +
 Sqrt[2]]]/(4*Sqrt[2*(1 + Sqrt[2])]) + ArcTanh[x/Sqrt[-1 + Sqrt[2]]]/(4*Sqr
t[2*(-1 + Sqrt[2])]) + ArcTanh[x/Sqrt[1 + Sqrt[2]]]/(4*Sqrt[2*(1 + Sqrt[2])
])
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 1107

```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(-1), x_Symbol] :> With[{q = Rt[b^
2 - 4*a*c, 2]}, Dist[c/q, Int[1/(b/2 - q/2 + c*x^2), x], x] - Dist[c/q, Int
[1/(b/2 + q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c,
0] && PosQ[b^2 - 4*a*c]
```

Rule 1433

```
Int[((d_) + (e_.)*(x_)^(n_))/((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_)), x
_Symbol] :> With[{q = Rt[2*(d/e) - b/c, 2]}, Dist[e/(2*c), Int[1/Simp[d/e +
 q*x^(n/2) + x^n, x], x], x] + Dist[e/(2*c), Int[1/Simp[d/e - q*x^(n/2) + x
^n, x], x], x]] /; FreeQ[{a, b, c, d, e}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4
*a*c, 0] && EqQ[c*d^2 - a*e^2, 0] && IGtQ[n/2, 0] && (GtQ[2*(d/e) - b/c, 0]
 || ( !LtQ[2*(d/e) - b/c, 0] && EqQ[d, e*Rt[a/c, 2]]))
```

Rubi steps

$$\begin{aligned}\int \frac{1-x^4}{1-6x^4+x^8}\,dx &= -\left(\frac{1}{2}\int \frac{1}{-1-2x^2+x^4}\,dx\right)-\frac{1}{2}\int \frac{1}{-1+2x^2+x^4}\,dx\\ &= -\frac{\int \frac{1}{-1-\sqrt{2}+x^2}\,dx}{4\sqrt{2}}-\frac{\int \frac{1}{1-\sqrt{2}+x^2}\,dx}{4\sqrt{2}}+\frac{\int \frac{1}{-1+\sqrt{2}+x^2}\,dx}{4\sqrt{2}}+\frac{\int \frac{1}{1+\sqrt{2}+x^2}\,dx}{4\sqrt{2}}\\ &= \frac{\tan^{-1}\left(\frac{x}{\sqrt{-1+\sqrt{2}}}\right)}{4\sqrt{2\left(-1+\sqrt{2}\right)}}+\frac{\tan^{-1}\left(\frac{x}{\sqrt{1+\sqrt{2}}}\right)}{4\sqrt{2\left(1+\sqrt{2}\right)}}+\frac{\tanh^{-1}\left(\frac{x}{\sqrt{-1+\sqrt{2}}}\right)}{4\sqrt{2\left(-1+\sqrt{2}\right)}}+\text{tan}\end{aligned}$$

Mathematica [A]
time = 0.03, size = 114, normalized size = 0.91

$$\frac{\sqrt{1+\sqrt{2}}\,\tan^{-1}\left(\frac{x}{\sqrt{-1+\sqrt{2}}}\right)+\sqrt{-1+\sqrt{2}}\,\tan^{-1}\left(\frac{x}{\sqrt{1+\sqrt{2}}}\right)+\sqrt{1+\sqrt{2}}\,\tanh^{-1}\left(\frac{x}{\sqrt{-1+\sqrt{2}}}\right)+\sqrt{-1+\sqrt{2}}\,\tanh^{-1}\left(\frac{x}{\sqrt{1+\sqrt{2}}}\right)}{4\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 - x^4)/(1 - 6*x^4 + x^8),x]
```

```
[Out] (Sqrt[1 + Sqrt[2]]*ArcTan[x/Sqrt[-1 + Sqrt[2]]] + Sqrt[-1 + Sqrt[2]]*ArcTan
[x/Sqrt[1 + Sqrt[2]]] + Sqrt[1 + Sqrt[2]]*ArcTanh[x/Sqrt[-1 + Sqrt[2]]] + S
qrt[-1 + Sqrt[2]]*ArcTanh[x/Sqrt[1 + Sqrt[2]]])/(4*Sqrt[2])
```

Maple [A]
time = 0.05, size = 90, normalized size = 0.72

method	result
risch	$\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(4\_Z^4-4\_Z^2-1\right)}\_R\ln\left(-2\_R^3+3\_R+x\right)\right)}{8}+\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(4\_Z^4+4\_Z^2-1\right)}\_R\ln\left(2\_R^3+3\_R+x\right)\right)}{8}$
default	$\frac{\sqrt{2}\,\arctan\left(\frac{x}{\sqrt{\sqrt{2}-1}}\right)}{8\sqrt{\sqrt{2}-1}}+\frac{\sqrt{2}\,\mathrm{arctanh}\left(\frac{x}{\sqrt{1+\sqrt{2}}}\right)}{8\sqrt{1+\sqrt{2}}}+\frac{\sqrt{2}\,\mathrm{arctanh}\left(\frac{x}{\sqrt{\sqrt{2}-1}}\right)}{8\sqrt{\sqrt{2}-1}}+\frac{\sqrt{2}\,\arctan\left(\ldots\right.}{8\sqrt{1\ldots}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-x^4+1)/(x^8-6*x^4+1),x,method=_RETURNVERBOSE)
```

```
[Out] 1/8*2^(1/2)*arctan(x/(2^(1/2)-1)^(1/2))/(2^(1/2)-1)^(1/2)+1/8*2^(1/2)*arcta
nh(x/(1+2^(1/2))^(1/2))/(1+2^(1/2))^(1/2)+1/8*2^(1/2)*arctanh(x/(2^(1/2)-1)
^(1/2))/(2^(1/2)-1)^(1/2)+1/8*2^(1/2)*arctan(x/(1+2^(1/2))^(1/2))/(1+2^(1/2
))^(1/2)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^4+1)/(x^8-6*x^4+1),x, algorithm="maxima")
```

```
[Out] -integrate((x^4 - 1)/(x^8 - 6*x^4 + 1), x)
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 199 vs. $2(85) = 170$.
time = 0.38, size = 199, normalized size = 1.59

$$-\frac{1}{4}\sqrt{2}\sqrt{\sqrt{2}+1}\arctan\left(-x\sqrt{\sqrt{2}+1}+\sqrt{x^2+\sqrt{2}-1}\sqrt{\sqrt{2}+1}\right)-\frac{1}{4}\sqrt{2}\sqrt{\sqrt{2}-1}\arctan\left(-x\sqrt{\sqrt{2}-1}+\sqrt{x^2+\sqrt{2}+1}\sqrt{\sqrt{2}-1}\right)+\frac{1}{16}\sqrt{2}\sqrt{\sqrt{2}-1}\log\left(\left(\sqrt{2}+1\right)\sqrt{\sqrt{2}-1}+x\right)-\frac{1}{16}\sqrt{2}\sqrt{\sqrt{2}-1}\log\left(-\left(\sqrt{2}+1\right)\sqrt{\sqrt{2}-1}+x\right)+\frac{1}{16}\sqrt{2}\sqrt{\sqrt{2}+1}\log\left(\sqrt{\sqrt{2}+1}\left(\sqrt{2}-1\right)+x\right)-\frac{1}{16}\sqrt{2}\sqrt{\sqrt{2}+1}\log\left(-\sqrt{\sqrt{2}+1}\left(\sqrt{2}-1\right)+x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^4+1)/(x^8-6*x^4+1),x, algorithm="fricas")
```

```
[Out] -1/4*sqrt(2)*sqrt(sqrt(2) + 1)*arctan(-x*sqrt(sqrt(2) + 1) + sqrt(x^2 + sqr
t(2) - 1)*sqrt(sqrt(2) + 1)) - 1/4*sqrt(2)*sqrt(sqrt(2) - 1)*arctan(-x*sqrt
(sqrt(2) - 1) + sqrt(x^2 + sqrt(2) + 1)*sqrt(sqrt(2) - 1)) + 1/16*sqrt(2)*s
qrt(sqrt(2) - 1)*log((sqrt(2) + 1)*sqrt(sqrt(2) - 1) + x) - 1/16*sqrt(2)*sq
rt(sqrt(2) - 1)*log(-(sqrt(2) + 1)*sqrt(sqrt(2) - 1) + x) + 1/16*sqrt(2)*sq
rt(sqrt(2) + 1)*log(sqrt(sqrt(2) + 1)*(sqrt(2) - 1) + x) - 1/16*sqrt(2)*sqr
t(sqrt(2) + 1)*log(-sqrt(sqrt(2) + 1)*(sqrt(2) - 1) + x)
```

Sympy [A]
time = 0.69, size = 51, normalized size = 0.41

$$-\operatorname{RootSum}\left(16384t^4-256t^2-1,\left(t\mapsto t\log\left(65536t^5-28t+x\right)\right)\right)-\operatorname{RootSum}\left(16384t^4+256t^2-1,\left(t\mapsto t\log\left(65536t^5-28t+x\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x**4+1)/(x**8-6*x**4+1),x)
```

```
[Out] -RootSum(16384*_t**4 - 256*_t**2 - 1, Lambda(_t, _t*log(65536*_t**5 - 28*_t
 + x))) - RootSum(16384*_t**4 + 256*_t**2 - 1, Lambda(_t, _t*log(65536*_t**
5 - 28*_t + x)))
```

Giac [A]
time = 3.76, size = 135, normalized size = 1.08

$$\frac{1}{8}\sqrt{2\sqrt{2}-2}\arctan\left(\frac{x}{\sqrt{\sqrt{2}+1}}\right)+\frac{1}{8}\sqrt{2\sqrt{2}+2}\arctan\left(\frac{x}{\sqrt{\sqrt{2}-1}}\right)+\frac{1}{16}\sqrt{2\sqrt{2}-2}\log\left(\left|x+\sqrt{\sqrt{2}+1}\right|\right)-\frac{1}{16}\sqrt{2\sqrt{2}-2}\log\left(\left|x-\sqrt{\sqrt{2}+1}\right|\right)+\frac{1}{16}\sqrt{2\sqrt{2}+2}\log\left(\left|x+\sqrt{\sqrt{2}-1}\right|\right)-\frac{1}{16}\sqrt{2\sqrt{2}+2}\log\left(\left|x-\sqrt{\sqrt{2}-1}\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^4+1)/(x^8-6*x^4+1),x, algorithm="giac")
```

```
[Out] 1/8*sqrt(2*sqrt(2) - 2)*arctan(x/sqrt(sqrt(2) + 1)) + 1/8*sqrt(2*sqrt(2) +
2)*arctan(x/sqrt(sqrt(2) - 1)) + 1/16*sqrt(2*sqrt(2) - 2)*log(abs(x + sqrt(
sqrt(2) + 1))) - 1/16*sqrt(2*sqrt(2) - 2)*log(abs(x - sqrt(sqrt(2) + 1))) +
 1/16*sqrt(2*sqrt(2) + 2)*log(abs(x + sqrt(sqrt(2) - 1))) - 1/16*sqrt(2*sqr
t(2) + 2)*log(abs(x - sqrt(sqrt(2) - 1)))
```

Mupad [B]
time = 0.20, size = 245, normalized size = 1.96

$$-\frac{\sqrt{2}\,\mathrm{atan}\left(\frac{x\,\sqrt{1-\sqrt{2}}\,4352{}\mathrm{i}}{3072\,\sqrt{2}-4352}-\frac{\sqrt{2}\,x\,\sqrt{1-\sqrt{2}}\,3072{}\mathrm{i}}{3072\,\sqrt{2}-4352}\right)\,\sqrt{1-\sqrt{2}}\,1{}\mathrm{i}}{8}+\frac{\sqrt{2}\,\mathrm{atan}\left(\frac{x\,\sqrt{-\sqrt{2}-1}\,4352{}\mathrm{i}}{3072\,\sqrt{2}+4352}+\frac{\sqrt{2}\,x\,\sqrt{-\sqrt{2}-1}\,3072{}\mathrm{i}}{3072\,\sqrt{2}+4352}\right)\,\sqrt{-\sqrt{2}-1}\,1{}\mathrm{i}}{8}+\frac{\sqrt{2}\,\mathrm{atan}\left(\frac{x\,\sqrt{\sqrt{2}-1}\,4352{}\mathrm{i}}{3072\,\sqrt{2}-4352}-\frac{\sqrt{2}\,x\,\sqrt{\sqrt{2}-1}\,3072{}\mathrm{i}}{3072\,\sqrt{2}-4352}\right)\,\sqrt{\sqrt{2}-1}\,1{}\mathrm{i}}{8}-\frac{\sqrt{2}\,\mathrm{atan}\left(\frac{x\,\sqrt{\sqrt{2}+1}\,4352{}\mathrm{i}}{3072\,\sqrt{2}+4352}+\frac{\sqrt{2}\,x\,\sqrt{\sqrt{2}+1}\,3072{}\mathrm{i}}{3072\,\sqrt{2}+4352}\right)\,\sqrt{\sqrt{2}+1}\,1{}\mathrm{i}}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(x^4 - 1)/(x^8 - 6*x^4 + 1),x)
```

```
[Out] (2^(1/2)*atan((x*(- 2^(1/2) - 1)^(1/2)*4352i)/(3072*2^(1/2) + 4352) + (2^(1
/2)*x*(- 2^(1/2) - 1)^(1/2)*3072i)/(3072*2^(1/2) + 4352))*(- 2^(1/2) - 1)^(
1/2)*1i)/8 - (2^(1/2)*atan((x*(1 - 2^(1/2))^(1/2)*4352i)/(3072*2^(1/2) - 43
52) - (2^(1/2)*x*(1 - 2^(1/2))^(1/2)*3072i)/(3072*2^(1/2) - 4352))*(1 - 2^(
1/2))^(1/2)*1i)/8 + (2^(1/2)*atan((x*(2^(1/2) - 1)^(1/2)*4352i)/(3072*2^(1/
2) - 4352) - (2^(1/2)*x*(2^(1/2) - 1)^(1/2)*3072i)/(3072*2^(1/2) - 4352))*(
2^(1/2) - 1)^(1/2)*1i)/8 - (2^(1/2)*atan((x*(2^(1/2) + 1)^(1/2)*4352i)/(307
2*2^(1/2) + 4352) + (2^(1/2)*x*(2^(1/2) + 1)^(1/2)*3072i)/(3072*2^(1/2) + 4
352))*(2^(1/2) + 1)^(1/2)*1i)/8
```

3.31 $\int \frac{-1+\sqrt{3}+2x^4}{1-x^4+x^8}\,dx$

Optimal. Leaf size=135

$$-\frac{\tan^{-1}\left(\frac{\sqrt{2+\sqrt{3}}-2x}{\sqrt{2-\sqrt{3}}}\right)}{\sqrt{2}}+\frac{\tan^{-1}\left(\frac{\sqrt{2+\sqrt{3}}+2x}{\sqrt{2-\sqrt{3}}}\right)}{\sqrt{2}}-\frac{\log\left(1-\sqrt{2-\sqrt{3}}\,x+x^2\right)}{2\sqrt{2}}+\frac{\log\left(1+\sqrt{2-\sqrt{3}}\right.}{2\sqrt{2}}$$

```
[Out] -1/2*arctan((-2*x+1/2*6^(1/2)+1/2*2^(1/2))/(1/2*6^(1/2)-1/2*2^(1/2)))*2^(1/
2)+1/2*arctan((2*x+1/2*6^(1/2)+1/2*2^(1/2))/(1/2*6^(1/2)-1/2*2^(1/2)))*2^(1
/2)-1/4*ln(1+x^2-x*(1/2*6^(1/2)-1/2*2^(1/2)))*2^(1/2)+1/4*ln(1+x^2+x*(1/2*6
^(1/2)-1/2*2^(1/2)))*2^(1/2)
```

Rubi [A]
time = 0.09, antiderivative size = 135, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.240$, Rules used = {1437, 1175, 632, 210, 1178, 642}

$$-\frac{\text{ArcTan}\left(\frac{\sqrt{2+\sqrt{3}}-2x}{\sqrt{2-\sqrt{3}}}\right)}{\sqrt{2}}+\frac{\text{ArcTan}\left(\frac{2x+\sqrt{2+\sqrt{3}}}{\sqrt{2-\sqrt{3}}}\right)}{\sqrt{2}}-\frac{\log\left(x^2-\sqrt{2-\sqrt{3}}\,x+1\right)}{2\sqrt{2}}+\frac{\log\left(x^2+\sqrt{2-\sqrt{3}}\,x+1\right)}{2\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[(-1 + Sqrt[3] + 2*x^4)/(1 - x^4 + x^8),x]
```

```
[Out] -(ArcTan[(Sqrt[2 + Sqrt[3]] - 2*x)/Sqrt[2 - Sqrt[3]]]/Sqrt[2]) + ArcTan[(Sq
rt[2 + Sqrt[3]] + 2*x)/Sqrt[2 - Sqrt[3]]]/Sqrt[2] - Log[1 - Sqrt[2 - Sqrt[3
]]*x + x^2]/(2*Sqrt[2]) + Log[1 + Sqrt[2 - Sqrt[3]]*x + x^2]/(2*Sqrt[2])
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
```

```
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1175

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[2*(d/e) - b/c, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2
, x], x], x] + Dist[e/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; Fre
eQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - a*e^2, 0] && (
GtQ[2*(d/e) - b/c, 0] || ( !LtQ[2*(d/e) - b/c, 0] && EqQ[d - e*Rt[a/c, 2],
0]))
```

Rule 1178

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[-2*(d/e) - b/c, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e
+ q*x - x^2, x], x], x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^
2, x], x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c
*d^2 - a*e^2, 0] &&  !GtQ[b^2 - 4*a*c, 0]
```

Rule 1437

```
Int[((d_) + (e_.)*(x_)^(n_))/((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_)), x
_Symbol] :> With[{q = Rt[a/c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Dist[1/(2*c
*q*r), Int[(d*r - (d - e*q)*x^(n/2))/(q - r*x^(n/2) + x^n), x], x] + Dist[1
/(2*c*q*r), Int[(d*r + (d - e*q)*x^(n/2))/(q + r*x^(n/2) + x^n), x], x]]] /
; FreeQ[{a, b, c, d, e}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && NeQ[c
*d^2 - b*d*e + a*e^2, 0] && IGtQ[n/2, 0] && NegQ[b^2 - 4*a*c]
```

Rubi steps

$$\int \frac{-1+\sqrt{3}+2x^4}{1-x^4+x^8}\,dx = \frac{\int \frac{\sqrt{3}\left(-1+\sqrt{3}\right)+\left(3-\sqrt{3}\right)x^2}{1-\sqrt{3}\,x^2+x^4}\,dx}{2\sqrt{3}} + \frac{\int \frac{\sqrt{3}\left(-1+\sqrt{3}\right)+\left(-3+\sqrt{3}\right)x^2}{1+\sqrt{3}\,x^2+x^4}\,dx}{2\sqrt{3}}$$

$$= -\frac{\int \frac{\sqrt{2-\sqrt{3}}+2x}{-1-\sqrt{2-\sqrt{3}}\,x-x^2}\,dx}{2\sqrt{2}} - \frac{\int \frac{\sqrt{2-\sqrt{3}}-2x}{-1+\sqrt{2-\sqrt{3}}\,x-x^2}\,dx}{2\sqrt{2}} + \frac{1}{4}\left(-1+\sqrt{3}\right)\int \frac{}{1-\sqrt{}}$$

$$= -\frac{\log\left(1-\sqrt{2-\sqrt{3}}\,x+x^2\right)}{2\sqrt{2}} + \frac{\log\left(1+\sqrt{2-\sqrt{3}}\,x+x^2\right)}{2\sqrt{2}} + \frac{1}{2}\left(1-\sqrt{3}\right)\text{Su}$$

$$= -\frac{\tan^{-1}\left(\frac{\sqrt{2+\sqrt{3}}-2x}{\sqrt{2-\sqrt{3}}}\right)}{\sqrt{2}} + \frac{\tan^{-1}\left(\frac{\sqrt{2+\sqrt{3}}+2x}{\sqrt{2-\sqrt{3}}}\right)}{\sqrt{2}} - \frac{\log\left(1-\sqrt{2-\sqrt{3}}\,x\right.}{2\sqrt{2}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.
time = 0.02, size = 71, normalized size = 0.53

$$\frac{1}{4}\text{RootSum}\left[1-\#1^4+\#1^8\&,\frac{-\log(x-\#1)+\sqrt{3}\,\log(x-\#1)+2\log(x-\#1)\#1^4}{-\#1^3+2\#1^7}\&\right]$$

Antiderivative was successfully verified.

```
[In] Integrate[(-1 + Sqrt[3] + 2*x^4)/(1 - x^4 + x^8),x]
```

```
[Out] RootSum[1 - #1^4 + #1^8 & , (-Log[x - #1] + Sqrt[3]*Log[x - #1] + 2*Log[x -
 #1]*#1^4)/(-#1^3 + 2*#1^7) & ]/4
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 0.11, size = 47, normalized size = 0.35

method	result
default	$\frac{\left(\sum_{\_R=\text{RootOf}\left(\_Z^8-\_Z^4+1\right)}\frac{\left(-1+2\_R^4+\sqrt{3}\right)\ln\left(x-\_R\right)}{2\_R^7-\_R^3}\right)}{4}$
risch	$\frac{\sqrt{2}\,\ln\left(2x^2+\left(\sqrt{2}\,\sqrt{3}-\sqrt{2}\right)x+2\right)}{4}-\frac{\sqrt{2}\,\ln\left(2x^2+\left(-\sqrt{2}\,\sqrt{3}+\sqrt{2}\right)x+2\right)}{4}+\frac{\sqrt{2}\,\arctan\left(\frac{\sqrt{2}\,x}{\sqrt{3}-1}\right)}{2}+\frac{\sqrt{2}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-1+2*x^4+3^(1/2))/(x^8-x^4+1),x,method=_RETURNVERBOSE)
```

```
[Out] 1/4*sum(1/(2*_R^7-_R^3)*(-1+2*_R^4+3^(1/2))*ln(x-_R),_R=RootOf(_Z^8-_Z^4+1)
)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+2*x^4+3^(1/2))/(x^8-x^4+1),x, algorithm="maxima")
```

```
[Out] integrate((2*x^4 + sqrt(3) - 1)/(x^8 - x^4 + 1), x)
```

Fricas [A]
time = 0.42, size = 153, normalized size = 1.13

$$\frac{1}{2}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{3}\sqrt{2}x^3+\frac{1}{2}\sqrt{2}(x^3-2x)\right)+\frac{1}{2}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{3}\sqrt{2}x+\frac{1}{2}\sqrt{2}x\right)+\frac{1}{4}\sqrt{2}\log\left(\frac{x^8+4x^6+5x^4+4x^2-\sqrt{2}(x^7+4x^5+4x^3+x)-\sqrt{3}\left(2x^6+4x^4+2x^2-\sqrt{2}(x^7+2x^5+2x^3+x)\right)+1}{x^8-x^4+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+2*x^4+3^(1/2))/(x^8-x^4+1),x, algorithm="fricas")
```

```
[Out] 1/2*sqrt(2)*arctan(1/2*sqrt(3)*sqrt(2)*x^3 + 1/2*sqrt(2)*(x^3 - 2*x)) + 1/2
*sqrt(2)*arctan(1/2*sqrt(3)*sqrt(2)*x + 1/2*sqrt(2)*x) + 1/4*sqrt(2)*log((x
^8 + 4*x^6 + 5*x^4 + 4*x^2 - sqrt(2)*(x^7 + 4*x^5 + 4*x^3 + x) - sqrt(3)*(2
*x^6 + 4*x^4 + 2*x^2 - sqrt(2)*(x^7 + 2*x^5 + 2*x^3 + x)) + 1)/(x^8 - x^4 +
 1))
```

Sympy [A]
time = 0.50, size = 163, normalized size = 1.21

$$\frac{\sqrt{2}\cdot\left(2\operatorname{atan}\left(x\left(\frac{\sqrt{6}}{1+\sqrt{3}}+\frac{2\sqrt{2}}{1+\sqrt{3}}\right)\right)+2\operatorname{atan}\left(x^3\left(\frac{\sqrt{6}}{1+\sqrt{3}}+\frac{2\sqrt{2}}{1+\sqrt{3}}\right)-\sqrt{2}x\right)\right)}{4}-\frac{\sqrt{2}\log\left(x^2-\frac{\sqrt{2}x\left(\frac{2}{\sqrt{3}+2}+\frac{2\sqrt{3}}{\sqrt{3}+2}\right)}{4}+1\right)}{4}+\frac{\sqrt{2}\log\left(x^2+\frac{\sqrt{2}x\left(\frac{2}{\sqrt{3}+2}+\frac{2\sqrt{3}}{\sqrt{3}+2}\right)}{4}+1\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+2*x**4+3**(1/2))/(x**8-x**4+1),x)
```

```
[Out] sqrt(2)*(2*atan(x*(sqrt(6)/(1 + sqrt(3)) + 2*sqrt(2)/(1 + sqrt(3)))) + 2*at
an(x**3*(sqrt(6)/(1 + sqrt(3)) + 2*sqrt(2)/(1 + sqrt(3))) - sqrt(2)*x))/4 -
 sqrt(2)*log(x**2 - sqrt(2)*x*(2/(sqrt(3) + 2) + 2*sqrt(3)/(sqrt(3) + 2))/4
 + 1)/4 + sqrt(2)*log(x**2 + sqrt(2)*x*(2/(sqrt(3) + 2) + 2*sqrt(3)/(sqrt(3
) + 2))/4 + 1)/4
```

Giac [A]
time = 3.85, size = 107, normalized size = 0.79

$$\frac{1}{2}\sqrt{2}\arctan\left(\frac{4x+\sqrt{6}+\sqrt{2}}{\sqrt{6}-\sqrt{2}}\right)+\frac{1}{2}\sqrt{2}\arctan\left(\frac{4x-\sqrt{6}-\sqrt{2}}{\sqrt{6}-\sqrt{2}}\right)+\frac{1}{4}\sqrt{2}\log\left(x^2+\frac{1}{2}x\left(\sqrt{6}-\sqrt{2}\right)+1\right)-\frac{1}{4}\sqrt{2}\log\left(x^2-\frac{1}{2}x\left(\sqrt{6}-\sqrt{2}\right)+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+2*x^4+3^(1/2))/(x^8-x^4+1),x, algorithm="giac")
```

```
[Out] 1/2*sqrt(2)*arctan((4*x + sqrt(6) + sqrt(2))/(sqrt(6) - sqrt(2))) + 1/2*sqr
t(2)*arctan((4*x - sqrt(6) - sqrt(2))/(sqrt(6) - sqrt(2))) + 1/4*sqrt(2)*lo
g(x^2 + 1/2*x*(sqrt(6) - sqrt(2)) + 1) - 1/4*sqrt(2)*log(x^2 - 1/2*x*(sqrt(
6) - sqrt(2)) + 1)
```

Mupad [B]
time = 2.24, size = 133, normalized size = 0.99

$$\frac{\sqrt{2}\,\mathrm{atan}\left(\frac{72\sqrt{2}\,x}{144\sqrt{3}-144\sqrt{3}\,x^2-288\,x^2+288}+\frac{72\sqrt{2}\,\sqrt{3}\,x}{144\sqrt{3}-144\sqrt{3}\,x^2-288\,x^2+288}\right)}{2}+\frac{\sqrt{2}\,\mathrm{atanh}\left(\frac{72\sqrt{2}\,x}{144\sqrt{3}+144\sqrt{3}\,x^2+288\,x^2+288}+\frac{72\sqrt{2}\,\sqrt{3}\,x}{144\sqrt{3}+144\sqrt{3}\,x^2+288\,x^2+288}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3^(1/2) + 2*x^4 - 1)/(x^8 - x^4 + 1),x)
```

```
[Out] (2^(1/2)*atan((72*2^(1/2)*x)/(144*3^(1/2) - 144*3^(1/2)*x^2 - 288*x^2 + 288
) + (72*2^(1/2)*3^(1/2)*x)/(144*3^(1/2) - 144*3^(1/2)*x^2 - 288*x^2 + 288))
)/2 + (2^(1/2)*atanh((72*2^(1/2)*x)/(144*3^(1/2) + 144*3^(1/2)*x^2 + 288*x^
2 + 288) + (72*2^(1/2)*3^(1/2)*x)/(144*3^(1/2) + 144*3^(1/2)*x^2 + 288*x^2
+ 288)))/2
```

3.32 $\int \frac{1+\left(1+\sqrt{3}\right)x^4}{1-x^4+x^8}\,dx$

Optimal. Leaf size=164

$$-\frac{1}{2}\sqrt{2+\sqrt{3}}\,\tan^{-1}\left(\frac{\sqrt{2+\sqrt{3}}-2x}{\sqrt{2-\sqrt{3}}}\right)+\frac{1}{2}\sqrt{2+\sqrt{3}}\,\tan^{-1}\left(\frac{\sqrt{2+\sqrt{3}}+2x}{\sqrt{2-\sqrt{3}}}\right)-\frac{1}{4}\sqrt{2+\sqrt{3}}\,\log\Big($$

```
[Out] -1/2*arctan((-2*x+1/2*6^(1/2)+1/2*2^(1/2))/(1/2*6^(1/2)-1/2*2^(1/2)))*(1/2*
6^(1/2)+1/2*2^(1/2))+1/2*arctan((2*x+1/2*6^(1/2)+1/2*2^(1/2))/(1/2*6^(1/2)-
1/2*2^(1/2)))*(1/2*6^(1/2)+1/2*2^(1/2))-1/4*ln(1+x^2-x*(1/2*6^(1/2)-1/2*2^(
1/2)))*(1/2*6^(1/2)+1/2*2^(1/2))+1/4*ln(1+x^2+x*(1/2*6^(1/2)-1/2*2^(1/2)))*
(1/2*6^(1/2)+1/2*2^(1/2))
```

Rubi [A]
time = 0.06, antiderivative size = 164, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 6, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$,
Rules used = {1437, 1175, 632, 210, 1178, 642}

$$-\frac{1}{2}\sqrt{2+\sqrt{3}}\,\text{ArcTan}\left(\frac{\sqrt{2+\sqrt{3}}-2x}{\sqrt{2-\sqrt{3}}}\right)+\frac{1}{2}\sqrt{2+\sqrt{3}}\,\text{ArcTan}\left(\frac{2x+\sqrt{2+\sqrt{3}}}{\sqrt{2-\sqrt{3}}}\right)-\frac{1}{4}\sqrt{2+\sqrt{3}}\,\log\left(x^2-\sqrt{2-\sqrt{3}}\,x+1\right)+\frac{1}{4}\sqrt{2+\sqrt{3}}\,\log\left(x^2+\sqrt{2-\sqrt{3}}\,x+1\right)$$

Antiderivative was successfully verified.

```
[In] Int[(1 + (1 + Sqrt[3])*x^4)/(1 - x^4 + x^8),x]
```

```
[Out] -1/2*(Sqrt[2 + Sqrt[3]]*ArcTan[(Sqrt[2 + Sqrt[3]] - 2*x)/Sqrt[2 - Sqrt[3]]]
) + (Sqrt[2 + Sqrt[3]]*ArcTan[(Sqrt[2 + Sqrt[3]] + 2*x)/Sqrt[2 - Sqrt[3]]])
/2 - (Sqrt[2 + Sqrt[3]]*Log[1 - Sqrt[2 - Sqrt[3]]*x + x^2])/4 + (Sqrt[2 + S
qrt[3]]*Log[1 + Sqrt[2 - Sqrt[3]]*x + x^2])/4
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
```

```
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1175

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[2*(d/e) - b/c, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2
, x], x], x] + Dist[e/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; Fre
eQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - a*e^2, 0] && (
GtQ[2*(d/e) - b/c, 0] || ( !LtQ[2*(d/e) - b/c, 0] && EqQ[d - e*Rt[a/c, 2],
0]))
```

Rule 1178

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[-2*(d/e) - b/c, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e
+ q*x - x^2, x], x], x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^
2, x], x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c
*d^2 - a*e^2, 0] &&  !GtQ[b^2 - 4*a*c, 0]
```

Rule 1437

```
Int[((d_) + (e_.)*(x_)^(n_))/((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_)), x
_Symbol] :> With[{q = Rt[a/c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Dist[1/(2*c
*q*r), Int[(d*r - (d - e*q)*x^(n/2))/(q - r*x^(n/2) + x^n), x], x] + Dist[1
/(2*c*q*r), Int[(d*r + (d - e*q)*x^(n/2))/(q + r*x^(n/2) + x^n), x], x]]] /
; FreeQ[{a, b, c, d, e}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && NeQ[c
*d^2 - b*d*e + a*e^2, 0] && IGtQ[n/2, 0] && NegQ[b^2 - 4*a*c]
```

Rubi steps

$$\int \frac{1+\left(1+\sqrt{3}\right)x^4}{1-x^4+x^8}\,dx = \frac{\int \frac{\sqrt{3}+\sqrt{3}\,x^2}{1-\sqrt{3}\,x^2+x^4}\,dx}{2\sqrt{3}} + \frac{\int \frac{\sqrt{3}-\sqrt{3}\,x^2}{1+\sqrt{3}\,x^2+x^4}\,dx}{2\sqrt{3}}$$

$$= \frac{1}{4}\int \frac{1}{1-\sqrt{2+\sqrt{3}}\,x+x^2}\,dx + \frac{1}{4}\int \frac{1}{1+\sqrt{2+\sqrt{3}}\,x+x^2}\,dx - \frac{1}{4}\sqrt{2+\sqrt{3}}$$

$$= -\frac{1}{4}\sqrt{2+\sqrt{3}}\,\log\left(1-\sqrt{2-\sqrt{3}}\,x+x^2\right) + \frac{1}{4}\sqrt{2+\sqrt{3}}\,\log\left(1+\sqrt{2-\sqrt{3}}\right.$$

$$= -\frac{\tan^{-1}\left(\frac{\sqrt{2+\sqrt{3}}-2x}{\sqrt{2-\sqrt{3}}}\right)}{2\sqrt{2-\sqrt{3}}} + \frac{\tan^{-1}\left(\frac{\sqrt{2+\sqrt{3}}+2x}{\sqrt{2-\sqrt{3}}}\right)}{2\sqrt{2-\sqrt{3}}} - \frac{1}{4}\sqrt{2+\sqrt{3}}\,\log\left(1-\right.$$

Mathematica [C] Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.
time = 0.02, size = 72, normalized size = 0.44

$$\frac{1}{4}\text{RootSum}\left[1-\#1^4+\#1^8\&,\frac{\log(x-\#1)+\log(x-\#1)\#1^4+\sqrt{3}\log(x-\#1)\#1^4}{-\#1^3+2\#1^7}\&\right]$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + (1 + Sqrt[3])*x^4)/(1 - x^4 + x^8),x]
```

```
[Out] RootSum[1 - #1^4 + #1^8 & , (Log[x - #1] + Log[x - #1]*#1^4 + Sqrt[3]*Log[x
 - #1]*#1^4)/(-#1^3 + 2*#1^7) & ]/4
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 0.10, size = 62, normalized size = 0.38

method	result	size
default	$\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(\_Z^8-\_Z^4+1\right)}\frac{\left(2\_R^4+2\sqrt{3}\,\_R^4+\left(1+\sqrt{3}\right)\left(\sqrt{3}-1\right)\right)\ln\left(x-\_R\right)}{2\_R^7-\_R^3}\right)}{8}$	62

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+x^4*(1+3^(1/2)))/(x^8-x^4+1),x,method=_RETURNVERBOSE)
```

```
[Out] 1/8*sum(1/(2*_R^7-_R^3)*(2*_R^4+2*3^(1/2)*_R^4+(1+3^(1/2))*(3^(1/2)-1))*ln(
x-_R),_R=RootOf(_Z^8-_Z^4+1))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x^4*(1+3^(1/2)))/(x^8-x^4+1),x, algorithm="maxima")
```

```
[Out] integrate((x^4*(sqrt(3) + 1) + 1)/(x^8 - x^4 + 1), x)
```

Fricas [A]
time = 0.34, size = 158, normalized size = 0.96

$$-\frac{1}{2}\sqrt{\sqrt{3}+2}\arctan\left(-\left(x^3-\sqrt{3}x+x\right)\sqrt{\sqrt{3}+2}\right)+\frac{1}{2}\sqrt{\sqrt{3}+2}\arctan\left(x\sqrt{\sqrt{3}+2}\right)+\frac{1}{4}\sqrt{\sqrt{3}+2}\log\left(\frac{x^8+4x^6+5x^4+4x^2-2\sqrt{3}\left(x^6+2x^4+x^2\right)+2\left(2x^7+5x^5+5x^3-\sqrt{3}\left(x^7+3x^5+3x^3+x\right)+2x\right)\sqrt{\sqrt{3}+2}+1}{x^8-x^4+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x^4*(1+3^(1/2)))/(x^8-x^4+1),x, algorithm="fricas")
```

```
[Out] -1/2*sqrt(sqrt(3) + 2)*arctan(-(x^3 - sqrt(3)*x + x)*sqrt(sqrt(3) + 2)) + 1
/2*sqrt(sqrt(3) + 2)*arctan(x*sqrt(sqrt(3) + 2)) + 1/4*sqrt(sqrt(3) + 2)*lo
g((x^8 + 4*x^6 + 5*x^4 + 4*x^2 - 2*sqrt(3)*(x^6 + 2*x^4 + x^2) + 2*(2*x^7 +
 5*x^5 + 5*x^3 - sqrt(3)*(x^7 + 3*x^5 + 3*x^3 + x) + 2*x)*sqrt(sqrt(3) + 2)
 + 1)/(x^8 - x^4 + 1))
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: PolynomialError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x**4*(1+3**(1/2)))/(x**8-x**4+1),x)
```

```
[Out] Exception raised: PolynomialError >> 1/(23946700083803759802903559826903258
1075191976715165250684200040290318941159424*_t**88 + 138256337395873345762
803423705330731641326126160751478072830556473063127384064*sqrt(3)*_t**88 - 5
732624312622
```

Giac [A]
time = 4.07, size = 123, normalized size = 0.75

$$\frac{1}{4}\left(\sqrt{6}+\sqrt{2}\right)\arctan\left(\frac{4x+\sqrt{6}+\sqrt{2}}{\sqrt{6}-\sqrt{2}}\right)+\frac{1}{4}\left(\sqrt{6}+\sqrt{2}\right)\arctan\left(\frac{4x-\sqrt{6}-\sqrt{2}}{\sqrt{6}-\sqrt{2}}\right)+\frac{1}{8}\left(\sqrt{6}+\sqrt{2}\right)\log\left(x^2+\frac{1}{2}x\left(\sqrt{6}-\sqrt{2}\right)+1\right)-\frac{1}{8}\left(\sqrt{6}+\sqrt{2}\right)\log\left(x^2-\frac{1}{2}x\left(\sqrt{6}-\sqrt{2}\right)+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x^4*(1+3^(1/2)))/(x^8-x^4+1),x, algorithm="giac")
```

```
[Out] 1/4*(sqrt(6) + sqrt(2))*arctan((4*x + sqrt(6) + sqrt(2))/(sqrt(6) - sqrt(2)
)) + 1/4*(sqrt(6) + sqrt(2))*arctan((4*x - sqrt(6) - sqrt(2))/(sqrt(6) - sq
rt(2))) + 1/8*(sqrt(6) + sqrt(2))*log(x^2 + 1/2*x*(sqrt(6) - sqrt(2)) + 1)
- 1/8*(sqrt(6) + sqrt(2))*log(x^2 - 1/2*x*(sqrt(6) - sqrt(2)) + 1)
```

Mupad [B]
time = 2.19, size = 1, normalized size = 0.01

$$0$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4*(3^(1/2) + 1) + 1)/(x^8 - x^4 + 1),x)
```

```
[Out] 0
```

3.33 $\int \frac{3-2\sqrt{3}+\left(-3+\sqrt{3}\right)x^4}{1-x^4+x^8}\,dx$

Optimal. Leaf size=180

$$\frac{1}{2}\sqrt{3\left(2-\sqrt{3}\right)}\tan^{-1}\left(\frac{\sqrt{2+\sqrt{3}}-2x}{\sqrt{2-\sqrt{3}}}\right)-\frac{1}{2}\sqrt{3\left(2-\sqrt{3}\right)}\tan^{-1}\left(\frac{\sqrt{2+\sqrt{3}}+2x}{\sqrt{2-\sqrt{3}}}\right)+\frac{1}{4}\sqrt{3\left(2-\right.}$$

```
[Out] 1/2*arctan((-2*x+1/2*6^(1/2)+1/2*2^(1/2))/(1/2*6^(1/2)-1/2*2^(1/2)))*(3/2*2
^(1/2)-1/2*6^(1/2))-1/2*arctan((2*x+1/2*6^(1/2)+1/2*2^(1/2))/(1/2*6^(1/2)-1
/2*2^(1/2)))*(3/2*2^(1/2)-1/2*6^(1/2))+1/4*ln(1+x^2-x*(1/2*6^(1/2)-1/2*2^(1
/2)))*(3/2*2^(1/2)-1/2*6^(1/2))-1/4*ln(1+x^2+x*(1/2*6^(1/2)-1/2*2^(1/2)))*(
3/2*2^(1/2)-1/2*6^(1/2))
```

Rubi [A]

time = 0.09, antiderivative size = 180, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 6, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {1437, 1175, 632, 210, 1178, 642}

$$\frac{1}{2}\sqrt{3\left(2-\sqrt{3}\right)}\,\text{ArcTan}\left(\frac{\sqrt{2+\sqrt{3}}-2x}{\sqrt{2-\sqrt{3}}}\right)-\frac{1}{2}\sqrt{3\left(2-\sqrt{3}\right)}\,\text{ArcTan}\left(\frac{2x+\sqrt{2+\sqrt{3}}}{\sqrt{2-\sqrt{3}}}\right)+\frac{1}{4}\sqrt{3\left(2-\sqrt{3}\right)}\log\left(x^2-\sqrt{2-\sqrt{3}}\,x+1\right)-\frac{1}{4}\sqrt{3\left(2-\sqrt{3}\right)}\log\left(x^2+\sqrt{2-\sqrt{3}}\,x+1\right)$$

Antiderivative was successfully verified.

```
[In] Int[(3 - 2*Sqrt[3] + (-3 + Sqrt[3])*x^4)/(1 - x^4 + x^8),x]
```

```
[Out] (Sqrt[3*(2 - Sqrt[3])]*ArcTan[(Sqrt[2 + Sqrt[3]] - 2*x)/Sqrt[2 - Sqrt[3]]])
/2 - (Sqrt[3*(2 - Sqrt[3])]*ArcTan[(Sqrt[2 + Sqrt[3]] + 2*x)/Sqrt[2 - Sqrt[
3]]])/2 + (Sqrt[3*(2 - Sqrt[3])]*Log[1 - Sqrt[2 - Sqrt[3]]*x + x^2])/4 - (S
qrt[3*(2 - Sqrt[3])]*Log[1 + Sqrt[2 - Sqrt[3]]*x + x^2])/4
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
```

```
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1175

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[2*(d/e) - b/c, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2
, x], x], x] + Dist[e/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; Fre
eQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - a*e^2, 0] && (
GtQ[2*(d/e) - b/c, 0] || ( !LtQ[2*(d/e) - b/c, 0] && EqQ[d - e*Rt[a/c, 2],
0]))
```

Rule 1178

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[-2*(d/e) - b/c, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e
+ q*x - x^2, x], x], x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^
2, x], x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c
*d^2 - a*e^2, 0] &&  !GtQ[b^2 - 4*a*c, 0]
```

Rule 1437

```
Int[((d_) + (e_.)*(x_)^(n_))/((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_)), x
_Symbol] :> With[{q = Rt[a/c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Dist[1/(2*c
*q*r), Int[(d*r - (d - e*q)*x^(n/2))/(q - r*x^(n/2) + x^n), x], x] + Dist[1
/(2*c*q*r), Int[(d*r + (d - e*q)*x^(n/2))/(q + r*x^(n/2) + x^n), x], x]]] /
; FreeQ[{a, b, c, d, e}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && NeQ[c
*d^2 - b*d*e + a*e^2, 0] && IGtQ[n/2, 0] && NegQ[b^2 - 4*a*c]
```

Rubi steps

$$\begin{aligned}\int \frac{3-2\sqrt{3}+\left(-3+\sqrt{3}\right)x^4}{1-x^4+x^8}\,dx &= \frac{\int \frac{\sqrt{3}\left(3-2\sqrt{3}\right)+\left(-6+3\sqrt{3}\right)x^2}{1-\sqrt{3}\,x^2+x^4}\,dx}{2\sqrt{3}}+\frac{\int \frac{\sqrt{3}\left(3-2\sqrt{3}\right)+\left(6-3\sqrt{3}\right)x^2}{1+\sqrt{3}\,x^2+x^4}\,dx}{2\sqrt{3}}\\ &= \frac{1}{4}\sqrt{3\left(2-\sqrt{3}\right)}\int \frac{\sqrt{2-\sqrt{3}}+2x}{-1-\sqrt{2-\sqrt{3}}\,x-x^2}\,dx+\frac{1}{4}\sqrt{3\left(2-\sqrt{3}\right)}\int\\ &= \frac{1}{4}\sqrt{3\left(2-\sqrt{3}\right)}\log\left(1-\sqrt{2-\sqrt{3}}\,x+x^2\right)-\frac{1}{4}\sqrt{3\left(2-\sqrt{3}\right)}\log\\ &= \frac{1}{2}\sqrt{6-3\sqrt{3}}\tan^{-1}\left(\frac{\sqrt{2+\sqrt{3}}-2x}{\sqrt{2-\sqrt{3}}}\right)-\frac{1}{2}\sqrt{6-3\sqrt{3}}\tan^{-1}\left(\sqrt{\right.\end{aligned}$$

Mathematica [C] Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.
time = 0.03, size = 89, normalized size = 0.49

$$\frac{1}{4}\text{RootSum}\left[1-\#1^4+\#1^8\&,\frac{3\log(x-\#1)-2\sqrt{3}\log(x-\#1)-3\log(x-\#1)\#1^4+\sqrt{3}\log(x-\#1)\#1^4}{-\#1^3+2\#1^7}\&\right]$$

Antiderivative was successfully verified.

```
[In] Integrate[(3 - 2*Sqrt[3] + (-3 + Sqrt[3])*x^4)/(1 - x^4 + x^8),x]
```

```
[Out] RootSum[1 - #1^4 + #1^8 & , (3*Log[x - #1] - 2*Sqrt[3]*Log[x - #1] - 3*Log[
x - #1]*#1^4 + Sqrt[3]*Log[x - #1]*#1^4)/(-#1^3 + 2*#1^7) & ]/4
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 0.09, size = 62, normalized size = 0.34

method	result	size
default	$\frac{\left(\sum_{\_R=\text{RootOf}\left(\_Z^8-\_Z^4+1\right)}\frac{\left(-6\_R^4+2\sqrt{3}\_R^4+\left(-3+\sqrt{3}\right)\left(\sqrt{3}-1\right)\right)\ln\left(x-\_R\right)}{2\_R^7-\_R^3}\right)}{8}$	62

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3+x^4*(-3+3^(1/2))-2*3^(1/2))/(x^8-x^4+1),x,method=_RETURNVERBOSE)
```

```
[Out] 1/8*sum(1/(2*_R^7-_R^3)*(-6*_R^4+2*3^(1/2)*_R^4+(-3+3^(1/2))*(3^(1/2)-1))*l
n(x-_R),_R=RootOf(_Z^8-_Z^4+1))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3+x^4*(-3+3^(1/2))-2*3^(1/2))/(x^8-x^4+1),x, algorithm="maxima")
```

```
[Out] integrate((x^4*(sqrt(3) - 3) - 2*sqrt(3) + 3)/(x^8 - x^4 + 1), x)
```

Fricas [A]
time = 0.37, size = 181, normalized size = 1.01

$$-\frac{1}{2}\sqrt{-3\sqrt{3}+6}\arctan\left(\frac{1}{3}\left(3x^3+\sqrt{3}\left(2x^3-x\right)-3x\right)\sqrt{-3\sqrt{3}+6}\right)-\frac{1}{2}\sqrt{-3\sqrt{3}+6}\arctan\left(\frac{1}{3}\left(2\sqrt{3}x+3x\right)\sqrt{-3\sqrt{3}+6}\right)+\frac{1}{4}\sqrt{-3\sqrt{3}+6}\log\left(\frac{3x^8+12x^6+15x^4+12x^2-6\sqrt{3}\left(x^6+2x^4+x^2\right)+2\left(3x^5+3x^3-\sqrt{3}\left(x^7+x^5+x^3+x\right)\right)\sqrt{-3\sqrt{3}+6}+3}{x^8-x^4+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3+x^4*(-3+3^(1/2))-2*3^(1/2))/(x^8-x^4+1),x, algorithm="fricas")
```

```
[Out] -1/2*sqrt(-3*sqrt(3) + 6)*arctan(1/3*(3*x^3 + sqrt(3)*(2*x^3 - x) - 3*x)*sq
rt(-3*sqrt(3) + 6)) - 1/2*sqrt(-3*sqrt(3) + 6)*arctan(1/3*(2*sqrt(3)*x + 3*
x)*sqrt(-3*sqrt(3) + 6)) + 1/4*sqrt(-3*sqrt(3) + 6)*log((3*x^8 + 12*x^6 + 1
5*x^4 + 12*x^2 - 6*sqrt(3)*(x^6 + 2*x^4 + x^2) + 2*(3*x^5 + 3*x^3 - sqrt(3)
*(x^7 + x^5 + x^3 + x))*sqrt(-3*sqrt(3) + 6) + 3)/(x^8 - x^4 + 1))
```

Sympy [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: PolynomialError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3+x**4*(-3+3**(1/2))-2*3**(1/2))/(x**8-x**4+1),x)
```

```
[Out] Exception raised: PolynomialError >> 1/(-369443695440637751966679695 36*_t**
32 + 21329841701306232282053345280*sqrt(3)*_t**32 - 1671110831730367838030 8
7978496*sqrt(3)*_t**28 + 28944488656356818274074021068 8*_t**28 - 9921139603
646460044679
```

Giac [**A**]
time = 3.95, size = 131, normalized size = 0.73

$$\frac{1}{4}\left(\sqrt{6}-3\sqrt{2}\right)\arctan\left(\frac{4x+\sqrt{6}+\sqrt{2}}{\sqrt{6}-\sqrt{2}}\right)+\frac{1}{4}\left(\sqrt{6}-3\sqrt{2}\right)\arctan\left(\frac{4x-\sqrt{6}-\sqrt{2}}{\sqrt{6}-\sqrt{2}}\right)+\frac{1}{8}\left(\sqrt{6}-3\sqrt{2}\right)\log\left(x^2+\frac{1}{2}x\left(\sqrt{6}-\sqrt{2}\right)+1\right)-\frac{1}{8}\left(\sqrt{6}-3\sqrt{2}\right)\log\left(x^2-\frac{1}{2}x\left(\sqrt{6}-\sqrt{2}\right)+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3+x^4*(-3+3^(1/2))-2*3^(1/2))/(x^8-x^4+1),x, algorithm="giac")
```

```
[Out] 1/4*(sqrt(6) - 3*sqrt(2))*arctan((4*x + sqrt(6) + sqrt(2))/(sqrt(6) - sqrt(
2))) + 1/4*(sqrt(6) - 3*sqrt(2))*arctan((4*x - sqrt(6) - sqrt(2))/(sqrt(6)
- sqrt(2))) + 1/8*(sqrt(6) - 3*sqrt(2))*log(x^2 + 1/2*x*(sqrt(6) - sqrt(2))
 + 1) - 1/8*(sqrt(6) - 3*sqrt(2))*log(x^2 - 1/2*x*(sqrt(6) - sqrt(2)) + 1)
```

Mupad [**B**]
time = 2.23, size = 1, normalized size = 0.01

$$0$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4*(3^(1/2) - 3) - 2*3^(1/2) + 3)/(x^8 - x^4 + 1),x)
```

```
[Out] 0
```

3.34 $\int \frac{d+\frac{e}{x}}{c+\frac{a}{x^2}}\,dx$

Optimal. Leaf size=49

$$\frac{dx}{c}-\frac{\sqrt{a}\,d\tan^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{a}}\right)}{c^{3/2}}+\frac{e\log\left(a+cx^2\right)}{2c}$$

```
[Out] d*x/c+1/2*e*ln(c*x^2+a)/c-d*arctan(x*c^(1/2)/a^(1/2))*a^(1/2)/c^(3/2)
```

Rubi [A]
time = 0.02, antiderivative size = 49, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.294, Rules used = {1408, 788, 649, 211, 266}

$$-\frac{\sqrt{a}\,d\text{ArcTan}\left(\frac{\sqrt{c}\,x}{\sqrt{a}}\right)}{c^{3/2}}+\frac{e\log\left(a+cx^2\right)}{2c}+\frac{dx}{c}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e/x)/(c + a/x^2),x]
```

```
[Out] (d*x)/c - (Sqrt[a]*d*ArcTan[(Sqrt[c]*x)/Sqrt[a]])/c^(3/2) + (e*Log[a + c*x^
2])/(2*c)
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 649

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[(-a)*c]
```

Rule 788

```
Int[(((d_.) + (e_.)*(x_))*((f_) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2), x_Sym
bol] :> Simp[e*g*(x/c), x] + Dist[1/c, Int[(c*d*f - a*e*g + c*(e*f + d*g)*x
)/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e, f, g}, x]
```

Rule 1408

```
Int[((a_) + (c_.)*(x_)^(n2_.))^(p_.)*((d_) + (e_.)*(x_)^(n_))^(q_.), x_Symb
ol] :> Int[x^(n*(2*p + q))*(e + d/x^n)^q*(c + a/x^(2*n))^p, x] /; FreeQ[{a,
 c, d, e, n}, x] && EqQ[n2, 2*n] && IntegersQ[p, q] && NegQ[n]
```

Rubi steps

$$
\begin{aligned}
\int \frac{d+\frac{e}{x}}{c+\frac{a}{x^2}}\,dx &= \int \frac{x(e+dx)}{a+cx^2}\,dx \\
&= \frac{dx}{c}+\frac{\int \frac{-ad+cex}{a+cx^2}\,dx}{c} \\
&= \frac{dx}{c}-\frac{(ad)\int \frac{1}{a+cx^2}\,dx}{c}+e\int \frac{x}{a+cx^2}\,dx \\
&= \frac{dx}{c}-\frac{\sqrt{a}\,d\tan^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{a}}\right)}{c^{3/2}}+\frac{e\log\left(a+cx^2\right)}{2c}
\end{aligned}
$$

Mathematica [A]
time = 0.02, size = 49, normalized size = 1.00

$$
\frac{dx}{c}-\frac{\sqrt{a}\,d\tan^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{a}}\right)}{c^{3/2}}+\frac{e\log\left(a+cx^2\right)}{2c}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e/x)/(c + a/x^2),x]
```

```
[Out] (d*x)/c - (Sqrt[a]*d*ArcTan[(Sqrt[c]*x)/Sqrt[a]])/c^(3/2) + (e*Log[a + c*x^
2])/(2*c)
```

Maple [A]
time = 0.02, size = 42, normalized size = 0.86

method	result	si
default	$\frac{dx}{c}+\frac{\frac{e\ln\left(c\,x^2+a\right)}{2}-\frac{ad\arctan\left(\frac{cx}{\sqrt{ac}}\right)}{\sqrt{ac}}}{c}$	4
risch	$\frac{dx}{c}+\frac{\ln\left(-\sqrt{-ac}\,x-a\right)d\sqrt{-ac}}{2c^2}+\frac{\ln\left(-\sqrt{-ac}\,x-a\right)e}{2c}-\frac{\ln\left(\sqrt{-ac}\,x-a\right)d\sqrt{-ac}}{2c^2}+\frac{\ln\left(\sqrt{-ac}\,x-a\right)e}{2c}$	9

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d+e/x)/(c+a/x^2),x,method=_RETURNVERBOSE)
```

```
[Out] d*x/c+1/c*(1/2*e*ln(c*x^2+a)-a*d/(a*c)^(1/2)*arctan(c*x/(a*c)^(1/2)))
```

Maxima [A]
time = 0.50, size = 43, normalized size = 0.88

$$-\frac{ad\arctan\left(\frac{cx}{\sqrt{ac}}\right)}{\sqrt{ac}\,c}+\frac{dx}{c}+\frac{e\log\left(cx^2+a\right)}{2\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+e/x)/(c+a/x^2),x, algorithm="maxima")
```

```
[Out] -a*d*arctan(c*x/sqrt(a*c))/(sqrt(a*c)*c) + d*x/c + 1/2*e*log(c*x^2 + a)/c
```

Fricas [A]
time = 0.34, size = 110, normalized size = 2.24

$$\left[\frac{d\sqrt{-\frac{a}{c}}\log\left(\frac{cx^2-2\,cx\sqrt{-\frac{a}{c}}-a}{cx^2+a}\right)+2\,dx+e\log\left(cx^2+a\right)}{2\,c},\,-\frac{2\,d\sqrt{\frac{a}{c}}\arctan\left(\frac{cx\sqrt{\frac{a}{c}}}{a}\right)-2\,dx-e\log\left(cx^2+a\right)}{2\,c}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+e/x)/(c+a/x^2),x, algorithm="fricas")
```

```
[Out] [1/2*(d*sqrt(-a/c)*log((c*x^2 - 2*c*x*sqrt(-a/c) - a)/(c*x^2 + a)) + 2*d*x
+ e*log(c*x^2 + a))/c, -1/2*(2*d*sqrt(a/c)*arctan(c*x*sqrt(a/c)/a) - 2*d*x
- e*log(c*x^2 + a))/c]
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 112 vs. $2(42) = 84$.
time = 0.13, size = 112, normalized size = 2.29

$$\left(\frac{e}{2c}-\frac{d\sqrt{-ac^3}}{2c^3}\right)\log\left(x+\frac{-2c\left(\frac{e}{2c}-\frac{d\sqrt{-ac^3}}{2c^3}\right)+e}{d}\right)+\left(\frac{e}{2c}+\frac{d\sqrt{-ac^3}}{2c^3}\right)\log\left(x+\frac{-2c\left(\frac{e}{2c}+\frac{d\sqrt{-ac^3}}{2c^3}\right)+e}{d}\right)+\frac{dx}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+e/x)/(c+a/x**2),x)
```

```
[Out] (e/(2*c) - d*sqrt(-a*c**3)/(2*c**3))*log(x + (-2*c*(e/(2*c) - d*sqrt(-a*c**
3)/(2*c**3)) + e)/d) + (e/(2*c) + d*sqrt(-a*c**3)/(2*c**3))*log(x + (-2*c*(
e/(2*c) + d*sqrt(-a*c**3)/(2*c**3)) + e)/d) + d*x/c
```

Giac [**A**]
time = 4.53, size = 43, normalized size = 0.88

$$-\frac{ad\arctan\left(\frac{cx}{\sqrt{ac}}\right)}{\sqrt{ac}\,c}+\frac{dx}{c}+\frac{e\log\left(cx^2+a\right)}{2\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+e/x)/(c+a/x^2),x, algorithm="giac")
```

```
[Out] -a*d*arctan(c*x/sqrt(a*c))/(sqrt(a*c)*c) + d*x/c + 1/2*e*log(c*x^2 + a)/c
```

Mupad [**B**]
time = 1.59, size = 39, normalized size = 0.80

$$\frac{e\,\ln\left(c\,x^2+a\right)}{2\,c}+\frac{d\,x}{c}-\frac{\sqrt{a}\,\,d\,\mathrm{atan}\left(\frac{\sqrt{c}\,\,x}{\sqrt{a}}\right)}{c^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e/x)/(c + a/x^2),x)
```

```
[Out] (e*log(a + c*x^2))/(2*c) + (d*x)/c - (a^(1/2)*d*atan((c^(1/2)*x)/a^(1/2)))/
c^(3/2)
```

3.35 $\int \frac{d+\frac{e}{x}}{c+\frac{a}{x^2}+\frac{b}{x}}\,dx$

Optimal. Leaf size=86

$$\frac{dx}{c}-\frac{(b^2d-2acd-bce)\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{c^2\sqrt{b^2-4ac}}-\frac{(bd-ce)\log(a+bx+cx^2)}{2c^2}$$

```
[Out] d*x/c-1/2*(b*d-c*e)*ln(c*x^2+b*x+a)/c^2-(-2*a*c*d+b^2*d-b*c*e)*arctanh((2*c
*x+b)/(-4*a*c+b^2)^(1/2))/c^2/(-4*a*c+b^2)^(1/2)
```

Rubi [A]
time = 0.06, antiderivative size = 86, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {1407, 787, 648, 632, 212, 642}

$$-\frac{(-2acd+b^2d-bce)\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{c^2\sqrt{b^2-4ac}}-\frac{(bd-ce)\log(a+bx+cx^2)}{2c^2}+\frac{dx}{c}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e/x)/(c + a/x^2 + b/x),x]
```

```
[Out] (d*x)/c - ((b^2*d - 2*a*c*d - b*c*e)*ArcTanh[(b + 2*c*x)/Sqrt[b^2 - 4*a*c]]
)/(c^2*Sqrt[b^2 - 4*a*c]) - ((b*d - c*e)*Log[a + b*x + c*x^2])/(2*c^2)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 787

```
Int[(((d_.) + (e_.)*(x_))*((f_) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) + (c_.)*
(x_)^2), x_Symbol] :> Simp[e*g*(x/c), x] + Dist[1/c, Int[(c*d*f - a*e*g + (
c*e*f + c*d*g - b*e*g)*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e
, f, g}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 1407

```
Int[((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.)*((d_) + (e_.)*(x_)^(
n_))^(q_.), x_Symbol] :> Int[x^(n*(2*p + q))*(e + d/x^n)^q*(c + b/x^n + a/x
^(2*n))^p, x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[n2, 2*n] && IntegersQ[
p, q] && NegQ[n]
```

Rubi steps

$$\begin{aligned}\int \frac{d+\frac{e}{x}}{c+\frac{a}{x^2}+\frac{b}{x}}\,dx &= \int \frac{x(e+dx)}{a+bx+cx^2}\,dx\\
&= \frac{dx}{c}+\frac{\int \frac{-ad+(-bd+ce)x}{a+bx+cx^2}\,dx}{c}\\
&= \frac{dx}{c}-\frac{(bd-ce)\int \frac{b+2cx}{a+bx+cx^2}\,dx}{2c^2}+\frac{(b^2d-2acd-bce)\int \frac{1}{a+bx+cx^2}\,dx}{2c^2}\\
&= \frac{dx}{c}-\frac{(bd-ce)\log\left(a+bx+cx^2\right)}{2c^2}-\frac{(b^2d-2acd-bce)\,\mathrm{Subst}\left(\int \frac{1}{b^2-4ac-x^2}\,dx,x,b+2cx\right)}{c^2}\\
&= \frac{dx}{c}-\frac{(b^2d-2acd-bce)\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{c^2\sqrt{b^2-4ac}}-\frac{(bd-ce)\log\left(a+bx+cx^2\right)}{2c^2}\end{aligned}$$

Mathematica [A]
time = 0.06, size = 86, normalized size = 1.00

$$\frac{2cdx+\frac{2(b^2d-2acd-bce)\tan^{-1}\left(\frac{b+2cx}{\sqrt{-b^2+4ac}}\right)}{\sqrt{-b^2+4ac}}+(-bd+ce)\log(a+x(b+cx))}{2c^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e/x)/(c + a/x^2 + b/x),x]
```

```
[Out] (2*c*d*x + (2*(b^2*d - 2*a*c*d - b*c*e)*ArcTan[(b + 2*c*x)/Sqrt[-b^2 + 4*a*
c]])/Sqrt[-b^2 + 4*a*c] + (-(b*d) + c*e)*Log[a + x*(b + c*x)])/(2*c^2)
```

Maple [A]
time = 0.06, size = 90, normalized size = 1.05

method	result
default	$\frac{dx}{c}+\frac{\frac{(-bd+ce)\ln\left(c\,x^{2}+bx+a\right)}{2c}+\frac{2\left(-ad-\frac{(-bd+ce)b}{2c}\right)\arctan\left(\frac{2cx+b}{\sqrt{4ac-b^{2}}}\right)}{\sqrt{4ac-b^{2}}}}{c}$
risch	$\frac{dx}{c}-\frac{2\ln\left(-8a^{2}c^{2}d+6a\,b^{2}cd-4ab\,c^{2}e-b^{4}d+b^{3}ce-2\sqrt{-\left(4ac-b^{2}\right)\left(2acd-b^{2}d+bce\right)^{2}}\,cx-\sqrt{-\left(4ac-b^{2}\right)\left(2\right.}\right.}{c\left(4ac-b^{2}\right)}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d+e/x)/(c+a/x^2+b/x),x,method=_RETURNVERBOSE)
```

```
[Out] d*x/c+1/c*(1/2*(-b*d+c*e)/c*ln(c*x^2+b*x+a)+2*(-a*d-1/2*(-b*d+c*e)*b/c)/(4*
a*c-b^2)^(1/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2)))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+e/x)/(c+a/x^2+b/x),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re deta
```

Fricas [A]
time = 0.39, size = 295, normalized size = 3.43

$$\left[\frac{2\,(b^{2}c-4\,ac^{2})dx+(bce-(b^{2}-2\,ac)d)\sqrt{b^{2}-4\,ac}\,\log\left(\frac{2\,c^{2}x^{2}+2\,bcx+b^{2}-2\,ac+\sqrt{b^{2}-4\,ac}\,(2\,cx+b)}{cx^{2}+bx+a}\right)-((b^{3}-4\,abc)d-(b^{2}c-4\,ac^{2})e)\log\left(cx^{2}+bx+a\right)}{2\,(b^{2}c^{2}-4\,ac^{3})},\ \frac{2\,(b^{2}c-4\,ac^{2})dx+2\,(bce-(b^{2}-2\,ac)d)\sqrt{-b^{2}+4\,ac}\,\arctan\left(-\frac{\sqrt{-b^{2}+4\,ac}\,(2\,cx+b)}{b^{2}-4\,ac}\right)-((b^{3}-4\,abc)d-(b^{2}c-4\,ac^{2})e)\log\left(cx^{2}+bx+a\right)}{2\,(b^{2}c^{2}-4\,ac^{3})}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+e/x)/(c+a/x^2+b/x),x, algorithm="fricas")
```

```
[Out] [1/2*(2*(b^2*c - 4*a*c^2)*d*x + (b*c*e - (b^2 - 2*a*c)*d)*sqrt(b^2 - 4*a*c)
*log((2*c^2*x^2 + 2*b*c*x + b^2 - 2*a*c + sqrt(b^2 - 4*a*c)*(2*c*x + b))/(c
```

```
*x^2 + b*x + a)) - ((b^3 - 4*a*b*c)*d - (b^2*c - 4*a*c^2)*e)*log(c*x^2 + b*
x + a))/(b^2*c^2 - 4*a*c^3), 1/2*(2*(b^2*c - 4*a*c^2)*d*x + 2*(b*c*e - (b^2
 - 2*a*c)*d)*sqrt(-b^2 + 4*a*c)*arctan(-sqrt(-b^2 + 4*a*c)*(2*c*x + b)/(b^2
 - 4*a*c)) - ((b^3 - 4*a*b*c)*d - (b^2*c - 4*a*c^2)*e)*log(c*x^2 + b*x + a)
)/(b^2*c^2 - 4*a*c^3)]
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 423 vs. $2(82) = 164$.
time = 0.78, size = 423, normalized size = 4.92

$$\left(-\frac{\sqrt{-4ac+b^2}\cdot(2acd-b^2d+bce)}{2c^2\cdot(4ac-b^2)}-\frac{bd-ce}{2c^2}\right)\log\left(x+\frac{-abd-4ac^2\left(-\frac{\sqrt{-4ac+b^2}\cdot(2acd-b^2d+bce)}{2c^2\cdot(4ac-b^2)}-\frac{bd-ce}{2c^2}\right)+2ace+b^2c\left(-\frac{\sqrt{-4ac+b^2}\cdot(2acd-b^2d+bce)}{2c^2\cdot(4ac-b^2)}-\frac{bd-ce}{2c^2}\right)}{2acd-b^2d+bce}\right)+\left(\frac{\sqrt{-4ac+b^2}\cdot(2acd-b^2d+bce)}{2c^2\cdot(4ac-b^2)}-\frac{bd-ce}{2c^2}\right)\log\left(x+\frac{-abd-4ac^2\left(\frac{\sqrt{-4ac+b^2}\cdot(2acd-b^2d+bce)}{2c^2\cdot(4ac-b^2)}-\frac{bd-ce}{2c^2}\right)+2ace+b^2c\left(\frac{\sqrt{-4ac+b^2}\cdot(2acd-b^2d+bce)}{2c^2\cdot(4ac-b^2)}-\frac{bd-ce}{2c^2}\right)}{2acd-b^2d+bce}\right)+\frac{dx}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+e/x)/(c+a/x**2+b/x),x)
```

```
[Out] (-sqrt(-4*a*c + b**2)*(2*a*c*d - b**2*d + b*c*e)/(2*c**2*(4*a*c - b**2)) -
(b*d - c*e)/(2*c**2))*log(x + (-a*b*d - 4*a*c**2*(-sqrt(-4*a*c + b**2)*(2*a
*c*d - b**2*d + b*c*e)/(2*c**2*(4*a*c - b**2)) - (b*d - c*e)/(2*c**2)) + 2*
a*c*e + b**2*c*(-sqrt(-4*a*c + b**2)*(2*a*c*d - b**2*d + b*c*e)/(2*c**2*(4*
a*c - b**2)) - (b*d - c*e)/(2*c**2)))/(2*a*c*d - b**2*d + b*c*e)) + (sqrt(-
4*a*c + b**2)*(2*a*c*d - b**2*d + b*c*e)/(2*c**2*(4*a*c - b**2)) - (b*d - c
*e)/(2*c**2))*log(x + (-a*b*d - 4*a*c**2*(sqrt(-4*a*c + b**2)*(2*a*c*d - b*
*2*d + b*c*e)/(2*c**2*(4*a*c - b**2)) - (b*d - c*e)/(2*c**2)) + 2*a*c*e + b
**2*c*(sqrt(-4*a*c + b**2)*(2*a*c*d - b**2*d + b*c*e)/(2*c**2*(4*a*c - b**2
)) - (b*d - c*e)/(2*c**2)))/(2*a*c*d - b**2*d + b*c*e)) + d*x/c
```

Giac [A]
time = 4.02, size = 85, normalized size = 0.99

$$\frac{dx}{c}-\frac{(bd-ce)\log\left(cx^2+bx+a\right)}{2\,c^2}+\frac{\left(b^2d-2\,acd-bce\right)\arctan\left(\frac{2\,cx+b}{\sqrt{-b^2+4\,ac}}\right)}{\sqrt{-b^2+4\,ac}\,c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+e/x)/(c+a/x^2+b/x),x, algorithm="giac")
```

```
[Out] d*x/c - 1/2*(b*d - c*e)*log(c*x^2 + b*x + a)/c^2 + (b^2*d - 2*a*c*d - b*c*e
)*arctan((2*c*x + b)/sqrt(-b^2 + 4*a*c))/(sqrt(-b^2 + 4*a*c)*c^2)
```

Mupad [B]
time = 1.77, size = 127, normalized size = 1.48

$$\frac{\ln\left(c\,x^2+b\,x+a\right)\,\left(d\,b^3-e\,b^2\,c-4\,a\,d\,b\,c+4\,a\,e\,c^2\right)}{2\,\left(4\,a\,c^3-b^2\,c^2\right)}+\frac{d\,x}{c}-\frac{\mathrm{atan}\left(\frac{b}{\sqrt{4\,a\,c-b^2}}+\frac{2\,c\,x}{\sqrt{4\,a\,c-b^2}}\right)\,\left(-d\,b^2+c\,e\,b+2\,a\,c\,d\right)}{c^2\,\sqrt{4\,a\,c-b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e/x)/(c + a/x^2 + b/x),x)
```

```
[Out] (log(a + b*x + c*x^2)*(b^3*d + 4*a*c^2*e - b^2*c*e - 4*a*b*c*d))/(2*(4*a*c^
3 - b^2*c^2)) + (d*x)/c - (atan(b/(4*a*c - b^2)^(1/2) + (2*c*x)/(4*a*c - b^
2)^(1/2))*(2*a*c*d - b^2*d + b*c*e))/(c^2*(4*a*c - b^2)^(1/2))
```

3.36 $\int \frac{d+\frac{e}{x^2}}{c+\frac{a}{x^4}}\,dx$

Optimal. Leaf size=253

$$\frac{dx}{c}+\frac{\left(\sqrt{a}\,d-\sqrt{c}\,e\right)\tan^{-1}\left(1-\frac{\sqrt{2}\,\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right)}{2\sqrt{2}\,\sqrt[4]{a}\,c^{5/4}}-\frac{\left(\sqrt{a}\,d-\sqrt{c}\,e\right)\tan^{-1}\left(1+\frac{\sqrt{2}\,\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right)}{2\sqrt{2}\,\sqrt[4]{a}\,c^{5/4}}+\frac{\left(\sqrt{a}\,d+\sqrt{c}\,e\right)\log\left(\right.}{4}$$

```
[Out] d*x/c-1/4*arctan(-1+c^(1/4)*x*2^(1/2)/a^(1/4))*(d*a^(1/2)-e*c^(1/2))/a^(1/4
)/c^(5/4)*2^(1/2)-1/4*arctan(1+c^(1/4)*x*2^(1/2)/a^(1/4))*(d*a^(1/2)-e*c^(1
/2))/a^(1/4)/c^(5/4)*2^(1/2)+1/8*ln(-a^(1/4)*c^(1/4)*x*2^(1/2)+a^(1/2)+x^2*
c^(1/2))*(d*a^(1/2)+e*c^(1/2))/a^(1/4)/c^(5/4)*2^(1/2)-1/8*ln(a^(1/4)*c^(1/
4)*x*2^(1/2)+a^(1/2)+x^2*c^(1/2))*(d*a^(1/2)+e*c^(1/2))/a^(1/4)/c^(5/4)*2^(
1/2)
```

Rubi [A]

time = 0.14, antiderivative size = 253, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 8, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.471$, Rules used = {1408, 1294, 1182, 1176, 631, 210, 1179, 642}

$$\frac{\text{ArcTan}\left(1-\frac{\sqrt{2}\,\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right)\left(\sqrt{a}\,d-\sqrt{c}\,e\right)}{2\sqrt{2}\,\sqrt[4]{a}\,c^{5/4}}-\frac{\text{ArcTan}\left(\frac{\sqrt{2}\,\sqrt[4]{c}\,x}{\sqrt[4]{a}}+1\right)\left(\sqrt{a}\,d-\sqrt{c}\,e\right)}{2\sqrt{2}\,\sqrt[4]{a}\,c^{5/4}}+\frac{\left(\sqrt{a}\,d+\sqrt{c}\,e\right)\log\left(-\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{c}\,x+\sqrt{a}+\sqrt{c}\,x^2\right)}{4\sqrt{2}\,\sqrt[4]{a}\,c^{5/4}}-\frac{\left(\sqrt{a}\,d+\sqrt{c}\,e\right)\log\left(\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{c}\,x+\sqrt{a}+\sqrt{c}\,x^2\right)}{4\sqrt{2}\,\sqrt[4]{a}\,c^{5/4}}+\frac{dx}{c}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e/x^2)/(c + a/x^4),x]
```

```
[Out] (d*x)/c + ((Sqrt[a]*d - Sqrt[c]*e)*ArcTan[1 - (Sqrt[2]*c^(1/4)*x)/a^(1/4)])
/(2*Sqrt[2]*a^(1/4)*c^(5/4)) - ((Sqrt[a]*d - Sqrt[c]*e)*ArcTan[1 + (Sqrt[2]
*c^(1/4)*x)/a^(1/4)])/(2*Sqrt[2]*a^(1/4)*c^(5/4)) + ((Sqrt[a]*d + Sqrt[c]*e
)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/(4*Sqrt[2]*a^(1/4
)*c^(5/4)) - ((Sqrt[a]*d + Sqrt[c]*e)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*c^(1/4)
*x + Sqrt[c]*x^2])/(4*Sqrt[2]*a^(1/4)*c^(5/4))
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1182

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
a*c, 2]}, Dist[(d*q + a*e)/(2*a*c), Int[(q + c*x^2)/(a + c*x^4), x], x] + D
ist[(d*q - a*e)/(2*a*c), Int[(q - c*x^2)/(a + c*x^4), x], x]] /; FreeQ[{a,
c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[(-a)
*c]
```

Rule 1294

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (c_.)*(x_)^4)^(p_), x_
Symbol] :> Simp[e*f*(f*x)^(m - 1)*((a + c*x^4)^(p + 1)/(c*(m + 4*p + 3))),
x] - Dist[f^2/(c*(m + 4*p + 3)), Int[(f*x)^(m - 2)*(a + c*x^4)^p*(a*e*(m -
1) - c*d*(m + 4*p + 3)*x^2), x], x] /; FreeQ[{a, c, d, e, f, p}, x] && GtQ[
m, 1] && NeQ[m + 4*p + 3, 0] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m
])
```

Rule 1408

```
Int[((a_) + (c_.)*(x_)^(n2_.))^(p_.)*((d_) + (e_.)*(x_)^(n_))^(q_.), x_Symb
ol] :> Int[x^(n*(2*p + q))*(e + d/x^n)^q*(c + a/x^(2*n))^p, x] /; FreeQ[{a,
 c, d, e, n}, x] && EqQ[n2, 2*n] && IntegersQ[p, q] && NegQ[n]
```

Rubi steps

$$\int \frac{d+\frac{e}{x^2}}{c+\frac{a}{x^4}}\,dx = \int \frac{x^2(e+dx^2)}{a+cx^4}\,dx$$

$$= \frac{dx}{c} - \frac{\int \frac{ad-cex^2}{a+cx^4}\,dx}{c}$$

$$= \frac{dx}{c} - \frac{\left(\frac{\sqrt{a}\,d}{\sqrt{c}}-e\right)\int \frac{\sqrt{a}\,\sqrt{c}+cx^2}{a+cx^4}\,dx}{2c} - \frac{\left(\frac{\sqrt{a}\,d}{\sqrt{c}}+e\right)\int \frac{\sqrt{a}\,\sqrt{c}-cx^2}{a+cx^4}\,dx}{2c}$$

$$= \frac{dx}{c} - \frac{\left(\frac{\sqrt{a}\,d}{\sqrt{c}}-e\right)\int \frac{1}{\frac{\sqrt{a}}{\sqrt{c}}-\frac{\sqrt{2}\,\sqrt[4]{a}\,x}{\sqrt[4]{c}}+x^2}\,dx}{4c} - \frac{\left(\frac{\sqrt{a}\,d}{\sqrt{c}}-e\right)\int \frac{1}{\frac{\sqrt{a}}{\sqrt{c}}+\frac{\sqrt{2}\,\sqrt[4]{a}\,x}{\sqrt[4]{c}}+x^2}\,dx}{4c} + \frac{(\sqrt{a}}{}$$

$$= \frac{dx}{c} + \frac{\left(\sqrt{a}\,d+\sqrt{c}\,e\right)\log\left(\sqrt{a}-\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{c}\,x+\sqrt{c}\,x^2\right)}{4\sqrt{2}\,\sqrt[4]{a}\,c^{5/4}} - \frac{\left(\sqrt{a}\,d+\sqrt{c}\,e\right)\log\left(\sqrt{a}+\sqrt{2}\right.}{4\sqrt{2}\,\sqrt[4]{a}\,c^{5}}$$

$$= \frac{dx}{c} + \frac{\left(\sqrt{a}\,d-\sqrt{c}\,e\right)\tan^{-1}\left(1-\frac{\sqrt{2}\,\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right)}{2\sqrt{2}\,\sqrt[4]{a}\,c^{5/4}} - \frac{\left(\sqrt{a}\,d-\sqrt{c}\,e\right)\tan^{-1}\left(1+\frac{\sqrt{2}\,\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right)}{2\sqrt{2}\,\sqrt[4]{a}\,c^{5/4}} + \frac{(}{}$$

Mathematica [A]
time = 0.06, size = 293, normalized size = 1.16

$$\frac{dx}{c} + \frac{\left(-a^{5/4}\sqrt{c}\,d+a^{3/4}ce\right)\tan^{-1}\left(\frac{-\sqrt{2}\,\sqrt[4]{a}+2\sqrt[4]{c}\,x}{\sqrt{2}\,\sqrt[4]{a}}\right)}{2\sqrt{2}\,ac^{7/4}} + \frac{\left(-a^{5/4}\sqrt{c}\,d+a^{3/4}ce\right)\tan^{-1}\left(\frac{\sqrt{2}\,\sqrt[4]{a}+2\sqrt[4]{c}\,x}{\sqrt{2}\,\sqrt[4]{a}}\right)}{2\sqrt{2}\,ac^{7/4}} + \frac{\left(a^{5/4}\sqrt{c}\,d+a^{3/4}ce\right)\log\left(\sqrt{a}-\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{c}\,x+\sqrt{c}\,x^2\right)}{4\sqrt{2}\,ac^{7/4}} - \frac{\left(a^{5/4}\sqrt{c}\,d+a^{3/4}ce\right)\log\left(\sqrt{a}+\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{c}\,x+\sqrt{c}\,x^2\right)}{4\sqrt{2}\,ac^{7/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e/x^2)/(c + a/x^4),x]
```

```
[Out] (d*x)/c + ((-(a^(5/4)*Sqrt[c]*d) + a^(3/4)*c*e)*ArcTan[(-(Sqrt[2]*a^(1/4))
+ 2*c^(1/4)*x)/(Sqrt[2]*a^(1/4))])/(2*Sqrt[2]*a*c^(7/4)) + ((-(a^(5/4)*Sqrt
[c]*d) + a^(3/4)*c*e)*ArcTan[(Sqrt[2]*a^(1/4) + 2*c^(1/4)*x)/(Sqrt[2]*a^(1/
4))])/(2*Sqrt[2]*a*c^(7/4)) + ((a^(5/4)*Sqrt[c]*d + a^(3/4)*c*e)*Log[Sqrt[a
] - Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/(4*Sqrt[2]*a*c^(7/4)) - ((a^(
5/4)*Sqrt[c]*d + a^(3/4)*c*e)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqr
t[c]*x^2])/(4*Sqrt[2]*a*c^(7/4))
```

Maple [A]
time = 0.04, size = 211, normalized size = 0.83

method	result

risch	$\frac{dx}{c}+\frac{\sum_{\_R=\mathrm{RootOf}\left(c\_Z^4+a\right)}\frac{\left(\_R^2 ce-ad\right)\ln\left(x-\_R\right)}{\_R^3}}{4c^2}$
default	$\frac{dx}{c}+\frac{-\frac{d\left(\frac{a}{c}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{x^2+\left(\frac{a}{c}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{a}{c}}}{x^2-\left(\frac{a}{c}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{a}{c}}}\right)+2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{c}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{c}\right)^{\frac{1}{4}}}-1\right)\right)}{8}+\frac{e\sqrt{2}\left(\ln\left(\frac{x^2-\left(\frac{a}{c}\right)^{\frac{1}{4}}x\sqrt{2}\ldots}{x^2+\left(\frac{a}{c}\right)^{\frac{1}{4}}x\sqrt{2}\ldots}\right.\right.}{\ldots}}{c}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d+e/x^2)/(c+a/x^4),x,method=_RETURNVERBOSE)
```

```
[Out] d*x/c+1/c*(-1/8*d*(a/c)^(1/4)*2^(1/2)*(ln((x^2+(a/c)^(1/4)*x*2^(1/2)+(a/c)^
(1/2))/(x^2-(a/c)^(1/4)*x*2^(1/2)+(a/c)^(1/2)))+2*arctan(2^(1/2)/(a/c)^(1/4
)*x+1)+2*arctan(2^(1/2)/(a/c)^(1/4)*x-1))+1/8*e/(a/c)^(1/4)*2^(1/2)*(ln((x^
2-(a/c)^(1/4)*x*2^(1/2)+(a/c)^(1/2))/(x^2+(a/c)^(1/4)*x*2^(1/2)+(a/c)^(1/2)
))+2*arctan(2^(1/2)/(a/c)^(1/4)*x+1)+2*arctan(2^(1/2)/(a/c)^(1/4)*x-1)))
```

<u>Maxima</u> [A]
time = 0.50, size = 244, normalized size = 0.96

$$\frac{dx}{c}-\frac{\frac{2\sqrt{2}\left(a\sqrt{c}\,d-\sqrt{a}\,ce\right)\arctan\left(\frac{\sqrt{2}\left(2\sqrt{c}\,x+\sqrt{2}\,a^{\frac{1}{4}}c^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{a}\sqrt{c}}}\right)}{\sqrt{a}\sqrt{\sqrt{a}\sqrt{c}}\sqrt{c}}+\frac{2\sqrt{2}\left(a\sqrt{c}\,d-\sqrt{a}\,ce\right)\arctan\left(\frac{\sqrt{2}\left(2\sqrt{c}\,x-\sqrt{2}\,a^{\frac{1}{4}}c^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{a}\sqrt{c}}}\right)}{\sqrt{a}\sqrt{\sqrt{a}\sqrt{c}}\sqrt{c}}+\frac{\sqrt{2}\left(a\sqrt{c}\,d+\sqrt{a}\,ce\right)\log\left(\sqrt{c}\,x^2+\sqrt{2}\,a^{\frac{1}{4}}c^{\frac{1}{4}}x+\sqrt{a}\right)}{a^{\frac{3}{4}}c^{\frac{3}{4}}}-\frac{\sqrt{2}\left(a\sqrt{c}\,d+\sqrt{a}\,ce\right)\log\left(\sqrt{c}\,x^2-\sqrt{2}\,a^{\frac{1}{4}}c^{\frac{1}{4}}x+\sqrt{a}\right)}{a^{\frac{3}{4}}c^{\frac{3}{4}}}}{8c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+e/x^2)/(c+a/x^4),x, algorithm="maxima")
```

```
[Out] d*x/c - 1/8*(2*sqrt(2)*(a*sqrt(c)*d - sqrt(a)*c*e)*arctan(1/2*sqrt(2)*(2*sq
rt(c)*x + sqrt(2)*a^(1/4)*c^(1/4))/sqrt(sqrt(a)*sqrt(c)))/(sqrt(a)*sqrt(sqr
t(a)*sqrt(c))*sqrt(c)) + 2*sqrt(2)*(a*sqrt(c)*d - sqrt(a)*c*e)*arctan(1/2*s
qrt(2)*(2*sqrt(c)*x - sqrt(2)*a^(1/4)*c^(1/4))/sqrt(sqrt(a)*sqrt(c)))/(sqrt
(a)*sqrt(sqrt(a)*sqrt(c))*sqrt(c)) + sqrt(2)*(a*sqrt(c)*d + sqrt(a)*c*e)*lo
g(sqrt(c)*x^2 + sqrt(2)*a^(1/4)*c^(1/4)*x + sqrt(a))/(a^(3/4)*c^(3/4)) - sq
rt(2)*(a*sqrt(c)*d + sqrt(a)*c*e)*log(sqrt(c)*x^2 - sqrt(2)*a^(1/4)*c^(1/4)
*x + sqrt(a))/(a^(3/4)*c^(3/4)))/c
```

<u>Fricas</u> [B] Leaf count of result is larger than twice the leaf count of optimal. 726 vs. $2(176) = 352$.
time = 0.36, size = 726, normalized size = 2.87

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+e/x^2)/(c+a/x^4),x, algorithm="fricas")
```

```
[Out] 1/4*(c*sqrt((c^2*sqrt(-(a^2*d^4 - 2*a*c*d^2*e^2 + c^2*e^4)/(a*c^5)) + 2*d*e
)/c^2)*log(-a^2*d^4*x + c^2*x*e^4 + (a*c^4*sqrt(-(a^2*d^4 - 2*a*c*d^2*e^2 +
 c^2*e^4)/(a*c^5))*e + a^2*c*d^3 - a*c^2*d*e^2)*sqrt((c^2*sqrt(-(a^2*d^4 -
2*a*c*d^2*e^2 + c^2*e^4)/(a*c^5)) + 2*d*e)/c^2)) - c*sqrt((c^2*sqrt(-(a^2*d
^4 - 2*a*c*d^2*e^2 + c^2*e^4)/(a*c^5)) + 2*d*e)/c^2)*log(-a^2*d^4*x + c^2*x
*e^4 - (a*c^4*sqrt(-(a^2*d^4 - 2*a*c*d^2*e^2 + c^2*e^4)/(a*c^5))*e + a^2*c*
d^3 - a*c^2*d*e^2)*sqrt((c^2*sqrt(-(a^2*d^4 - 2*a*c*d^2*e^2 + c^2*e^4)/(a*c
^5)) + 2*d*e)/c^2)) - c*sqrt(-(c^2*sqrt(-(a^2*d^4 - 2*a*c*d^2*e^2 + c^2*e^4
)/(a*c^5)) - 2*d*e)/c^2)*log(-a^2*d^4*x + c^2*x*e^4 + (a*c^4*sqrt(-(a^2*d^4
 - 2*a*c*d^2*e^2 + c^2*e^4)/(a*c^5))*e - a^2*c*d^3 + a*c^2*d*e^2)*sqrt(-(c^
2*sqrt(-(a^2*d^4 - 2*a*c*d^2*e^2 + c^2*e^4)/(a*c^5)) - 2*d*e)/c^2)) + c*sqr
t(-(c^2*sqrt(-(a^2*d^4 - 2*a*c*d^2*e^2 + c^2*e^4)/(a*c^5)) - 2*d*e)/c^2)*lo
g(-a^2*d^4*x + c^2*x*e^4 - (a*c^4*sqrt(-(a^2*d^4 - 2*a*c*d^2*e^2 + c^2*e^4)
/(a*c^5))*e - a^2*c*d^3 + a*c^2*d*e^2)*sqrt(-(c^2*sqrt(-(a^2*d^4 - 2*a*c*d^
2*e^2 + c^2*e^4)/(a*c^5)) - 2*d*e)/c^2)) + 4*d*x)/c
```

Sympy [A]

time = 0.37, size = 109, normalized size = 0.43

$$\operatorname{RootSum}\left(256t^4ac^5 - 64t^2ac^3de + a^2d^4 + 2acd^2e^2 + c^2e^4, \left(t \mapsto t\log\left(x + \frac{-64t^3ac^4e - 4ta^2cd^3 + 12tac^2de^2}{a^2d^4 - c^2e^4}\right)\right)\right) + \frac{dx}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+e/x**2)/(c+a/x**4),x)
```

```
[Out] RootSum(256*_t**4*a*c**5 - 64*_t**2*a*c**3*d*e + a**2*d**4 + 2*a*c*d**2*e**
2 + c**2*e**4, Lambda(_t, _t*log(x + (-64*_t**3*a*c**4*e - 4*_t*a**2*c*d**3
 + 12*_t*a*c**2*d*e**2)/(a**2*d**4 - c**2*e**4)))) + d*x/c
```

Giac [A]

time = 3.59, size = 247, normalized size = 0.98

$$\frac{dx}{c} - \frac{\sqrt{2}\left((ac^3)^{\frac{1}{4}}acd - (ac^3)^{\frac{3}{4}}e\right)\arctan\left(\frac{\sqrt{2}\left(2x+\sqrt{2}\left(\frac{a}{c}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{c}\right)^{\frac{1}{4}}}\right)}{4ac^3} - \frac{\sqrt{2}\left((ac^3)^{\frac{1}{4}}acd - (ac^3)^{\frac{3}{4}}e\right)\arctan\left(\frac{\sqrt{2}\left(2x-\sqrt{2}\left(\frac{a}{c}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{c}\right)^{\frac{1}{4}}}\right)}{4ac^3} - \frac{\sqrt{2}\left((ac^3)^{\frac{1}{4}}acd + (ac^3)^{\frac{3}{4}}e\right)\log\left(x^2 + \sqrt{2}x\left(\frac{a}{c}\right)^{\frac{1}{4}} + \sqrt{\frac{a}{c}}\right)}{8ac^3} + \frac{\sqrt{2}\left((ac^3)^{\frac{1}{4}}acd + (ac^3)^{\frac{3}{4}}e\right)\log\left(x^2 - \sqrt{2}x\left(\frac{a}{c}\right)^{\frac{1}{4}} + \sqrt{\frac{a}{c}}\right)}{8ac^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+e/x^2)/(c+a/x^4),x, algorithm="giac")
```

```
[Out] d*x/c - 1/4*sqrt(2)*((a*c^3)^(1/4)*a*c*d - (a*c^3)^(3/4)*e)*arctan(1/2*sqrt
(2)*(2*x + sqrt(2)*(a/c)^(1/4))/(a/c)^(1/4))/(a*c^3) - 1/4*sqrt(2)*((a*c^3)
^(1/4)*a*c*d - (a*c^3)^(3/4)*e)*arctan(1/2*sqrt(2)*(2*x - sqrt(2)*(a/c)^(1/
4))/(a/c)^(1/4))/(a*c^3) - 1/8*sqrt(2)*((a*c^3)^(1/4)*a*c*d + (a*c^3)^(3/4)
*e)*log(x^2 + sqrt(2)*x*(a/c)^(1/4) + sqrt(a/c))/(a*c^3) + 1/8*sqrt(2)*((a*
c^3)^(1/4)*a*c*d + (a*c^3)^(3/4)*e)*log(x^2 - sqrt(2)*x*(a/c)^(1/4) + sqrt(
a/c))/(a*c^3)
```

Mupad [B]

time = 0.31, size = 555, normalized size = 2.19

$$\frac{d\,x}{c} - 2\,\mathrm{atanh}\left(\frac{8\,a^2\,c\,d^2\,x\,\sqrt{\frac{d^2\,\sqrt{-a\,c^5}}{16\,c^5}+\frac{d\,e}{8\,c^2}-\frac{e^2\,\sqrt{-a\,c^5}}{16\,a\,c^4}}}{2\,a^2\,d^2\,e-2\,a\,c\,e^3+\frac{2\,a^2\,d^3\,\sqrt{-a\,c^5}}{c^3}-\frac{2\,a\,d\,e^2\,\sqrt{-a\,c^5}}{c^2}} - \frac{8\,a\,c^2\,e^2\,x\,\sqrt{\frac{d^2\,\sqrt{-a\,c^5}}{16\,c^5}+\frac{d\,e}{8\,c^2}-\frac{e^2\,\sqrt{-a\,c^5}}{16\,a\,c^4}}}{2\,a^2\,d^2\,e-2\,a\,c\,e^3+\frac{2\,a^2\,d^3\,\sqrt{-a\,c^5}}{c^3}-\frac{2\,a\,d\,e^2\,\sqrt{-a\,c^5}}{c^2}}\right)\sqrt{\frac{a\,d^2\,\sqrt{-a\,c^5}-c\,e^2\,\sqrt{-a\,c^5}+2\,a\,c^3\,d\,e}{16\,a\,c^5}} - 2\,\mathrm{atanh}\left(\frac{8\,a^2\,c\,d^2\,x\,\sqrt{\frac{d\,e}{8\,c^2}-\frac{d^2\,\sqrt{-a\,c^5}}{16\,c^5}+\frac{e^2\,\sqrt{-a\,c^5}}{16\,a\,c^4}}}{2\,a^2\,d^2\,e-2\,a\,c\,e^3-\frac{2\,a^2\,d^3\,\sqrt{-a\,c^5}}{c^3}+\frac{2\,a\,d\,e^2\,\sqrt{-a\,c^5}}{c^2}} - \frac{8\,a\,c^2\,e^2\,x\,\sqrt{\frac{d\,e}{8\,c^2}-\frac{d^2\,\sqrt{-a\,c^5}}{16\,c^5}+\frac{e^2\,\sqrt{-a\,c^5}}{16\,a\,c^4}}}{2\,a^2\,d^2\,e-2\,a\,c\,e^3-\frac{2\,a^2\,d^3\,\sqrt{-a\,c^5}}{c^3}+\frac{2\,a\,d\,e^2\,\sqrt{-a\,c^5}}{c^2}}\right)\sqrt{\frac{c\,e^2\,\sqrt{-a\,c^5}-a\,d^2\,\sqrt{-a\,c^5}+2\,a\,c^3\,d\,e}{16\,a\,c^5}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d + e/x^2)/(c + a/x^4),x)

[Out] (d*x)/c - 2*atanh((8*a^2*c*d^2*x*((d^2*(-a*c^5)^(1/2))/(16*c^5) + (d*e)/(8*c^2) - (e^2*(-a*c^5)^(1/2))/(16*a*c^4))^(1/2))/(2*a^2*d^2*e - 2*a*c*e^3 + (2*a^2*d^3*(-a*c^5)^(1/2))/c^3 - (2*a*d*e^2*(-a*c^5)^(1/2))/c^2) - (8*a*c^2*e^2*x*((d^2*(-a*c^5)^(1/2))/(16*c^5) + (d*e)/(8*c^2) - (e^2*(-a*c^5)^(1/2))/(16*a*c^4))^(1/2))/(2*a^2*d^2*e - 2*a*c*e^3 + (2*a^2*d^3*(-a*c^5)^(1/2))/c^3 - (2*a*d*e^2*(-a*c^5)^(1/2))/c^2))*((a*d^2*(-a*c^5)^(1/2) - c*e^2*(-a*c^5)^(1/2) + 2*a*c^3*d*e)/(16*a*c^5))^(1/2) - 2*atanh((8*a^2*c*d^2*x*((d*e)/(8*c^2) - (d^2*(-a*c^5)^(1/2))/(16*c^5) + (e^2*(-a*c^5)^(1/2))/(16*a*c^4))^(1/2))/(2*a^2*d^2*e - 2*a*c*e^3 - (2*a^2*d^3*(-a*c^5)^(1/2))/c^3 + (2*a*d*e^2*(-a*c^5)^(1/2))/c^2) - (8*a*c^2*e^2*x*((d*e)/(8*c^2) - (d^2*(-a*c^5)^(1/2))/(16*c^5) + (e^2*(-a*c^5)^(1/2))/(16*a*c^4))^(1/2))/(2*a^2*d^2*e - 2*a*c*e^3 - (2*a^2*d^3*(-a*c^5)^(1/2))/c^3 + (2*a*d*e^2*(-a*c^5)^(1/2))/c^2))*((c*e^2*(-a*c^5)^(1/2) - a*d^2*(-a*c^5)^(1/2) + 2*a*c^3*d*e)/(16*a*c^5))^(1/2)

3.37 $\int \frac{d+\frac{e}{x^2}}{c+\frac{a}{x^4}+\frac{b}{x^2}}\,dx$

Optimal. Leaf size=208

$$\frac{dx}{c}-\frac{\left(bd-ce-\frac{b^2d-2acd-bce}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\,c^{3/2}\sqrt{b-\sqrt{b^2-4ac}}}-\frac{\left(bd-ce+\frac{b^2d-2acd-bce}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\,c^{3/2}\sqrt{b+\sqrt{b^2-4ac}}}$$

```
[Out] d*x/c-1/2*arctan(x*2^(1/2)*c^(1/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2))*(b*d-c*e+(
2*a*c*d-b^2*d+b*c*e)/(-4*a*c+b^2)^(1/2))/c^(3/2)*2^(1/2)/(b-(-4*a*c+b^2)^(1
/2))^(1/2)-1/2*arctan(x*2^(1/2)*c^(1/2)/(b+(-4*a*c+b^2)^(1/2))^(1/2))*(b*d-
c*e+(-2*a*c*d+b^2*d-b*c*e)/(-4*a*c+b^2)^(1/2))/c^(3/2)*2^(1/2)/(b+(-4*a*c+b
^2)^(1/2))^(1/2)
```

Rubi [A]
time = 0.37, antiderivative size = 208, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {1407, 1293, 1180, 211}

$$-\frac{\text{ArcTan}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)\left(-\frac{-2acd+b^2d-bce}{\sqrt{b^2-4ac}}+bd-ce\right)}{\sqrt{2}\,c^{3/2}\sqrt{b-\sqrt{b^2-4ac}}}-\frac{\text{ArcTan}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)\left(\frac{-2acd+b^2d-bce}{\sqrt{b^2-4ac}}+bd-ce\right)}{\sqrt{2}\,c^{3/2}\sqrt{\sqrt{b^2-4ac}+b}}+\frac{dx}{c}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e/x^2)/(c + a/x^4 + b/x^2),x]
```

```
[Out] (d*x)/c - ((b*d - c*e - (b^2*d - 2*a*c*d - b*c*e)/Sqrt[b^2 - 4*a*c])*ArcTan
[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*c^(3/2)*Sqrt[b
- Sqrt[b^2 - 4*a*c]]) - ((b*d - c*e + (b^2*d - 2*a*c*d - b*c*e)/Sqrt[b^2 -
4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*c
^(3/2)*Sqrt[b + Sqrt[b^2 - 4*a*c]])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 1180

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
```

```
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1293

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(
x_)^4)^(p_), x_Symbol] :> Simp[e*f*(f*x)^(m - 1)*((a + b*x^2 + c*x^4)^(p +
1)/(c*(m + 4*p + 3))), x] - Dist[f^2/(c*(m + 4*p + 3)), Int[(f*x)^(m - 2)*(
a + b*x^2 + c*x^4)^p*Simp[a*e*(m - 1) + (b*e*(m + 2*p + 1) - c*d*(m + 4*p +
 3))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && NeQ[b^2 - 4*a*c,
 0] && GtQ[m, 1] && NeQ[m + 4*p + 3, 0] && IntegerQ[2*p] && (IntegerQ[p] ||
 IntegerQ[m])
```

Rule 1407

```
Int[((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.)*((d_) + (e_.)*(x_)^(
n_))^(q_.), x_Symbol] :> Int[x^(n*(2*p + q))*(e + d/x^n)^q*(c + b/x^n + a/x
^(2*n))^p, x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[n2, 2*n] && IntegersQ[
p, q] && NegQ[n]
```

Rubi steps

$$\begin{aligned}\int \frac{d+\frac{e}{x^2}}{c+\frac{a}{x^4}+\frac{b}{x^2}}\,dx &= \int \frac{x^2(e+dx^2)}{a+bx^2+cx^4}\,dx\\ &= \frac{dx}{c}-\frac{\int \frac{ad+(bd-ce)x^2}{a+bx^2+cx^4}\,dx}{c}\\ &= \frac{dx}{c}-\frac{\left(bd-ce-\frac{b^2d-2acd-bce}{\sqrt{b^2-4ac}}\right)\int \frac{1}{\frac{b}{2}-\frac{1}{2}\sqrt{b^2-4ac}+cx^2}\,dx}{2c}-\frac{\left(bd-ce+\frac{b^2d-2acd-bce}{\sqrt{b^2-4ac}}\right)\int}{2c}\\ &= \frac{dx}{c}-\frac{\left(bd-ce-\frac{b^2d-2acd-bce}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\,c^{3/2}\sqrt{b-\sqrt{b^2-4ac}}}-\frac{\left(bd-ce+\frac{b^2d-2acd-b}{\sqrt{b^2-4a}}\right.}{\sqrt{2}\,c^{3/}}\end{aligned}$$

Mathematica [A]
time = 0.12, size = 251, normalized size = 1.21

$$\frac{dx}{c}-\frac{\left(-b^2d+2acd+b\sqrt{b^2-4ac}\,d+bce-c\sqrt{b^2-4ac}\,e\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\,c^{3/2}\sqrt{b^2-4ac}\,\sqrt{b-\sqrt{b^2-4ac}}}-\frac{\left(b^2d-2acd+b\sqrt{b^2-4ac}\,d-bce-c\sqrt{b^2-4ac}\,e\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\,c^{3/2}\sqrt{b^2-4ac}\,\sqrt{b+\sqrt{b^2-4ac}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e/x^2)/(c + a/x^4 + b/x^2),x]
```

```
[Out] (d*x)/c - ((-(b^2*d) + 2*a*c*d + b*Sqrt[b^2 - 4*a*c]*d + b*c*e - c*Sqrt[b^2
 - 4*a*c]*e)*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(Sqrt
[2]*c^(3/2)*Sqrt[b^2 - 4*a*c]*Sqrt[b - Sqrt[b^2 - 4*a*c]]) - ((b^2*d - 2*a*
c*d + b*Sqrt[b^2 - 4*a*c]*d - b*c*e - c*Sqrt[b^2 - 4*a*c]*e)*ArcTan[(Sqrt[2
]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*c^(3/2)*Sqrt[b^2 - 4*a*
c]*Sqrt[b + Sqrt[b^2 - 4*a*c]])
```

Maple [A]
time = 0.05, size = 212, normalized size = 1.02

method	result
risch	$\frac{dx}{c}+\frac{\sum_{\_R=\mathrm{RootOf}\left(c\_Z^4+\_Z^2b+a\right)}\frac{\left((-bd+ce)\_R^2-ad\right)\ln\left(x-\_R\right)}{2c\_R^3+\_Rb}}{2c}$
default	$\frac{dx}{c}-\frac{\left(-bd\sqrt{-4ac+b^2}+\sqrt{-4ac+b^2}\,ce-2acd+b^2d-bce\right)\sqrt{2}\,\operatorname{arctanh}\left(\frac{cx\sqrt{2}}{\sqrt{\left(-b+\sqrt{-4ac+b^2}\right)c}}\right)}{2\sqrt{-4ac+b^2}\,c\sqrt{\left(-b+\sqrt{-4ac+b^2}\right)c}}+$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d+e/x^2)/(a/x^4+b/x^2+c),x,method=_RETURNVERBOSE)
```

```
[Out] d*x/c-1/2*(-b*d*(-4*a*c+b^2)^(1/2)+(-4*a*c+b^2)^(1/2)*c*e-2*a*c*d+b^2*d-b*c
*e)/(-4*a*c+b^2)^(1/2)/c*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(
c*x*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2))+1/2*(-b*d*(-4*a*c+b^2)^(1/2)
+(-4*a*c+b^2)^(1/2)*c*e+2*a*c*d-b^2*d+b*c*e)/(-4*a*c+b^2)^(1/2)/c*2^(1/2)/(
(b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(c*x*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*
c)^(1/2))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+e/x^2)/(c+a/x^4+b/x^2),x, algorithm="maxima")
```

```
[Out] d*x/c - integrate(((b*d - c*e)*x^2 + a*d)/(c*x^4 + b*x^2 + a), x)/c
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 2524 vs. 2(176) = 352.
time = 0.45, size = 2524, normalized size = 12.13

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d+e/x^2)/(c+a/x^4+b/x^2),x, algorithm="fricas")

[Out] 1/2*(sqrt(1/2)*c*sqrt(-(b*c^2*e^2 + (b^3 - 3*a*b*c)*d^2 - 2*(b^2*c - 2*a*c^2)*d*e + (b^2*c^3 - 4*a*c^4)*sqrt(-(4*b*c^3*d*e^3 - (b^4 - 2*a*b^2*c + a^2*c^2)*d^4 - c^4*e^4 + 4*(b^3*c - a*b*c^2)*d^3*e - 2*(3*b^2*c^2 - a*c^3)*d^2*e^2)/(b^2*c^6 - 4*a*c^7)))/(b^2*c^3 - 4*a*c^4))*log(6*b^2*c*d^2*x*e^2 + 2*(a*b^2 - a^2*c)*d^4*x - 6*b*c^2*d*x*e^3 - 2*(b^3 + a*b*c)*d^3*x*e + 2*c^3*x*e^4 + sqrt(1/2)*((b^4 - 5*a*b^2*c + 4*a^2*c^2)*d^3 - 2*(b^3*c - 4*a*b*c^2)*d^2*e + (b^2*c^2 - 4*a*c^3)*d*e^2 - ((b^3*c^3 - 4*a*b*c^4)*d - 2*(b^2*c^4 - 4*a*c^5)*e)*sqrt(-(4*b*c^3*d*e^3 - (b^4 - 2*a*b^2*c + a^2*c^2)*d^4 - c^4*e^4 + 4*(b^3*c - a*b*c^2)*d^3*e - 2*(3*b^2*c^2 - a*c^3)*d^2*e^2)/(b^2*c^6 - 4*a*c^7)))*sqrt(-(b*c^2*e^2 + (b^3 - 3*a*b*c)*d^2 - 2*(b^2*c - 2*a*c^2)*d*e + (b^2*c^3 - 4*a*c^4)*sqrt(-(4*b*c^3*d*e^3 - (b^4 - 2*a*b^2*c + a^2*c^2)*d^4 - c^4*e^4 + 4*(b^3*c - a*b*c^2)*d^3*e - 2*(3*b^2*c^2 - a*c^3)*d^2*e^2)/(b^2*c^6 - 4*a*c^7)))/(b^2*c^3 - 4*a*c^4))) - sqrt(1/2)*c*sqrt(-(b*c^2*e^2 + (b^3 - 3*a*b*c)*d^2 - 2*(b^2*c - 2*a*c^2)*d*e + (b^2*c^3 - 4*a*c^4)*sqrt(-(4*b*c^3*d*e^3 - (b^4 - 2*a*b^2*c + a^2*c^2)*d^4 - c^4*e^4 + 4*(b^3*c - a*b*c^2)*d^3*e - 2*(3*b^2*c^2 - a*c^3)*d^2*e^2)/(b^2*c^6 - 4*a*c^7)))/(b^2*c^3 - 4*a*c^4))*log(6*b^2*c*d^2*x*e^2 + 2*(a*b^2 - a^2*c)*d^4*x - 6*b*c^2*d*x*e^3 - 2*(b^3 + a*b*c)*d^3*x*e + 2*c^3*x*e^4 - sqrt(1/2)*((b^4 - 5*a*b^2*c + 4*a^2*c^2)*d^3 - 2*(b^3*c - 4*a*b*c^2)*d^2*e + (b^2*c^2 - 4*a*c^3)*d*e^2 - ((b^3*c^3 - 4*a*b*c^4)*d - 2*(b^2*c^4 - 4*a*c^5)*e)*sqrt(-(4*b*c^3*d*e^3 - (b^4 - 2*a*b^2*c + a^2*c^2)*d^4 - c^4*e^4 + 4*(b^3*c - a*b*c^2)*d^3*e - 2*(3*b^2*c^2 - a*c^3)*d^2*e^2)/(b^2*c^6 - 4*a*c^7)))*sqrt(-(b*c^2*e^2 + (b^3 - 3*a*b*c)*d^2 - 2*(b^2*c - 2*a*c^2)*d*e + (b^2*c^3 - 4*a*c^4)*sqrt(-(4*b*c^3*d*e^3 - (b^4 - 2*a*b^2*c + a^2*c^2)*d^4 - c^4*e^4 + 4*(b^3*c - a*b*c^2)*d^3*e - 2*(3*b^2*c^2 - a*c^3)*d^2*e^2)/(b^2*c^6 - 4*a*c^7)))/(b^2*c^3 - 4*a*c^4))) + sqrt(1/2)*c*sqrt(-(b*c^2*e^2 + (b^3 - 3*a*b*c)*d^2 - 2*(b^2*c - 2*a*c^2)*d*e - (b^2*c^3 - 4*a*c^4)*sqrt(-(4*b*c^3*d*e^3 - (b^4 - 2*a*b^2*c + a^2*c^2)*d^4 - c^4*e^4 + 4*(b^3*c - a*b*c^2)*d^3*e - 2*(3*b^2*c^2 - a*c^3)*d^2*e^2)/(b^2*c^6 - 4*a*c^7)))/(b^2*c^3 - 4*a*c^4))*log(6*b^2*c*d^2*x*e^2 + 2*(a*b^2 - a^2*c)*d^4*x - 6*b*c^2*d*x*e^3 - 2*(b^3 + a*b*c)*d^3*x*e + 2*c^3*x*e^4 + sqrt(1/2)*((b^4 - 5*a*b^2*c + 4*a^2*c^2)*d^3 - 2*(b^3*c - 4*a*b*c^2)*d^2*e + (b^2*c^2 - 4*a*c^3)*d*e^2 + ((b^3*c^3 - 4*a*b*c^4)*d - 2*(b^2*c^4 - 4*a*c^5)*e)*sqrt(-(4*b*c^3*d*e^3 - (b^4 - 2*a*b^2*c + a^2*c^2)*d^4 - c^4*e^4 + 4*(b^3*c - a*b*c^2)*d^3*e - 2*(3*b^2*c^2 - a*c^3)*d^2*e^2)/(b^2*c^6 - 4*a*c^7)))*sqrt(-(b*c^2*e^2 + (b^3 - 3*a*b*c)*d^2 - 2*(b^2*c - 2*a*c^2)*d*e - (b^2*c^3 - 4*a*c^4)*sqrt(-(4*b*c^3*d*e^3 - (b^4 - 2*a*b^2*c + a^2*c^2)*d^4 - c^4*e^4 + 4*(b^3*c - a*b*c^2)*d^3*e - 2*(3*b^2*c^2 - a*c^3)*d^2*e^2)/(b^2*c^6 - 4*a*c^7)))/(b^2*c^3 - 4*a*c^4))) - sqrt(1/2)*c*sqrt(-(b*c^2*e^2 + (b^3 - 3*a*b*c)*d^2 - 2*(b^2*c - 2*a*c^2)*d*e - (b^2*c^3 - 4*a*c^4)*sqrt(-(4*b*c^3*d*e^3 - (b^4 - 2*a*b^2*c + a^2*c^2)*d^4 - c^4*e^4 + 4*(b^3*c - a*b*c^2)*d^3*e - 2*(3*b^2*c^2 - a*c^3)*d^2*e^2)/(b^2*c^6 - 4*a*c^7)))/(b^

```
2*c^3 - 4*a*c^4))*log(6*b^2*c*d^2*x*e^2 + 2*(a*b^2 - a^2*c)*d^4*x - 6*b*c^2
*d*x*e^3 - 2*(b^3 + a*b*c)*d^3*x*e + 2*c^3*x*e^4 - sqrt(1/2)*((b^4 - 5*a*b^
2*c + 4*a^2*c^2)*d^3 - 2*(b^3*c - 4*a*b*c^2)*d^2*e + (b^2*c^2 - 4*a*c^3)*d*
e^2 + ((b^3*c^3 - 4*a*b*c^4)*d - 2*(b^2*c^4 - 4*a*c^5)*e)*sqrt(-(4*b*c^3*d*
e^3 - (b^4 - 2*a*b^2*c + a^2*c^2)*d^4 - c^4*e^4 + 4*(b^3*c - a*b*c^2)*d^3*e
 - 2*(3*b^2*c^2 - a*c^3)*d^2*e^2)/(b^2*c^6 - 4*a*c^7)))*sqrt(-(b*c^2*e^2 +
(b^3 - 3*a*b*c)*d^2 - 2*(b^2*c - 2*a*c^2)*d*e - (b^2*c^3 - 4*a*c^4)*sqrt(-(
4*b*c^3*d*e^3 - (b^4 - 2*a*b^2*c + a^2*c^2)*d^4 - c^4*e^4 + 4*(b^3*c - a*b*
c^2)*d^3*e - 2*(3*b^2*c^2 - a*c^3)*d^2*e^2)/(b^2*c^6 - 4*a*c^7)))/(b^2*c^3
- 4*a*c^4))) + 2*d*x)/c
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+e/x**2)/(c+a/x**4+b/x**2),x)
```

```
[Out] Timed out
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 3183 vs. 2(176) = 352.
time = 4.64, size = 3183, normalized size = 15.30

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+e/x^2)/(c+a/x^4+b/x^2),x, algorithm="giac")
```

```
[Out] d*x/c - 1/8*((2*b^5*c^2 - 16*a*b^3*c^3 + 32*a^2*b*c^4 - sqrt(2)*sqrt(b^2 -
4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^5 + 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sq
rt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^3*c + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*
c - sqrt(b^2 - 4*a*c)*c)*b^4*c - 16*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sq
rt(b^2 - 4*a*c)*c)*a^2*b*c^2 - 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(
b^2 - 4*a*c)*c)*a*b^2*c^2 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 -
 4*a*c)*c)*b^3*c^2 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*
c)*c)*a*b*c^3 - 2*(b^2 - 4*a*c)*b^3*c^2 + 8*(b^2 - 4*a*c)*a*b*c^3)*c^2*d -
(2*b^4*c^3 - 16*a*b^2*c^4 + 32*a^2*c^5 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c
 - sqrt(b^2 - 4*a*c)*c)*b^4*c + 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt
(b^2 - 4*a*c)*c)*a*b^2*c^2 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^
2 - 4*a*c)*c)*b^3*c^2 - 16*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 -
4*a*c)*c)*a^2*c^3 - 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c
)*c)*a*b*c^3 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^
2*c^3 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*c^4 -
 2*(b^2 - 4*a*c)*b^2*c^3 + 8*(b^2 - 4*a*c)*a*c^4)*c^2*e + 2*(sqrt(2)*sqrt(b
```

*c - sqrt(b^2 - 4*a*c)*c)*a*b^4*c^2 - 8*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c
)*c)*a^2*b^2*c^3 - 2*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^3*c^3 + 2*
a*b^4*c^3 + 16*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*c^4 + 8*sqrt(2)*
sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b*c^4 + sqrt(2)*sqrt(b*c - sqrt(b^2 - 4
*a*c)*c)*a*b^2*c^4 - 16*a^2*b^2*c^4 - 4*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c
)*c)*a^2*c^5 + 32*a^3*c^5 - 2*(b^2 - 4*a*c)*a*b^2*c^3 + 8*(b^2 - 4*a*c)*a^2
*c^4)*d*abs(c) - (2*b^5*c^4 - 12*a*b^3*c^5 + 16*a^2*b*c^6 - sqrt(2)*sqrt(b^
2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^5*c^2 + 6*sqrt(2)*sqrt(b^2 - 4
*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^3*c^3 + 2*sqrt(2)*sqrt(b^2 - 4*a*
c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^4*c^3 - 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sq
rt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b*c^4 - 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(
b*c - sqrt(b^2 - 4*a*c)*c)*a*b^2*c^4 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c -
 sqrt(b^2 - 4*a*c)*c)*b^3*c^4 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt
(b^2 - 4*a*c)*c)*a*b*c^5 - 2*(b^2 - 4*a*c)*b^3*c^4 + 4*(b^2 - 4*a*c)*a*b*c^
5)*d + (2*b^4*c^5 - 8*a*b^2*c^6 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt
(b^2 - 4*a*c)*c)*b^4*c^3 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2
- 4*a*c)*c)*a*b^2*c^4 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4
*a*c)*c)*b^3*c^4 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c
)*b^2*c^5 - 2*(b^2 - 4*a*c)*b^2*c^5)*e)*arctan(2*sqrt(1/2)*x/sqrt((b*c + sq
rt(b^2*c^2 - 4*a*c^3))/c^2))/((a*b^4*c^3 - 8*a^2*b^2*c^4 - 2*a*b^3*c^4 + 16
*a^3*c^5 + 8*a^2*b*c^5 + a*b^2*c^5 - 4*a^2*c^6)*c^2) + 1/8*((2*b^5*c^2 - 16
*a*b^3*c^3 + 32*a^2*b*c^4 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 -
 4*a*c)*c)*b^5 + 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c
)*a*b^3*c + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^4
*c - 16*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b*c^2
 - 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^2*c^2 -
sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^3*c^2 + 4*sqrt(
2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b*c^3 - 2*(b^2 - 4*a
*c)*b^3*c^2 + 8*(b^2 - 4*a*c)*a*b*c^3)*c^2*d - (2*b^4*c^3 - 16*a*b^2*c^4 +
32*a^2*c^5 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^4*
c + 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^2*c^2 +
 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^3*c^2 - 16*s
qrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*c^3 - 8*sqrt(2
)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b*c^3 - sqrt(2)*sqrt(
b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^2*c^3 + 4*sqrt(2)*sqrt(b^2 -
 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*c^4 - 2*(b^2 - 4*a*c)*b^2*c^3 + 8
*(b^2 - 4*a*c)*a*c^4)*c^2*e - 2*(sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*
b^4*c^2 - 8*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^3 - 2*sqrt(2)
*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^3*c^3 - 2*a*b^4*c^3 + 16*sqrt(2)*sqrt(
b*c + sqrt(b^2 - 4*a*c)*c)*a^3*c^4 + 8*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)
*c)*a^2*b*c^4 + sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^2*c^4 + 16*a^2*
b^2*c^4 - 4*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*c^5 - 32*a^3*c^5 +
2*(b^2 - 4*a*c)*a*b^2*c^3 - 8*(b^2 - 4*a*c)*a^2*c^4)*d*abs(c) - (2*b^5*c^4
- 12*a*b^3*c^5 + 16*a^2*b*c^6 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b
^2 - 4*a*c)*c)*b^5*c^2 + 6*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 -

```
4*a*c)*c)*a*b^3*c^3 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a
*c)*c)*b^4*c^3 - 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c
)*a^2*b*c^4 - 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a
*b^2*c^4 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^3*c^
4 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b*c^5 - 2
*(b^2 - 4*a*c)*b^3*c^4 + 4*(b^2 - 4*a*c)*a*b*c^...
```

Mupad [B]
time = 2.85, size = 2500, normalized size = 12.02

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e/x^2)/(c + a/x^4 + b/x^2),x)
```

```
[Out] (d*x)/c - atan(((((16*a^2*c^3*d - 4*a*b^2*c^2*d)/c - (2*x*(4*b^3*c^3 - 16*a
*b*c^4)*(-(b^5*d^2 - b^2*d^2*(-(4*a*c - b^2)^3)^(1/2) + b^3*c^2*e^2 - c^2*e
^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2*d^2 - 2*b^4*c*d*e - 7*a*b^3*c*d^
2 + a*c*d^2*(-(4*a*c - b^2)^3)^(1/2) - 4*a*b*c^3*e^2 - 16*a^2*c^3*d*e + 12*
a*b^2*c^2*d*e + 2*b*c*d*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^5 + b^4*c^
3 - 8*a*b^2*c^4)))^(1/2))/c)*(-(b^5*d^2 - b^2*d^2*(-(4*a*c - b^2)^3)^(1/2)
+ b^3*c^2*e^2 - c^2*e^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2*d^2 - 2*b^4
*c*d*e - 7*a*b^3*c*d^2 + a*c*d^2*(-(4*a*c - b^2)^3)^(1/2) - 4*a*b*c^3*e^2 -
 16*a^2*c^3*d*e + 12*a*b^2*c^2*d*e + 2*b*c*d*e*(-(4*a*c - b^2)^3)^(1/2))/(8
*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4)))^(1/2) - (2*x*(b^4*d^2 - 2*a*c^3*e^2
 + 2*a^2*c^2*d^2 + b^2*c^2*e^2 - 2*b^3*c*d*e - 4*a*b^2*c*d^2 + 6*a*b*c^2*d*
e))/c)*(-(b^5*d^2 - b^2*d^2*(-(4*a*c - b^2)^3)^(1/2) + b^3*c^2*e^2 - c^2*e^
2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2*d^2 - 2*b^4*c*d*e - 7*a*b^3*c*d^2
 + a*c*d^2*(-(4*a*c - b^2)^3)^(1/2) - 4*a*b*c^3*e^2 - 16*a^2*c^3*d*e + 12*a
*b^2*c^2*d*e + 2*b*c*d*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^5 + b^4*c^3
 - 8*a*b^2*c^4)))^(1/2)*1i - (((16*a^2*c^3*d - 4*a*b^2*c^2*d)/c + (2*x*(4*b
^3*c^3 - 16*a*b*c^4)*(-(b^5*d^2 - b^2*d^2*(-(4*a*c - b^2)^3)^(1/2) + b^3*c^
2*e^2 - c^2*e^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2*d^2 - 2*b^4*c*d*e -
 7*a*b^3*c*d^2 + a*c*d^2*(-(4*a*c - b^2)^3)^(1/2) - 4*a*b*c^3*e^2 - 16*a^2*
c^3*d*e + 12*a*b^2*c^2*d*e + 2*b*c*d*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2
*c^5 + b^4*c^3 - 8*a*b^2*c^4)))^(1/2))/c)*(-(b^5*d^2 - b^2*d^2*(-(4*a*c - b
^2)^3)^(1/2) + b^3*c^2*e^2 - c^2*e^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^
2*d^2 - 2*b^4*c*d*e - 7*a*b^3*c*d^2 + a*c*d^2*(-(4*a*c - b^2)^3)^(1/2) - 4*
a*b*c^3*e^2 - 16*a^2*c^3*d*e + 12*a*b^2*c^2*d*e + 2*b*c*d*e*(-(4*a*c - b^2)
^3)^(1/2))/(8*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4)))^(1/2) + (2*x*(b^4*d^2
- 2*a*c^3*e^2 + 2*a^2*c^2*d^2 + b^2*c^2*e^2 - 2*b^3*c*d*e - 4*a*b^2*c*d^2 +
 6*a*b*c^2*d*e))/c)*(-(b^5*d^2 - b^2*d^2*(-(4*a*c - b^2)^3)^(1/2) + b^3*c^2
*e^2 - c^2*e^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2*d^2 - 2*b^4*c*d*e -
7*a*b^3*c*d^2 + a*c*d^2*(-(4*a*c - b^2)^3)^(1/2) - 4*a*b*c^3*e^2 - 16*a^2*c
^3*d*e + 12*a*b^2*c^2*d*e + 2*b*c*d*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*
```

```
c^5 + b^4*c^3 - 8*a*b^2*c^4)))^(1/2)*1i)/(((((16*a^2*c^3*d - 4*a*b^2*c^2*d)/
c - (2*x*(4*b^3*c^3 - 16*a*b*c^4)*(-(b^5*d^2 - b^2*d^2*(-(4*a*c - b^2)^3)^(
1/2) + b^3*c^2*e^2 - c^2*e^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2*d^2 -
2*b^4*c*d*e - 7*a*b^3*c*d^2 + a*c*d^2*(-(4*a*c - b^2)^3)^(1/2) - 4*a*b*c^3*
e^2 - 16*a^2*c^3*d*e + 12*a*b^2*c^2*d*e + 2*b*c*d*e*(-(4*a*c - b^2)^3)^(1/2
))/(8*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4)))^(1/2))/c)*(-(b^5*d^2 - b^2*d^2
*(-(4*a*c - b^2)^3)^(1/2) + b^3*c^2*e^2 - c^2*e^2*(-(4*a*c - b^2)^3)^(1/2)
+ 12*a^2*b*c^2*d^2 - 2*b^4*c*d*e - 7*a*b^3*c*d^2 + a*c*d^2*(-(4*a*c - b^2)^
3)^(1/2) - 4*a*b*c^3*e^2 - 16*a^2*c^3*d*e + 12*a*b^2*c^2*d*e + 2*b*c*d*e*(-
(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4)))^(1/2) - (
2*x*(b^4*d^2 - 2*a*c^3*e^2 + 2*a^2*c^2*d^2 + b^2*c^2*e^2 - 2*b^3*c*d*e - 4*
a*b^2*c*d^2 + 6*a*b*c^2*d*e))/c)*(-(b^5*d^2 - b^2*d^2*(-(4*a*c - b^2)^3)^(1
/2) + b^3*c^2*e^2 - c^2*e^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2*d^2 - 2
*b^4*c*d*e - 7*a*b^3*c*d^2 + a*c*d^2*(-(4*a*c - b^2)^3)^(1/2) - 4*a*b*c^3*e
^2 - 16*a^2*c^3*d*e + 12*a*b^2*c^2*d*e + 2*b*c*d*e*(-(4*a*c - b^2)^3)^(1/2)
)/(8*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4)))^(1/2) - (2*(a*c^2*e^3 - a^2*b*d
^3 + a*b^2*d^2*e + a^2*c*d^2*e - 2*a*b*c*d*e^2))/c + (((16*a^2*c^3*d - 4*a*
b^2*c^2*d)/c + (2*x*(4*b^3*c^3 - 16*a*b*c^4)*(-(b^5*d^2 - b^2*d^2*(-(4*a*c
- b^2)^3)^(1/2) + b^3*c^2*e^2 - c^2*e^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b
*c^2*d^2 - 2*b^4*c*d*e - 7*a*b^3*c*d^2 + a*c*d^2*(-(4*a*c - b^2)^3)^(1/2) -
 4*a*b*c^3*e^2 - 16*a^2*c^3*d*e + 12*a*b^2*c^2*d*e + 2*b*c*d*e*(-(4*a*c - b
^2)^3)^(1/2))/(8*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4)))^(1/2))/c)*(-(b^5*d^
2 - b^2*d^2*(-(4*a*c - b^2)^3)^(1/2) + b^3*c^2*e^2 - c^2*e^2*(-(4*a*c - b^2
)^3)^(1/2) + 12*a^2*b*c^2*d^2 - 2*b^4*c*d*e - 7*a*b^3*c*d^2 + a*c*d^2*(-(4*
a*c - b^2)^3)^(1/2) - 4*a*b*c^3*e^2 - 16*a^2*c^3*d*e + 12*a*b^2*c^2*d*e + 2
*b*c*d*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4))
)^(1/2) + (2*x*(b^4*d^2 - 2*a*c^3*e^2 + 2*a^2*c^2*d^2 + b^2*c^2*e^2 - 2*b^3
*c*d*e - 4*a*b^2*c*d^2 + 6*a*b*c^2*d*e))/c)*(-(b^5*d^2 - b^2*d^2*(-(4*a*c -
 b^2)^3)^(1/2) + b^3*c^2*e^2 - c^2*e^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*
c^2*d^2 - 2*b^4*c*d*e - 7*a*b^3*c*d^2 + a*c*d^2*(-(4*a*c - b^2)^3)^(1/2) -
4*a*b*c^3*e^2 - 16*a^2*c^3*d*e + 12*a*b^2*c^2*d*e + 2*b*c*d*e*(-(4*a*c - b^
2)^3)^(1/2))/(8*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4)))^(1/2)))*(-(b^5*d^2 -
 b^2*d^2*(-(4*a*c - b^2)^3)^(1/2) + b^3*c^2*e^2 - c^2*e^2*(-(4*a*c - b^2)^3
)^(1/2) + 12*a^2*b*c^2*d^2 - 2*b^4*c*d*e - 7*a*b^3*c*d^2 + a*c*d^2*(-(4*a*c
 - b^2)^3)^(1/2) - 4*a*b*c^3*e^2 - 16*a^2*c^3*d*e + 12*a*b^2*c^2*d*e + 2*b*
c*d*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4)))^(
1/2)*2i - atan(((((16*a^2*c^3*d - 4*a*b^2*c^2*d...
```

3.38 $\int \frac{d+\frac{e}{x^3}}{c+\frac{a}{x^6}}\,dx$

Optimal. Leaf size=311

$$\frac{dx}{c}-\frac{\sqrt[6]{a}\,d\tan^{-1}\left(\frac{\sqrt[6]{c}\,x}{\sqrt[6]{a}}\right)}{3c^{7/6}}+\frac{\left(\sqrt{a}\,d-\sqrt{3}\,\sqrt{c}\,e\right)\tan^{-1}\left(\sqrt{3}-\frac{2\sqrt[6]{c}\,x}{\sqrt[6]{a}}\right)}{6\sqrt[3]{a}\,c^{7/6}}-\frac{\left(\sqrt{a}\,d+\sqrt{3}\,\sqrt{c}\,e\right)\tan^{-1}\left(\sqrt{3}\right.}{6\sqrt[3]{a}\,c^{7/6}}$$

```
[Out] d*x/c-1/3*a^(1/6)*d*arctan(c^(1/6)*x/a^(1/6))/c^(7/6)-1/6*e*ln(a^(1/3)+c^(1
/3)*x^2)/a^(1/3)/c^(2/3)-1/12*ln(a^(1/3)+c^(1/3)*x^2+a^(1/6)*c^(1/6)*x*3^(1
/2))*(d*3^(1/2)*a^(1/2)-e*c^(1/2))/a^(1/3)/c^(7/6)+1/12*ln(a^(1/3)+c^(1/3)*
x^2-a^(1/6)*c^(1/6)*x*3^(1/2))*(d*3^(1/2)*a^(1/2)+e*c^(1/2))/a^(1/3)/c^(7/6
)-1/6*arctan(2*c^(1/6)*x/a^(1/6)-3^(1/2))*(d*a^(1/2)-e*3^(1/2)*c^(1/2))/a^(
1/3)/c^(7/6)-1/6*arctan(2*c^(1/6)*x/a^(1/6)+3^(1/2))*(d*a^(1/2)+e*3^(1/2)*c
^(1/2))/a^(1/3)/c^(7/6)
```

Rubi [A]
time = 0.20, antiderivative size = 311, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 10, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.588$, Rules used = {1408, 1517, 1430, 649, 209, 266, 648, 631, 210, 642}

$$\frac{\text{ArcTan}\left(\sqrt{3}-\frac{2\sqrt[6]{c}\,x}{\sqrt[6]{a}}\right)\left(\sqrt{a}\,d-\sqrt{3}\,\sqrt{c}\,e\right)}{6\sqrt[3]{a}\,c^{7/6}}-\frac{\text{ArcTan}\left(\frac{2\sqrt[6]{c}\,x}{\sqrt[6]{a}}+\sqrt{3}\right)\left(\sqrt{a}\,d+\sqrt{3}\,\sqrt{c}\,e\right)}{6\sqrt[3]{a}\,c^{7/6}}-\frac{\sqrt[6]{a}\,d\text{ArcTan}\left(\frac{\sqrt[6]{c}\,x}{\sqrt[6]{a}}\right)}{3c^{7/6}}+\frac{\left(\sqrt{3}\,\sqrt{a}\,d+\sqrt{c}\,e\right)\log\left(-\sqrt{3}\,\sqrt[6]{a}\,\sqrt[6]{c}\,x+\sqrt[3]{a}+\sqrt[3]{c}\,x^2\right)}{12\sqrt[3]{a}\,c^{7/6}}-\frac{\left(\sqrt{3}\,\sqrt{a}\,d-\sqrt{c}\,e\right)\log\left(\sqrt{3}\,\sqrt[6]{a}\,\sqrt[6]{c}\,x+\sqrt[3]{a}+\sqrt[3]{c}\,x^2\right)}{12\sqrt[3]{a}\,c^{7/6}}-\frac{e\log\left(\sqrt[3]{a}+\sqrt[3]{c}\,x^2\right)}{6\sqrt[3]{a}\,c^{2/3}}+\frac{dx}{c}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e/x^3)/(c + a/x^6),x]
```

```
[Out] (d*x)/c - (a^(1/6)*d*ArcTan[(c^(1/6)*x)/a^(1/6)])/(3*c^(7/6)) + ((Sqrt[a]*d
 - Sqrt[3]*Sqrt[c]*e)*ArcTan[Sqrt[3] - (2*c^(1/6)*x)/a^(1/6)])/(6*a^(1/3)*c
^(7/6)) - ((Sqrt[a]*d + Sqrt[3]*Sqrt[c]*e)*ArcTan[Sqrt[3] + (2*c^(1/6)*x)/a
^(1/6)])/(6*a^(1/3)*c^(7/6)) - (e*Log[a^(1/3) + c^(1/3)*x^2])/(6*a^(1/3)*c^
(2/3)) + ((Sqrt[3]*Sqrt[a]*d + Sqrt[c]*e)*Log[a^(1/3) - Sqrt[3]*a^(1/6)*c^(
1/6)*x + c^(1/3)*x^2])/(12*a^(1/3)*c^(7/6)) - ((Sqrt[3]*Sqrt[a]*d - Sqrt[c]
*e)*Log[a^(1/3) + Sqrt[3]*a^(1/6)*c^(1/6)*x + c^(1/3)*x^2])/(12*a^(1/3)*c^(
7/6))
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
```

```
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 649

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[(-a)*c]
```

Rule 1408

```
Int[((a_) + (c_.)*(x_)^(n2_.))^(p_.)*((d_) + (e_.)*(x_)^(n_))^(q_.), x_Symb
ol] :> Int[x^(n*(2*p + q))*(e + d/x^n)^q*(c + a/x^(2*n))^p, x] /; FreeQ[{a,
 c, d, e, n}, x] && EqQ[n2, 2*n] && IntegersQ[p, q] && NegQ[n]
```

Rule 1430

```
Int[((d_) + (e_.)*(x_)^3)/((a_) + (c_.)*(x_)^6), x_Symbol] :> With[{q = Rt[
c/a, 6]}, Dist[1/(3*a*q^2), Int[(q^2*d - e*x)/(1 + q^2*x^2), x], x] + (Dist
[1/(6*a*q^2), Int[(2*q^2*d - (Sqrt[3]*q^3*d - e)*x)/(1 - Sqrt[3]*q*x + q^2*
x^2), x], x] + Dist[1/(6*a*q^2), Int[(2*q^2*d + (Sqrt[3]*q^3*d + e)*x)/(1 +
 Sqrt[3]*q*x + q^2*x^2), x], x])] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 +
```

```
a*e^2, 0] && PosQ[c/a]
```

Rule 1517

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^(n_))*((a_) + (c_.)*(x_)^(n2_))^(
p_), x_Symbol] :> Simp[e*f^(n - 1)*(f*x)^(m - n + 1)*((a + c*x^(2*n))^(p +
1)/(c*(m + n*(2*p + 1) + 1))), x] - Dist[f^n/(c*(m + n*(2*p + 1) + 1)), Int
[(f*x)^(m - n)*(a + c*x^(2*n))^p*(a*e*(m - n + 1) - c*d*(m + n*(2*p + 1) +
1)*x^n), x], x] /; FreeQ[{a, c, d, e, f, p}, x] && EqQ[n2, 2*n] && IGtQ[n,
0] && GtQ[m, n - 1] && NeQ[m + n*(2*p + 1) + 1, 0] && IntegerQ[p]
```

Rubi steps

$$\begin{aligned}
\int \frac{d+\frac{e}{x^3}}{c+\frac{a}{x^6}}\,dx &= \int \frac{x^3(e+dx^3)}{a+cx^6}\,dx \\
&= \frac{dx}{c} - \frac{\int \frac{ad-cex^3}{a+cx^6}\,dx}{c} \\
&= \frac{dx}{c} - \frac{\int \frac{2a^{2/3}\sqrt[3]{c}\,d-\left(\sqrt{3}\,\sqrt{a}\,\sqrt{c}\,d+ce\right)x}{1-\frac{\sqrt{3}\,\sqrt[6]{c}\,x}{\sqrt[6]{a}}+\frac{\sqrt[3]{c}\,x^2}{\sqrt[3]{a}}}\,dx}{6a^{2/3}c^{4/3}} - \frac{\int \frac{2a^{2/3}\sqrt[3]{c}\,d+\left(\sqrt{3}\,\sqrt{a}\,\sqrt{c}\,d-ce\right)x}{1+\frac{\sqrt{3}\,\sqrt[6]{c}\,x}{\sqrt[6]{a}}+\frac{\sqrt[3]{c}\,x^2}{\sqrt[3]{a}}}\,dx}{6a^{2/3}c^{4/3}} - \frac{\int \frac{a^{2/3}\sqrt[3]{c}\,d \cdots}{1+\frac{\sqrt[3]{c}\cdots}{\sqrt[3]{a}\cdots}}}{3a^{2/3}c\cdots} \\
&= \frac{dx}{c} - \frac{d\int \frac{1}{1+\frac{\sqrt[3]{c}\,x^2}{\sqrt[3]{a}}}\,dx}{3c} - \frac{e\int \frac{x}{1+\frac{\sqrt[3]{c}\,x^2}{\sqrt[3]{a}}}\,dx}{3a^{2/3}\sqrt[3]{c}} - \frac{\left(\sqrt{3}\,\sqrt{a}\,d-\sqrt{c}\,e\right)\int \frac{\frac{\sqrt{3}\,\sqrt[6]{c}}{\sqrt[6]{a}}+\frac{2\sqrt[3]{c}\,x}{\sqrt[3]{a}}}{1+\frac{\sqrt{3}\,\sqrt[6]{c}\,x}{\sqrt[6]{a}}+\frac{\sqrt[3]{c}\,x^2}{\sqrt[3]{a}}}\,dx}{12\sqrt[3]{a}\,c^{7/6}} + \cdots \\
&= \frac{dx}{c} - \frac{\sqrt[6]{a}\,d\tan^{-1}\left(\frac{\sqrt[6]{c}\,x}{\sqrt[6]{a}}\right)}{3c^{7/6}} - \frac{e\log\left(\sqrt[3]{a}+\sqrt[3]{c}\,x^2\right)}{6\sqrt[3]{a}\,c^{2/3}} + \frac{\left(\sqrt{3}\,\sqrt{a}\,d+\sqrt{c}\,e\right)\log\left(\sqrt[3]{a}-\sqrt{3}\,\sqrt[6]{\cdots}\right.}{12\sqrt[3]{a}\,c^{7/6}} \\
&= \frac{dx}{c} - \frac{\sqrt[6]{a}\,d\tan^{-1}\left(\frac{\sqrt[6]{c}\,x}{\sqrt[6]{a}}\right)}{3c^{7/6}} + \frac{\left(\sqrt{a}\,d-\sqrt{3}\,\sqrt{c}\,e\right)\tan^{-1}\left(\sqrt{3}-\frac{2\sqrt[6]{c}\,x}{\sqrt[6]{a}}\right)}{6\sqrt[3]{a}\,c^{7/6}} - \frac{\left(\sqrt{a}\,d+\sqrt{3}\,\cdots\right.}{\cdots}
\end{aligned}$$

Mathematica [A]
time = 0.08, size = 346, normalized size = 1.11

$$\frac{dx}{c}-\frac{\sqrt[6]{a}\,d\tan^{-1}\left(\frac{\sqrt[6]{c}\,x}{\sqrt[6]{a}}\right)}{3c^{7/6}}+\frac{\left(-a^{7/6}\sqrt{c}\,d+\sqrt{3}\,a^{2/3}ce\right)\tan^{-1}\left(\frac{-\sqrt{3}\,\sqrt[6]{a}+2\sqrt[6]{c}\,x}{\sqrt[6]{a}}\right)}{6ac^{5/3}}+\frac{\left(-a^{7/6}\sqrt{c}\,d-\sqrt{3}\,a^{2/3}ce\right)\tan^{-1}\left(\frac{\sqrt{3}\,\sqrt[6]{a}+2\sqrt[6]{c}\,x}{\sqrt[6]{a}}\right)}{6ac^{5/3}}-\frac{e\log\left(\sqrt[3]{a}+\sqrt[3]{c}\,x^2\right)}{6\sqrt[3]{a}\,c^{2/3}}-\frac{\left(-\sqrt{3}\,a^{7/6}\sqrt{c}\,d-a^{2/3}ce\right)\log\left(\sqrt[3]{a}-\sqrt{3}\,\sqrt[6]{a}\,\sqrt[6]{c}\,x+\sqrt[3]{c}\,x^2\right)}{12ac^{5/3}}-\frac{\left(\sqrt{3}\,a^{7/6}\sqrt{c}\,d-a^{2/3}ce\right)\log\left(\sqrt[3]{a}+\sqrt{3}\,\sqrt[6]{a}\,\sqrt[6]{c}\,x+\sqrt[3]{c}\,x^2\right)}{12ac^{5/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e/x^3)/(c + a/x^6),x]
```

```
[Out] (d*x)/c - (a^(1/6)*d*ArcTan[(c^(1/6)*x)/a^(1/6)])/(3*c^(7/6)) + ((-(a^(7/6)
*Sqrt[c]*d) + Sqrt[3]*a^(2/3)*c*e)*ArcTan[(-(Sqrt[3]*a^(1/6)) + 2*c^(1/6)*x
)/a^(1/6)])/(6*a*c^(5/3)) + ((-(a^(7/6)*Sqrt[c]*d) - Sqrt[3]*a^(2/3)*c*e)*A
rcTan[(Sqrt[3]*a^(1/6) + 2*c^(1/6)*x)/a^(1/6)])/(6*a*c^(5/3)) - (e*Log[a^(1
/3) + c^(1/3)*x^2])/(6*a^(1/3)*c^(2/3)) - ((-(Sqrt[3]*a^(7/6)*Sqrt[c]*d) -
a^(2/3)*c*e)*Log[a^(1/3) - Sqrt[3]*a^(1/6)*c^(1/6)*x + c^(1/3)*x^2])/(12*a*
c^(5/3)) - ((Sqrt[3]*a^(7/6)*Sqrt[c]*d - a^(2/3)*c*e)*Log[a^(1/3) + Sqrt[3]
*a^(1/6)*c^(1/6)*x + c^(1/3)*x^2])/(12*a*c^(5/3))
```

Maple [A]
time = 0.09, size = 339, normalized size = 1.09

method	result
risch	$\frac{dx}{c}+\frac{\sum_{\_R=\mathrm{RootOf}\left(\_Z^{6}c+a\right)}\frac{\left(\_R^{3}ce-ad\right)\ln\left(x-\_R\right)}{\_R^{5}}}{6c^{2}}$
default	$\frac{dx}{c}+\frac{-\frac{c\left(\frac{a}{c}\right)^{\frac{7}{6}}\ln\left(x^{2}+\sqrt{3}\left(\frac{a}{c}\right)^{\frac{1}{6}}x+\left(\frac{a}{c}\right)^{\frac{1}{3}}\right)\sqrt{3}\,d}{12a}+\frac{c\left(\frac{a}{c}\right)^{\frac{2}{3}}\ln\left(x^{2}+\sqrt{3}\left(\frac{a}{c}\right)^{\frac{1}{6}}x+\left(\frac{a}{c}\right)^{\frac{1}{3}}\right)e}{12a}-\frac{\left(\frac{a}{c}\right)^{\frac{1}{6}}\arctan\left(\frac{2x}{\left(\frac{a}{c}\right)^{\frac{1}{6}}}+\sqrt{3}\right)d}{6}-\frac{c\left(\frac{a}{c}\right)\cdots}{}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d+e/x^3)/(c+a/x^6),x,method=_RETURNVERBOSE)
```

```
[Out] d*x/c+(-1/12*c*(a/c)^(7/6)/a*ln(x^2+3^(1/2)*(a/c)^(1/6)*x+(a/c)^(1/3))*3^(1
/2)*d+1/12*c*(a/c)^(2/3)/a*ln(x^2+3^(1/2)*(a/c)^(1/6)*x+(a/c)^(1/3))*e-1/6*
(a/c)^(1/6)*arctan(2*x/(a/c)^(1/6)+3^(1/2))*d-1/6*c*(a/c)^(2/3)/a*arctan(2*
x/(a/c)^(1/6)+3^(1/2))*3^(1/2)*e-1/6*c*(a/c)^(2/3)/a*e*ln(x^2+(a/c)^(1/3))-
1/3*(a/c)^(1/6)*d*arctan(x/(a/c)^(1/6))+1/12*ln(-x^2+3^(1/2)*(a/c)^(1/6)*x-
(a/c)^(1/3))*(a/c)^(1/6)*3^(1/2)*d+1/12*c/a*ln(-x^2+3^(1/2)*(a/c)^(1/6)*x-(
a/c)^(1/3))*(a/c)^(2/3)*e-1/6*(a/c)^(1/6)*d*arctan(2*x/(a/c)^(1/6)-3^(1/2))
+1/6*c/a*(a/c)^(2/3)*3^(1/2)*e*arctan(2*x/(a/c)^(1/6)-3^(1/2)))/c
```

Maxima [A]
time = 0.51, size = 300, normalized size = 0.96

$$\frac{dx}{c}-\frac{\frac{2c^{\frac{1}{3}}e\log\left(c^{\frac{1}{3}}x^{2}+a^{\frac{1}{3}}\right)}{a^{\frac{1}{3}}}+\frac{4a^{\frac{1}{3}}d\arctan\left(\frac{c^{\frac{1}{3}}x}{\sqrt{a^{\frac{1}{3}}c^{\frac{1}{3}}}}\right)}{\sqrt{a^{\frac{1}{3}}c^{\frac{1}{3}}}}+\frac{\left(\sqrt{3}a^{\frac{7}{6}}\sqrt{c}d-a^{\frac{2}{3}}ce\right)\log\left(c^{\frac{1}{3}}x^{2}+\sqrt{3}a^{\frac{1}{6}}c^{\frac{1}{6}}x+a^{\frac{1}{3}}\right)}{ac^{\frac{2}{3}}}-\frac{\left(\sqrt{3}a^{\frac{7}{6}}\sqrt{c}d+a^{\frac{2}{3}}ce\right)\log\left(c^{\frac{1}{3}}x^{2}-\sqrt{3}a^{\frac{1}{6}}c^{\frac{1}{6}}x+a^{\frac{1}{3}}\right)}{ac^{\frac{2}{3}}}+\frac{2\left(\sqrt{3}a^{\frac{5}{6}}c^{\frac{7}{6}}e+a^{\frac{4}{3}}c^{\frac{2}{3}}d\right)\arctan\left(\frac{2c^{\frac{1}{3}}x+\sqrt{3}a^{\frac{1}{6}}c^{\frac{1}{6}}}{\sqrt{a^{\frac{1}{3}}c^{\frac{1}{3}}}}\right)}{ac^{\frac{2}{3}}\sqrt{a^{\frac{1}{3}}c^{\frac{1}{3}}}}-\frac{2\left(\sqrt{3}a^{\frac{5}{6}}c^{\frac{7}{6}}e-a^{\frac{4}{3}}c^{\frac{2}{3}}d\right)\arctan\left(\frac{2c^{\frac{1}{3}}x-\sqrt{3}a^{\frac{1}{6}}c^{\frac{1}{6}}}{\sqrt{a^{\frac{1}{3}}c^{\frac{1}{3}}}}\right)}{ac^{\frac{2}{3}}\sqrt{a^{\frac{1}{3}}c^{\frac{1}{3}}}}}{12c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+e/x^3)/(c+a/x^6),x, algorithm="maxima")
```

```
[Out] d*x/c - 1/12*(2*c^(1/3)*e*log(c^(1/3)*x^2 + a^(1/3))/a^(1/3) + 4*a^(1/3)*d*
arctan(c^(1/3)*x/sqrt(a^(1/3)*c^(1/3)))/sqrt(a^(1/3)*c^(1/3)) + (sqrt(3)*a^
(7/6)*sqrt(c)*d - a^(2/3)*c*e)*log(c^(1/3)*x^2 + sqrt(3)*a^(1/6)*c^(1/6)*x
+ a^(1/3))/(a*c^(2/3)) - (sqrt(3)*a^(7/6)*sqrt(c)*d + a^(2/3)*c*e)*log(c^(1
```

```
/3)*x^2 - sqrt(3)*a^(1/6)*c^(1/6)*x + a^(1/3))/(a*c^(2/3)) + 2*(sqrt(3)*a^(
5/6)*c^(7/6)*e + a^(4/3)*c^(2/3)*d)*arctan((2*c^(1/3)*x + sqrt(3)*a^(1/6)*c
^(1/6))/sqrt(a^(1/3)*c^(1/3)))/(a*c^(2/3)*sqrt(a^(1/3)*c^(1/3))) - 2*(sqrt(
3)*a^(5/6)*c^(7/6)*e - a^(4/3)*c^(2/3)*d)*arctan((2*c^(1/3)*x - sqrt(3)*a^(
1/6)*c^(1/6))/sqrt(a^(1/3)*c^(1/3)))/(a*c^(2/3)*sqrt(a^(1/3)*c^(1/3))))/c
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 3543 vs. $2(218) = 436$.
time = 1.45, size = 3543, normalized size = 11.39

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+e/x^3)/(c+a/x^6),x, algorithm="fricas")
```

```
[Out] -1/12*(4*sqrt(3)*c*((a*c^3*sqrt(-(a^2*d^6 - 6*a*c*d^4*e^2 + 9*c^2*d^2*e^4)/
(a*c^7)) + 3*a*d^2*e - c*e^3)/(a*c^3))^(1/3)*arctan(-1/3*(2*sqrt(a^4*d^10*x
^2 - 4*a^3*c*d^8*x^2*e^2 - 2*a^2*c^2*d^6*x^2*e^4 + 12*a*c^3*d^4*x^2*e^6 + 9
*c^4*d^2*x^2*e^8 + (a^4*c^2*d^8 - 7*a^3*c^3*d^6*e^2 + 15*a^2*c^4*d^4*e^4 -
9*a*c^5*d^2*e^6 + 2*(a^3*c^6*d^4*e - 3*a^2*c^7*d^2*e^3)*sqrt(-(a^2*d^6 - 6*
a*c*d^4*e^2 + 9*c^2*d^2*e^4)/(a*c^7)))*((a*c^3*sqrt(-(a^2*d^6 - 6*a*c*d^4*e
^2 + 9*c^2*d^2*e^4)/(a*c^7)) + 3*a*d^2*e - c*e^3)/(a*c^3))^(2/3) + (a^4*c*d
^9*x - 5*a^3*c^2*d^7*x*e^2 + 3*a^2*c^3*d^5*x*e^4 + 9*a*c^4*d^3*x*e^6 + (a^3
*c^5*d^5*x*e - 2*a^2*c^6*d^3*x*e^3 - 3*a*c^7*d*x*e^5)*sqrt(-(a^2*d^6 - 6*a*
c*d^4*e^2 + 9*c^2*d^2*e^4)/(a*c^7)))*((a*c^3*sqrt(-(a^2*d^6 - 6*a*c*d^4*e^2
 + 9*c^2*d^2*e^4)/(a*c^7)) + 3*a*d^2*e - c*e^3)/(a*c^3))^(1/3))*(sqrt(3)*(a
^2*c^6*d^2 - a*c^7*e^2)*sqrt(-(a^2*d^6 - 6*a*c*d^4*e^2 + 9*c^2*d^2*e^4)/(a*
c^7)) - 2*sqrt(3)*(a^2*c^3*d^4*e - 3*a*c^4*d^2*e^3))*((a*c^3*sqrt(-(a^2*d^6
 - 6*a*c*d^4*e^2 + 9*c^2*d^2*e^4)/(a*c^7)) + 3*a*d^2*e - c*e^3)/(a*c^3))^(2
/3) + 2*(sqrt(3)*(a^4*c^6*d^7*x - 3*a^3*c^7*d^5*x*e^2 - a^2*c^8*d^3*x*e^4 +
 3*a*c^9*d*x*e^6)*sqrt(-(a^2*d^6 - 6*a*c*d^4*e^2 + 9*c^2*d^2*e^4)/(a*c^7))
- 2*sqrt(3)*(a^4*c^3*d^9*x*e - 5*a^3*c^4*d^7*x*e^3 + 3*a^2*c^5*d^5*x*e^5 +
9*a*c^6*d^3*x*e^7))*((a*c^3*sqrt(-(a^2*d^6 - 6*a*c*d^4*e^2 + 9*c^2*d^2*e^4)
/(a*c^7)) + 3*a*d^2*e - c*e^3)/(a*c^3))^(2/3) - sqrt(3)*(a^5*d^12 - 3*a^4*c
*d^10*e^2 - 6*a^3*c^2*d^8*e^4 + 10*a^2*c^3*d^6*e^6 + 21*a*c^4*d^4*e^8 + 9*c
^5*d^2*e^10))/(a^5*d^12 - 3*a^4*c*d^10*e^2 - 6*a^3*c^2*d^8*e^4 + 10*a^2*c^3
*d^6*e^6 + 21*a*c^4*d^4*e^8 + 9*c^5*d^2*e^10)) - 4*sqrt(3)*c*(-(a*c^3*sqrt(
-(a^2*d^6 - 6*a*c*d^4*e^2 + 9*c^2*d^2*e^4)/(a*c^7)) - 3*a*d^2*e + c*e^3)/(a
*c^3))^(1/3)*arctan(-1/3*(2*sqrt(a^4*d^10*x^2 - 4*a^3*c*d^8*x^2*e^2 - 2*a^2
*c^2*d^6*x^2*e^4 + 12*a*c^3*d^4*x^2*e^6 + 9*c^4*d^2*x^2*e^8 + (a^4*c^2*d^8
- 7*a^3*c^3*d^6*e^2 + 15*a^2*c^4*d^4*e^4 - 9*a*c^5*d^2*e^6 - 2*(a^3*c^6*d^4
*e - 3*a^2*c^7*d^2*e^3)*sqrt(-(a^2*d^6 - 6*a*c*d^4*e^2 + 9*c^2*d^2*e^4)/(a*
c^7)))*(-(a*c^3*sqrt(-(a^2*d^6 - 6*a*c*d^4*e^2 + 9*c^2*d^2*e^4)/(a*c^7)) -
3*a*d^2*e + c*e^3)/(a*c^3))^(2/3) + (a^4*c*d^9*x - 5*a^3*c^2*d^7*x*e^2 + 3*
a^2*c^3*d^5*x*e^4 + 9*a*c^4*d^3*x*e^6 - (a^3*c^5*d^5*x*e - 2*a^2*c^6*d^3*x*
```

```
e^3 - 3*a*c^7*d*x*e^5)*sqrt(-(a^2*d^6 - 6*a*c*d^4*e^2 + 9*c^2*d^2*e^4)/(a*c
^7)))*(-(a*c^3*sqrt(-(a^2*d^6 - 6*a*c*d^4*e^2 + 9*c^2*d^2*e^4)/(a*c^7)) - 3
*a*d^2*e + c*e^3)/(a*c^3))^(1/3))*(sqrt(3)*(a^2*c^6*d^2 - a*c^7*e^2)*sqrt(-
(a^2*d^6 - 6*a*c*d^4*e^2 + 9*c^2*d^2*e^4)/(a*c^7)) + 2*sqrt(3)*(a^2*c^3*d^4
*e - 3*a*c^4*d^2*e^3))*(-(a*c^3*sqrt(-(a^2*d^6 - 6*a*c*d^4*e^2 + 9*c^2*d^2*
e^4)/(a*c^7)) - 3*a*d^2*e + c*e^3)/(a*c^3))^(2/3) + 2*(sqrt(3)*(a^4*c^6*d^7
*x - 3*a^3*c^7*d^5*x*e^2 - a^2*c^8*d^3*x*e^4 + 3*a*c^9*d*x*e^6)*sqrt(-(a^2*
d^6 - 6*a*c*d^4*e^2 + 9*c^2*d^2*e^4)/(a*c^7)) + 2*sqrt(3)*(a^4*c^3*d^9*x*e
- 5*a^3*c^4*d^7*x*e^3 + 3*a^2*c^5*d^5*x*e^5 + 9*a*c^6*d^3*x*e^7))*(-(a*c^3*
sqrt(-(a^2*d^6 - 6*a*c*d^4*e^2 + 9*c^2*d^2*e^4)/(a*c^7)) - 3*a*d^2*e + c*e^
3)/(a*c^3))^(2/3) + sqrt(3)*(a^5*d^12 - 3*a^4*c*d^10*e^2 - 6*a^3*c^2*d^8*e^
4 + 10*a^2*c^3*d^6*e^6 + 21*a*c^4*d^4*e^8 + 9*c^5*d^2*e^10))/(a^5*d^12 - 3*
a^4*c*d^10*e^2 - 6*a^3*c^2*d^8*e^4 + 10*a^2*c^3*d^6*e^6 + 21*a*c^4*d^4*e^8
+ 9*c^5*d^2*e^10)) + c*((a*c^3*sqrt(-(a^2*d^6 - 6*a*c*d^4*e^2 + 9*c^2*d^2*e
^4)/(a*c^7)) + 3*a*d^2*e - c*e^3)/(a*c^3))^(1/3)*log(a^4*d^10*x^2 - 4*a^3*c
*d^8*x^2*e^2 - 2*a^2*c^2*d^6*x^2*e^4 + 12*a*c^3*d^4*x^2*e^6 + 9*c^4*d^2*x^2
*e^8 + (a^4*c^2*d^8 - 7*a^3*c^3*d^6*e^2 + 15*a^2*c^4*d^4*e^4 - 9*a*c^5*d^2*
e^6 + 2*(a^3*c^6*d^4*e - 3*a^2*c^7*d^2*e^3)*sqrt(-(a^2*d^6 - 6*a*c*d^4*e^2
+ 9*c^2*d^2*e^4)/(a*c^7)))*((a*c^3*sqrt(-(a^2*d^6 - 6*a*c*d^4*e^2 + 9*c^2*d
^2*e^4)/(a*c^7)) + 3*a*d^2*e - c*e^3)/(a*c^3))^(2/3) + (a^4*c*d^9*x - 5*a^3
*c^2*d^7*x*e^2 + 3*a^2*c^3*d^5*x*e^4 + 9*a*c^4*d^3*x*e^6 + (a^3*c^5*d^5*x*e
 - 2*a^2*c^6*d^3*x*e^3 - 3*a*c^7*d*x*e^5)*sqrt(-(a^2*d^6 - 6*a*c*d^4*e^2 +
9*c^2*d^2*e^4)/(a*c^7)))*((a*c^3*sqrt(-(a^2*d^6 - 6*a*c*d^4*e^2 + 9*c^2*d^2
*e^4)/(a*c^7)) + 3*a*d^2*e - c*e^3)/(a*c^3))^(1/3)) + c*(-(a*c^3*sqrt(-(a^2
*d^6 - 6*a*c*d^4*e^2 + 9*c^2*d^2*e^4)/(a*c^7)) - 3*a*d^2*e + c*e^3)/(a*c^3)
)^(1/3)*log(a^4*d^10*x^2 - 4*a^3*c*d^8*x^2*e^2 - 2*a^2*c^2*d^6*x^2*e^4 + 12
*a*c^3*d^4*x^2*e^6 + 9*c^4*d^2*x^2*e^8 + (a^4*c^2*d^8 - 7*a^3*c^3*d^6*e^2 +
 15*a^2*c^4*d^4*e^4 - 9*a*c^5*d^2*e^6 - 2*(a^3*c^6*d^4*e - 3*a^2*c^7*d^2*e^
3)*sqrt(-(a^2*d^6 - 6*a*c*d^4*e^2 + 9*c^2*d^2*e^4)/(a*c^7)))*(-(a*c^3*sqrt(
-(a^2*d^6 - 6*a*c*d^4*e^2 + 9*c^2*d^2*e^4)/(a*c^7)) - 3*a*d^2*e + c*e^3)/(a
*c^3))^(2/3) + (a^4*c*d^9*x - 5*a^3*c^2*d^7*x*e^2 + 3*a^2*c^3*d^5*x*e^4 + 9
*a*c^4*d^3*x*e^6 - (a^3*c^5*d^5*x*e - 2*a^2*c^6*d^3*x*e^3 - 3*a*c^7*d*x*e^5
)*sqrt(-(a^2*d^6 - 6*a*c*d^4*e^2 + 9*c^2*d^2*e^4)/(a*c^7)))*(-(a*c^3*sqrt(-
(a^2*d^6 - 6*a*c*d^4*e^2 + 9*c^2*d^2*e^4)/(a*c^7)) - 3*a*d^2*e + c*e^3)/(a*
c^3))^(1/3)) - 2*c*((a*c^3*sqrt(-(a^2*d^6 - 6*a*c*d^4*e^2 + 9*c^2*d^2*e^4)/
(a*c^7)) + 3*a*d^2*e - c*e^3)/(a*c^3))^(1/3)*log(-a^2*d^5*x + 2*a*c*d^3*x*e
^2 + 3*c^2*d*x*e^4 + (a*c^5*sqrt(-(a^2*d^6 - 6*...
```

Sympy [A]
time = 5.93, size = 167, normalized size = 0.54

$$\operatorname{RootSum}\left(46656t^{6}a^{2}c^{7}+t^{3}\left(-1296a^{2}c^{4}d^{2}e+432ac^{5}e^{3}\right)+a^{3}d^{6}+3a^{2}cd^{4}e^{2}+3ac^{2}d^{2}e^{4}+c^{3}e^{6},\left(t\mapsto t\log\left(x+\frac{-1296t^{4}ac^{5}e-6ta^{2}cd^{4}+36tac^{2}d^{2}e^{2}-6tc^{3}e^{4}}{a^{2}d^{5}-2acd^{3}e^{2}-3c^{2}de^{4}}\right)\right)\right)+\frac{dx}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+e/x**3)/(c+a/x**6),x)
```

```
[Out] RootSum(46656*_t**6*a**2*c**7 + _t**3*(-1296*a**2*c**4*d**2*e + 432*a*c**5*
e**3) + a**3*d**6 + 3*a**2*c*d**4*e**2 + 3*a*c**2*d**2*e**4 + c**3*e**6, La
mbda(_t, _t*log(x + (-1296*_t**4*a*c**5*e - 6*_t*a**2*c*d**4 + 36*_t*a*c**2
*d**2*e**2 - 6*_t*c**3*e**4)/(a**2*d**5 - 2*a*c*d**3*e**2 - 3*c**2*d*e**4))
)) + d*x/c
```

<u>Giac</u> [A]
time = 4.54, size = 295, normalized size = 0.95

$$-\frac{|c|e\log\left(x^2+\left(\frac{a}{c}\right)^{\frac{1}{3}}\right)}{6\left(ac^5\right)^{\frac{1}{3}}}+\frac{dx}{c}-\frac{\left(ac^5\right)^{\frac{1}{6}}d\arctan\left(\frac{x}{\left(\frac{a}{c}\right)^{\frac{1}{6}}}\right)}{3c^2}-\frac{\left(\left(ac^5\right)^{\frac{1}{6}}ac^2d+\sqrt{3}\left(ac^5\right)^{\frac{2}{3}}e\right)\arctan\left(\frac{2x+\sqrt{3}\left(\frac{a}{c}\right)^{\frac{1}{6}}}{\left(\frac{a}{c}\right)^{\frac{1}{6}}}\right)}{6ac^4}-\frac{\left(\left(ac^5\right)^{\frac{1}{6}}ac^2d-\sqrt{3}\left(ac^5\right)^{\frac{2}{3}}e\right)\arctan\left(\frac{2x-\sqrt{3}\left(\frac{a}{c}\right)^{\frac{1}{6}}}{\left(\frac{a}{c}\right)^{\frac{1}{6}}}\right)}{6ac^4}-\frac{\left(\sqrt{3}\left(ac^5\right)^{\frac{1}{6}}ac^2d-\left(ac^5\right)^{\frac{2}{3}}e\right)\log\left(x^2+\sqrt{3}x\left(\frac{a}{c}\right)^{\frac{1}{6}}+\left(\frac{a}{c}\right)^{\frac{1}{3}}\right)}{12ac^4}+\frac{\left(\sqrt{3}\left(ac^5\right)^{\frac{1}{6}}ac^2d+\left(ac^5\right)^{\frac{2}{3}}e\right)\log\left(x^2-\sqrt{3}x\left(\frac{a}{c}\right)^{\frac{1}{6}}+\left(\frac{a}{c}\right)^{\frac{1}{3}}\right)}{12ac^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+e/x^3)/(c+a/x^6),x, algorithm="giac")
```

```
[Out] -1/6*abs(c)*e*log(x^2 + (a/c)^(1/3))/(a*c^5)^(1/3) + d*x/c - 1/3*(a*c^5)^(1
/6)*d*arctan(x/(a/c)^(1/6))/c^2 - 1/6*((a*c^5)^(1/6)*a*c^2*d + sqrt(3)*(a*c
^5)^(2/3)*e)*arctan((2*x + sqrt(3)*(a/c)^(1/6))/(a/c)^(1/6))/(a*c^4) - 1/6*
((a*c^5)^(1/6)*a*c^2*d - sqrt(3)*(a*c^5)^(2/3)*e)*arctan((2*x - sqrt(3)*(a/
c)^(1/6))/(a/c)^(1/6))/(a*c^4) - 1/12*(sqrt(3)*(a*c^5)^(1/6)*a*c^2*d - (a*c
^5)^(2/3)*e)*log(x^2 + sqrt(3)*x*(a/c)^(1/6) + (a/c)^(1/3))/(a*c^4) + 1/12*
(sqrt(3)*(a*c^5)^(1/6)*a*c^2*d + (a*c^5)^(2/3)*e)*log(x^2 - sqrt(3)*x*(a/c)
^(1/6) + (a/c)^(1/3))/(a*c^4)
```

<u>Mupad</u> [B]
time = 3.10, size = 1308, normalized size = 4.21

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e/x^3)/(c + a/x^6),x)
```

```
[Out] log(e*x*(-a^3*c^7)^(1/2) - a^2*c^4*(-(a*c^5*e^3 + a*d^3*(-a^3*c^7)^(1/2) -
3*a^2*c^4*d^2*e - 3*c*d*e^2*(-a^3*c^7)^(1/2))/(a^2*c^7))^(1/3) + a^2*c^3*d*
x)*(-(a*c^5*e^3 + a*d^3*(-a^3*c^7)^(1/2) - 3*a^2*c^4*d^2*e - 3*c*d*e^2*(-a^
3*c^7)^(1/2))/(216*a^2*c^7))^(1/3) + log(e*x*(-a^3*c^7)^(1/2) + a^2*c^4*(-(
a*c^5*e^3 - a*d^3*(-a^3*c^7)^(1/2) - 3*a^2*c^4*d^2*e + 3*c*d*e^2*(-a^3*c^7)
^(1/2))/(a^2*c^7))^(1/3) - a^2*c^3*d*x)*(-(a*c^5*e^3 - a*d^3*(-a^3*c^7)^(1/
2) - 3*a^2*c^4*d^2*e + 3*c*d*e^2*(-a^3*c^7)^(1/2))/(216*a^2*c^7))^(1/3) + l
og(2*e*x*(-a^3*c^7)^(1/2) + a^2*c^4*(-(a*c^5*e^3 + a*d^3*(-a^3*c^7)^(1/2) -
 3*a^2*c^4*d^2*e - 3*c*d*e^2*(-a^3*c^7)^(1/2))/(a^2*c^7))^(1/3) - 3^(1/2)*a
^2*c^4*(-(a*c^5*e^3 + a*d^3*(-a^3*c^7)^(1/2) - 3*a^2*c^4*d^2*e - 3*c*d*e^2*
(-a^3*c^7)^(1/2))/(a^2*c^7))^(1/3)*1i + 2*a^2*c^3*d*x)*((3^(1/2)*1i)/2 - 1/
2)*(-(a*c^5*e^3 + a*d^3*(-a^3*c^7)^(1/2) - 3*a^2*c^4*d^2*e - 3*c*d*e^2*(-a^
3*c^7)^(1/2))/(216*a^2*c^7))^(1/3) - log(2*e*x*(-a^3*c^7)^(1/2) + a^2*c^4*(
-(a*c^5*e^3 + a*d^3*(-a^3*c^7)^(1/2) - 3*a^2*c^4*d^2*e - 3*c*d*e^2*(-a^3*c^
7)^(1/2))/(a^2*c^7))^(1/3) + 3^(1/2)*a^2*c^4*(-(a*c^5*e^3 + a*d^3*(-a^3*c^7
```

```
)^(1/2) - 3*a^2*c^4*d^2*e - 3*c*d*e^2*(-a^3*c^7)^(1/2))/(a^2*c^7))^(1/3)*1i
 + 2*a^2*c^3*d*x)*((3^(1/2)*1i)/2 + 1/2)*(-(a*c^5*e^3 + a*d^3*(-a^3*c^7)^(1
/2) - 3*a^2*c^4*d^2*e - 3*c*d*e^2*(-a^3*c^7)^(1/2))/(216*a^2*c^7))^(1/3) -
log(a^2*c^4*(-(a*c^5*e^3 - a*d^3*(-a^3*c^7)^(1/2) - 3*a^2*c^4*d^2*e + 3*c*d
*e^2*(-a^3*c^7)^(1/2))/(a^2*c^7))^(1/3) - 2*e*x*(-a^3*c^7)^(1/2) + 3^(1/2)*
a^2*c^4*(-(a*c^5*e^3 - a*d^3*(-a^3*c^7)^(1/2) - 3*a^2*c^4*d^2*e + 3*c*d*e^2
*(-a^3*c^7)^(1/2))/(a^2*c^7))^(1/3)*1i + 2*a^2*c^3*d*x)*((3^(1/2)*1i)/2 + 1
/2)*(-(a*c^5*e^3 - a*d^3*(-a^3*c^7)^(1/2) - 3*a^2*c^4*d^2*e + 3*c*d*e^2*(-a
^3*c^7)^(1/2))/(216*a^2*c^7))^(1/3) + log(2*e*x*(-a^3*c^7)^(1/2) - a^2*c^4*
(-(a*c^5*e^3 - a*d^3*(-a^3*c^7)^(1/2) - 3*a^2*c^4*d^2*e + 3*c*d*e^2*(-a^3*c
^7)^(1/2))/(a^2*c^7))^(1/3) + 3^(1/2)*a^2*c^4*(-(a*c^5*e^3 - a*d^3*(-a^3*c^
7)^(1/2) - 3*a^2*c^4*d^2*e + 3*c*d*e^2*(-a^3*c^7)^(1/2))/(a^2*c^7))^(1/3)*1
i - 2*a^2*c^3*d*x)*((3^(1/2)*1i)/2 - 1/2)*(-(a*c^5*e^3 - a*d^3*(-a^3*c^7)^(
1/2) - 3*a^2*c^4*d^2*e + 3*c*d*e^2*(-a^3*c^7)^(1/2))/(216*a^2*c^7))^(1/3) +
 (d*x)/c
```

3.39 $\int \frac{d+\frac{e}{x^3}}{c+\frac{a}{x^6}+\frac{b}{x^3}}\,dx$

Optimal. Leaf size=716

$$\frac{dx}{c}+\frac{\left(bd-ce-\frac{b^2d-2acd-bce}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{2}\,\sqrt[3]{c}\,x}{\sqrt[3]{b-\sqrt{b^2-4ac}}}}{\sqrt{3}}\right)}{\sqrt[3]{2}\,\sqrt{3}\,c^{4/3}\left(b-\sqrt{b^2-4ac}\right)^{2/3}}+\frac{\left(bd-ce+\frac{b^2d-2acd-bce}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{2}}{\sqrt[3]{b+}}}{}\right)}{\sqrt[3]{2}\,\sqrt{3}\,c^{4/3}\left(b+\sqrt{b^2-4ac}\right)^{2}}$$

```
[Out] d*x/c-1/6*ln(2^(1/3)*c^(1/3)*x+(b-(-4*a*c+b^2)^(1/2))^(1/3))*(b*d-c*e+(2*a*
c*d-b^2*d+b*c*e)/(-4*a*c+b^2)^(1/2))*2^(2/3)/c^(4/3)/(b-(-4*a*c+b^2)^(1/2))
^(2/3)+1/12*ln(2^(2/3)*c^(2/3)*x^2-2^(1/3)*c^(1/3)*x*(b-(-4*a*c+b^2)^(1/2))
^(1/3)+(b-(-4*a*c+b^2)^(1/2))^(2/3))*(b*d-c*e+(2*a*c*d-b^2*d+b*c*e)/(-4*a*c
+b^2)^(1/2))*2^(2/3)/c^(4/3)/(b-(-4*a*c+b^2)^(1/2))^(2/3)+1/6*arctan(1/3*(1
-2*2^(1/3)*c^(1/3)*x/(b-(-4*a*c+b^2)^(1/2))^(1/3))*3^(1/2))*(b*d-c*e+(2*a*c
*d-b^2*d+b*c*e)/(-4*a*c+b^2)^(1/2))*2^(2/3)/c^(4/3)*3^(1/2)/(b-(-4*a*c+b^2)
^(1/2))^(2/3)-1/6*ln(2^(1/3)*c^(1/3)*x+(b+(-4*a*c+b^2)^(1/2))^(1/3))*(b*d-c
*e+(-2*a*c*d+b^2*d-b*c*e)/(-4*a*c+b^2)^(1/2))*2^(2/3)/c^(4/3)/(b+(-4*a*c+b^
2)^(1/2))^(2/3)+1/12*ln(2^(2/3)*c^(2/3)*x^2-2^(1/3)*c^(1/3)*x*(b+(-4*a*c+b^
2)^(1/2))^(1/3)+(b+(-4*a*c+b^2)^(1/2))^(2/3))*(b*d-c*e+(-2*a*c*d+b^2*d-b*c*
e)/(-4*a*c+b^2)^(1/2))*2^(2/3)/c^(4/3)/(b+(-4*a*c+b^2)^(1/2))^(2/3)+1/6*arc
tan(1/3*(1-2*2^(1/3)*c^(1/3)*x/(b+(-4*a*c+b^2)^(1/2))^(1/3))*3^(1/2))*(b*d-
c*e+(-2*a*c*d+b^2*d-b*c*e)/(-4*a*c+b^2)^(1/2))*2^(2/3)/c^(4/3)*3^(1/2)/(b+(
-4*a*c+b^2)^(1/2))^(2/3)
```

Rubi [A]
time = 1.07, antiderivative size = 716, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 9, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.409$, Rules used = {1407, 1516, 1436, 206, 31, 648, 631, 210, 642}

$$\frac{\text{ArcTan}\left(\frac{1-\frac{2\sqrt[3]{2}\sqrt[3]{c}x}{\sqrt[3]{b-\sqrt{b^2-4ac}}}}{\sqrt{3}}\right)\left(-\frac{-2acd+b^2d-bce}{\sqrt{b^2-4ac}}+bd-ce\right)}{\sqrt[3]{2}\sqrt{3}c^{4/3}\left(b-\sqrt{b^2-4ac}\right)^{2/3}}+\frac{\text{ArcTan}\left(\frac{1-\frac{2\sqrt[3]{2}\sqrt[3]{c}x}{\sqrt[3]{\sqrt{b^2-4ac}+b}}}{\sqrt{3}}\right)\left(\frac{-2acd+b^2d-bce}{\sqrt{b^2-4ac}}+bd-ce\right)}{\sqrt[3]{2}\sqrt{3}c^{4/3}\left(\sqrt{b^2-4ac}+b\right)^{2/3}}+\frac{\left(-\frac{-2acd+b^2d-bce}{\sqrt{b^2-4ac}}+bd-ce\right)\log\left(-\sqrt[3]{2}\sqrt[3]{c}x\sqrt[3]{b-\sqrt{b^2-4ac}}+\left(b-\sqrt{b^2-4ac}\right)^{2/3}+2^{2/3}c^{2/3}x^2\right)}{6\sqrt[3]{2}c^{4/3}\left(b-\sqrt{b^2-4ac}\right)^{2/3}}+\frac{\left(\frac{-2acd+b^2d-bce}{\sqrt{b^2-4ac}}+bd-ce\right)\log\left(-\sqrt[3]{2}\sqrt[3]{c}x\sqrt[3]{\sqrt{b^2-4ac}+b}+\left(\sqrt{b^2-4ac}+b\right)^{2/3}+2^{2/3}c^{2/3}x^2\right)}{6\sqrt[3]{2}c^{4/3}\left(\sqrt{b^2-4ac}+b\right)^{2/3}}-\frac{\left(-\frac{-2acd+b^2d-bce}{\sqrt{b^2-4ac}}+bd-ce\right)\log\left(\sqrt[3]{b-\sqrt{b^2-4ac}}+\sqrt[3]{2}\sqrt[3]{c}x\right)}{3\sqrt[3]{2}c^{4/3}\left(b-\sqrt{b^2-4ac}\right)^{2/3}}-\frac{\left(\frac{-2acd+b^2d-bce}{\sqrt{b^2-4ac}}+bd-ce\right)\log\left(\sqrt[3]{\sqrt{b^2-4ac}+b}+\sqrt[3]{2}\sqrt[3]{c}x\right)}{3\sqrt[3]{2}c^{4/3}\left(\sqrt{b^2-4ac}+b\right)^{2/3}}+\frac{dx}{c}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e/x^3)/(c + a/x^6 + b/x^3),x]
```

```
[Out] (d*x)/c + ((b*d - c*e - (b^2*d - 2*a*c*d - b*c*e)/Sqrt[b^2 - 4*a*c])*ArcTan
[(1 - (2*2^(1/3)*c^(1/3)*x)/(b - Sqrt[b^2 - 4*a*c])^(1/3))/Sqrt[3]])/(2^(1/
3)*Sqrt[3]*c^(4/3)*(b - Sqrt[b^2 - 4*a*c])^(2/3)) + ((b*d - c*e + (b^2*d -
2*a*c*d - b*c*e)/Sqrt[b^2 - 4*a*c])*ArcTan[(1 - (2*2^(1/3)*c^(1/3)*x)/(b +
Sqrt[b^2 - 4*a*c])^(1/3))/Sqrt[3]])/(2^(1/3)*Sqrt[3]*c^(4/3)*(b + Sqrt[b^2
```

```
- 4*a*c])^(2/3)) - ((b*d - c*e - (b^2*d - 2*a*c*d - b*c*e)/Sqrt[b^2 - 4*a*c
])*Log[(b - Sqrt[b^2 - 4*a*c])^(1/3) + 2^(1/3)*c^(1/3)*x])/(3*2^(1/3)*c^(4/
3)*(b - Sqrt[b^2 - 4*a*c])^(2/3)) - ((b*d - c*e + (b^2*d - 2*a*c*d - b*c*e)
/Sqrt[b^2 - 4*a*c])*Log[(b + Sqrt[b^2 - 4*a*c])^(1/3) + 2^(1/3)*c^(1/3)*x])
/(3*2^(1/3)*c^(4/3)*(b + Sqrt[b^2 - 4*a*c])^(2/3)) + ((b*d - c*e - (b^2*d -
 2*a*c*d - b*c*e)/Sqrt[b^2 - 4*a*c])*Log[(b - Sqrt[b^2 - 4*a*c])^(2/3) - 2^
(1/3)*c^(1/3)*(b - Sqrt[b^2 - 4*a*c])^(1/3)*x + 2^(2/3)*c^(2/3)*x^2])/(6*2^
(1/3)*c^(4/3)*(b - Sqrt[b^2 - 4*a*c])^(2/3)) + ((b*d - c*e + (b^2*d - 2*a*c
*d - b*c*e)/Sqrt[b^2 - 4*a*c])*Log[(b + Sqrt[b^2 - 4*a*c])^(2/3) - 2^(1/3)*
c^(1/3)*(b + Sqrt[b^2 - 4*a*c])^(1/3)*x + 2^(2/3)*c^(2/3)*x^2])/(6*2^(1/3)*
c^(4/3)*(b + Sqrt[b^2 - 4*a*c])^(2/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
```

```
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1407

```
Int[((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.)*((d_) + (e_.)*(x_)^(
n_))^(q_.), x_Symbol] :> Int[x^(n*(2*p + q))*(e + d/x^n)^q*(c + b/x^n + a/x
^(2*n))^p, x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[n2, 2*n] && IntegersQ[
p, q] && NegQ[n]
```

Rule 1436

```
Int[((d_) + (e_.)*(x_)^(n_))/((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_)), x
_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q),
Int[1/(b/2 - q/2 + c*x^n), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(
b/2 + q/2 + c*x^n), x], x]] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[n2, 2*n]
 && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && (PosQ[b^2 - 4*a
*c] ||  !IGtQ[n/2, 0])
```

Rule 1516

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^(n_))*((a_) + (b_.)*(x_)^(n_) + (
c_.)*(x_)^(n2_))^(p_), x_Symbol] :> Simp[e*f^(n - 1)*(f*x)^(m - n + 1)*((a
+ b*x^n + c*x^(2*n))^(p + 1)/(c*(m + n*(2*p + 1) + 1))), x] - Dist[f^n/(c*(
m + n*(2*p + 1) + 1)), Int[(f*x)^(m - n)*(a + b*x^n + c*x^(2*n))^p*Simp[a*e
*(m - n + 1) + (b*e*(m + n*p + 1) - c*d*(m + n*(2*p + 1) + 1))*x^n, x], x],
 x] /; FreeQ[{a, b, c, d, e, f, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c,
0] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*(2*p + 1) + 1, 0] && Integer
Q[p]
```

Rubi steps

$$\int \frac{d+\frac{e}{x^3}}{c+\frac{a}{x^6}+\frac{b}{x^3}}\,dx = \int \frac{x^3(e+dx^3)}{a+bx^3+cx^6}\,dx$$

$$= \frac{dx}{c} - \frac{\int \frac{ad+(bd-ce)x^3}{a+bx^3+cx^6}\,dx}{c}$$

$$= \frac{dx}{c} - \frac{\left(bd-ce-\frac{b^2d-2acd-bce}{\sqrt{b^2-4ac}}\right)\int \frac{1}{\frac{b}{2}-\frac{1}{2}\sqrt{b^2-4ac}+cx^3}\,dx}{2c} - \frac{\left(bd-ce+\frac{b^2d-2acd-bce}{\sqrt{b^2-4ac}}\right)\int}{2c}$$

$$= \frac{dx}{c} - \frac{\left(bd-ce-\frac{b^2d-2acd-bce}{\sqrt{b^2-4ac}}\right)\int \frac{1}{\frac{\sqrt[3]{b-\sqrt{b^2-4ac}}}{\sqrt[3]{2}}+\sqrt[3]{c}\,x}\,dx}{3\sqrt[3]{2}\,c\left(b-\sqrt{b^2-4ac}\right)^{2/3}} - \frac{\left(bd-ce-\frac{b^2d-2acd-}{\sqrt{b^2-4}}\right.}{}$$

$$= \frac{dx}{c} - \frac{\left(bd-ce-\frac{b^2d-2acd-bce}{\sqrt{b^2-4ac}}\right)\log\left(\sqrt[3]{b-\sqrt{b^2-4ac}}+\sqrt[3]{2}\,\sqrt[3]{c}\,x\right)}{3\sqrt[3]{2}\,c^{4/3}\left(b-\sqrt{b^2-4ac}\right)^{2/3}} - \frac{\left(bd-ce+\right.}{}$$

$$= \frac{dx}{c} - \frac{\left(bd-ce-\frac{b^2d-2acd-bce}{\sqrt{b^2-4ac}}\right)\log\left(\sqrt[3]{b-\sqrt{b^2-4ac}}+\sqrt[3]{2}\,\sqrt[3]{c}\,x\right)}{3\sqrt[3]{2}\,c^{4/3}\left(b-\sqrt{b^2-4ac}\right)^{2/3}} - \frac{\left(bd-ce+\right.}{}$$

$$= \frac{dx}{c} + \frac{\left(bd-ce-\frac{b^2d-2acd-bce}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{2}\,\sqrt[3]{c}\,x}{\sqrt[3]{b-\sqrt{b^2-4ac}}}}{\sqrt{3}}\right)}{\sqrt[3]{2}\,\sqrt{3}\,c^{4/3}\left(b-\sqrt{b^2-4ac}\right)^{2/3}} + \frac{\left(bd-ce+\frac{b^2d-2acd}{\sqrt{b^2-}}\right.}{\sqrt[3]{2}\,\sqrt{3}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.
time = 0.04, size = 88, normalized size = 0.12

$$\frac{dx}{c} - \frac{\text{RootSum}\left[a+b\#1^3+c\#1^6\&, \frac{ad\log(x-\#1)+bd\log(x-\#1)\#1^3-ce\log(x-\#1)\#1^3}{b\#1^2+2c\#1^5}\&\right]}{3c}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e/x^3)/(c + a/x^6 + b/x^3),x]
```

```
[Out] (d*x)/c - RootSum[a + b*#1^3 + c*#1^6 & , (a*d*Log[x - #1] + b*d*Log[x - #1
]*#1^3 - c*e*Log[x - #1]*#1^3)/(b*#1^2 + 2*c*#1^5) & ]/(3*c)
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 0.23, size = 67, normalized size = 0.09

method	result	size
default	$\frac{dx}{c}+\frac{\sum_{\_R=\mathrm{RootOf}\left(\_Z^{6}c+\_Z^{3}b+a\right)}\frac{\left(\left(-bd+ce\right)\_R^{3}-ad\right)\ln\left(x-\_R\right)}{2\_R^{5}c+\_R^{2}b}}{3c}$	67
risch	$\frac{dx}{c}+\frac{\sum_{\_R=\mathrm{RootOf}\left(\_Z^{6}c+\_Z^{3}b+a\right)}\frac{\left(\left(-bd+ce\right)\_R^{3}-ad\right)\ln\left(x-\_R\right)}{2\_R^{5}c+\_R^{2}b}}{3c}$	67

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d+e/x^3)/(c+a/x^6+b/x^3),x,method=_RETURNVERBOSE)
```

```
[Out] d*x/c+1/3/c*sum(((-b*d+c*e)*_R^3-a*d)/(2*_R^5*c+_R^2*b)*ln(x-_R),_R=RootOf(
_Z^6*c+_Z^3*b+a))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+e/x^3)/(c+a/x^6+b/x^3),x, algorithm="maxima")
```

```
[Out] d*x/c - integrate(((b*d - c*e)*x^3 + a*d)/(c*x^6 + b*x^3 + a), x)/c
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+e/x^3)/(c+a/x^6+b/x^3),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+e/x**3)/(c+a/x**6+b/x**3),x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+e/x^3)/(c+a/x^6+b/x^3),x, algorithm="giac")
```

```
[Out] integrate((d + e/x^3)/(c + b/x^3 + a/x^6), x)
```

Mupad [B]
time = 29.42, size = 2500, normalized size = 3.49

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e/x^3)/(c + a/x^6 + b/x^3),x)
```

```
[Out] log((3*a*x*(a*b^4*d^4 - 2*a*c^4*e^4 - b^5*d^3*e + 2*a^3*c^2*d^4 + b^2*c^3*e
^4 - 4*a^2*b^2*c*d^4 - 3*b^3*c^2*d*e^3 + 3*b^4*c*d^2*e^2 + 8*a*b*c^3*d*e^3
+ 2*a*b^3*c*d^3*e + 4*a^2*b*c^2*d^3*e - 9*a*b^2*c^2*d^2*e^2))/c - (2^(2/3)*
((2^(1/3)*(81*a*c^3*e*x*(4*a*c - b^2)^2 - (81*2^(2/3)*a*b*c^3*(4*a*c - b^2)
^2*((b^7*d^3 + b^4*d^3*(-(4*a*c - b^2)^3)^(1/2) - 16*a^2*c^5*e^3 - b^4*c^3*
e^3 - 32*a^3*b*c^3*d^3 + 8*a*b^2*c^4*e^3 - b*c^3*e^3*(-(4*a*c - b^2)^3)^(1/
2) + 48*a^3*c^4*d^2*e + 3*b^5*c^2*d*e^2 + 32*a^2*b^3*c^2*d^3 + 2*a^2*c^2*d^
3*(-(4*a*c - b^2)^3)^(1/2) - 10*a*b^5*c*d^3 - 3*b^6*c*d^2*e - 4*a*b^2*c*d^3
*(-(4*a*c - b^2)^3)^(1/2) - 24*a*b^3*c^3*d*e^2 + 27*a*b^4*c^2*d^2*e + 48*a^
2*b*c^4*d*e^2 - 6*a*c^3*d*e^2*(-(4*a*c - b^2)^3)^(1/2) - 3*b^3*c*d^2*e*(-(4
*a*c - b^2)^3)^(1/2) - 72*a^2*b^2*c^3*d^2*e + 3*b^2*c^2*d*e^2*(-(4*a*c - b^
2)^3)^(1/2) + 9*a*b*c^2*d^2*e*(-(4*a*c - b^2)^3)^(1/2))/(c^4*(4*a*c - b^2)^
3))^(1/3))/2)*((b^7*d^3 + b^4*d^3*(-(4*a*c - b^2)^3)^(1/2) - 16*a^2*c^5*e^3
 - b^4*c^3*e^3 - 32*a^3*b*c^3*d^3 + 8*a*b^2*c^4*e^3 - b*c^3*e^3*(-(4*a*c -
b^2)^3)^(1/2) + 48*a^3*c^4*d^2*e + 3*b^5*c^2*d*e^2 + 32*a^2*b^3*c^2*d^3 + 2
*a^2*c^2*d^3*(-(4*a*c - b^2)^3)^(1/2) - 10*a*b^5*c*d^3 - 3*b^6*c*d^2*e - 4*
a*b^2*c*d^3*(-(4*a*c - b^2)^3)^(1/2) - 24*a*b^3*c^3*d*e^2 + 27*a*b^4*c^2*d^
2*e + 48*a^2*b*c^4*d*e^2 - 6*a*c^3*d*e^2*(-(4*a*c - b^2)^3)^(1/2) - 3*b^3*c
*d^2*e*(-(4*a*c - b^2)^3)^(1/2) - 72*a^2*b^2*c^3*d^2*e + 3*b^2*c^2*d*e^2*(-
(4*a*c - b^2)^3)^(1/2) + 9*a*b*c^2*d^2*e*(-(4*a*c - b^2)^3)^(1/2))/(c^4*(4*
a*c - b^2)^3))^(2/3))/18 + (9*a*(4*a*c - b^2)*(b^4*d^3 - b*c^3*e^3 + a^2*c^
2*d^3 + 3*b^2*c^2*d*e^2 - 3*a*b^2*c*d^3 - 3*a*c^3*d*e^2 - 3*b^3*c*d^2*e + 6
```

```
*a*b*c^2*d^2*e))/c)*((b^7*d^3 + b^4*d^3*(-(4*a*c - b^2)^3)^(1/2) - 16*a^2*c
^5*e^3 - b^4*c^3*e^3 - 32*a^3*b*c^3*d^3 + 8*a*b^2*c^4*e^3 - b*c^3*e^3*(-(4*
a*c - b^2)^3)^(1/2) + 48*a^3*c^4*d^2*e + 3*b^5*c^2*d*e^2 + 32*a^2*b^3*c^2*d
^3 + 2*a^2*c^2*d^3*(-(4*a*c - b^2)^3)^(1/2) - 10*a*b^5*c*d^3 - 3*b^6*c*d^2*
e - 4*a*b^2*c*d^3*(-(4*a*c - b^2)^3)^(1/2) - 24*a*b^3*c^3*d*e^2 + 27*a*b^4*
c^2*d^2*e + 48*a^2*b*c^4*d*e^2 - 6*a*c^3*d*e^2*(-(4*a*c - b^2)^3)^(1/2) - 3
*b^3*c*d^2*e*(-(4*a*c - b^2)^3)^(1/2) - 72*a^2*b^2*c^3*d^2*e + 3*b^2*c^2*d*
e^2*(-(4*a*c - b^2)^3)^(1/2) + 9*a*b*c^2*d^2*e*(-(4*a*c - b^2)^3)^(1/2))/(c
^4*(4*a*c - b^2)^3))^(1/3))/6)*((b^7*d^3 + b^4*d^3*(-(4*a*c - b^2)^3)^(1/2)
 - 16*a^2*c^5*e^3 - b^4*c^3*e^3 - 32*a^3*b*c^3*d^3 + 8*a*b^2*c^4*e^3 - b*c^
3*e^3*(-(4*a*c - b^2)^3)^(1/2) + 48*a^3*c^4*d^2*e + 3*b^5*c^2*d*e^2 + 32*a^
2*b^3*c^2*d^3 + 2*a^2*c^2*d^3*(-(4*a*c - b^2)^3)^(1/2) - 10*a*b^5*c*d^3 - 3
*b^6*c*d^2*e - 4*a*b^2*c*d^3*(-(4*a*c - b^2)^3)^(1/2) - 24*a*b^3*c^3*d*e^2
+ 27*a*b^4*c^2*d^2*e + 48*a^2*b*c^4*d*e^2 - 6*a*c^3*d*e^2*(-(4*a*c - b^2)^3
)^(1/2) - 3*b^3*c*d^2*e*(-(4*a*c - b^2)^3)^(1/2) - 72*a^2*b^2*c^3*d^2*e + 3
*b^2*c^2*d*e^2*(-(4*a*c - b^2)^3)^(1/2) + 9*a*b*c^2*d^2*e*(-(4*a*c - b^2)^3
)^(1/2))/(54*(64*a^3*c^7 - b^6*c^4 + 12*a*b^4*c^5 - 48*a^2*b^2*c^6)))^(1/3)
 + log((3*a*x*(a*b^4*d^4 - 2*a*c^4*e^4 - b^5*d^3*e + 2*a^3*c^2*d^4 + b^2*c^
3*e^4 - 4*a^2*b^2*c*d^4 - 3*b^3*c^2*d*e^3 + 3*b^4*c*d^2*e^2 + 8*a*b*c^3*d*e
^3 + 2*a*b^3*c*d^3*e + 4*a^2*b*c^2*d^3*e - 9*a*b^2*c^2*d^2*e^2))/c - (2^(2/
3)*((2^(1/3)*(81*a*c^3*e*x*(4*a*c - b^2)^2 - (81*2^(2/3)*a*b*c^3*(4*a*c - b
^2)^2*((b^7*d^3 - b^4*d^3*(-(4*a*c - b^2)^3)^(1/2) - 16*a^2*c^5*e^3 - b^4*c
^3*e^3 - 32*a^3*b*c^3*d^3 + 8*a*b^2*c^4*e^3 + b*c^3*e^3*(-(4*a*c - b^2)^3)^
(1/2) + 48*a^3*c^4*d^2*e + 3*b^5*c^2*d*e^2 + 32*a^2*b^3*c^2*d^3 - 2*a^2*c^2
*d^3*(-(4*a*c - b^2)^3)^(1/2) - 10*a*b^5*c*d^3 - 3*b^6*c*d^2*e + 4*a*b^2*c*
d^3*(-(4*a*c - b^2)^3)^(1/2) - 24*a*b^3*c^3*d*e^2 + 27*a*b^4*c^2*d^2*e + 48
*a^2*b*c^4*d*e^2 + 6*a*c^3*d*e^2*(-(4*a*c - b^2)^3)^(1/2) + 3*b^3*c*d^2*e*(
-(4*a*c - b^2)^3)^(1/2) - 72*a^2*b^2*c^3*d^2*e - 3*b^2*c^2*d*e^2*(-(4*a*c -
 b^2)^3)^(1/2) - 9*a*b*c^2*d^2*e*(-(4*a*c - b^2)^3)^(1/2))/(c^4*(4*a*c - b^
2)^3))^(1/3))/2)*((b^7*d^3 - b^4*d^3*(-(4*a*c - b^2)^3)^(1/2) - 16*a^2*c^5*
e^3 - b^4*c^3*e^3 - 32*a^3*b*c^3*d^3 + 8*a*b^2*c^4*e^3 + b*c^3*e^3*(-(4*a*c
 - b^2)^3)^(1/2) + 48*a^3*c^4*d^2*e + 3*b^5*c^2*d*e^2 + 32*a^2*b^3*c^2*d^3
- 2*a^2*c^2*d^3*(-(4*a*c - b^2)^3)^(1/2) - 10*a*b^5*c*d^3 - 3*b^6*c*d^2*e +
 4*a*b^2*c*d^3*(-(4*a*c - b^2)^3)^(1/2) - 24*a*b^3*c^3*d*e^2 + 27*a*b^4*c^2
*d^2*e + 48*a^2*b*c^4*d*e^2 + 6*a*c^3*d*e^2*(-(4*a*c - b^2)^3)^(1/2) + 3*b^
3*c*d^2*e*(-(4*a*c - b^2)^3)^(1/2) - 72*a^2*b^2*c^3*d^2*e - 3*b^2*c^2*d*e^2
*(-(4*a*c - b^2)^3)^(1/2) - 9*a*b*c^2*d^2*e*(-(4*a*c - b^2)^3)^(1/2))/(c^4*
(4*a*c - b^2)^3))^(2/3))/18 + (9*a*(4*a*c - b^2)*(b^4*d^3 - b*c^3*e^3 + a^2
*c^2*d^3 + 3*b^2*c^2*d*e^2 - 3*a*b^2*c*d^3 - 3*a*c^3*d*e^2 - 3*b^3*c*d^2*e
+ 6*a*b*c^2*d^2*e))/c)*((b^7*d^3 - b^4*d^3*(-(4*a*c - b^2)^3)^(1/2) - 16*a^
2*c^5*e^3 - b^4*c^3*e^3 - 32*a^3*b*c^3*d^3 + 8*a*b^2*c^4*e^3 + b*c^3*e^3*(-
(4*a*c - b^2)^3)^(1/2) + 48*a^3*c^4*d^2*e + 3*b^5*c^2*d*e^2 + 32*a^2*b^3*c^
2*d^3 - 2*a^2*c^2*d^3*(-(4*a*c - b^2)^3)^(1/2) - 10*a*b^5*c*d^3 - 3*b^6*c*d
^2*e + 4*a*b^2*c*d^3*(-(4*a*c - b^2)^3)^(1/2) - 24*a*b^3*c^3*d*e^2 + 27*a*b
^4*c^2*d^2*e + 48*a^2*b*c^4*d*e^2 + 6*a*c^3*d*e...
```

3.40 $\int \frac{d+\frac{e}{x^4}}{c+\frac{a}{x^8}} dx$

Optimal. Leaf size=753

$$\frac{dx}{c}+\frac{\sqrt{2-\sqrt{2}}\left(\left(1+\sqrt{2}\right)\sqrt{a}\,d+\sqrt{c}\,e\right)\tan^{-1}\left(\frac{\sqrt{2-\sqrt{2}}\,\sqrt[8]{a}-2\sqrt[8]{c}\,x}{\sqrt{2+\sqrt{2}}\,\sqrt[8]{a}}\right)}{8a^{3/8}c^{9/8}}-\frac{\sqrt{2+\sqrt{2}}\left(\sqrt{a}\left(d-\sqrt{2}\right.\right.}{}$$

```
[Out] d*x/c+1/8*arctan((-2*c^(1/8)*x+a^(1/8)*(2-2^(1/2))^(1/2))/a^(1/8)/(2+2^(1/2
))^(1/2))*(d*(1+2^(1/2))*a^(1/2)+e*c^(1/2))*(2-2^(1/2))^(1/2)/a^(3/8)/c^(9/
8)-1/8*arctan((2*c^(1/8)*x+a^(1/8)*(2-2^(1/2))^(1/2))/a^(1/8)/(2+2^(1/2))^(
1/2))*(d*(1+2^(1/2))*a^(1/2)+e*c^(1/2))*(2-2^(1/2))^(1/2)/a^(3/8)/c^(9/8)-1
/8*ln(a^(1/4)+c^(1/4)*x^2-a^(1/8)*c^(1/8)*x*(2-2^(1/2))^(1/2))*((d-d*2^(1/2
))*a^(1/2)+e*c^(1/2))/a^(3/8)/c^(9/8)/(4-2*2^(1/2))^(1/2)+1/8*ln(a^(1/4)+c^
(1/4)*x^2+a^(1/8)*c^(1/8)*x*(2-2^(1/2))^(1/2))*((d-d*2^(1/2))*a^(1/2)+e*c^(
1/2))/a^(3/8)/c^(9/8)/(4-2*2^(1/2))^(1/2)-1/8*arctan((-2*c^(1/8)*x+a^(1/8)*
(2+2^(1/2))^(1/2))/a^(1/8)/(2-2^(1/2))^(1/2))*((d-d*2^(1/2))*a^(1/2)+e*c^(1
/2))*(2+2^(1/2))^(1/2)/a^(3/8)/c^(9/8)+1/8*arctan((2*c^(1/8)*x+a^(1/8)*(2+2
^(1/2))^(1/2))/a^(1/8)/(2-2^(1/2))^(1/2))*((d-d*2^(1/2))*a^(1/2)+e*c^(1/2))
*(2+2^(1/2))^(1/2)/a^(3/8)/c^(9/8)+1/8*ln(a^(1/4)+c^(1/4)*x^2-a^(1/8)*c^(1/
8)*x*(2+2^(1/2))^(1/2))*(d*(1+2^(1/2))*a^(1/2)+e*c^(1/2))/a^(3/8)/c^(9/8)/(
4+2*2^(1/2))^(1/2)-1/8*ln(a^(1/4)+c^(1/4)*x^2+a^(1/8)*c^(1/8)*x*(2+2^(1/2))
^(1/2))*(d*(1+2^(1/2))*a^(1/2)+e*c^(1/2))/a^(3/8)/c^(9/8)/(4+2*2^(1/2))^(1/
2)
```

Rubi [A]
time = 0.99, antiderivative size = 753, normalized size of antiderivative = 1.00, number of steps used = 21, number of rules used = 8, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.471$, Rules used = {1408, 1517, 1429, 1183, 648, 632, 210, 642}

$$\frac{\sqrt{2-\sqrt{2}}\,\text{ArcTan}\left(\frac{\sqrt{2-\sqrt{2}}\,\sqrt[8]{a}-2\sqrt[8]{c}\,x}{\sqrt{2+\sqrt{2}}\,\sqrt[8]{a}}\right)\left(\left(1+\sqrt{2}\right)\sqrt{a}\,d+\sqrt{c}\,e\right)}{8a^{3/8}c^{9/8}}-\frac{\sqrt{2+\sqrt{2}}\,\text{ArcTan}\left(\frac{\sqrt{2+\sqrt{2}}\,\sqrt[8]{a}-2\sqrt[8]{c}\,x}{\sqrt{2-\sqrt{2}}\,\sqrt[8]{a}}\right)\left(\sqrt{a}\left(d-\sqrt{2}\,d\right)+\sqrt{c}\,e\right)}{8a^{3/8}c^{9/8}}-\frac{\sqrt{2-\sqrt{2}}\,\text{ArcTan}\left(\frac{\sqrt{2-\sqrt{2}}\,\sqrt[8]{a}+2\sqrt[8]{c}\,x}{\sqrt{2+\sqrt{2}}\,\sqrt[8]{a}}\right)\left(\left(1+\sqrt{2}\right)\sqrt{a}\,d+\sqrt{c}\,e\right)}{8a^{3/8}c^{9/8}}+\frac{\sqrt{2+\sqrt{2}}\,\text{ArcTan}\left(\frac{\sqrt{2+\sqrt{2}}\,\sqrt[8]{a}+2\sqrt[8]{c}\,x}{\sqrt{2-\sqrt{2}}\,\sqrt[8]{a}}\right)\left(\sqrt{a}\left(d-\sqrt{2}\,d\right)+\sqrt{c}\,e\right)}{8a^{3/8}c^{9/8}}-\frac{\left(\sqrt{a}\left(d-\sqrt{2}\,d\right)+\sqrt{c}\,e\right)\log\left(-\sqrt{2-\sqrt{2}}\,\sqrt[8]{a}\,\sqrt[8]{c}\,x+\sqrt[4]{a}+\sqrt[4]{c}\,x^2\right)}{8\sqrt{2\left(2-\sqrt{2}\right)}\,a^{3/8}c^{9/8}}+\frac{\left(\sqrt{a}\left(d-\sqrt{2}\,d\right)+\sqrt{c}\,e\right)\log\left(\sqrt{2-\sqrt{2}}\,\sqrt[8]{a}\,\sqrt[8]{c}\,x+\sqrt[4]{a}+\sqrt[4]{c}\,x^2\right)}{8\sqrt{2\left(2-\sqrt{2}\right)}\,a^{3/8}c^{9/8}}+\frac{\left(\left(1+\sqrt{2}\right)\sqrt{a}\,d+\sqrt{c}\,e\right)\log\left(-\sqrt{2+\sqrt{2}}\,\sqrt[8]{a}\,\sqrt[8]{c}\,x+\sqrt[4]{a}+\sqrt[4]{c}\,x^2\right)}{8\sqrt{2\left(2+\sqrt{2}\right)}\,a^{3/8}c^{9/8}}-\frac{\left(\left(1+\sqrt{2}\right)\sqrt{a}\,d+\sqrt{c}\,e\right)\log\left(\sqrt{2+\sqrt{2}}\,\sqrt[8]{a}\,\sqrt[8]{c}\,x+\sqrt[4]{a}+\sqrt[4]{c}\,x^2\right)}{8\sqrt{2\left(2+\sqrt{2}\right)}\,a^{3/8}c^{9/8}}+\frac{dx}{c}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e/x^4)/(c + a/x^8),x]
```

```
[Out] (d*x)/c + (Sqrt[2 - Sqrt[2]]*((1 + Sqrt[2])*Sqrt[a]*d + Sqrt[c]*e)*ArcTan[(
Sqrt[2 - Sqrt[2]]*a^(1/8) - 2*c^(1/8)*x)/(Sqrt[2 + Sqrt[2]]*a^(1/8))])/(8*a
^(3/8)*c^(9/8)) - (Sqrt[2 + Sqrt[2]]*(Sqrt[a]*(d - Sqrt[2]*d) + Sqrt[c]*e)*
ArcTan[(Sqrt[2 + Sqrt[2]]*a^(1/8) - 2*c^(1/8)*x)/(Sqrt[2 - Sqrt[2]]*a^(1/8)
)])/(8*a^(3/8)*c^(9/8)) - (Sqrt[2 - Sqrt[2]]*((1 + Sqrt[2])*Sqrt[a]*d + Sqr
t[c]*e)*ArcTan[(Sqrt[2 - Sqrt[2]]*a^(1/8) + 2*c^(1/8)*x)/(Sqrt[2 + Sqrt[2]]
*a^(1/8))])/(8*a^(3/8)*c^(9/8)) + (Sqrt[2 + Sqrt[2]]*(Sqrt[a]*(d - Sqrt[2]*
```

```
d) + Sqrt[c]*e)*ArcTan[(Sqrt[2 + Sqrt[2]]*a^(1/8) + 2*c^(1/8)*x)/(Sqrt[2 -
Sqrt[2]]*a^(1/8))]])/(8*a^(3/8)*c^(9/8)) - ((Sqrt[a]*(d - Sqrt[2]*d) + Sqrt[
c]*e)*Log[a^(1/4) - Sqrt[2 - Sqrt[2]]*a^(1/8)*c^(1/8)*x + c^(1/4)*x^2])/(8*
Sqrt[2*(2 - Sqrt[2])]*a^(3/8)*c^(9/8)) + ((Sqrt[a]*(d - Sqrt[2]*d) + Sqrt[c
]*e)*Log[a^(1/4) + Sqrt[2 - Sqrt[2]]*a^(1/8)*c^(1/8)*x + c^(1/4)*x^2])/(8*S
qrt[2*(2 - Sqrt[2])]*a^(3/8)*c^(9/8)) + (((1 + Sqrt[2])*Sqrt[a]*d + Sqrt[c]
*e)*Log[a^(1/4) - Sqrt[2 + Sqrt[2]]*a^(1/8)*c^(1/8)*x + c^(1/4)*x^2])/(8*Sq
rt[2*(2 + Sqrt[2])]*a^(3/8)*c^(9/8)) - (((1 + Sqrt[2])*Sqrt[a]*d + Sqrt[c]*
e)*Log[a^(1/4) + Sqrt[2 + Sqrt[2]]*a^(1/8)*c^(1/8)*x + c^(1/4)*x^2])/(8*Sqr
t[2*(2 + Sqrt[2])]*a^(3/8)*c^(9/8))
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1183

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[a/c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Dist[1/(2*c*q*r), Int
[(d*r - (d - e*q)*x)/(q - r*x + x^2), x], x] + Dist[1/(2*c*q*r), Int[(d*r +
 (d - e*q)*x)/(q + r*x + x^2), x], x]]] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NegQ[b^2 - 4*a*c]
```

Rule 1408

```
Int[((a_) + (c_.)*(x_)^(n2_.))^(p_.)*((d_) + (e_.)*(x_)^(n_))^(q_.), x_Symb
ol] :> Int[x^(n*(2*p + q))*(e + d/x^n)^q*(c + a/x^(2*n))^p, x] /; FreeQ[{a,
```

```
 c, d, e, n}, x] && EqQ[n2, 2*n] && IntegersQ[p, q] && NegQ[n]
```

Rule 1429

```
Int[((d_) + (e_.)*(x_)^(n_))/((a_) + (c_.)*(x_)^(n2_)), x_Symbol] :> With[{
q = Rt[a/c, 4]}, Dist[1/(2*Sqrt[2]*c*q^3), Int[(Sqrt[2]*d*q - (d - e*q^2)*x
^(n/2))/(q^2 - Sqrt[2]*q*x^(n/2) + x^n), x], x] + Dist[1/(2*Sqrt[2]*c*q^3),
 Int[(Sqrt[2]*d*q + (d - e*q^2)*x^(n/2))/(q^2 + Sqrt[2]*q*x^(n/2) + x^n), x
], x]] /; FreeQ[{a, c, d, e}, x] && EqQ[n2, 2*n] && NeQ[c*d^2 + a*e^2, 0] &
& NeQ[c*d^2 - a*e^2, 0] && IGtQ[n/2, 0] && PosQ[a*c]
```

Rule 1517

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^(n_))*((a_) + (c_.)*(x_)^(n2_))^(
p_), x_Symbol] :> Simp[e*f^(n - 1)*(f*x)^(m - n + 1)*((a + c*x^(2*n))^(p +
1)/(c*(m + n*(2*p + 1) + 1))), x] - Dist[f^n/(c*(m + n*(2*p + 1) + 1)), Int
[(f*x)^(m - n)*(a + c*x^(2*n))^p*(a*e*(m - n + 1) - c*d*(m + n*(2*p + 1) +
1)*x^n), x], x] /; FreeQ[{a, c, d, e, f, p}, x] && EqQ[n2, 2*n] && IGtQ[n,
0] && GtQ[m, n - 1] && NeQ[m + n*(2*p + 1) + 1, 0] && IntegerQ[p]
```

Rubi steps

$$\int \frac{d+\frac{e}{x^4}}{c+\frac{a}{x^8}}\,dx = \int \frac{x^4(e+dx^4)}{a+cx^8}\,dx$$

$$= \frac{dx}{c} - \frac{\int \frac{ad-cex^4}{a+cx^8}\,dx}{c}$$

$$= \frac{dx}{c} - \frac{\int \frac{\frac{\sqrt{2}\,a^{5/4}d}{\sqrt[4]{c}}+\left(-ad-\sqrt{a}\,\sqrt{c}\,e\right)x^2}{\frac{\sqrt{a}}{\sqrt{c}}-\frac{\sqrt{2}\,\sqrt[4]{a}\,x^2}{\sqrt[4]{c}}+x^4}\,dx}{2\sqrt{2}\,a^{3/4}c^{5/4}} - \frac{\int \frac{\frac{\sqrt{2}\,a^{5/4}d}{\sqrt[4]{c}}+\left(ad+\sqrt{a}\,\sqrt{c}\,e\right)x^2}{\frac{\sqrt{a}}{\sqrt{c}}+\frac{\sqrt{2}\,\sqrt[4]{a}\,x^2}{\sqrt[4]{c}}+x^4}\,dx}{2\sqrt{2}\,a^{3/4}c^{5/4}}$$

$$= \frac{dx}{c} - \frac{\int \frac{\frac{\sqrt{2\left(2-\sqrt{2}\right)}\,a^{11/8}d}{c^{3/8}}-\left(\frac{\sqrt{2}\,a^{5/4}d}{\sqrt[4]{c}}-\frac{\sqrt[4]{a}\left(ad+\sqrt{a}\,\sqrt{c}\,e\right)}{\sqrt[4]{c}}\right)x}{\frac{\sqrt[4]{a}}{\sqrt[4]{c}}-\frac{\sqrt{2-\sqrt{2}}\,\sqrt[8]{a}\,x}{\sqrt[8]{c}}+x^2}\,dx}{4\sqrt{2\left(2-\sqrt{2}\right)}\,a^{9/8}c^{7/8}} - \frac{\int \frac{\frac{\sqrt{2\left(2-\sqrt{2}\right)}\,a^{11/8}d}{c^{3/8}} \cdots}{\frac{\sqrt[4]{a}}{\sqrt[4]{c}}+ \cdots}}{4\sqrt{2\left(\cdots\right.}}$$

$$= \frac{dx}{c} - \frac{\left(\left(1+\sqrt{2}\right)\sqrt{a}\,d+\sqrt{c}\,e\right)\int \frac{1}{\frac{\sqrt[4]{a}}{\sqrt[4]{c}}-\frac{\sqrt{2-\sqrt{2}}\,\sqrt[8]{a}\,x}{\sqrt[8]{c}}+x^2}\,dx}{8\sqrt{2}\,\sqrt[4]{a}\,c^{5/4}} - \frac{\left(\left(1+\sqrt{2}\right)\sqrt{a}\,d+\sqrt{\cdots}\right.}{8 \cdots}$$

$$= \frac{dx}{c} - \frac{\left(\left(1-\sqrt{2}\right)\sqrt{a}\,d+\sqrt{c}\,e\right)\log\left(\sqrt[4]{a}-\sqrt{2-\sqrt{2}}\,\sqrt[8]{a}\,\sqrt[8]{c}\,x+\sqrt[4]{c}\,x^2\right)}{8\sqrt{2\left(2-\sqrt{2}\right)}\,a^{3/8}c^{9/8}} + \frac{\left(\left(1-\sqrt{2}\cdots\right.\right.}{\cdots}$$

$$= \frac{dx}{c} + \frac{\left(\left(1+\sqrt{2}\right)\sqrt{a}\,d+\sqrt{c}\,e\right)\tan^{-1}\left(\frac{\sqrt{2-\sqrt{2}}\,\sqrt[8]{a}-2\sqrt[8]{c}\,x}{\sqrt{2+\sqrt{2}}\,\sqrt[8]{a}}\right)}{4\sqrt{2\left(2+\sqrt{2}\right)}\,a^{3/8}c^{9/8}} - \frac{\sqrt{2+\sqrt{2}}\left(\left(1-\sqrt{2}\cdots\right.\right.}{\cdots}$$

Mathematica [A]

time = 0.62, size = 551, normalized size = 0.73

$$\frac{8ac^{5/8}dx + 2\tan^{-1}\left(\cot\left(\frac{\pi}{8}\right) + \frac{\sqrt[8]{c}\,x\csc\left(\frac{\pi}{8}\right)}{\sqrt[8]{a}}\right)\left(a^{5/8}ce\cos\left(\frac{\pi}{8}\right) - a^{9/8}\sqrt{c}\,d\sin\left(\frac{\pi}{8}\right)\right) + \log\left(\sqrt[4]{a} + \sqrt[4]{c}\,x^2 + 2\sqrt[8]{a}\,\sqrt[8]{c}\,x\sin\left(\frac{\pi}{8}\right)\right)\left(a^{5/8}ce\cos\left(\frac{\pi}{8}\right) - a^{9/8}\sqrt{c}\,d\sin\left(\frac{\pi}{8}\right)\right) + 2\tan^{-1}\left(\cot\left(\frac{\pi}{8}\right) - \frac{\sqrt[8]{c}\,x\csc\left(\frac{\pi}{8}\right)}{\sqrt[8]{a}}\right)\left(-a^{5/8}ce\cos\left(\frac{\pi}{8}\right) + a^{9/8}\sqrt{c}\,d\sin\left(\frac{\pi}{8}\right)\right) + \log\left(\sqrt[4]{a} + \sqrt[4]{c}\,x^2 - 2\sqrt[8]{a}\,\sqrt[8]{c}\,x\sin\left(\frac{\pi}{8}\right)\right)\left(-a^{5/8}ce\cos\left(\frac{\pi}{8}\right) + a^{9/8}\sqrt{c}\,d\sin\left(\frac{\pi}{8}\right)\right) - 2\tan^{-1}\left(\frac{\sqrt[8]{c}\,x\sec\left(\frac{\pi}{8}\right)}{\sqrt[8]{a}} - \tan\left(\frac{\pi}{8}\right)\right)\left(a^{9/8}\sqrt{c}\,d\cos\left(\frac{\pi}{8}\right) + a^{5/8}ce\sin\left(\frac{\pi}{8}\right)\right) - 2\tan^{-1}\left(\frac{\sqrt[8]{c}\,x\sec\left(\frac{\pi}{8}\right)}{\sqrt[8]{a}} + \tan\left(\frac{\pi}{8}\right)\right)\left(a^{9/8}\sqrt{c}\,d\cos\left(\frac{\pi}{8}\right) + a^{5/8}ce\sin\left(\frac{\pi}{8}\right)\right) + \log\left(\sqrt[4]{a} + \sqrt[4]{c}\,x^2 - 2\sqrt[8]{a}\,\sqrt[8]{c}\,x\cos\left(\frac{\pi}{8}\right)\right)\left(a^{9/8}\sqrt{c}\,d\cos\left(\frac{\pi}{8}\right) + a^{5/8}ce\sin\left(\frac{\pi}{8}\right)\right) - \log\left(\sqrt[4]{a} + \sqrt[4]{c}\,x^2 + 2\sqrt[8]{a}\,\sqrt[8]{c}\,x\cos\left(\frac{\pi}{8}\right)\right)\left(a^{9/8}\sqrt{c}\,d\cos\left(\frac{\pi}{8}\right) + a^{5/8}ce\sin\left(\frac{\pi}{8}\right)\right)}{8ac^{13/8}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e/x^4)/(c + a/x^8),x]
```

```
[Out] (8*a*c^(5/8)*d*x + 2*ArcTan[Cot[Pi/8] + (c^(1/8)*x*Csc[Pi/8])/a^(1/8)]*(a^(
5/8)*c*e*Cos[Pi/8] - a^(9/8)*Sqrt[c]*d*Sin[Pi/8]) + Log[a^(1/4) + c^(1/4)*x
^2 + 2*a^(1/8)*c^(1/8)*x*Sin[Pi/8]]*(a^(5/8)*c*e*Cos[Pi/8] - a^(9/8)*Sqrt[c
]*d*Sin[Pi/8]) + 2*ArcTan[Cot[Pi/8] - (c^(1/8)*x*Csc[Pi/8])/a^(1/8)]*(-(a^(
5/8)*c*e*Cos[Pi/8]) + a^(9/8)*Sqrt[c]*d*Sin[Pi/8]) + Log[a^(1/4) + c^(1/4)*
x^2 - 2*a^(1/8)*c^(1/8)*x*Sin[Pi/8]]*(-(a^(5/8)*c*e*Cos[Pi/8]) + a^(9/8)*Sq
rt[c]*d*Sin[Pi/8]) - 2*ArcTan[(c^(1/8)*x*Sec[Pi/8])/a^(1/8) - Tan[Pi/8]]*(a
^(9/8)*Sqrt[c]*d*Cos[Pi/8] + a^(5/8)*c*e*Sin[Pi/8]) - 2*ArcTan[(c^(1/8)*x*S
ec[Pi/8])/a^(1/8) + Tan[Pi/8]]*(a^(9/8)*Sqrt[c]*d*Cos[Pi/8] + a^(5/8)*c*e*S
in[Pi/8]) + Log[a^(1/4) + c^(1/4)*x^2 - 2*a^(1/8)*c^(1/8)*x*Cos[Pi/8]]*(a^(
9/8)*Sqrt[c]*d*Cos[Pi/8] + a^(5/8)*c*e*Sin[Pi/8]) - Log[a^(1/4) + c^(1/4)*x
^2 + 2*a^(1/8)*c^(1/8)*x*Cos[Pi/8]]*(a^(9/8)*Sqrt[c]*d*Cos[Pi/8] + a^(5/8)*
c*e*Sin[Pi/8]))/(8*a*c^(13/8))
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 0.04, size = 45, normalized size = 0.06

method	result	size
default	$\frac{dx}{c}+\frac{\sum\limits_{\_R=\mathrm{RootOf}\left(c\_Z^8+a\right)}\frac{\left(\_R^4ce-ad\right)\ln\left(x-\_R\right)}{\_R^7}}{8c^2}$	45
risch	$\frac{dx}{c}+\frac{\sum\limits_{\_R=\mathrm{RootOf}\left(c\_Z^8+a\right)}\frac{\left(\_R^4ce-ad\right)\ln\left(x-\_R\right)}{\_R^7}}{8c^2}$	45

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d+e/x^4)/(c+a/x^8),x,method=_RETURNVERBOSE)
```

```
[Out] d*x/c+1/8/c^2*sum((_R^4*c*e-a*d)/_R^7*ln(x-_R),_R=RootOf(_Z^8*c+a))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+e/x^4)/(c+a/x^8),x, algorithm="maxima")
```

[Out] d*x/c + integrate((c*x^4*e - a*d)/(c*x^8 + a), x)/c

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 3552 vs. $2(537) = 1074$.
time = 2.06, size = 3552, normalized size = 4.72

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+e/x^4)/(c+a/x^8),x, algorithm="fricas")
```

```
[Out] -1/8*(4*c*(-(a*c^4*sqrt(-(a^4*d^8 - 12*a^3*c*d^6*e^2 + 38*a^2*c^2*d^4*e^4 -
 12*a*c^3*d^2*e^6 + c^4*e^8)/(a^3*c^9)) - 4*a*d^3*e + 4*c*d*e^3)/(a*c^4))^(
1/4)*arctan(-(sqrt(a^6*d^12*x^2 - 10*a^5*c*d^10*x^2*e^2 + 15*a^4*c^2*d^8*x^
2*e^4 + 52*a^3*c^3*d^6*x^2*e^6 + 15*a^2*c^4*d^4*x^2*e^8 - 10*a*c^5*d^2*x^2*
e^10 + c^6*x^2*e^12 + (a^6*c^2*d^10 - 13*a^5*c^3*d^8*e^2 + 50*a^4*c^4*d^6*e
^4 - 50*a^3*c^5*d^4*e^6 + 13*a^2*c^6*d^2*e^8 - a*c^7*e^10 - 2*(a^5*c^7*d^5*
e - 6*a^4*c^8*d^3*e^3 + a^3*c^9*d*e^5)*sqrt(-(a^4*d^8 - 12*a^3*c*d^6*e^2 +
38*a^2*c^2*d^4*e^4 - 12*a*c^3*d^2*e^6 + c^4*e^8)/(a^3*c^9)))*sqrt(-(a*c^4*s
qrt(-(a^4*d^8 - 12*a^3*c*d^6*e^2 + 38*a^2*c^2*d^4*e^4 - 12*a*c^3*d^2*e^6 +
c^4*e^8)/(a^3*c^9)) - 4*a*d^3*e + 4*c*d*e^3)/(a*c^4)))*(3*a^4*c^4*d^6*e - 1
9*a^3*c^5*d^4*e^3 + 9*a^2*c^6*d^2*e^5 - a*c^7*e^7 + (a^4*c^8*d^3 - 3*a^3*c^
9*d*e^2)*sqrt(-(a^4*d^8 - 12*a^3*c*d^6*e^2 + 38*a^2*c^2*d^4*e^4 - 12*a*c^3*
d^2*e^6 + c^4*e^8)/(a^3*c^9)))*sqrt(-(a*c^4*sqrt(-(a^4*d^8 - 12*a^3*c*d^6*e
^2 + 38*a^2*c^2*d^4*e^4 - 12*a*c^3*d^2*e^6 + c^4*e^8)/(a^3*c^9)) - 4*a*d^3*
e + 4*c*d*e^3)/(a*c^4)) - (3*a^7*c^4*d^12*x*e - 34*a^6*c^5*d^10*x*e^3 + 89*
a^5*c^6*d^8*x*e^5 + 52*a^4*c^7*d^6*x*e^7 - 59*a^3*c^8*d^4*x*e^9 + 14*a^2*c^
9*d^2*x*e^11 - a*c^10*x*e^13 + (a^7*c^8*d^9*x - 8*a^6*c^9*d^7*x*e^2 + 10*a^
5*c^10*d^5*x*e^4 + 16*a^4*c^11*d^3*x*e^6 - 3*a^3*c^12*d*x*e^8)*sqrt(-(a^4*d
^8 - 12*a^3*c*d^6*e^2 + 38*a^2*c^2*d^4*e^4 - 12*a*c^3*d^2*e^6 + c^4*e^8)/(a
^3*c^9)))*sqrt(-(a*c^4*sqrt(-(a^4*d^8 - 12*a^3*c*d^6*e^2 + 38*a^2*c^2*d^4*e
^4 - 12*a*c^3*d^2*e^6 + c^4*e^8)/(a^3*c^9)) - 4*a*d^3*e + 4*c*d*e^3)/(a*c^4
)))*(-(a*c^4*sqrt(-(a^4*d^8 - 12*a^3*c*d^6*e^2 + 38*a^2*c^2*d^4*e^4 - 12*a*
c^3*d^2*e^6 + c^4*e^8)/(a^3*c^9)) - 4*a*d^3*e + 4*c*d*e^3)/(a*c^4))^(1/4)/(
a^8*d^16 - 8*a^7*c*d^14*e^2 - 4*a^6*c^2*d^12*e^4 + 72*a^5*c^3*d^10*e^6 + 13
4*a^4*c^4*d^8*e^8 + 72*a^3*c^5*d^6*e^10 - 4*a^2*c^6*d^4*e^12 - 8*a*c^7*d^2*
e^14 + c^8*e^16)) - 4*c*((a*c^4*sqrt(-(a^4*d^8 - 12*a^3*c*d^6*e^2 + 38*a^2*
c^2*d^4*e^4 - 12*a*c^3*d^2*e^6 + c^4*e^8)/(a^3*c^9)) + 4*a*d^3*e - 4*c*d*e^
3)/(a*c^4))^(1/4)*arctan((sqrt(a^6*d^12*x^2 - 10*a^5*c*d^10*x^2*e^2 + 15*a^
4*c^2*d^8*x^2*e^4 + 52*a^3*c^3*d^6*x^2*e^6 + 15*a^2*c^4*d^4*x^2*e^8 - 10*a*
c^5*d^2*x^2*e^10 + c^6*x^2*e^12 + (a^6*c^2*d^10 - 13*a^5*c^3*d^8*e^2 + 50*a
^4*c^4*d^6*e^4 - 50*a^3*c^5*d^4*e^6 + 13*a^2*c^6*d^2*e^8 - a*c^7*e^10 + 2*(
a^5*c^7*d^5*e - 6*a^4*c^8*d^3*e^3 + a^3*c^9*d*e^5)*sqrt(-(a^4*d^8 - 12*a^3*
c*d^6*e^2 + 38*a^2*c^2*d^4*e^4 - 12*a*c^3*d^2*e^6 + c^4*e^8)/(a^3*c^9)))*sq
rt((a*c^4*sqrt(-(a^4*d^8 - 12*a^3*c*d^6*e^2 + 38*a^2*c^2*d^4*e^4 - 12*a*c^3
```

```
*d^2*e^6 + c^4*e^8)/(a^3*c^9)) + 4*a*d^3*e - 4*c*d*e^3)/(a*c^4)))*(3*a^4*c^
4*d^6*e - 19*a^3*c^5*d^4*e^3 + 9*a^2*c^6*d^2*e^5 - a*c^7*e^7 - (a^4*c^8*d^3
 - 3*a^3*c^9*d*e^2)*sqrt(-(a^4*d^8 - 12*a^3*c*d^6*e^2 + 38*a^2*c^2*d^4*e^4
- 12*a*c^3*d^2*e^6 + c^4*e^8)/(a^3*c^9)))*((a*c^4*sqrt(-(a^4*d^8 - 12*a^3*c
*d^6*e^2 + 38*a^2*c^2*d^4*e^4 - 12*a*c^3*d^2*e^6 + c^4*e^8)/(a^3*c^9)) + 4*
a*d^3*e - 4*c*d*e^3)/(a*c^4))^(3/4) - (3*a^7*c^4*d^12*x*e - 34*a^6*c^5*d^10
*x*e^3 + 89*a^5*c^6*d^8*x*e^5 + 52*a^4*c^7*d^6*x*e^7 - 59*a^3*c^8*d^4*x*e^9
 + 14*a^2*c^9*d^2*x*e^11 - a*c^10*x*e^13 - (a^7*c^8*d^9*x - 8*a^6*c^9*d^7*x
*e^2 + 10*a^5*c^10*d^5*x*e^4 + 16*a^4*c^11*d^3*x*e^6 - 3*a^3*c^12*d*x*e^8)*
sqrt(-(a^4*d^8 - 12*a^3*c*d^6*e^2 + 38*a^2*c^2*d^4*e^4 - 12*a*c^3*d^2*e^6 +
 c^4*e^8)/(a^3*c^9)))*((a*c^4*sqrt(-(a^4*d^8 - 12*a^3*c*d^6*e^2 + 38*a^2*c^
2*d^4*e^4 - 12*a*c^3*d^2*e^6 + c^4*e^8)/(a^3*c^9)) + 4*a*d^3*e - 4*c*d*e^3)
/(a*c^4))^(3/4))/(a^8*d^16 - 8*a^7*c*d^14*e^2 - 4*a^6*c^2*d^12*e^4 + 72*a^5
*c^3*d^10*e^6 + 134*a^4*c^4*d^8*e^8 + 72*a^3*c^5*d^6*e^10 - 4*a^2*c^6*d^4*e
^12 - 8*a*c^7*d^2*e^14 + c^8*e^16)) + c*((a*c^4*sqrt(-(a^4*d^8 - 12*a^3*c*d
^6*e^2 + 38*a^2*c^2*d^4*e^4 - 12*a*c^3*d^2*e^6 + c^4*e^8)/(a^3*c^9)) + 4*a*
d^3*e - 4*c*d*e^3)/(a*c^4))^(1/4)*log(a^3*d^6*x - 5*a^2*c*d^4*x*e^2 - 5*a*c
^2*d^2*x*e^4 + c^3*x*e^6 + (a^2*c^6*sqrt(-(a^4*d^8 - 12*a^3*c*d^6*e^2 + 38*
a^2*c^2*d^4*e^4 - 12*a*c^3*d^2*e^6 + c^4*e^8)/(a^3*c^9))*e + a^3*c*d^5 - 6*
a^2*c^2*d^3*e^2 + a*c^3*d*e^4)*((a*c^4*sqrt(-(a^4*d^8 - 12*a^3*c*d^6*e^2 +
38*a^2*c^2*d^4*e^4 - 12*a*c^3*d^2*e^6 + c^4*e^8)/(a^3*c^9)) + 4*a*d^3*e - 4
*c*d*e^3)/(a*c^4))^(1/4)) - c*((a*c^4*sqrt(-(a^4*d^8 - 12*a^3*c*d^6*e^2 + 3
8*a^2*c^2*d^4*e^4 - 12*a*c^3*d^2*e^6 + c^4*e^8)/(a^3*c^9)) + 4*a*d^3*e - 4*
c*d*e^3)/(a*c^4))^(1/4)*log(a^3*d^6*x - 5*a^2*c*d^4*x*e^2 - 5*a*c^2*d^2*x*e
^4 + c^3*x*e^6 - (a^2*c^6*sqrt(-(a^4*d^8 - 12*a^3*c*d^6*e^2 + 38*a^2*c^2*d^
4*e^4 - 12*a*c^3*d^2*e^6 + c^4*e^8)/(a^3*c^9))*e + a^3*c*d^5 - 6*a^2*c^2*d^
3*e^2 + a*c^3*d*e^4)*((a*c^4*sqrt(-(a^4*d^8 - 12*a^3*c*d^6*e^2 + 38*a^2*c^2
*d^4*e^4 - 12*a*c^3*d^2*e^6 + c^4*e^8)/(a^3*c^9)) + 4*a*d^3*e - 4*c*d*e^3)/
(a*c^4))^(1/4)) - c*(-(a*c^4*sqrt(-(a^4*d^8 - 12*a^3*c*d^6*e^2 + 38*a^2*c^2
*d^4*e^4 - 12*a*c^3*d^2*e^6 + c^4*e^8)/(a^3*c^9)) - 4*a*d^3*e + 4*c*d*e^3)/
(a*c^4))^(1/4)*log(a^3*d^6*x - 5*a^2*c*d^4*x*e^2 - 5*a*c^2*d^2*x*e^4 + c^3*
x*e^6 + (a^2*c^6*sqrt(-(a^4*d^8 - 12*a^3*c*d^6*e^2 + 38*a^2*c^2*d^4*e^4 - 1
2*a*c^3*d^2*e^6 + c^4*e^8)/(a^3*c^9))*e - a^3*c...
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+e/x**4)/(c+a/x**8),x)
```

```
[Out] Timed out
```

Giac [**A**]

time = 3.65, size = 647, normalized size = 0.86

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+e/x^4)/(c+a/x^8),x, algorithm="giac")
```

```
[Out] d*x/c - 1/8*(c*sqrt(-sqrt(2) + 2)*(a/c)^(5/8)*e + a*d*sqrt(sqrt(2) + 2)*(a/
c)^(1/8))*arctan((2*x + sqrt(-sqrt(2) + 2)*(a/c)^(1/8))/(sqrt(sqrt(2) + 2)*
(a/c)^(1/8)))/(a*c) - 1/8*(c*sqrt(-sqrt(2) + 2)*(a/c)^(5/8)*e + a*d*sqrt(sq
rt(2) + 2)*(a/c)^(1/8))*arctan((2*x - sqrt(-sqrt(2) + 2)*(a/c)^(1/8))/(sqrt
(sqrt(2) + 2)*(a/c)^(1/8)))/(a*c) + 1/8*(c*sqrt(sqrt(2) + 2)*(a/c)^(5/8)*e
- a*d*sqrt(-sqrt(2) + 2)*(a/c)^(1/8))*arctan((2*x + sqrt(sqrt(2) + 2)*(a/c)
^(1/8))/(sqrt(-sqrt(2) + 2)*(a/c)^(1/8)))/(a*c) + 1/8*(c*sqrt(sqrt(2) + 2)*
(a/c)^(5/8)*e - a*d*sqrt(-sqrt(2) + 2)*(a/c)^(1/8))*arctan((2*x - sqrt(sqrt
(2) + 2)*(a/c)^(1/8))/(sqrt(-sqrt(2) + 2)*(a/c)^(1/8)))/(a*c) - 1/16*(c*sqr
t(-sqrt(2) + 2)*(a/c)^(5/8)*e + a*d*sqrt(sqrt(2) + 2)*(a/c)^(1/8))*log(x^2
+ x*sqrt(sqrt(2) + 2)*(a/c)^(1/8) + (a/c)^(1/4))/(a*c) + 1/16*(c*sqrt(-sqrt
(2) + 2)*(a/c)^(5/8)*e + a*d*sqrt(sqrt(2) + 2)*(a/c)^(1/8))*log(x^2 - x*sqr
t(sqrt(2) + 2)*(a/c)^(1/8) + (a/c)^(1/4))/(a*c) + 1/16*(c*sqrt(sqrt(2) + 2)
*(a/c)^(5/8)*e - a*d*sqrt(-sqrt(2) + 2)*(a/c)^(1/8))*log(x^2 + x*sqrt(-sqrt
(2) + 2)*(a/c)^(1/8) + (a/c)^(1/4))/(a*c) - 1/16*(c*sqrt(sqrt(2) + 2)*(a/c)
^(5/8)*e - a*d*sqrt(-sqrt(2) + 2)*(a/c)^(1/8))*log(x^2 - x*sqrt(-sqrt(2) +
2)*(a/c)^(1/8) + (a/c)^(1/4))/(a*c)
```

Mupad [B]
time = 1.22, size = 2520, normalized size = 3.35

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e/x^4)/(c + a/x^8),x)
```

```
[Out] (atan((a^3*d^6*x - c^3*e^6*x - a*c^2*d^2*e^4*x + a^2*c*d^4*e^2*x + (2*d*e*x
*(a^2*d^4*(-a^3*c^9)^(1/2) + c^2*e^4*(-a^3*c^9)^(1/2) + 4*a^2*c^6*d*e^3 - 4
*a^3*c^5*d^3*e - 6*a*c*d^2*e^2*(-a^3*c^9)^(1/2)))/(a*c^4))/(a^2*c^6*e*(-(a^
2*d^4*(-a^3*c^9)^(1/2) + c^2*e^4*(-a^3*c^9)^(1/2) + 4*a^2*c^6*d*e^3 - 4*a^3
*c^5*d^3*e - 6*a*c*d^2*e^2*(-a^3*c^9)^(1/2))/(a^3*c^9))^(5/4) - a^3*c*d^5*(
-(a^2*d^4*(-a^3*c^9)^(1/2) + c^2*e^4*(-a^3*c^9)^(1/2) + 4*a^2*c^6*d*e^3 - 4
*a^3*c^5*d^3*e - 6*a*c*d^2*e^2*(-a^3*c^9)^(1/2))/(a^3*c^9))^(1/4) + 2*a^2*c
^2*d^3*e^2*(-(a^2*d^4*(-a^3*c^9)^(1/2) + c^2*e^4*(-a^3*c^9)^(1/2) + 4*a^2*c
^6*d*e^3 - 4*a^3*c^5*d^3*e - 6*a*c*d^2*e^2*(-a^3*c^9)^(1/2))/(a^3*c^9))^(1/
4) + 3*a*c^3*d*e^4*(-(a^2*d^4*(-a^3*c^9)^(1/2) + c^2*e^4*(-a^3*c^9)^(1/2) +
 4*a^2*c^6*d*e^3 - 4*a^3*c^5*d^3*e - 6*a*c*d^2*e^2*(-a^3*c^9)^(1/2))/(a^3*c
^9))^(1/4))*(-(a^2*d^4*(-a^3*c^9)^(1/2) + c^2*e^4*(-a^3*c^9)^(1/2) + 4*a^2
```

```
*c^6*d*e^3 - 4*a^3*c^5*d^3*e - 6*a*c*d^2*e^2*(-a^3*c^9)^(1/2))/(a^3*c^9))^(
1/4))/4 - (atan((c^3*e^6*x - a^3*d^6*x + a*c^2*d^2*e^4*x - a^2*c*d^4*e^2*x
+ (2*d*e*x*(a^2*d^4*(-a^3*c^9)^(1/2) + c^2*e^4*(-a^3*c^9)^(1/2) - 4*a^2*c^6
*d*e^3 + 4*a^3*c^5*d^3*e - 6*a*c*d^2*e^2*(-a^3*c^9)^(1/2)))/(a*c^4))/(a^2*c
^6*e*((a^2*d^4*(-a^3*c^9)^(1/2) + c^2*e^4*(-a^3*c^9)^(1/2) - 4*a^2*c^6*d*e^
3 + 4*a^3*c^5*d^3*e - 6*a*c*d^2*e^2*(-a^3*c^9)^(1/2))/(a^3*c^9))^(5/4) - a^
3*c*d^5*((a^2*d^4*(-a^3*c^9)^(1/2) + c^2*e^4*(-a^3*c^9)^(1/2) - 4*a^2*c^6*d
*e^3 + 4*a^3*c^5*d^3*e - 6*a*c*d^2*e^2*(-a^3*c^9)^(1/2))/(a^3*c^9))^(1/4) +
 2*a^2*c^2*d^3*e^2*((a^2*d^4*(-a^3*c^9)^(1/2) + c^2*e^4*(-a^3*c^9)^(1/2) -
4*a^2*c^6*d*e^3 + 4*a^3*c^5*d^3*e - 6*a*c*d^2*e^2*(-a^3*c^9)^(1/2))/(a^3*c^
9))^(1/4) + 3*a*c^3*d*e^4*((a^2*d^4*(-a^3*c^9)^(1/2) + c^2*e^4*(-a^3*c^9)^(
1/2) - 4*a^2*c^6*d*e^3 + 4*a^3*c^5*d^3*e - 6*a*c*d^2*e^2*(-a^3*c^9)^(1/2))/
(a^3*c^9))^(1/4)))*((a^2*d^4*(-a^3*c^9)^(1/2) + c^2*e^4*(-a^3*c^9)^(1/2) -
4*a^2*c^6*d*e^3 + 4*a^3*c^5*d^3*e - 6*a*c*d^2*e^2*(-a^3*c^9)^(1/2))/(a^3*c^
9))^(1/4))/4 + atan((c^3*e^6*x*1i - a^3*d^6*x*1i + a*c^2*d^2*e^4*x*1i - a^2
*c*d^4*e^2*x*1i + (d*e*x*(a^2*d^4*(-a^3*c^9)^(1/2) + c^2*e^4*(-a^3*c^9)^(1/
2) - 4*a^2*c^6*d*e^3 + 4*a^3*c^5*d^3*e - 6*a*c*d^2*e^2*(-a^3*c^9)^(1/2))*2i
)/(a*c^4))/(a^2*c^6*e*((a^2*d^4*(-a^3*c^9)^(1/2) + c^2*e^4*(-a^3*c^9)^(1/2)
 - 4*a^2*c^6*d*e^3 + 4*a^3*c^5*d^3*e - 6*a*c*d^2*e^2*(-a^3*c^9)^(1/2))/(a^3
*c^9))^(5/4) - a^3*c*d^5*((a^2*d^4*(-a^3*c^9)^(1/2) + c^2*e^4*(-a^3*c^9)^(1
/2) - 4*a^2*c^6*d*e^3 + 4*a^3*c^5*d^3*e - 6*a*c*d^2*e^2*(-a^3*c^9)^(1/2))/(
a^3*c^9))^(1/4) + 2*a^2*c^2*d^3*e^2*((a^2*d^4*(-a^3*c^9)^(1/2) + c^2*e^4*(-
a^3*c^9)^(1/2) - 4*a^2*c^6*d*e^3 + 4*a^3*c^5*d^3*e - 6*a*c*d^2*e^2*(-a^3*c^
9)^(1/2))/(a^3*c^9))^(1/4) + 3*a*c^3*d*e^4*((a^2*d^4*(-a^3*c^9)^(1/2) + c^2
*e^4*(-a^3*c^9)^(1/2) - 4*a^2*c^6*d*e^3 + 4*a^3*c^5*d^3*e - 6*a*c*d^2*e^2*(
-a^3*c^9)^(1/2))/(a^3*c^9))^(1/4)))*((a^2*d^4*(-a^3*c^9)^(1/2) + c^2*e^4*(-
a^3*c^9)^(1/2) - 4*a^2*c^6*d*e^3 + 4*a^3*c^5*d^3*e - 6*a*c*d^2*e^2*(-a^3*c^
9)^(1/2))/(4096*a^3*c^9))^(1/4)*2i - atan((a^3*d^6*x*1i - c^3*e^6*x*1i - a*
c^2*d^2*e^4*x*1i + a^2*c*d^4*e^2*x*1i + (d*e*x*(a^2*d^4*(-a^3*c^9)^(1/2) +
c^2*e^4*(-a^3*c^9)^(1/2) + 4*a^2*c^6*d*e^3 - 4*a^3*c^5*d^3*e - 6*a*c*d^2*e^
2*(-a^3*c^9)^(1/2))*2i)/(a*c^4))/(a^2*c^6*e*(-(a^2*d^4*(-a^3*c^9)^(1/2) + c
^2*e^4*(-a^3*c^9)^(1/2) + 4*a^2*c^6*d*e^3 - 4*a^3*c^5*d^3*e - 6*a*c*d^2*e^2
*(-a^3*c^9)^(1/2))/(a^3*c^9))^(5/4) - a^3*c*d^5*(-(a^2*d^4*(-a^3*c^9)^(1/2)
 + c^2*e^4*(-a^3*c^9)^(1/2) + 4*a^2*c^6*d*e^3 - 4*a^3*c^5*d^3*e - 6*a*c*d^2
*e^2*(-a^3*c^9)^(1/2))/(a^3*c^9))^(1/4) + 2*a^2*c^2*d^3*e^2*(-(a^2*d^4*(-a^
3*c^9)^(1/2) + c^2*e^4*(-a^3*c^9)^(1/2) + 4*a^2*c^6*d*e^3 - 4*a^3*c^5*d^3*e
 - 6*a*c*d^2*e^2*(-a^3*c^9)^(1/2))/(a^3*c^9))^(1/4) + 3*a*c^3*d*e^4*(-(a^2*
d^4*(-a^3*c^9)^(1/2) + c^2*e^4*(-a^3*c^9)^(1/2) + 4*a^2*c^6*d*e^3 - 4*a^3*c
^5*d^3*e - 6*a*c*d^2*e^2*(-a^3*c^9)^(1/2))/(a^3*c^9))^(1/4)))*(-(a^2*d^4*(-
a^3*c^9)^(1/2) + c^2*e^4*(-a^3*c^9)^(1/2) + 4*a^2*c^6*d*e^3 - 4*a^3*c^5*d^3
*e - 6*a*c*d^2*e^2*(-a^3*c^9)^(1/2))/(4096*a^3*c^9))^(1/4)*2i + (d*x)/c
```

3.41 $\int \frac{d+\frac{e}{x^4}}{c+\frac{a}{x^8}+\frac{b}{x^4}}\,dx$

Optimal. Leaf size=433

$$\frac{dx}{c}+\frac{\left(bd-ce+\frac{b^2d-2acd-bce}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt[4]{2}\,\sqrt[4]{c}\,x}{\sqrt[4]{-b-\sqrt{b^2-4ac}}}\right)}{2\sqrt[4]{2}\,c^{5/4}\left(-b-\sqrt{b^2-4ac}\right)^{3/4}}+\frac{\left(bd-ce-\frac{b^2d-2acd-bce}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt[4]{2}}{\sqrt[4]{-b+\sqrt{}}}\right.}{2\sqrt[4]{2}\,c^{5/4}\left(-b+\sqrt{b^2-4ac}\right)^{3/4}}$$

```
[Out] d*x/c+1/4*arctan(2^(1/4)*c^(1/4)*x/(-b-(-4*a*c+b^2)^(1/2))^(1/4))*(b*d-c*e+
(-2*a*c*d+b^2*d-b*c*e)/(-4*a*c+b^2)^(1/2))*2^(3/4)/c^(5/4)/(-b-(-4*a*c+b^2)
^(1/2))^(3/4)+1/4*arctanh(2^(1/4)*c^(1/4)*x/(-b-(-4*a*c+b^2)^(1/2))^(1/4))*
(b*d-c*e+(-2*a*c*d+b^2*d-b*c*e)/(-4*a*c+b^2)^(1/2))*2^(3/4)/c^(5/4)/(-b-(-4
*a*c+b^2)^(1/2))^(3/4)+1/4*arctan(2^(1/4)*c^(1/4)*x/(-b+(-4*a*c+b^2)^(1/2))
^(1/4))*(b*d-c*e+(2*a*c*d-b^2*d+b*c*e)/(-4*a*c+b^2)^(1/2))*2^(3/4)/c^(5/4)/
(-b+(-4*a*c+b^2)^(1/2))^(3/4)+1/4*arctanh(2^(1/4)*c^(1/4)*x/(-b+(-4*a*c+b^2
)^(1/2))^(1/4))*(b*d-c*e+(2*a*c*d-b^2*d+b*c*e)/(-4*a*c+b^2)^(1/2))*2^(3/4)/
c^(5/4)/(-b+(-4*a*c+b^2)^(1/2))^(3/4)
```

Rubi [A]

time = 0.57, antiderivative size = 433, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 6, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {1407, 1516, 1436, 218, 214, 211}

$$\frac{\text{ArcTan}\left(\frac{\sqrt[4]{2}\,\sqrt[4]{c}\,x}{\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)\left(\frac{-2acd+b^2d-bce}{\sqrt{b^2-4ac}}+bd-ce\right)}{2\sqrt[4]{2}\,c^{5/4}\left(-\sqrt{b^2-4ac}-b\right)^{3/4}}+\frac{\text{ArcTan}\left(\frac{\sqrt[4]{2}\,\sqrt[4]{c}\,x}{\sqrt[4]{\sqrt{b^2-4ac}-b}}\right)\left(-\frac{-2acd+b^2d-bce}{\sqrt{b^2-4ac}}+bd-ce\right)}{2\sqrt[4]{2}\,c^{5/4}\left(\sqrt{b^2-4ac}-b\right)^{3/4}}+\frac{\left(\frac{-2acd+b^2d-bce}{\sqrt{b^2-4ac}}+bd-ce\right)\tanh^{-1}\left(\frac{\sqrt[4]{2}\,\sqrt[4]{c}\,x}{\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)}{2\sqrt[4]{2}\,c^{5/4}\left(-\sqrt{b^2-4ac}-b\right)^{3/4}}+\frac{\left(-\frac{-2acd+b^2d-bce}{\sqrt{b^2-4ac}}+bd-ce\right)\tanh^{-1}\left(\frac{\sqrt[4]{2}\,\sqrt[4]{c}\,x}{\sqrt[4]{\sqrt{b^2-4ac}-b}}\right)}{2\sqrt[4]{2}\,c^{5/4}\left(\sqrt{b^2-4ac}-b\right)^{3/4}}+\frac{dx}{c}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e/x^4)/(c + a/x^8 + b/x^4),x]
```

```
[Out] (d*x)/c + ((b*d - c*e + (b^2*d - 2*a*c*d - b*c*e)/Sqrt[b^2 - 4*a*c])*ArcTan
[(2^(1/4)*c^(1/4)*x)/(-b - Sqrt[b^2 - 4*a*c])^(1/4)])/(2*2^(1/4)*c^(5/4)*(-
b - Sqrt[b^2 - 4*a*c])^(3/4)) + ((b*d - c*e - (b^2*d - 2*a*c*d - b*c*e)/Sqr
t[b^2 - 4*a*c])*ArcTan[(2^(1/4)*c^(1/4)*x)/(-b + Sqrt[b^2 - 4*a*c])^(1/4)])
/(2*2^(1/4)*c^(5/4)*(-b + Sqrt[b^2 - 4*a*c])^(3/4)) + ((b*d - c*e + (b^2*d
- 2*a*c*d - b*c*e)/Sqrt[b^2 - 4*a*c])*ArcTanh[(2^(1/4)*c^(1/4)*x)/(-b - Sqr
t[b^2 - 4*a*c])^(1/4)])/(2*2^(1/4)*c^(5/4)*(-b - Sqrt[b^2 - 4*a*c])^(3/4))
+ ((b*d - c*e - (b^2*d - 2*a*c*d - b*c*e)/Sqrt[b^2 - 4*a*c])*ArcTanh[(2^(1/
4)*c^(1/4)*x)/(-b + Sqrt[b^2 - 4*a*c])^(1/4)])/(2*2^(1/4)*c^(5/4)*(-b + Sqr
t[b^2 - 4*a*c])^(3/4))
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 218

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x], x]
+ Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a/b
, 0]
```

Rule 1407

```
Int[((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.)*((d_) + (e_.)*(x_)^(
n_))^(q_.), x_Symbol] :> Int[x^(n*(2*p + q))*(e + d/x^n)^q*(c + b/x^n + a/x
^(2*n))^p, x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[n2, 2*n] && IntegersQ[
p, q] && NegQ[n]
```

Rule 1436

```
Int[((d_) + (e_.)*(x_)^(n_))/((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_)), x
_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q),
Int[1/(b/2 - q/2 + c*x^n), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(
b/2 + q/2 + c*x^n), x], x]] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[n2, 2*n]
 && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && (PosQ[b^2 - 4*a
*c] ||  !IGtQ[n/2, 0])
```

Rule 1516

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^(n_))*((a_) + (b_.)*(x_)^(n_) + (
c_.)*(x_)^(n2_))^(p_), x_Symbol] :> Simp[e*f^(n - 1)*(f*x)^(m - n + 1)*((a
+ b*x^n + c*x^(2*n))^(p + 1)/(c*(m + n*(2*p + 1) + 1))), x] - Dist[f^n/(c*(
m + n*(2*p + 1) + 1)), Int[(f*x)^(m - n)*(a + b*x^n + c*x^(2*n))^p*Simp[a*e
*(m - n + 1) + (b*e*(m + n*p + 1) - c*d*(m + n*(2*p + 1) + 1))*x^n, x], x],
 x] /; FreeQ[{a, b, c, d, e, f, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c,
0] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*(2*p + 1) + 1, 0] && Integer
Q[p]
```

Rubi steps

$$\int \frac{d+\frac{e}{x^4}}{c+\frac{a}{x^8}+\frac{b}{x^4}}\,dx = \int \frac{x^4(e+dx^4)}{a+bx^4+cx^8}\,dx$$
$$= \frac{dx}{c} - \frac{\int \frac{ad+(bd-ce)x^4}{a+bx^4+cx^8}\,dx}{c}$$
$$= \frac{dx}{c} - \frac{\left(bd-ce-\frac{b^2d-2acd-bce}{\sqrt{b^2-4ac}}\right)\int \frac{1}{\frac{b}{2}-\frac{1}{2}\sqrt{b^2-4ac}+cx^4}\,dx}{2c} - \frac{\left(bd-ce+\frac{b^2d-2acd-bce}{\sqrt{b^2-4ac}}\right)\int}{2c}$$
$$= \frac{dx}{c} + \frac{\left(bd-ce-\frac{b^2d-2acd-bce}{\sqrt{b^2-4ac}}\right)\int \frac{1}{\sqrt{-b+\sqrt{b^2-4ac}}-\sqrt{2}\sqrt{c}\,x^2}\,dx}{2c\sqrt{-b+\sqrt{b^2-4ac}}} + \frac{\left(bd-ce-\frac{b^2d-}{\sqrt{b}}\right.}{}$$
$$= \frac{dx}{c} + \frac{\left(bd-ce+\frac{b^2d-2acd-bce}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\,x}{\sqrt[4]{-b-\sqrt{b^2-4ac}}}\right)}{2\sqrt[4]{2}\,c^{5/4}\left(-b-\sqrt{b^2-4ac}\right)^{3/4}} + \frac{\left(bd-ce-\frac{b^2d-2acd-b}{\sqrt{b^2-4a}}\right.}{2\sqrt[4]{2}\,c^{5/4}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.
time = 0.05, size = 88, normalized size = 0.20

$$\frac{dx}{c} - \frac{\text{RootSum}\left[a+b\#1^4+c\#1^8\&, \frac{ad\log(x-\#1)+bd\log(x-\#1)\#1^4-ce\log(x-\#1)\#1^4}{b\#1^3+2c\#1^7}\&\right]}{4c}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e/x^4)/(c + a/x^8 + b/x^4),x]
```

```
[Out] (d*x)/c - RootSum[a + b*#1^4 + c*#1^8 & , (a*d*Log[x - #1] + b*d*Log[x - #1
]*#1^4 - c*e*Log[x - #1]*#1^4)/(b*#1^3 + 2*c*#1^7) & ]/(4*c)
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 0.05, size = 67, normalized size = 0.15

method	result	size
default	$\frac{dx}{c}+\frac{\sum_{\_R=\text{RootOf}\left(c\_Z^8+\_Z^4b+a\right)}\frac{\left((-bd+ce)\_R^4-ad\right)\ln\left(x-\_R\right)}{2\_R^7c+\_R^3b}}{4c}$	67

risch	$\frac{dx}{c} + \frac{\sum_{\_R=\mathrm{RootOf}(c\_Z^8+\_Z^4b+a)} \frac{((-bd+ce)\_R^4-ad)\ln(x-\_R)}{2\_R^7c+\_R^3b}}{4c}$	67

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d+e/x^4)/(c+a/x^8+b/x^4),x,method=_RETURNVERBOSE)
```

```
[Out] d*x/c+1/4/c*sum(((-b*d+c*e)*_R^4-a*d)/(2*_R^7*c+_R^3*b)*ln(x-_R),_R=RootOf(
_Z^8*c+_Z^4*b+a))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+e/x^4)/(c+a/x^8+b/x^4),x, algorithm="maxima")
```

```
[Out] d*x/c - integrate(((b*d - c*e)*x^4 + a*d)/(c*x^8 + b*x^4 + a), x)/c
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 22075 vs. $2(361) = 722$.
time = 137.29, size = 22075, normalized size = 50.98

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+e/x^4)/(c+a/x^8+b/x^4),x, algorithm="fricas")
```

```
[Out] -1/4*(4*c*sqrt(sqrt(1/2)*sqrt(-(b*c^4*e^4 + (b^5 - 5*a*b^3*c + 5*a^2*b*c^2)
*d^4 - 4*(b^4*c - 4*a*b^2*c^2 + 2*a^2*c^3)*d^3*e + 6*(b^3*c^2 - 3*a*b*c^3)*
d^2*e^2 - 4*(b^2*c^3 - 2*a*c^4)*d*e^3 + (b^4*c^5 - 8*a*b^2*c^6 + 16*a^2*c^7
)*sqrt(-(8*b*c^7*d*e^7 - (b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3
+ a^4*c^4)*d^8 - c^8*e^8 + 8*(b^7*c - 5*a*b^5*c^2 + 7*a^2*b^3*c^3 - 2*a^3*b
*c^4)*d^7*e - 4*(7*b^6*c^2 - 28*a*b^4*c^3 + 28*a^2*b^2*c^4 - 3*a^3*c^5)*d^6
*e^2 + 8*(7*b^5*c^3 - 21*a*b^3*c^4 + 13*a^2*b*c^5)*d^5*e^3 - 2*(35*b^4*c^4
- 71*a*b^2*c^5 + 19*a^2*c^6)*d^4*e^4 + 8*(7*b^3*c^5 - 8*a*b*c^6)*d^3*e^5 -
4*(7*b^2*c^6 - 3*a*c^7)*d^2*e^6)/(b^6*c^10 - 12*a*b^4*c^11 + 48*a^2*b^2*c^1
2 - 64*a^3*c^13)))/(b^4*c^5 - 8*a*b^2*c^6 + 16*a^2*c^7)))*arctan(-1/2*(sqrt
(1/2)*sqrt((a^2*b^8 - 6*a^3*b^6*c + 11*a^4*b^4*c^2 - 6*a^5*b^2*c^3 + a^6*c^
4)*d^12*x^2 - 2*(a*b^9 - 2*a^2*b^7*c - 9*a^3*b^5*c^2 + 22*a^4*b^3*c^3 - 7*a
^5*b*c^4)*d^11*x^2*e - 10*b*c^9*d*x^2*e^11 + (b^10 + 12*a*b^8*c - 53*a^2*b^
6*c^2 + 16*a^3*b^4*c^3 + 69*a^4*b^2*c^4 - 10*a^5*c^5)*d^10*x^2*e^2 + c^10*x
^2*e^12 - 10*(b^9*c + 2*a*b^7*c^2 - 17*a^2*b^5*c^3 + 14*a^3*b^3*c^4 + 5*a^4
*b*c^5)*d^9*x^2*e^3 + 15*(3*b^8*c^2 - 2*a*b^6*c^3 - 17*a^2*b^4*c^4 + 16*a^3
```

*b^2*c^5 + a^4*c^6)*d^8*x^2*e^4 - 60*(2*b^7*c^3 - 3*a*b^5*c^4 - 3*a^2*b^3*c
^5 + 3*a^3*b*c^6)*d^7*x^2*e^5 + 2*(105*b^6*c^4 - 169*a*b^4*c^5 - 13*a^2*b^2
*c^6 + 26*a^3*c^7)*d^6*x^2*e^6 - 12*(21*b^5*c^5 - 29*a*b^3*c^6 + 3*a^2*b*c^
7)*d^5*x^2*e^7 + 15*(14*b^4*c^6 - 14*a*b^2*c^7 + a^2*c^8)*d^4*x^2*e^8 - 10*
(12*b^3*c^7 - 7*a*b*c^8)*d^3*x^2*e^9 + 5*(9*b^2*c^8 - 2*a*c^9)*d^2*x^2*e^10
 + 1/2*sqrt(1/2)*((b^12 - 12*a*b^10*c + 55*a^2*b^8*c^2 - 120*a^3*b^6*c^3 +
125*a^4*b^4*c^4 - 54*a^5*b^2*c^5 + 8*a^6*c^6)*d^10 - 2*(5*b^11*c - 54*a*b^9
*c^2 + 215*a^2*b^7*c^3 - 386*a^3*b^5*c^4 + 297*a^4*b^3*c^5 - 68*a^5*b*c^6)*
d^9*e + 2*(23*b^10*c^2 - 222*a*b^8*c^3 + 755*a^2*b^6*c^4 - 1080*a^3*b^4*c^5
 + 573*a^4*b^2*c^6 - 52*a^5*c^7)*d^8*e^2 - 8*(16*b^9*c^3 - 137*a*b^7*c^4 +
389*a^2*b^5*c^5 - 421*a^3*b^3*c^6 + 132*a^4*b*c^7)*d^7*e^3 + 2*(119*b^8*c^4
 - 897*a*b^6*c^5 + 2061*a^2*b^4*c^6 - 1558*a^3*b^2*c^7 + 200*a^4*c^8)*d^6*e
^4 - 4*(77*b^7*c^5 - 507*a*b^5*c^6 + 899*a^2*b^3*c^7 - 412*a^3*b*c^8)*d^5*e
^5 + 20*(14*b^6*c^6 - 80*a*b^4*c^7 + 101*a^2*b^2*c^8 - 20*a^3*c^9)*d^4*e^6
 - 8*(22*b^5*c^7 - 109*a*b^3*c^8 + 84*a^2*b*c^9)*d^3*e^7 + (73*b^4*c^8 - 318
*a*b^2*c^9 + 104*a^2*c^10)*d^2*e^8 - 18*(b^3*c^9 - 4*a*b*c^10)*d*e^9 + 2*(b
^2*c^10 - 4*a*c^11)*e^10 - ((b^11*c^5 - 15*a*b^9*c^6 + 85*a^2*b^7*c^7 - 220
*a^3*b^5*c^8 + 240*a^4*b^3*c^9 - 64*a^5*b*c^10)*d^6 - 2*(3*b^10*c^6 - 43*a*
b^8*c^7 + 229*a^2*b^6*c^8 - 540*a^3*b^4*c^9 + 496*a^4*b^2*c^10 - 64*a^5*c^1
1)*d^5*e + 2*(7*b^9*c^7 - 95*a*b^7*c^8 + 468*a^2*b^5*c^9 - 976*a^3*b^3*c^10
 + 704*a^4*b*c^11)*d^4*e^2 - 4*(4*b^8*c^8 - 51*a*b^6*c^9 + 228*a^2*b^4*c^10
 - 400*a^3*b^2*c^11 + 192*a^4*c^12)*d^3*e^3 + 9*(b^7*c^9 - 12*a*b^5*c^10 +
48*a^2*b^3*c^11 - 64*a^3*b*c^12)*d^2*e^4 - 2*(b^6*c^10 - 12*a*b^4*c^11 + 48
*a^2*b^2*c^12 - 64*a^3*c^13)*d*e^5)*sqrt(-(8*b*c^7*d*e^7 - (b^8 - 6*a*b^6*c
 + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)*d^8 - c^8*e^8 + 8*(b^7*c - 5*a
*b^5*c^2 + 7*a^2*b^3*c^3 - 2*a^3*b*c^4)*d^7*e - 4*(7*b^6*c^2 - 28*a*b^4*c^3
 + 28*a^2*b^2*c^4 - 3*a^3*c^5)*d^6*e^2 + 8*(7*b^5*c^3 - 21*a*b^3*c^4 + 13*a
^2*b*c^5)*d^5*e^3 - 2*(35*b^4*c^4 - 71*a*b^2*c^5 + 19*a^2*c^6)*d^4*e^4 + 8*
(7*b^3*c^5 - 8*a*b*c^6)*d^3*e^5 - 4*(7*b^2*c^6 - 3*a*c^7)*d^2*e^6)/(b^6*c^1
0 - 12*a*b^4*c^11 + 48*a^2*b^2*c^12 - 64*a^3*c^13)))*sqrt(-(b*c^4*e^4 + (b^
5 - 5*a*b^3*c + 5*a^2*b*c^2)*d^4 - 4*(b^4*c - 4*a*b^2*c^2 + 2*a^2*c^3)*d^3*
e + 6*(b^3*c^2 - 3*a*b*c^3)*d^2*e^2 - 4*(b^2*c^3 - 2*a*c^4)*d*e^3 + (b^4*c^
5 - 8*a*b^2*c^6 + 16*a^2*c^7)*sqrt(-(8*b*c^7*d*e^7 - (b^8 - 6*a*b^6*c + 11*
a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)*d^8 - c^8*e^8 + 8*(b^7*c - 5*a*b^5*c
^2 + 7*a^2*b^3*c^3 - 2*a^3*b*c^4)*d^7*e - 4*(7*b^6*c^2 - 28*a*b^4*c^3 + 28*
a^2*b^2*c^4 - 3*a^3*c^5)*d^6*e^2 + 8*(7*b^5*c^3 - 21*a*b^3*c^4 + 13*a^2*b*c
^5)*d^5*e^3 - 2*(35*b^4*c^4 - 71*a*b^2*c^5 + 19*a^2*c^6)*d^4*e^4 + 8*(7*b^3
*c^5 - 8*a*b*c^6)*d^3*e^5 - 4*(7*b^2*c^6 - 3*a*c^7)*d^2*e^6)/(b^6*c^10 - 12
*a*b^4*c^11 + 48*a^2*b^2*c^12 - 64*a^3*c^13)))/(b^4*c^5 - 8*a*b^2*c^6 + 16*
a^2*c^7)))*((b^11 - 13*a*b^9*c + 63*a^2*b^7*c^2 - 138*a^3*b^5*c^3 + 128*a^4
*b^3*c^4 - 32*a^5*b*c^5)*d^7 - (7*b^10*c - 84*a*b^8*c^2 + 364*a^2*b^6*c^3 -
 675*a^3*b^4*c^4 + 472*a^4*b^2*c^5 - 48*a^5*c^6)*d^6*e + 3*(7*b^9*c^2 - 77*
a*b^7*c^3 + 293*a^2*b^5*c^4 - 440*a^3*b^3*c^5 + 208*a^4*b*c^6)*d^5*e^2 - (3
5*b^8*c^3 - 351*a*b^6*c^4 + 1147*a^2*b^4*c^5 - 1288*a^3*b^2*c^6 + 304*a^4*c
^7)*d^4*e^3 + 5*(7*b^7*c^4 - 64*a*b^5*c^5 + 176*a^2*b^3*c^6 - 128*a^3*b*c^7

```
)*d^3*e^4 - 3*(7*b^6*c^5 - 59*a*b^4*c^6 + 136*a^2*b^2*c^7 - 48*a^3*c^8)*d^2
*e^5 + 7*(b^5*c^6 - 8*a*b^3*c^7 + 16*a^2*b*c^8)*d*e^6 - (b^4*c^7 - 8*a*b^2*
c^8 + 16*a^2*c^9)*e^7 - ((b^10*c^5 - 16*a*b^8*c^6 + 98*a^2*b^6*c^7 - 280*a^
3*b^4*c^8 + 352*a^4*b^2*c^9 - 128*a^5*c^10)*d^3 - 3*(b^9*c^6 - 15*a*b^7*c^7
 + 84*a^2*b^5*c^8 - 208*a^3*b^3*c^9 + 192*a^4*b*c^10)*d^2*e + 3*(b^8*c^7 -
14*a*b^6*c^8 + 72*a^2*b^4*c^9 - 160*a^3*b^2*c^1...
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+e/x**4)/(c+a/x**8+b/x**4),x)
```

```
[Out] Timed out
```

Giac [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+e/x^4)/(c+a/x^8+b/x^4),x, algorithm="giac")
```

```
[Out] Timed out
```

Mupad [**B**]
time = 9.24, size = 2500, normalized size = 5.77

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e/x^4)/(c + a/x^8 + b/x^4),x)
```

```
[Out] atan((((((4*x*(4096*a^5*b*c^6*d^2 + 4096*a^4*b*c^7*e^2 + 256*a^3*b^5*c^4*d^
2 - 2048*a^4*b^3*c^5*d^2 + 256*a^2*b^5*c^5*e^2 - 2048*a^3*b^3*c^6*e^2 - 163
84*a^5*c^7*d*e - 1024*a^3*b^4*c^5*d*e + 8192*a^4*b^2*c^6*d*e))/c - (16*(-(b
^9*d^4 + b^4*d^4*(-(4*a*c - b^2)^5)^(1/2) + b^5*c^4*e^4 + c^4*e^4*(-(4*a*c
- b^2)^5)^(1/2) + 80*a^4*b*c^4*d^4 - 8*a*b^3*c^5*e^4 + 16*a^2*b*c^6*e^4 + 1
28*a^3*c^6*d*e^3 - 128*a^4*c^5*d^3*e - 4*b^6*c^3*d*e^3 + 61*a^2*b^5*c^2*d^4
 - 120*a^3*b^3*c^3*d^4 + a^2*c^2*d^4*(-(4*a*c - b^2)^5)^(1/2) + 6*b^7*c^2*d
^2*e^2 - 13*a*b^7*c*d^4 - 4*b^8*c*d^3*e + 240*a^2*b^3*c^4*d^2*e^2 + 6*b^2*c
^2*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) - 3*a*b^2*c*d^4*(-(4*a*c - b^2)^5)^(1/2
) + 40*a*b^4*c^4*d*e^3 + 48*a*b^6*c^2*d^3*e - 4*b*c^3*d*e^3*(-(4*a*c - b^2)
^5)^(1/2) - 4*b^3*c*d^3*e*(-(4*a*c - b^2)^5)^(1/2) - 66*a*b^5*c^3*d^2*e^2 -
```

128*a^2*b^2*c^5*d*e^3 - 200*a^2*b^4*c^3*d^3*e - 288*a^3*b*c^5*d^2*e^2 + 320*a^3*b^2*c^4*d^3*e - 6*a*c^3*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) + 8*a*b*c^2*d^3*e*(-(4*a*c - b^2)^5)^(1/2))/(512*(256*a^4*c^9 + b^8*c^5 - 16*a*b^6*c^6 + 96*a^2*b^4*c^7 - 256*a^3*b^2*c^8)))^(1/4)*(16384*a^5*c^8*e - 256*a^2*b^6*c^5*e + 3072*a^3*b^4*c^6*e - 12288*a^4*b^2*c^7*e))/c)*(-(b^9*d^4 + b^4*d^4*(-(4*a*c - b^2)^5)^(1/2) + b^5*c^4*e^4 + c^4*e^4*(-(4*a*c - b^2)^5)^(1/2) + 80*a^4*b*c^4*d^4 - 8*a*b^3*c^5*e^4 + 16*a^2*b*c^6*e^4 + 128*a^3*c^6*d*e^3 - 128*a^4*c^5*d^3*e - 4*b^6*c^3*d*e^3 + 61*a^2*b^5*c^2*d^4 - 120*a^3*b^3*c^3*d^4 + a^2*c^2*d^4*(-(4*a*c - b^2)^5)^(1/2) + 6*b^7*c^2*d^2*e^2 - 13*a*b^7*c*d^4 - 4*b^8*c*d^3*e + 240*a^2*b^3*c^4*d^2*e^2 + 6*b^2*c^2*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) - 3*a*b^2*c*d^4*(-(4*a*c - b^2)^5)^(1/2) + 40*a*b^4*c^4*d*e^3 + 48*a*b^6*c^2*d^3*e - 4*b*c^3*d*e^3*(-(4*a*c - b^2)^5)^(1/2) - 4*b^3*c*d^3*e*(-(4*a*c - b^2)^5)^(1/2) - 66*a*b^5*c^3*d^2*e^2 - 128*a^2*b^2*c^5*d*e^3 - 200*a^2*b^4*c^3*d^3*e - 288*a^3*b*c^5*d^2*e^2 + 320*a^3*b^2*c^4*d^3*e - 6*a*c^3*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) + 8*a*b*c^2*d^3*e*(-(4*a*c - b^2)^5)^(1/2))/(512*(256*a^4*c^9 + b^8*c^5 - 16*a*b^6*c^6 + 96*a^2*b^4*c^7 - 256*a^3*b^2*c^8)))^(3/4) - (16*(a^3*b^6*d^5 - 4*a^6*c^3*d^5 - 7*a^4*b^4*c*d^5 + 4*a^3*b*c^5*e^5 - a^2*b^7*d^4*e + 12*a^4*c^5*d*e^4 + 13*a^5*b^2*c^2*d^5 - a^2*b^3*c^4*e^5 + 8*a^5*c^4*d^3*e^2 - 6*a^2*b^5*c^2*d^2*e^3 + 32*a^3*b^3*c^3*d^2*e^3 - 22*a^3*b^4*c^2*d^3*e^2 + 22*a^4*b^2*c^3*d^3*e^2 + 4*a^3*b^5*c*d^4*e - 20*a^5*b*c^3*d^4*e + 4*a^2*b^4*c^3*d*e^4 + 4*a^2*b^6*c*d^3*e^2 - 19*a^3*b^2*c^4*d*e^4 - 32*a^4*b*c^4*d^2*e^3 + 5*a^4*b^3*c^2*d^4*e))/c)*(-(b^9*d^4 + b^4*d^4*(-(4*a*c - b^2)^5)^(1/2) + b^5*c^4*e^4 + c^4*e^4*(-(4*a*c - b^2)^5)^(1/2) + 80*a^4*b*c^4*d^4 - 8*a*b^3*c^5*e^4 + 16*a^2*b*c^6*e^4 + 128*a^3*c^6*d*e^3 - 128*a^4*c^5*d^3*e - 4*b^6*c^3*d*e^3 + 61*a^2*b^5*c^2*d^4 - 120*a^3*b^3*c^3*d^4 + a^2*c^2*d^4*(-(4*a*c - b^2)^5)^(1/2) + 6*b^7*c^2*d^2*e^2 - 13*a*b^7*c*d^4 - 4*b^8*c*d^3*e + 240*a^2*b^3*c^4*d^2*e^2 + 6*b^2*c^2*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) - 3*a*b^2*c*d^4*(-(4*a*c - b^2)^5)^(1/2) + 40*a*b^4*c^4*d*e^3 + 48*a*b^6*c^2*d^3*e - 4*b*c^3*d*e^3*(-(4*a*c - b^2)^5)^(1/2) - 4*b^3*c*d^3*e*(-(4*a*c - b^2)^5)^(1/2) - 66*a*b^5*c^3*d^2*e^2 - 128*a^2*b^2*c^5*d*e^3 - 200*a^2*b^4*c^3*d^3*e - 288*a^3*b*c^5*d^2*e^2 + 320*a^3*b^2*c^4*d^3*e - 6*a*c^3*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) + 8*a*b*c^2*d^3*e*(-(4*a*c - b^2)^5)^(1/2))/(512*(256*a^4*c^9 + b^8*c^5 - 16*a*b^6*c^6 + 96*a^2*b^4*c^7 - 256*a^3*b^2*c^8)))^(1/4) + (4*x*(a^4*b^4*d^6 + 2*a^6*c^2*d^6 - 2*a^3*c^5*e^6 - 4*a^5*b^2*c*d^6 - 2*a^3*b^5*d^5*e + a^2*b^2*c^4*e^6 + a^2*b^6*d^4*e^2 - 2*a^4*c^4*d^2*e^4 + 2*a^5*c^3*d^4*e^2 + 6*a^2*b^4*c^2*d^2*e^4 - 16*a^3*b^2*c^3*d^2*e^4 + 8*a^3*b^3*c^2*d^3*e^3 - 17*a^4*b^2*c^2*d^4*e^2 + 10*a^3*b*c^4*d*e^5 + 6*a^4*b^3*c*d^5*e + 2*a^5*b*c^2*d^5*e - 4*a^2*b^3*c^3*d*e^5 - 4*a^2*b^5*c*d^3*e^3 + 2*a^3*b^4*c*d^4*e^2 + 12*a^4*b*c^3*d^3*e^3))/c)*(-(b^9*d^4 + b^4*d^4*(-(4*a*c - b^2)^5)^(1/2) + b^5*c^4*e^4 + c^4*e^4*(-(4*a*c - b^2)^5)^(1/2) + 80*a^4*b*c^4*d^4 - 8*a*b^3*c^5*e^4 + 16*a^2*b*c^6*e^4 + 128*a^3*c^6*d*e^3 - 128*a^4*c^5*d^3*e - 4*b^6*c^3*d*e^3 + 61*a^2*b^5*c^2*d^4 - 120*a^3*b^3*c^3*d^4 + a^2*c^2*d^4*(-(4*a*c - b^2)^5)^(1/2) + 6*b^7*c^2*d^2*e^2 - 13*a*b^7*c*d^4 - 4*b^8*c*d^3*e + 240*a^2*b^3*c^4*d^2*e^2 + 6*b^2*c^2*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) - 3*a*b^2*c*d^4*(-(4*

```
a*c - b^2)^5)^(1/2) + 40*a*b^4*c^4*d*e^3 + 48*a*b^6*c^2*d^3*e - 4*b*c^3*d*e
^3*(-(4*a*c - b^2)^5)^(1/2) - 4*b^3*c*d^3*e*(-(4*a*c - b^2)^5)^(1/2) - 66*a
*b^5*c^3*d^2*e^2 - 128*a^2*b^2*c^5*d*e^3 - 200*a^2*b^4*c^3*d^3*e - 288*a^3*
b*c^5*d^2*e^2 + 320*a^3*b^2*c^4*d^3*e - 6*a*c^3*d^2*e^2*(-(4*a*c - b^2)^5)^
(1/2) + 8*a*b*c^2*d^3*e*(-(4*a*c - b^2)^5)^(1/2))/(512*(256*a^4*c^9 + b^8*c
^5 - 16*a*b^6*c^6 + 96*a^2*b^4*c^7 - 256*a^3*b^2*c^8)))^(1/4)*1i + ((((4*x*
(4096*a^5*b*c^6*d^2 + 4096*a^4*b*c^7*e^2 + 256*a^3*b^5*c^4*d^2 - 2048*a^4*b
^3*c^5*d^2 + 256*a^2*b^5*c^5*e^2 - 2048*a^3*b^3*c^6*e^2 - 16384*a^5*c^7*d*e
 - 1024*a^3*b^4*c^5*d*e + 8192*a^4*b^2*c^6*d*e))/c + (16*(-(b^9*d^4 + b^4*d
^4*(-(4*a*c - b^2)^5)^(1/2) + b^5*c^4*e^4 + c^4*e^4*(-(4*a*c - b^2)^5)^(1/2
) + 80*a^4*b*c^4*d^4 - 8*a*b^3*c^5*e^4 + 16*a^2...
```

3.42 $\int \frac{(d+ex^n)^3}{a+cx^{2n}}\,dx$

Optimal. Leaf size=141

$$\frac{3de^2x}{c}+\frac{e^3x^{1+n}}{c(1+n)}+\frac{d(cd^2-3ae^2)\,x\,{}_2F_1\left(1,\frac{1}{2n};\frac{1}{2}\left(2+\frac{1}{n}\right);-\frac{cx^{2n}}{a}\right)}{ac}+\frac{e(3cd^2-ae^2)\,x^{1+n}\,{}_2F_1\left(1,\frac{1+n}{2n};\frac{1}{2}\left(3+\frac{1}{n}\right);-\right.}{ac(1+n)}$$

```
[Out] 3*d*e^2*x/c+e^3*x^(1+n)/c/(1+n)+d*(-3*a*e^2+c*d^2)*x*hypergeom([1, 1/2/n],[
1+1/2/n],-c*x^(2*n)/a)/a/c+e*(-a*e^2+3*c*d^2)*x^(1+n)*hypergeom([1, 1/2*(1+
n)/n],[3/2+1/2/n],-c*x^(2*n)/a)/a/c/(1+n)
```

Rubi [A]
time = 0.10, antiderivative size = 141, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {1439, 1432, 251, 371}

$$\frac{ex^{n+1}(3cd^2-ae^2)\,{}_2F_1\left(1,\frac{n+1}{2n};\frac{1}{2}\left(3+\frac{1}{n}\right);-\frac{cx^{2n}}{a}\right)}{ac(n+1)}+\frac{dx(cd^2-3ae^2)\,{}_2F_1\left(1,\frac{1}{2n};\frac{1}{2}\left(2+\frac{1}{n}\right);-\frac{cx^{2n}}{a}\right)}{ac}+\frac{3de^2x}{c}+\frac{e^3x^{n+1}}{c(n+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x^n)^3/(a + c*x^(2*n)),x]
```

```
[Out] (3*d*e^2*x)/c + (e^3*x^(1 + n))/(c*(1 + n)) + (d*(c*d^2 - 3*a*e^2)*x*Hyperg
eometric2F1[1, 1/(2*n), (2 + n^(-1))/2, -((c*x^(2*n))/a)])/(a*c) + (e*(3*c*
d^2 - a*e^2)*x^(1 + n)*Hypergeometric2F1[1, (1 + n)/(2*n), (3 + n^(-1))/2,
-((c*x^(2*n))/a)])/(a*c*(1 + n))
```

Rule 251

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, (-b)*(x^n/a)], x] /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p
, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 1432

```
Int[((d_) + (e_.)*(x_)^(n_))/((a_) + (c_.)*(x_)^(n2_)), x_Symbol] :> Dist[d
, Int[1/(a + c*x^(2*n)), x], x] + Dist[e, Int[x^n/(a + c*x^(2*n)), x], x] /
```

```
; FreeQ[{a, c, d, e, n}, x] && EqQ[n2, 2*n] && NeQ[c*d^2 + a*e^2, 0] && (Po
sQ[a*c] ||  !IntegerQ[n])
```

Rule 1439

```
Int[((d_) + (e_.)*(x_)^(n_))^(q_)/((a_) + (c_.)*(x_)^(n2_)), x_Symbol] :> I
nt[ExpandIntegrand[(d + e*x^n)^q/(a + c*x^(2*n)), x], x] /; FreeQ[{a, c, d,
 e, n}, x] && EqQ[n2, 2*n] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[q]
```

Rubi steps

$$\begin{aligned}\int \frac{(d+ex^n)^3}{a+cx^{2n}}\,dx &= \int \left(\frac{3de^2}{c}+\frac{e^3x^n}{c}+\frac{cd^3-3ade^2+(3cd^2e-ae^3)\,x^n}{c\,(a+cx^{2n})}\right)dx\\ &= \frac{3de^2x}{c}+\frac{e^3x^{1+n}}{c(1+n)}+\frac{\int \frac{cd^3-3ade^2+(3cd^2e-ae^3)x^n}{a+cx^{2n}}\,dx}{c}\\ &= \frac{3de^2x}{c}+\frac{e^3x^{1+n}}{c(1+n)}+\frac{(d(cd^2-3ae^2))\int \frac{1}{a+cx^{2n}}\,dx}{c}+\frac{(e(3cd^2-ae^2))\int \frac{x^n}{a+cx^{2n}}\,dx}{c}\\ &= \frac{3de^2x}{c}+\frac{e^3x^{1+n}}{c(1+n)}+\frac{d(cd^2-3ae^2)\,x\,{}_2F_1\left(1,\frac{1}{2n};\frac{1}{2}\left(2+\frac{1}{n}\right);-\frac{cx^{2n}}{a}\right)}{ac}+\frac{e(3cd^2-ae^2)\,x^{1+n}}{}\end{aligned}$$

Mathematica [A]
time = 0.43, size = 128, normalized size = 0.91

$$\frac{d(cd^2-3ae^2)\,(1+n)x\,{}_2F_1\left(1,\frac{1}{2n};1+\frac{1}{2n};-\frac{cx^{2n}}{a}\right)+ex\left(ae(3d(1+n)+ex^n)+(3cd^2-ae^2)\,x^n\,{}_2F_1\left(1,\frac{1+n}{2n};\frac{1}{2}\left(3+\frac{1}{n}\right);-\frac{cx^{2n}}{a}\right)\right)}{ac(1+n)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x^n)^3/(a + c*x^(2*n)),x]
[Out] (d*(c*d^2 - 3*a*e^2)*(1 + n)*x*Hypergeometric2F1[1, 1/(2*n), 1 + 1/(2*n), -
((c*x^(2*n))/a)] + e*x*(a*e*(3*d*(1 + n) + e*x^n) + (3*c*d^2 - a*e^2)*x^n*H
ypergeometric2F1[1, (1 + n)/(2*n), (3 + n^(-1))/2, -((c*x^(2*n))/a)]))/(a*c
*(1 + n))
```

Maple [F]
time = 0.01, size = 0, normalized size = 0.00

$$\int \frac{(d+e\,x^n)^3}{a+c\,x^{2n}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d+e*x^n)^3/(a+c*x^(2*n)),x)
```

```
[Out] int((d+e*x^n)^3/(a+c*x^(2*n)),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+e*x^n)^3/(a+c*x^(2*n)),x, algorithm="maxima")
```

```
[Out] (3*d*(n + 1)*x*e^2 + x*e^(n*log(x) + 3))/(c*(n + 1)) - integrate(-(c*d^3 -
3*a*d*e^2 + (3*c*d^2*e - a*e^3)*x^n)/(c^2*x^(2*n) + a*c), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+e*x^n)^3/(a+c*x^(2*n)),x, algorithm="fricas")
```

```
[Out] integral((3*d^2*x^n*e + d^3 + 3*d*x^(2*n)*e^2 + x^(3*n)*e^3)/(c*x^(2*n) + a
), x)
```

Sympy [C] Result contains complex when optimal does not.
time = 4.34, size = 337, normalized size = 2.39

$$-\frac{3de^2x\Phi\left(\frac{ax^{-2n}e^{i\pi}}{c},1,\frac{e^{i\pi}}{2n}\right)\Gamma\left(\frac{1}{2n}\right)}{4cn^2\Gamma\left(1+\frac{1}{2n}\right)}+\frac{d^3x\Phi\left(\frac{cx^{2n}e^{i\pi}}{a},1,\frac{1}{2n}\right)\Gamma\left(\frac{1}{2n}\right)}{4an^2\Gamma\left(1+\frac{1}{2n}\right)}+\frac{3d^2exx^n\Phi\left(\frac{cx^{2n}e^{i\pi}}{a},1,\frac{1}{2}+\frac{1}{2n}\right)\Gamma\left(\frac{1}{2}+\frac{1}{2n}\right)}{4an\Gamma\left(\frac{3}{2}+\frac{1}{2n}\right)}+\frac{3d^2exx^n\Phi\left(\frac{cx^{2n}e^{i\pi}}{a},1,\frac{1}{2}+\frac{1}{2n}\right)\Gamma\left(\frac{1}{2}+\frac{1}{2n}\right)}{4an^2\Gamma\left(\frac{3}{2}+\frac{1}{2n}\right)}+\frac{3e^3xx^{3n}\Phi\left(\frac{cx^{2n}e^{i\pi}}{a},1,\frac{3}{2}+\frac{1}{2n}\right)\Gamma\left(\frac{3}{2}+\frac{1}{2n}\right)}{4an\Gamma\left(\frac{5}{2}+\frac{1}{2n}\right)}+\frac{e^3xx^{3n}\Phi\left(\frac{cx^{2n}e^{i\pi}}{a},1,\frac{3}{2}+\frac{1}{2n}\right)\Gamma\left(\frac{3}{2}+\frac{1}{2n}\right)}{4an^2\Gamma\left(\frac{5}{2}+\frac{1}{2n}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+e*x**n)**3/(a+c*x**(2*n)),x)
```

```
[Out] -3*d*e**2*x*lerchphi(a*exp_polar(I*pi)/(c*x**(2*n)), 1, exp_polar(I*pi)/(2*
n))*gamma(1/(2*n))/(4*c*n**2*gamma(1 + 1/(2*n))) + d**3*x*lerchphi(c*x**(2*
n)*exp_polar(I*pi)/a, 1, 1/(2*n))*gamma(1/(2*n))/(4*a*n**2*gamma(1 + 1/(2*n
))) + 3*d**2*e*x*x**n*lerchphi(c*x**(2*n)*exp_polar(I*pi)/a, 1, 1/2 + 1/(2*
n))*gamma(1/2 + 1/(2*n))/(4*a*n*gamma(3/2 + 1/(2*n))) + 3*d**2*e*x*x**n*ler
chphi(c*x**(2*n)*exp_polar(I*pi)/a, 1, 1/2 + 1/(2*n))*gamma(1/2 + 1/(2*n))/
(4*a*n**2*gamma(3/2 + 1/(2*n))) + 3*e**3*x*x**(3*n)*lerchphi(c*x**(2*n)*exp
_polar(I*pi)/a, 1, 3/2 + 1/(2*n))*gamma(3/2 + 1/(2*n))/(4*a*n*gamma(5/2 + 1
/(2*n))) + e**3*x*x**(3*n)*lerchphi(c*x**(2*n)*exp_polar(I*pi)/a, 1, 3/2 +
1/(2*n))*gamma(3/2 + 1/(2*n))/(4*a*n**2*gamma(5/2 + 1/(2*n)))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+e*x^n)^3/(a+c*x^(2*n)),x, algorithm="giac")
```

```
[Out] integrate((x^n*e + d)^3/(c*x^(2*n) + a), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(d+e\,x^n)^3}{a+c\,x^{2\,n}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x^n)^3/(a + c*x^(2*n)),x)
```

```
[Out] int((d + e*x^n)^3/(a + c*x^(2*n)), x)
```

3.43 $\int \frac{(d+ex^n)^2}{a+cx^{2n}}\,dx$

Optimal. Leaf size=107

$$\frac{e^2x}{c}+\frac{(cd^2-ae^2)\,x\,{}_2F_1\left(1,\frac{1}{2n};\frac{1}{2}\left(2+\frac{1}{n}\right);-\frac{cx^{2n}}{a}\right)}{ac}+\frac{2dex^{1+n}\,{}_2F_1\left(1,\frac{1+n}{2n};\frac{1}{2}\left(3+\frac{1}{n}\right);-\frac{cx^{2n}}{a}\right)}{a(1+n)}$$

```
[Out] e^2*x/c+(-a*e^2+c*d^2)*x*hypergeom([1, 1/2/n],[1+1/2/n],-c*x^(2*n)/a)/a/c+2
*d*e*x^(1+n)*hypergeom([1, 1/2*(1+n)/n],[3/2+1/2/n],-c*x^(2*n)/a)/a/(1+n)
```

Rubi [A]
time = 0.07, antiderivative size = 107, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {1439, 1432, 251, 371}

$$\frac{x(cd^2-ae^2)\,{}_2F_1\left(1,\frac{1}{2n};\frac{1}{2}\left(2+\frac{1}{n}\right);-\frac{cx^{2n}}{a}\right)}{ac}+\frac{2dex^{n+1}\,{}_2F_1\left(1,\frac{n+1}{2n};\frac{1}{2}\left(3+\frac{1}{n}\right);-\frac{cx^{2n}}{a}\right)}{a(n+1)}+\frac{e^2x}{c}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x^n)^2/(a + c*x^(2*n)),x]
```

```
[Out] (e^2*x)/c + ((c*d^2 - a*e^2)*x*Hypergeometric2F1[1, 1/(2*n), (2 + n^(-1))/2
, -((c*x^(2*n))/a)])/(a*c) + (2*d*e*x^(1 + n)*Hypergeometric2F1[1, (1 + n)/
(2*n), (3 + n^(-1))/2, -((c*x^(2*n))/a)])/(a*(1 + n))
```

Rule 251

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, (-b)*(x^n/a)], x] /; FreeQ[{a, b, n, p}, x] && !IGtQ[p
, 0] && !IntegerQ[1/n] && !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 1432

```
Int[((d_) + (e_.)*(x_)^(n_))/((a_) + (c_.)*(x_)^(n2_)), x_Symbol] :> Dist[d
, Int[1/(a + c*x^(2*n)), x], x] + Dist[e, Int[x^n/(a + c*x^(2*n)), x], x] /
; FreeQ[{a, c, d, e, n}, x] && EqQ[n2, 2*n] && NeQ[c*d^2 + a*e^2, 0] && (Po
```

```
sQ[a*c] ||  !IntegerQ[n])
```

Rule 1439

```
Int[((d_) + (e_.)*(x_)^(n_))^(q_)/((a_) + (c_.)*(x_)^(n2_)), x_Symbol] :> I
nt[ExpandIntegrand[(d + e*x^n)^q/(a + c*x^(2*n)), x], x] /; FreeQ[{a, c, d,
 e, n}, x] && EqQ[n2, 2*n] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[q]
```

Rubi steps

$$\begin{aligned}\int \frac{(d+ex^n)^2}{a+cx^{2n}}\,dx &= \int \left(\frac{e^2}{c}+\frac{cd^2-ae^2+2cdex^n}{c\,(a+cx^{2n})}\right)dx\\ &= \frac{e^2x}{c}+\frac{\int \frac{cd^2-ae^2+2cdex^n}{a+cx^{2n}}\,dx}{c}\\ &= \frac{e^2x}{c}+(2de)\int \frac{x^n}{a+cx^{2n}}\,dx+\frac{(cd^2-ae^2)\int \frac{1}{a+cx^{2n}}\,dx}{c}\\ &= \frac{e^2x}{c}+\frac{(cd^2-ae^2)\,x\,{}_2F_1\left(1,\frac{1}{2n};\frac{1}{2}\left(2+\frac{1}{n}\right);-\frac{cx^{2n}}{a}\right)}{ac}+\frac{2dex^{1+n}\,{}_2F_1\left(1,\frac{1+n}{2n};\frac{1}{2}\left(3+\frac{1}{n}\right);-\frac{cx^{2n}}{a}\right)}{a(1+n)}\end{aligned}$$

Mathematica [A]
time = 0.17, size = 107, normalized size = 1.00

$$\frac{x\left((cd^2-ae^2)\,(1+n)\,{}_2F_1\left(1,\frac{1}{2n};1+\frac{1}{2n};-\frac{cx^{2n}}{a}\right)+e\left(ae(1+n)+2cdx^n\,{}_2F_1\left(1,\frac{1+n}{2n};\frac{1}{2}\left(3+\frac{1}{n}\right);-\frac{cx^{2n}}{a}\right)\right)\right)}{ac(1+n)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x^n)^2/(a + c*x^(2*n)),x]
```

```
[Out] (x*((c*d^2 - a*e^2)*(1 + n)*Hypergeometric2F1[1, 1/(2*n), 1 + 1/(2*n), -((c
*x^(2*n))/a)] + e*(a*e*(1 + n) + 2*c*d*x^n*Hypergeometric2F1[1, (1 + n)/(2*
n), (3 + n^(-1))/2, -((c*x^(2*n))/a)])))/(a*c*(1 + n))
```

Maple [F]
time = 0.01, size = 0, normalized size = 0.00

$$\int \frac{(d+e\,x^n)^2}{a+c\,x^{2n}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d+e*x^n)^2/(a+c*x^(2*n)),x)
```

```
[Out] int((d+e*x^n)^2/(a+c*x^(2*n)),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+e*x^n)^2/(a+c*x^(2*n)),x, algorithm="maxima")
```

```
[Out] x*e^2/c + integrate((c*d^2 + 2*c*d*e^(n*log(x) + 1) - a*e^2)/(c^2*x^(2*n) +
 a*c), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+e*x^n)^2/(a+c*x^(2*n)),x, algorithm="fricas")
```

```
[Out] integral((2*d*x^n*e + d^2 + x^(2*n)*e^2)/(c*x^(2*n) + a), x)
```

Sympy [C] Result contains complex when optimal does not.
time = 3.17, size = 207, normalized size = 1.93

$$-\frac{e^{2}x\Phi\left(\frac{ax^{-2n}e^{i\pi}}{c},1,\frac{e^{i\pi}}{2n}\right)\Gamma\left(\frac{1}{2n}\right)}{4cn^{2}\Gamma\left(1+\frac{1}{2n}\right)}+\frac{d^{2}x\Phi\left(\frac{cx^{2n}e^{i\pi}}{a},1,\frac{1}{2n}\right)\Gamma\left(\frac{1}{2n}\right)}{4an^{2}\Gamma\left(1+\frac{1}{2n}\right)}+\frac{dexx^{n}\Phi\left(\frac{cx^{2n}e^{i\pi}}{a},1,\frac{1}{2}+\frac{1}{2n}\right)\Gamma\left(\frac{1}{2}+\frac{1}{2n}\right)}{2an\Gamma\left(\frac{3}{2}+\frac{1}{2n}\right)}+\frac{dexx^{n}\Phi\left(\frac{cx^{2n}e^{i\pi}}{a},1,\frac{1}{2}+\frac{1}{2n}\right)\Gamma\left(\frac{1}{2}+\frac{1}{2n}\right)}{2an^{2}\Gamma\left(\frac{3}{2}+\frac{1}{2n}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+e*x**n)**2/(a+c*x**(2*n)),x)
```

```
[Out] -e**2*x*lerchphi(a*exp_polar(I*pi)/(c*x**(2*n)), 1, exp_polar(I*pi)/(2*n))*
gamma(1/(2*n))/(4*c*n**2*gamma(1 + 1/(2*n))) + d**2*x*lerchphi(c*x**(2*n)*e
xp_polar(I*pi)/a, 1, 1/(2*n))*gamma(1/(2*n))/(4*a*n**2*gamma(1 + 1/(2*n)))
+ d*e*x*x**n*lerchphi(c*x**(2*n)*exp_polar(I*pi)/a, 1, 1/2 + 1/(2*n))*gamma
(1/2 + 1/(2*n))/(2*a*n*gamma(3/2 + 1/(2*n))) + d*e*x*x**n*lerchphi(c*x**(2*
n)*exp_polar(I*pi)/a, 1, 1/2 + 1/(2*n))*gamma(1/2 + 1/(2*n))/(2*a*n**2*gamm
a(3/2 + 1/(2*n)))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+e*x^n)^2/(a+c*x^(2*n)),x, algorithm="giac")
```

```
[Out] integrate((x^n*e + d)^2/(c*x^(2*n) + a), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(d+e\,x^n)^2}{a+c\,x^{2\,n}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x^n)^2/(a + c*x^(2*n)),x)
```

```
[Out] int((d + e*x^n)^2/(a + c*x^(2*n)), x)
```

3.44 $\int \frac{d+ex^n}{a+cx^{2n}}\,dx$

Optimal. Leaf size=83

$$\frac{dx\,{}_2F_1\left(1,\frac{1}{2n};\frac{1}{2}\left(2+\frac{1}{n}\right);-\frac{cx^{2n}}{a}\right)}{a}+\frac{ex^{1+n}\,{}_2F_1\left(1,\frac{1+n}{2n};\frac{1}{2}\left(3+\frac{1}{n}\right);-\frac{cx^{2n}}{a}\right)}{a(1+n)}$$

```
[Out] d*x*hypergeom([1, 1/2/n],[1+1/2/n],-c*x^(2*n)/a)/a+e*x^(1+n)*hypergeom([1,
1/2*(1+n)/n],[3/2+1/2/n],-c*x^(2*n)/a)/a/(1+n)
```

Rubi [A]
time = 0.02, antiderivative size = 83, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {1432, 251, 371}

$$\frac{dx\,{}_2F_1\left(1,\frac{1}{2n};\frac{1}{2}\left(2+\frac{1}{n}\right);-\frac{cx^{2n}}{a}\right)}{a}+\frac{ex^{n+1}\,{}_2F_1\left(1,\frac{n+1}{2n};\frac{1}{2}\left(3+\frac{1}{n}\right);-\frac{cx^{2n}}{a}\right)}{a(n+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x^n)/(a + c*x^(2*n)),x]
```

```
[Out] (d*x*Hypergeometric2F1[1, 1/(2*n), (2 + n^(-1))/2, -((c*x^(2*n))/a)])/a + (
e*x^(1 + n)*Hypergeometric2F1[1, (1 + n)/(2*n), (3 + n^(-1))/2, -((c*x^(2*n
))/a)])/(a*(1 + n))
```

Rule 251

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, (-b)*(x^n/a)], x] /; FreeQ[{a, b, n, p}, x] && !IGtQ[p
, 0] && !IntegerQ[1/n] && !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 1432

```
Int[((d_) + (e_.)*(x_)^(n_))/((a_) + (c_.)*(x_)^(n2_)), x_Symbol] :> Dist[d
, Int[1/(a + c*x^(2*n)), x], x] + Dist[e, Int[x^n/(a + c*x^(2*n)), x], x] /
; FreeQ[{a, c, d, e, n}, x] && EqQ[n2, 2*n] && NeQ[c*d^2 + a*e^2, 0] && (Po
sQ[a*c] || !IntegerQ[n])
```

Rubi steps

$$\int \frac{d+ex^n}{a+cx^{2n}}\,dx = d\int \frac{1}{a+cx^{2n}}\,dx + e\int \frac{x^n}{a+cx^{2n}}\,dx$$
$$= \frac{dx\,{}_2F_1\left(1,\frac{1}{2n};\frac{1}{2}\left(2+\frac{1}{n}\right);-\frac{cx^{2n}}{a}\right)}{a} + \frac{ex^{1+n}\,{}_2F_1\left(1,\frac{1+n}{2n};\frac{1}{2}\left(3+\frac{1}{n}\right);-\frac{cx^{2n}}{a}\right)}{a(1+n)}$$

Mathematica [A]
time = 0.06, size = 82, normalized size = 0.99

$$\frac{x\left(d(1+n)\,{}_2F_1\left(1,\frac{1}{2n};1+\frac{1}{2n};-\frac{cx^{2n}}{a}\right)+ex^n\,{}_2F_1\left(1,\frac{1+n}{2n};\frac{1}{2}\left(3+\frac{1}{n}\right);-\frac{cx^{2n}}{a}\right)\right)}{a(1+n)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x^n)/(a + c*x^(2*n)),x]
```

```
[Out] (x*(d*(1 + n)*Hypergeometric2F1[1, 1/(2*n), 1 + 1/(2*n), -((c*x^(2*n))/a)]
+ e*x^n*Hypergeometric2F1[1, (1 + n)/(2*n), (3 + n^(-1))/2, -((c*x^(2*n))/a
)]))/(a*(1 + n))
```

Maple [F]
time = 0.04, size = 0, normalized size = 0.00

$$\int \frac{d+e\,x^n}{a+c\,x^{2n}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d+e*x^n)/(a+c*x^(2*n)),x)
```

```
[Out] int((d+e*x^n)/(a+c*x^(2*n)),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+e*x^n)/(a+c*x^(2*n)),x, algorithm="maxima")
```

```
[Out] integrate((x^n*e + d)/(c*x^(2*n) + a), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+e*x^n)/(a+c*x^(2*n)),x, algorithm="fricas")
```

```
[Out] integral((x^n*e + d)/(c*x^(2*n) + a), x)
```

Sympy [C] Result contains complex when optimal does not.
time = 2.38, size = 153, normalized size = 1.84

$$\frac{dx\Phi\left(\frac{cx^{2n}e^{i\pi}}{a},1,\frac{1}{2n}\right)\Gamma\left(\frac{1}{2n}\right)}{4an^2\Gamma\left(1+\frac{1}{2n}\right)}+\frac{exx^n\Phi\left(\frac{cx^{2n}e^{i\pi}}{a},1,\frac{1}{2}+\frac{1}{2n}\right)\Gamma\left(\frac{1}{2}+\frac{1}{2n}\right)}{4an\Gamma\left(\frac{3}{2}+\frac{1}{2n}\right)}+\frac{exx^n\Phi\left(\frac{cx^{2n}e^{i\pi}}{a},1,\frac{1}{2}+\frac{1}{2n}\right)\Gamma\left(\frac{1}{2}+\frac{1}{2n}\right)}{4an^2\Gamma\left(\frac{3}{2}+\frac{1}{2n}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+e*x**n)/(a+c*x**(2*n)),x)
```

```
[Out] d*x*lerchphi(c*x**(2*n)*exp_polar(I*pi)/a, 1, 1/(2*n))*gamma(1/(2*n))/(4*a*
n**2*gamma(1 + 1/(2*n))) + e*x*x**n*lerchphi(c*x**(2*n)*exp_polar(I*pi)/a,
1, 1/2 + 1/(2*n))*gamma(1/2 + 1/(2*n))/(4*a*n*gamma(3/2 + 1/(2*n))) + e*x*x
**n*lerchphi(c*x**(2*n)*exp_polar(I*pi)/a, 1, 1/2 + 1/(2*n))*gamma(1/2 + 1/
(2*n))/(4*a*n**2*gamma(3/2 + 1/(2*n)))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+e*x^n)/(a+c*x^(2*n)),x, algorithm="giac")
```

```
[Out] integrate((x^n*e + d)/(c*x^(2*n) + a), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{d+e\,x^n}{a+c\,x^{2\,n}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x^n)/(a + c*x^(2*n)),x)
```

```
[Out] int((d + e*x^n)/(a + c*x^(2*n)), x)
```

3.45 $\int \frac{1}{(d+ex^n)(a+cx^{2n})}\,dx$

Optimal. Leaf size=152

$$\frac{cdx\,{}_2F_1\left(1,\frac{1}{2n};\frac{1}{2}\left(2+\frac{1}{n}\right);-\frac{cx^{2n}}{a}\right)}{a\left(cd^2+ae^2\right)}+\frac{e^2x\,{}_2F_1\left(1,\frac{1}{n};1+\frac{1}{n};-\frac{ex^n}{d}\right)}{d\left(cd^2+ae^2\right)}-\frac{cex^{1+n}\,{}_2F_1\left(1,\frac{1+n}{2n};\frac{1}{2}\left(3+\frac{1}{n}\right);-\frac{cx^{2n}}{a}\right)}{a\left(cd^2+ae^2\right)(1+n)}$$

```
[Out] c*d*x*hypergeom([1, 1/2/n],[1+1/2/n],-c*x^(2*n)/a)/a/(a*e^2+c*d^2)+e^2*x*hy
pergeom([1, 1/n],[1+1/n],-e*x^n/d)/d/(a*e^2+c*d^2)-c*e*x^(1+n)*hypergeom([1
, 1/2*(1+n)/n],[3/2+1/2/n],-c*x^(2*n)/a)/a/(a*e^2+c*d^2)/(1+n)
```

Rubi [A]
time = 0.08, antiderivative size = 152, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {1439, 251, 1432, 371}

$$-\frac{cex^{n+1}\,{}_2F_1\left(1,\frac{n+1}{2n};\frac{1}{2}\left(3+\frac{1}{n}\right);-\frac{cx^{2n}}{a}\right)}{a(n+1)\left(ae^2+cd^2\right)}+\frac{cdx\,{}_2F_1\left(1,\frac{1}{2n};\frac{1}{2}\left(2+\frac{1}{n}\right);-\frac{cx^{2n}}{a}\right)}{a\left(ae^2+cd^2\right)}+\frac{e^2x\,{}_2F_1\left(1,\frac{1}{n};1+\frac{1}{n};-\frac{ex^n}{d}\right)}{d\left(ae^2+cd^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[1/((d + e*x^n)*(a + c*x^(2*n))),x]
```

```
[Out] (c*d*x*Hypergeometric2F1[1, 1/(2*n), (2 + n^(-1))/2, -((c*x^(2*n))/a)])/(a*
(c*d^2 + a*e^2)) + (e^2*x*Hypergeometric2F1[1, n^(-1), 1 + n^(-1), -((e*x^n
)/d)])/(d*(c*d^2 + a*e^2)) - (c*e*x^(1 + n)*Hypergeometric2F1[1, (1 + n)/(2
*n), (3 + n^(-1))/2, -((c*x^(2*n))/a)])/(a*(c*d^2 + a*e^2)*(1 + n))
```

Rule 251

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, (-b)*(x^n/a)], x] /; FreeQ[{a, b, n, p}, x] && !IGtQ[p
, 0] && !IntegerQ[1/n] && !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 1432

```
Int[((d_) + (e_.)*(x_)^(n_))/((a_) + (c_.)*(x_)^(n2_)), x_Symbol] :> Dist[d
, Int[1/(a + c*x^(2*n)), x], x] + Dist[e, Int[x^n/(a + c*x^(2*n)), x], x] /
```

```
; FreeQ[{a, c, d, e, n}, x] && EqQ[n2, 2*n] && NeQ[c*d^2 + a*e^2, 0] && (Po
sQ[a*c] ||  !IntegerQ[n])
```

Rule 1439

```
Int[((d_) + (e_.)*(x_)^(n_))^(q_)/((a_) + (c_.)*(x_)^(n2_)), x_Symbol] :> I
nt[ExpandIntegrand[(d + e*x^n)^q/(a + c*x^(2*n)), x], x] /; FreeQ[{a, c, d,
 e, n}, x] && EqQ[n2, 2*n] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[q]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{(d+ex^n)(a+cx^{2n})}\,dx &= \int \left(\frac{e^2}{(cd^2+ae^2)(d+ex^n)} - \frac{c(-d+ex^n)}{(cd^2+ae^2)(a+cx^{2n})}\right)dx\\ &= -\frac{c\int \frac{-d+ex^n}{a+cx^{2n}}\,dx}{cd^2+ae^2} + \frac{e^2\int \frac{1}{d+ex^n}\,dx}{cd^2+ae^2}\\ &= \frac{e^2x\,{}_2F_1\left(1,\frac{1}{n};1+\frac{1}{n};-\frac{ex^n}{d}\right)}{d(cd^2+ae^2)} + \frac{(cd)\int \frac{1}{a+cx^{2n}}\,dx}{cd^2+ae^2} - \frac{(ce)\int \frac{x^n}{a+cx^{2n}}\,dx}{cd^2+ae^2}\\ &= \frac{cdx\,{}_2F_1\left(1,\frac{1}{2n};\frac{1}{2}\left(2+\frac{1}{n}\right);-\frac{cx^{2n}}{a}\right)}{a(cd^2+ae^2)} + \frac{e^2x\,{}_2F_1\left(1,\frac{1}{n};1+\frac{1}{n};-\frac{ex^n}{d}\right)}{d(cd^2+ae^2)} - \frac{cex^{1+n}\,{}_2F_1\left(1,\right.}{a(cd^2}\end{aligned}$$

Mathematica [A]
time = 0.14, size = 131, normalized size = 0.86

$$\frac{x\left(cd^2(1+n)\,{}_2F_1\left(1,\frac{1}{2n};1+\frac{1}{2n};-\frac{cx^{2n}}{a}\right)+e\left(ae(1+n)\,{}_2F_1\left(1,\frac{1}{n};1+\frac{1}{n};-\frac{ex^n}{d}\right)-cdx^n\,{}_2F_1\left(1,\frac{1+n}{2n};\frac{1}{2}\left(3+\frac{1}{n}\right);-\frac{cx^{2n}}{a}\right)\right)\right)}{ad\left(cd^2+ae^2\right)(1+n)}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((d + e*x^n)*(a + c*x^(2*n))),x]
```

```
[Out] (x*(c*d^2*(1 + n)*Hypergeometric2F1[1, 1/(2*n), 1 + 1/(2*n), -((c*x^(2*n))/
a)] + e*(a*e*(1 + n)*Hypergeometric2F1[1, n^(-1), 1 + n^(-1), -((e*x^n)/d)]
 - c*d*x^n*Hypergeometric2F1[1, (1 + n)/(2*n), (3 + n^(-1))/2, -((c*x^(2*n)
)/a)])))/(a*d*(c*d^2 + a*e^2)*(1 + n))
```

Maple [F]
time = 0.08, size = 0, normalized size = 0.00

$$\int \frac{1}{(d+e\,x^n)(a+c\,x^{2n})}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(d+e*x^n)/(a+c*x^(2*n)),x)
[Out] int(1/(d+e*x^n)/(a+c*x^(2*n)),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d+e*x^n)/(a+c*x^(2*n)),x, algorithm="maxima")
[Out] integrate(1/((c*x^(2*n) + a)*(x^n*e + d)), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d+e*x^n)/(a+c*x^(2*n)),x, algorithm="fricas")
[Out] integral(1/(a*x^n*e + a*d + (c*x^n*e + c*d)*x^(2*n)), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(a + cx^{2n})(d + ex^n)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d+e*x**n)/(a+c*x**(2*n)),x)
[Out] Integral(1/((a + c*x**(2*n))*(d + e*x**n)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d+e*x^n)/(a+c*x^(2*n)),x, algorithm="giac")
[Out] integrate(1/((c*x^(2*n) + a)*(x^n*e + d)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{(a + c\, x^{2\,n})\,(d + e\, x^n)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + c*x^(2*n))*(d + e*x^n)),x)
[Out] int(1/((a + c*x^(2*n))*(d + e*x^n)), x)
```

3.46 $\int \frac{1}{(d+ex^n)^2(a+cx^{2n})}\,dx$

Optimal. Leaf size=205

$$\frac{c(cd^2-ae^2)\,x\,{}_2F_1\left(1,\frac{1}{2n};\frac{1}{2}\left(2+\frac{1}{n}\right);-\frac{cx^{2n}}{a}\right)}{a\left(cd^2+ae^2\right)^2}+\frac{2ce^2x\,{}_2F_1\left(1,\frac{1}{n};1+\frac{1}{n};-\frac{ex^n}{d}\right)}{\left(cd^2+ae^2\right)^2}-\frac{2c^2dex^{1+n}\,{}_2F_1\left(1,\frac{1+n}{2n};\frac{1}{2}\left(3+\frac{1}{n}\right)\right.}{a\left(cd^2+ae^2\right)^2(1+n)}$$

```
[Out] c*(-a*e^2+c*d^2)*x*hypergeom([1, 1/2/n],[1+1/2/n],-c*x^(2*n)/a)/a/(a*e^2+c*
d^2)^2+2*c*e^2*x*hypergeom([1, 1/n],[1+1/n],-e*x^n/d)/(a*e^2+c*d^2)^2-2*c^2
*d*e*x^(1+n)*hypergeom([1, 1/2*(1+n)/n],[3/2+1/2/n],-c*x^(2*n)/a)/a/(a*e^2+
c*d^2)^2/(1+n)+e^2*x*hypergeom([2, 1/n],[1+1/n],-e*x^n/d)/d^2/(a*e^2+c*d^2)
```

Rubi [A]
time = 0.12, antiderivative size = 205, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {1439, 251, 1432, 371}

$$-\frac{2c^2dex^{n+1}\,{}_2F_1\left(1,\frac{n+1}{2n};\frac{1}{2}\left(3+\frac{1}{n}\right);-\frac{cx^{2n}}{a}\right)}{a(n+1)\left(ae^2+cd^2\right)^2}+\frac{cx(cd^2-ae^2)\,{}_2F_1\left(1,\frac{1}{2n};\frac{1}{2}\left(2+\frac{1}{n}\right);-\frac{cx^{2n}}{a}\right)}{a\left(ae^2+cd^2\right)^2}+\frac{2ce^2x\,{}_2F_1\left(1,\frac{1}{n};1+\frac{1}{n};-\frac{ex^n}{d}\right)}{\left(ae^2+cd^2\right)^2}+\frac{e^2x\,{}_2F_1\left(2,\frac{1}{n};1+\frac{1}{n};-\frac{ex^n}{d}\right)}{d^2\left(ae^2+cd^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[1/((d + e*x^n)^2*(a + c*x^(2*n))),x]
```

```
[Out] (c*(c*d^2 - a*e^2)*x*Hypergeometric2F1[1, 1/(2*n), (2 + n^(-1))/2, -((c*x^(
2*n))/a)])/(a*(c*d^2 + a*e^2)^2) + (2*c*e^2*x*Hypergeometric2F1[1, n^(-1),
1 + n^(-1), -((e*x^n)/d)])/(c*d^2 + a*e^2)^2 - (2*c^2*d*e*x^(1 + n)*Hyperge
ometric2F1[1, (1 + n)/(2*n), (3 + n^(-1))/2, -((c*x^(2*n))/a)])/(a*(c*d^2 +
 a*e^2)^2*(1 + n)) + (e^2*x*Hypergeometric2F1[2, n^(-1), 1 + n^(-1), -((e*x
^n)/d)])/(d^2*(c*d^2 + a*e^2))
```

Rule 251

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, (-b)*(x^n/a)], x] /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p
, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 1432

```
Int[((d_) + (e_.)*(x_)^(n_))/((a_) + (c_.)*(x_)^(n2_)), x_Symbol] :> Dist[d
, Int[1/(a + c*x^(2*n)), x], x] + Dist[e, Int[x^n/(a + c*x^(2*n)), x], x] /
; FreeQ[{a, c, d, e, n}, x] && EqQ[n2, 2*n] && NeQ[c*d^2 + a*e^2, 0] && (Po
sQ[a*c] ||  !IntegerQ[n])
```

Rule 1439

```
Int[((d_) + (e_.)*(x_)^(n_))^(q_)/((a_) + (c_.)*(x_)^(n2_)), x_Symbol] :> I
nt[ExpandIntegrand[(d + e*x^n)^q/(a + c*x^(2*n)), x], x] /; FreeQ[{a, c, d,
 e, n}, x] && EqQ[n2, 2*n] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[q]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{(d+ex^n)^2(a+cx^{2n})}\,dx &= \int \left(\frac{e^2}{(cd^2+ae^2)(d+ex^n)^2}+\frac{2cde^2}{(cd^2+ae^2)^2(d+ex^n)}-\frac{c(-cd^2+ae^2+2cdex^n}{(cd^2+ae^2)^2(a+cx^{2n}}\right.\\ &= -\frac{c\int \frac{-cd^2+ae^2+2cdex^n}{a+cx^{2n}}\,dx}{(cd^2+ae^2)^2}+\frac{(2cde^2)\int \frac{1}{d+ex^n}\,dx}{(cd^2+ae^2)^2}+\frac{e^2\int \frac{1}{(d+ex^n)^2}\,dx}{cd^2+ae^2}\\ &= \frac{2ce^2x\,{}_2F_1\left(1,\frac{1}{n};1+\frac{1}{n};-\frac{ex^n}{d}\right)}{(cd^2+ae^2)^2}+\frac{e^2x\,{}_2F_1\left(2,\frac{1}{n};1+\frac{1}{n};-\frac{ex^n}{d}\right)}{d^2(cd^2+ae^2)}-\frac{(2c^2de)\int \frac{x^n}{a+cx^{2n}}\,c}{(cd^2+ae^2)^2}\\ &= \frac{c(cd^2-ae^2)\,x\,{}_2F_1\left(1,\frac{1}{2n};\frac{1}{2}\left(2+\frac{1}{n}\right);-\frac{cx^{2n}}{a}\right)}{a(cd^2+ae^2)^2}+\frac{2ce^2x\,{}_2F_1\left(1,\frac{1}{n};1+\frac{1}{n};-\frac{ex^n}{d}\right)}{(cd^2+ae^2)^2}-\end{aligned}$$

Mathematica [A]
time = 0.39, size = 188, normalized size = 0.92

$$\frac{x\left(\frac{c(cd^2-ae^2)\,{}_2F_1\left(1,\frac{1}{2n};1+\frac{1}{2n};-\frac{cx^{2n}}{a}\right)}{a}+e\left(\frac{cd^2e+ae^3}{d^2n+denx^n}+\frac{(ae^3(-1+n)+cd^2e(-1+3n))\,{}_2F_1\left(1,\frac{1}{n};1+\frac{1}{n};-\frac{ex^n}{d}\right)}{d^2n}-\frac{2c^2dx^n\,{}_2F_1\left(1,\frac{1+n}{2n};\frac{1}{2}\left(3+\frac{1}{n}\right);-\frac{cx^{2n}}{a}\right)}{a(1+n)}\right)\right)}{(cd^2+ae^2)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((d + e*x^n)^2*(a + c*x^(2*n))),x]
[Out] (x*((c*(c*d^2 - a*e^2)*Hypergeometric2F1[1, 1/(2*n), 1 + 1/(2*n), -((c*x^(2
*n))/a)])/a + e*((c*d^2*e + a*e^3)/(d^2*n + d*e*n*x^n) + ((a*e^3*(-1 + n) +
 c*d^2*e*(-1 + 3*n))*Hypergeometric2F1[1, n^(-1), 1 + n^(-1), -((e*x^n)/d)]
)/(d^2*n) - (2*c^2*d*x^n*Hypergeometric2F1[1, (1 + n)/(2*n), (3 + n^(-1))/2
, -((c*x^(2*n))/a)])/(a*(1 + n)))))/(c*d^2 + a*e^2)^2
```

Maple [F]
time = 0.02, size = 0, normalized size = 0.00

$$\int \frac{1}{(d+e\,x^n)^2(a+c\,x^{2n})}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(d+e*x^n)^2/(a+c*x^(2*n)),x)
```

```
[Out] int(1/(d+e*x^n)^2/(a+c*x^(2*n)),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d+e*x^n)^2/(a+c*x^(2*n)),x, algorithm="maxima")
```

```
[Out] (c*d^2*(3*n - 1)*e^2 + a*(n - 1)*e^4)*integrate(1/(c^2*d^6*n + 2*a*c*d^4*n*
e^2 + a^2*d^2*n*e^4 + (c^2*d^5*n*e + 2*a*c*d^3*n*e^3 + a^2*d*n*e^5)*x^n), x
) + x*e^2/(c*d^4*n + a*d^2*n*e^2 + (c*d^3*n*e + a*d*n*e^3)*x^n) - integrate
(-(c^2*d^2 - 2*c^2*d*e^(n*log(x) + 1) - a*c*e^2)/(a*c^2*d^4 + 2*a^2*c*d^2*e
^2 + a^3*e^4 + (c^3*d^4 + 2*a*c^2*d^2*e^2 + a^2*c*e^4)*x^(2*n)), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d+e*x^n)^2/(a+c*x^(2*n)),x, algorithm="fricas")
```

```
[Out] integral(1/(2*a*d*x^n*e + a*d^2 + a*x^(2*n)*e^2 + (2*c*d*x^n*e + c*d^2 + c*
x^(2*n)*e^2)*x^(2*n)), x)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: HeuristicGCDFailed

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d+e*x**n)**2/(a+c*x**(2*n)),x)
```

```
[Out] Exception raised: HeuristicGCDFailed >> no luck
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d+e*x^n)^2/(a+c*x^(2*n)),x, algorithm="giac")
```

```
[Out] integrate(1/((c*x^(2*n) + a)*(x^n*e + d)^2), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{(a+c\,x^{2\,n})\,{(d+e\,x^n)}^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + c*x^(2*n))*(d + e*x^n)^2),x)
```

```
[Out] int(1/((a + c*x^(2*n))*(d + e*x^n)^2), x)
```

3.47 $\int \frac{d+ex^n}{a-cx^{2n}}\,dx$

Optimal. Leaf size=81

$$\frac{dx\,{}_2F_1\left(1,\frac{1}{2n};\frac{1}{2}\left(2+\frac{1}{n}\right);\frac{cx^{2n}}{a}\right)}{a}+\frac{ex^{1+n}\,{}_2F_1\left(1,\frac{1+n}{2n};\frac{1}{2}\left(3+\frac{1}{n}\right);\frac{cx^{2n}}{a}\right)}{a(1+n)}$$

```
[Out] d*x*hypergeom([1, 1/2/n],[1+1/2/n],c*x^(2*n)/a)/a+e*x^(1+n)*hypergeom([1, 1
/2*(1+n)/n],[3/2+1/2/n],c*x^(2*n)/a)/a/(1+n)
```

Rubi [A]
time = 0.02, antiderivative size = 81, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.150$, Rules used = {1432, 251, 371}

$$\frac{dx\,{}_2F_1\left(1,\frac{1}{2n};\frac{1}{2}\left(2+\frac{1}{n}\right);\frac{cx^{2n}}{a}\right)}{a}+\frac{ex^{n+1}\,{}_2F_1\left(1,\frac{n+1}{2n};\frac{1}{2}\left(3+\frac{1}{n}\right);\frac{cx^{2n}}{a}\right)}{a(n+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x^n)/(a - c*x^(2*n)),x]
```

```
[Out] (d*x*Hypergeometric2F1[1, 1/(2*n), (2 + n^(-1))/2, (c*x^(2*n))/a])/a + (e*x
^(1 + n)*Hypergeometric2F1[1, (1 + n)/(2*n), (3 + n^(-1))/2, (c*x^(2*n))/a]
)/(a*(1 + n))
```

Rule 251

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, (-b)*(x^n/a)], x] /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p
, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 1432

```
Int[((d_) + (e_.)*(x_)^(n_))/((a_) + (c_.)*(x_)^(n2_)), x_Symbol] :> Dist[d
, Int[1/(a + c*x^(2*n)), x], x] + Dist[e, Int[x^n/(a + c*x^(2*n)), x], x] /
; FreeQ[{a, c, d, e, n}, x] && EqQ[n2, 2*n] && NeQ[c*d^2 + a*e^2, 0] && (Po
sQ[a*c] ||  !IntegerQ[n])
```

Rubi steps

$$\int \frac{d+ex^n}{a-cx^{2n}}\,dx = d\int \frac{1}{a-cx^{2n}}\,dx + e\int \frac{x^n}{a-cx^{2n}}\,dx$$
$$= \frac{dx\,{}_2F_1\left(1,\frac{1}{2n};\frac{1}{2}\left(2+\frac{1}{n}\right);\frac{cx^{2n}}{a}\right)}{a} + \frac{ex^{1+n}\,{}_2F_1\left(1,\frac{1+n}{2n};\frac{1}{2}\left(3+\frac{1}{n}\right);\frac{cx^{2n}}{a}\right)}{a(1+n)}$$

Mathematica [A]
time = 0.08, size = 80, normalized size = 0.99

$$\frac{x\left(d(1+n)\,{}_2F_1\left(1,\frac{1}{2n};1+\frac{1}{2n};\frac{cx^{2n}}{a}\right)+ex^n\,{}_2F_1\left(1,\frac{1+n}{2n};\frac{1}{2}\left(3+\frac{1}{n}\right);\frac{cx^{2n}}{a}\right)\right)}{a(1+n)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x^n)/(a - c*x^(2*n)),x]
```

```
[Out] (x*(d*(1 + n)*Hypergeometric2F1[1, 1/(2*n), 1 + 1/(2*n), (c*x^(2*n))/a] + e
*x^n*Hypergeometric2F1[1, (1 + n)/(2*n), (3 + n^(-1))/2, (c*x^(2*n))/a]))/(
a*(1 + n))
```

Maple [F]
time = 0.04, size = 0, normalized size = 0.00

$$\int \frac{d+e\,x^n}{a-c\,x^{2n}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d+e*x^n)/(a-c*x^(2*n)),x)
```

```
[Out] int((d+e*x^n)/(a-c*x^(2*n)),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+e*x^n)/(a-c*x^(2*n)),x, algorithm="maxima")
```

```
[Out] -integrate((x^n*e + d)/(c*x^(2*n) - a), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+e*x^n)/(a-c*x^(2*n)),x, algorithm="fricas")
```

```
[Out] integral(-(x^n*e + d)/(c*x^(2*n) - a), x)
```

Sympy [C] Result contains complex when optimal does not.
time = 2.45, size = 158, normalized size = 1.95

$$\frac{dx\Phi\left(\frac{cx^{2n}e^{2i\pi}}{a},1,\frac{1}{2n}\right)\Gamma\left(\frac{1}{2n}\right)}{4an^2\Gamma\left(1+\frac{1}{2n}\right)}+\frac{exx^n\Phi\left(\frac{cx^{2n}e^{2i\pi}}{a},1,\frac{1}{2}+\frac{1}{2n}\right)\Gamma\left(\frac{1}{2}+\frac{1}{2n}\right)}{4an\Gamma\left(\frac{3}{2}+\frac{1}{2n}\right)}+\frac{exx^n\Phi\left(\frac{cx^{2n}e^{2i\pi}}{a},1,\frac{1}{2}+\frac{1}{2n}\right)\Gamma\left(\frac{1}{2}+\frac{1}{2n}\right)}{4an^2\Gamma\left(\frac{3}{2}+\frac{1}{2n}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+e*x**n)/(a-c*x**(2*n)),x)
```

```
[Out] d*x*lerchphi(c*x**(2*n)*exp_polar(2*I*pi)/a, 1, 1/(2*n))*gamma(1/(2*n))/(4*
a*n**2*gamma(1 + 1/(2*n))) + e*x*x**n*lerchphi(c*x**(2*n)*exp_polar(2*I*pi)
/a, 1, 1/2 + 1/(2*n))*gamma(1/2 + 1/(2*n))/(4*a*n*gamma(3/2 + 1/(2*n))) + e
*x*x**n*lerchphi(c*x**(2*n)*exp_polar(2*I*pi)/a, 1, 1/2 + 1/(2*n))*gamma(1/
2 + 1/(2*n))/(4*a*n**2*gamma(3/2 + 1/(2*n)))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+e*x^n)/(a-c*x^(2*n)),x, algorithm="giac")
```

```
[Out] integrate(-(x^n*e + d)/(c*x^(2*n) - a), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{d+e\,x^n}{a-c\,x^{2\,n}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x^n)/(a - c*x^(2*n)),x)
```

```
[Out] int((d + e*x^n)/(a - c*x^(2*n)), x)
```

3.48 $\int \frac{(d+ex^n)^3}{(a+cx^{2n})^2} \, dx$

Optimal. Leaf size=288

$$\frac{x(d(cd^2-3ae^2)+e(3cd^2-ae^2)\,x^n)}{2acn\,(a+cx^{2n})}+\frac{3de^2x\,{}_2F_1\left(1,\frac{1}{2n};\frac{1}{2}\left(2+\frac{1}{n}\right);-\frac{cx^{2n}}{a}\right)}{ac}-\frac{d(cd^2-3ae^2)\,(1-2n)x\,{}_2F_1\left(1,\frac{1}{2n}\right.}{2a^2cn}$$

```
[Out] 1/2*x*(d*(-3*a*e^2+c*d^2)+e*(-a*e^2+3*c*d^2)*x^n)/a/c/n/(a+c*x^(2*n))+3*d*e
^2*x*hypergeom([1, 1/2/n],[1+1/2/n],-c*x^(2*n)/a)/a/c-1/2*d*(-3*a*e^2+c*d^2
)*(1-2*n)*x*hypergeom([1, 1/2/n],[1+1/2/n],-c*x^(2*n)/a)/a^2/c/n+e^3*x^(1+n
)*hypergeom([1, 1/2*(1+n)/n],[3/2+1/2/n],-c*x^(2*n)/a)/a/c/(1+n)-1/2*e*(-a*
e^2+3*c*d^2)*(1-n)*x^(1+n)*hypergeom([1, 1/2*(1+n)/n],[3/2+1/2/n],-c*x^(2*n
)/a)/a^2/c/n/(1+n)
```

Rubi [A]

time = 0.17, antiderivative size = 288, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used = {1451, 1445, 1432, 251, 371}

$$-\frac{e(1-n)x^{n+1}(3cd^2-ae^2)\,{}_2F_1\left(1,\frac{n+1}{2n};\frac{1}{2}\left(3+\frac{1}{n}\right);-\frac{cx^{2n}}{a}\right)}{2a^2cn(n+1)}-\frac{d(1-2n)x(cd^2-3ae^2)\,{}_2F_1\left(1,\frac{1}{2n};\frac{1}{2}\left(2+\frac{1}{n}\right);-\frac{cx^{2n}}{a}\right)}{2a^2cn}+\frac{x(ex^n(3cd^2-ae^2)+d(cd^2-3ae^2))}{2acn\,(a+cx^{2n})}+\frac{3de^2x\,{}_2F_1\left(1,\frac{1}{2n};\frac{1}{2}\left(2+\frac{1}{n}\right);-\frac{cx^{2n}}{a}\right)}{ac}+\frac{e^3x^{n+1}\,{}_2F_1\left(1,\frac{n+1}{2n};\frac{1}{2}\left(3+\frac{1}{n}\right);-\frac{cx^{2n}}{a}\right)}{ac(n+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x^n)^3/(a + c*x^(2*n))^2,x]
```

```
[Out] (x*(d*(c*d^2 - 3*a*e^2) + e*(3*c*d^2 - a*e^2)*x^n))/(2*a*c*n*(a + c*x^(2*n)
)) + (3*d*e^2*x*Hypergeometric2F1[1, 1/(2*n), (2 + n^(-1))/2, -((c*x^(2*n))
/a)])/(a*c) - (d*(c*d^2 - 3*a*e^2)*(1 - 2*n)*x*Hypergeometric2F1[1, 1/(2*n)
, (2 + n^(-1))/2, -((c*x^(2*n))/a)])/(2*a^2*c*n) + (e^3*x^(1 + n)*Hypergeom
etric2F1[1, (1 + n)/(2*n), (3 + n^(-1))/2, -((c*x^(2*n))/a)])/(a*c*(1 + n))
 - (e*(3*c*d^2 - a*e^2)*(1 - n)*x^(1 + n)*Hypergeometric2F1[1, (1 + n)/(2*n
), (3 + n^(-1))/2, -((c*x^(2*n))/a)])/(2*a^2*c*n*(1 + n))
```

Rule 251

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, (-b)*(x^n/a)], x] /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p
, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
```

```
Q[p, 0] || GtQ[a, 0])
```

Rule 1432

```
Int[((d_) + (e_.)*(x_)^(n_))/((a_) + (c_.)*(x_)^(n2_)), x_Symbol] :> Dist[d
, Int[1/(a + c*x^(2*n)), x], x] + Dist[e, Int[x^n/(a + c*x^(2*n)), x], x] /
; FreeQ[{a, c, d, e, n}, x] && EqQ[n2, 2*n] && NeQ[c*d^2 + a*e^2, 0] && (Po
sQ[a*c] ||  !IntegerQ[n])
```

Rule 1445

```
Int[((d_) + (e_.)*(x_)^(n_))*((a_) + (c_.)*(x_)^(n2_))^(p_), x_Symbol] :> S
imp[(-x)*(d + e*x^n)*((a + c*x^(2*n))^(p + 1)/(2*a*n*(p + 1))), x] + Dist[1
/(2*a*n*(p + 1)), Int[(d*(2*n*p + 2*n + 1) + e*(2*n*p + 3*n + 1)*x^n)*(a +
c*x^(2*n))^(p + 1), x], x] /; FreeQ[{a, c, d, e, n}, x] && EqQ[n2, 2*n] &&
ILtQ[p, -1]
```

Rule 1451

```
Int[((d_) + (e_.)*(x_)^(n_))^(q_)*((a_) + (c_.)*(x_)^(n2_))^(p_), x_Symbol]
 :> Int[ExpandIntegrand[(d + e*x^n)^q*(a + c*x^(2*n))^p, x], x] /; FreeQ[{a
, c, d, e, n, p, q}, x] && EqQ[n2, 2*n] && NeQ[c*d^2 + a*e^2, 0] && ((Integ
ersQ[p, q] &&  !IntegerQ[n]) || IGtQ[p, 0] || (IGtQ[q, 0] &&  !IntegerQ[n])
)
```

Rubi steps

$$\int \frac{(d+ex^n)^3}{(a+cx^{2n})^2}\,dx = \int \left(\frac{cd^3-3ade^2+(3cd^2e-ae^3)\,x^n}{c\,(a+cx^{2n})^2}+\frac{e^2(3d+ex^n)}{c\,(a+cx^{2n})}\right)dx$$

$$= \frac{\int \frac{cd^3-3ade^2+(3cd^2e-ae^3)x^n}{(a+cx^{2n})^2}\,dx}{c}+\frac{e^2\int \frac{3d+ex^n}{a+cx^{2n}}\,dx}{c}$$

$$= \frac{x(d(cd^2-3ae^2)+e(3cd^2-ae^2)\,x^n)}{2acn\,(a+cx^{2n})}+\frac{(3de^2)\int \frac{1}{a+cx^{2n}}\,dx}{c}+\frac{e^3\int \frac{x^n}{a+cx^{2n}}\,dx}{c}-\frac{\int \frac{(cd^3-3a}{}}{}$$

$$= \frac{x(d(cd^2-3ae^2)+e(3cd^2-ae^2)\,x^n)}{2acn\,(a+cx^{2n})}+\frac{3de^2x\,{}_2F_1\left(1,\frac{1}{2n};\frac{1}{2}\left(2+\frac{1}{n}\right);-\frac{cx^{2n}}{a}\right)}{ac}+\frac{e^3x^{1+n}\,{}_2F}{}$$

$$= \frac{x(d(cd^2-3ae^2)+e(3cd^2-ae^2)\,x^n)}{2acn\,(a+cx^{2n})}+\frac{3de^2x\,{}_2F_1\left(1,\frac{1}{2n};\frac{1}{2}\left(2+\frac{1}{n}\right);-\frac{cx^{2n}}{a}\right)}{ac}-\frac{d(cd^2-3}{}$$

Mathematica [A]

time = 0.37, size = 165, normalized size = 0.57

$$\frac{x\left(\frac{a(-ae^2(3d+ex^n)+cd^2(d+3ex^n))}{a+cx^{2n}}+(3ade^2+cd^3(-1+2n))\,{}_2F_1\left(1,\frac{1}{2n};1+\frac{1}{2n};-\frac{cx^{2n}}{a}\right)+\frac{e(3cd^2(-1+n)+ae^2(1+n))x^n\,{}_2F_1\left(1,\frac{1+n}{2n};\frac{1}{2}\left(3+\frac{1}{n}\right);-\frac{cx^{2n}}{a}\right)}{1+n}\right)}{2a^2cn}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x^n)^3/(a + c*x^(2*n))^2,x]
```

```
[Out] (x*((a*(-(a*e^2*(3*d + e*x^n)) + c*d^2*(d + 3*e*x^n)))/(a + c*x^(2*n)) + (3
*a*d*e^2 + c*d^3*(-1 + 2*n))*Hypergeometric2F1[1, 1/(2*n), 1 + 1/(2*n), -((
c*x^(2*n))/a)] + (e*(3*c*d^2*(-1 + n) + a*e^2*(1 + n))*x^n*Hypergeometric2F
1[1, (1 + n)/(2*n), (3 + n^(-1))/2, -((c*x^(2*n))/a)])/(1 + n)))/(2*a^2*c*n
)
```

Maple [F]
time = 0.01, size = 0, normalized size = 0.00

$$\int \frac{(d+e\,x^n)^3}{(a+c\,x^{2n})^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d+e*x^n)^3/(a+c*x^(2*n))^2,x)
```

```
[Out] int((d+e*x^n)^3/(a+c*x^(2*n))^2,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+e*x^n)^3/(a+c*x^(2*n))^2,x, algorithm="maxima")
```

```
[Out] 1/2*((3*c*d^2*e - a*e^3)*x*x^n + (c*d^3 - 3*a*d*e^2)*x)/(a*c^2*n*x^(2*n) +
a^2*c*n) + integrate(1/2*(c*d^3*(2*n - 1) + 3*a*d*e^2 + (3*c*d^2*(n - 1)*e
+ a*(n + 1)*e^3)*x^n)/(a*c^2*n*x^(2*n) + a^2*c*n), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+e*x^n)^3/(a+c*x^(2*n))^2,x, algorithm="fricas")
```

```
[Out] integral((3*d^2*x^n*e + d^3 + 3*d*x^(2*n)*e^2 + x^(3*n)*e^3)/(c^2*x^(4*n) +
 2*a*c*x^(2*n) + a^2), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(d+ex^n)^3}{(a+cx^{2n})^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+e*x**n)**3/(a+c*x**(2*n))**2,x)
```

```
[Out] Integral((d + e*x**n)**3/(a + c*x**(2*n))**2, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+e*x^n)^3/(a+c*x^(2*n))^2,x, algorithm="giac")
```

```
[Out] integrate((x^n*e + d)^3/(c*x^(2*n) + a)^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(d+e\,x^n)^3}{(a+c\,x^{2\,n})^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x^n)^3/(a + c*x^(2*n))^2,x)
```

```
[Out] int((d + e*x^n)^3/(a + c*x^(2*n))^2, x)
```

3.49 $\int \frac{(d+ex^n)^2}{(a+cx^{2n})^2}\,dx$

Optimal. Leaf size=203

$$\frac{x(cd^2-ae^2+2cdex^n)}{2acn\,(a+cx^{2n})}+\frac{e^2x\,{}_2F_1\left(1,\frac{1}{2n};\frac{1}{2}\left(2+\frac{1}{n}\right);-\frac{cx^{2n}}{a}\right)}{ac}-\frac{(cd^2-ae^2)\,(1-2n)x\,{}_2F_1\left(1,\frac{1}{2n};\frac{1}{2}\left(2+\frac{1}{n}\right);-\frac{cx^{2n}}{a}\right)}{2a^2cn}$$

```
[Out] 1/2*x*(c*d^2-a*e^2+2*c*d*e*x^n)/a/c/n/(a+c*x^(2*n))+e^2*x*hypergeom([1, 1/2
/n],[1+1/2/n],-c*x^(2*n)/a)/a/c-1/2*(-a*e^2+c*d^2)*(1-2*n)*x*hypergeom([1,
1/2/n],[1+1/2/n],-c*x^(2*n)/a)/a^2/c/n-d*e*(1-n)*x^(1+n)*hypergeom([1, 1/2*
(1+n)/n],[3/2+1/2/n],-c*x^(2*n)/a)/a^2/n/(1+n)
```

Rubi [A]
time = 0.11, antiderivative size = 203, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used = {1451, 1445, 1432, 251, 371}

$$-\frac{(1-2n)x(cd^2-ae^2)\,{}_2F_1\left(1,\frac{1}{2n};\frac{1}{2}\left(2+\frac{1}{n}\right);-\frac{cx^{2n}}{a}\right)}{2a^2cn}-\frac{de(1-n)x^{n+1}\,{}_2F_1\left(1,\frac{n+1}{2n};\frac{1}{2}\left(3+\frac{1}{n}\right);-\frac{cx^{2n}}{a}\right)}{a^2n(n+1)}+\frac{x(-ae^2+cd^2+2cdex^n)}{2acn\,(a+cx^{2n})}+\frac{e^2x\,{}_2F_1\left(1,\frac{1}{2n};\frac{1}{2}\left(2+\frac{1}{n}\right);-\frac{cx^{2n}}{a}\right)}{ac}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x^n)^2/(a + c*x^(2*n))^2,x]
```

```
[Out] (x*(c*d^2 - a*e^2 + 2*c*d*e*x^n))/(2*a*c*n*(a + c*x^(2*n))) + (e^2*x*Hyperg
eometric2F1[1, 1/(2*n), (2 + n^(-1))/2, -((c*x^(2*n))/a)])/(a*c) - ((c*d^2
- a*e^2)*(1 - 2*n)*x*Hypergeometric2F1[1, 1/(2*n), (2 + n^(-1))/2, -((c*x^(
2*n))/a)])/(2*a^2*c*n) - (d*e*(1 - n)*x^(1 + n)*Hypergeometric2F1[1, (1 + n
)/(2*n), (3 + n^(-1))/2, -((c*x^(2*n))/a)])/(a^2*n*(1 + n))
```

Rule 251

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, (-b)*(x^n/a)], x] /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p
, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 1432

```
Int[((d_) + (e_.)*(x_)^(n_))/((a_) + (c_.)*(x_)^(n2_)), x_Symbol] :> Dist[d
, Int[1/(a + c*x^(2*n)), x], x] + Dist[e, Int[x^n/(a + c*x^(2*n)), x], x] /
; FreeQ[{a, c, d, e, n}, x] && EqQ[n2, 2*n] && NeQ[c*d^2 + a*e^2, 0] && (Po
sQ[a*c] ||  !IntegerQ[n])
```

Rule 1445

```
Int[((d_) + (e_.)*(x_)^(n_))*((a_) + (c_.)*(x_)^(n2_))^(p_), x_Symbol] :> S
imp[(-x)*(d + e*x^n)*((a + c*x^(2*n))^(p + 1)/(2*a*n*(p + 1))), x] + Dist[1
/(2*a*n*(p + 1)), Int[(d*(2*n*p + 2*n + 1) + e*(2*n*p + 3*n + 1)*x^n)*(a +
c*x^(2*n))^(p + 1), x], x] /; FreeQ[{a, c, d, e, n}, x] && EqQ[n2, 2*n] &&
ILtQ[p, -1]
```

Rule 1451

```
Int[((d_) + (e_.)*(x_)^(n_))^(q_)*((a_) + (c_.)*(x_)^(n2_))^(p_), x_Symbol]
 :> Int[ExpandIntegrand[(d + e*x^n)^q*(a + c*x^(2*n))^p, x], x] /; FreeQ[{a
, c, d, e, n, p, q}, x] && EqQ[n2, 2*n] && NeQ[c*d^2 + a*e^2, 0] && ((Integ
ersQ[p, q] &&  !IntegerQ[n]) || IGtQ[p, 0] || (IGtQ[q, 0] &&  !IntegerQ[n])
)
```

Rubi steps

$$\begin{aligned}\int \frac{(d+ex^n)^2}{(a+cx^{2n})^2}\,dx &= \int \left(\frac{cd^2-ae^2+2cdex^n}{c\,(a+cx^{2n})^2}+\frac{e^2}{c\,(a+cx^{2n})}\right)dx\\
&= \frac{\int \frac{cd^2-ae^2+2cdex^n}{(a+cx^{2n})^2}\,dx}{c}+\frac{e^2\int \frac{1}{a+cx^{2n}}\,dx}{c}\\
&= \frac{x(cd^2-ae^2+2cdex^n)}{2acn\,(a+cx^{2n})}+\frac{e^2x\,{}_2F_1\left(1,\frac{1}{2n};\frac{1}{2}\left(2+\frac{1}{n}\right);-\frac{cx^{2n}}{a}\right)}{ac}-\frac{\int \frac{(cd^2-ae^2)(1-2n)+2cde(1-n)}{a+cx^{2n}}}{2acn}\\
&= \frac{x(cd^2-ae^2+2cdex^n)}{2acn\,(a+cx^{2n})}+\frac{e^2x\,{}_2F_1\left(1,\frac{1}{2n};\frac{1}{2}\left(2+\frac{1}{n}\right);-\frac{cx^{2n}}{a}\right)}{ac}-\frac{((cd^2-ae^2)(1-2n))\int}{2acn}\\
&= \frac{x(cd^2-ae^2+2cdex^n)}{2acn\,(a+cx^{2n})}+\frac{e^2x\,{}_2F_1\left(1,\frac{1}{2n};\frac{1}{2}\left(2+\frac{1}{n}\right);-\frac{cx^{2n}}{a}\right)}{ac}-\frac{(cd^2-ae^2)(1-2n)x\,{}_2F}{2a}\end{aligned}$$

Mathematica [A]
time = 0.28, size = 142, normalized size = 0.70

$$\frac{x\left(\frac{a(-ae^2+cd(d+2ex^n))}{c(a+cx^{2n})}+\frac{(ae^2+cd^2(-1+2n))\,{}_2F_1\left(1,\frac{1}{2n};1+\frac{1}{2n};-\frac{cx^{2n}}{a}\right)}{c}+\frac{2de(-1+n)x^n\,{}_2F_1\left(1,\frac{1+n}{2n};\frac{1}{2}\left(3+\frac{1}{n}\right);-\frac{cx^{2n}}{a}\right)}{1+n}\right)}{2a^2n}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x^n)^2/(a + c*x^(2*n))^2,x]
```

```
[Out] (x*((a*(-(a*e^2) + c*d*(d + 2*e*x^n)))/(c*(a + c*x^(2*n))) + ((a*e^2 + c*d^
2*(-1 + 2*n))*Hypergeometric2F1[1, 1/(2*n), 1 + 1/(2*n), -((c*x^(2*n))/a)])
/c + (2*d*e*(-1 + n)*x^n*Hypergeometric2F1[1, (1 + n)/(2*n), (3 + n^(-1))/2
, -((c*x^(2*n))/a)])/(1 + n)))/(2*a^2*n)
```

Maple [F]
time = 0.01, size = 0, normalized size = 0.00

$$\int \frac{(d + e\,x^{n})^{2}}{(a + c\,x^{2n})^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d+e*x^n)^2/(a+c*x^(2*n))^2,x)
```

```
[Out] int((d+e*x^n)^2/(a+c*x^(2*n))^2,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+e*x^n)^2/(a+c*x^(2*n))^2,x, algorithm="maxima")
```

```
[Out] 1/2*(2*c*d*x*e^(n*log(x) + 1) + (c*d^2 - a*e^2)*x)/(a*c^2*n*x^(2*n) + a^2*c
*n) + integrate(1/2*(c*d^2*(2*n - 1) + 2*c*d*(n - 1)*e^(n*log(x) + 1) + a*e
^2)/(a*c^2*n*x^(2*n) + a^2*c*n), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+e*x^n)^2/(a+c*x^(2*n))^2,x, algorithm="fricas")
```

```
[Out] integral((2*d*x^n*e + d^2 + x^(2*n)*e^2)/(c^2*x^(4*n) + 2*a*c*x^(2*n) + a^2
), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(d + ex^{n})^{2}}{(a + cx^{2n})^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+e*x**n)**2/(a+c*x**(2*n))**2,x)
```

```
[Out] Integral((d + e*x**n)**2/(a + c*x**(2*n))**2, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+e*x^n)^2/(a+c*x^(2*n))^2,x, algorithm="giac")
```

```
[Out] integrate((x^n*e + d)^2/(c*x^(2*n) + a)^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(d+e\,x^n)^2}{(a+c\,x^{2\,n})^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x^n)^2/(a + c*x^(2*n))^2,x)
```

```
[Out] int((d + e*x^n)^2/(a + c*x^(2*n))^2, x)
```

3.50 $\int \frac{d+ex^n}{(a+cx^{2n})^2} \, dx$

Optimal. Leaf size=134

$$\frac{x(d+ex^n)}{2an(a+cx^{2n})}-\frac{d(1-2n)x\,{}_2F_1\left(1,\frac{1}{2n};\frac{1}{2}\left(2+\frac{1}{n}\right);-\frac{cx^{2n}}{a}\right)}{2a^2n}-\frac{e(1-n)x^{1+n}\,{}_2F_1\left(1,\frac{1+n}{2n};\frac{1}{2}\left(3+\frac{1}{n}\right);-\frac{cx^{2n}}{a}\right)}{2a^2n(1+n)}$$

```
[Out] 1/2*x*(d+e*x^n)/a/n/(a+c*x^(2*n))-1/2*d*(1-2*n)*x*hypergeom([1, 1/2/n],[1+1
/2/n],-c*x^(2*n)/a)/a^2/n-1/2*e*(1-n)*x^(1+n)*hypergeom([1, 1/2*(1+n)/n],[3
/2+1/2/n],-c*x^(2*n)/a)/a^2/n/(1+n)
```

Rubi [A]
time = 0.04, antiderivative size = 134, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.210$, Rules used = {1445, 1432, 251, 371}

$$-\frac{d(1-2n)x\,{}_2F_1\left(1,\frac{1}{2n};\frac{1}{2}\left(2+\frac{1}{n}\right);-\frac{cx^{2n}}{a}\right)}{2a^2n}-\frac{e(1-n)x^{n+1}\,{}_2F_1\left(1,\frac{n+1}{2n};\frac{1}{2}\left(3+\frac{1}{n}\right);-\frac{cx^{2n}}{a}\right)}{2a^2n(n+1)}+\frac{x(d+ex^n)}{2an(a+cx^{2n})}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x^n)/(a + c*x^(2*n))^2,x]
```

```
[Out] (x*(d + e*x^n))/(2*a*n*(a + c*x^(2*n))) - (d*(1 - 2*n)*x*Hypergeometric2F1[
1, 1/(2*n), (2 + n^(-1))/2, -((c*x^(2*n))/a)])/(2*a^2*n) - (e*(1 - n)*x^(1
+ n)*Hypergeometric2F1[1, (1 + n)/(2*n), (3 + n^(-1))/2, -((c*x^(2*n))/a)])
/(2*a^2*n*(1 + n))
```

Rule 251

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, (-b)*(x^n/a)], x] /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p
, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 1432

```
Int[((d_) + (e_.)*(x_)^(n_))/((a_) + (c_.)*(x_)^(n2_)), x_Symbol] :> Dist[d
, Int[1/(a + c*x^(2*n)), x], x] + Dist[e, Int[x^n/(a + c*x^(2*n)), x], x] /
```

```
; FreeQ[{a, c, d, e, n}, x] && EqQ[n2, 2*n] && NeQ[c*d^2 + a*e^2, 0] && (Po
sQ[a*c] ||  !IntegerQ[n])
```

Rule 1445

```
Int[((d_) + (e_.)*(x_)^(n_))*((a_) + (c_.)*(x_)^(n2_))^(p_), x_Symbol] :> S
imp[(-x)*(d + e*x^n)*((a + c*x^(2*n))^(p + 1)/(2*a*n*(p + 1))), x] + Dist[1
/(2*a*n*(p + 1)), Int[(d*(2*n*p + 2*n + 1) + e*(2*n*p + 3*n + 1)*x^n)*(a +
c*x^(2*n))^(p + 1), x], x] /; FreeQ[{a, c, d, e, n}, x] && EqQ[n2, 2*n] &&
ILtQ[p, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{d+ex^n}{(a+cx^{2n})^2}\,dx &= \frac{x(d+ex^n)}{2an\,(a+cx^{2n})} - \frac{\int \frac{d(1-2n)+e(1-n)x^n}{a+cx^{2n}}\,dx}{2an}\\ &= \frac{x(d+ex^n)}{2an\,(a+cx^{2n})} - \frac{(d(1-2n))\int \frac{1}{a+cx^{2n}}\,dx}{2an} - \frac{(e(1-n))\int \frac{x^n}{a+cx^{2n}}\,dx}{2an}\\ &= \frac{x(d+ex^n)}{2an\,(a+cx^{2n})} - \frac{d(1-2n)x\,{}_2F_1\left(1,\frac{1}{2n};\frac{1}{2}\left(2+\frac{1}{n}\right);-\frac{cx^{2n}}{a}\right)}{2a^2n} - \frac{e(1-n)x^{1+n}\,{}_2F_1\left(1,\frac{1+n}{2n};\right.}{2a^2n(1+}\end{aligned}$$

Mathematica [A]
time = 0.11, size = 137, normalized size = 1.02

$$\frac{x\left(a(1+n)\left(d+ex^n\right)+d\left(-1+n+2n^2\right)\left(a+cx^{2n}\right)\,{}_2F_1\left(1,\frac{1}{2n};1+\frac{1}{2n};-\frac{cx^{2n}}{a}\right)+e(-1+n)x^n\left(a+cx^{2n}\right)\,{}_2F_1\left(1,\frac{1+n}{2n};\frac{1}{2}\left(3+\frac{1}{n}\right);-\frac{cx^{2n}}{a}\right)\right)}{2a^2n(1+n)\left(a+cx^{2n}\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x^n)/(a + c*x^(2*n))^2,x]
```

```
[Out] (x*(a*(1 + n)*(d + e*x^n) + d*(-1 + n + 2*n^2)*(a + c*x^(2*n))*Hypergeometr
ic2F1[1, 1/(2*n), 1 + 1/(2*n), -((c*x^(2*n))/a)] + e*(-1 + n)*x^n*(a + c*x^
(2*n))*Hypergeometric2F1[1, (1 + n)/(2*n), (3 + n^(-1))/2, -((c*x^(2*n))/a)
]))/(2*a^2*n*(1 + n)*(a + c*x^(2*n)))
```

Maple [F]
time = 0.01, size = 0, normalized size = 0.00

$$\int \frac{d+e\,x^n}{\left(a+c\,x^{2n}\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d+e*x^n)/(a+c*x^(2*n))^2,x)
```

```
[Out] int((d+e*x^n)/(a+c*x^(2*n))^2,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+e*x^n)/(a+c*x^(2*n))^2,x, algorithm="maxima")
```

```
[Out] 1/2*(d*x + x*e^(n*log(x) + 1))/(a*c*n*x^(2*n) + a^2*n) + integrate(1/2*(d*(
2*n - 1) + (n - 1)*e^(n*log(x) + 1))/(a*c*n*x^(2*n) + a^2*n), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+e*x^n)/(a+c*x^(2*n))^2,x, algorithm="fricas")
```

```
[Out] integral((x^n*e + d)/(c^2*x^(4*n) + 2*a*c*x^(2*n) + a^2), x)
```

Sympy [C] Result contains complex when optimal does not.
time = 180.69, size = 784, normalized size = 5.85

$$d\left(\frac{2nx\Phi\left(\frac{cx^{2n}e^{i\pi}}{a},1,\frac{1}{2n}\right)\Gamma\left(\frac{1}{2n}\right)}{a\left(8an^3\Gamma\left(1+\frac{1}{2n}\right)+8cn^3x^{2n}\Gamma\left(1+\frac{1}{2n}\right)\right)}+\frac{2nx\Gamma\left(\frac{1}{2n}\right)}{a\left(8an^3\Gamma\left(1+\frac{1}{2n}\right)+8cn^3x^{2n}\Gamma\left(1+\frac{1}{2n}\right)\right)}-\frac{x\Phi\left(\frac{cx^{2n}e^{i\pi}}{a},1,\frac{1}{2n}\right)\Gamma\left(\frac{1}{2n}\right)}{a\left(8an^3\Gamma\left(1+\frac{1}{2n}\right)+8cn^3x^{2n}\Gamma\left(1+\frac{1}{2n}\right)\right)}+\frac{2cnxx^{2n}\Phi\left(\frac{cx^{2n}e^{i\pi}}{a},1,\frac{1}{2n}\right)\Gamma\left(\frac{1}{2n}\right)}{a^2\cdot\left(8an^3\Gamma\left(1+\frac{1}{2n}\right)+8cn^3x^{2n}\Gamma\left(1+\frac{1}{2n}\right)\right)}-\frac{cxx^{2n}\Phi\left(\frac{cx^{2n}e^{i\pi}}{a},1,\frac{1}{2n}\right)\Gamma\left(\frac{1}{2n}\right)}{a^2\cdot\left(8an^3\Gamma\left(1+\frac{1}{2n}\right)+8cn^3x^{2n}\Gamma\left(1+\frac{1}{2n}\right)\right)}\right)+e\left(\frac{n^2xx^n\Phi\left(\frac{cx^{2n}e^{i\pi}}{a},1,\frac{1}{2}+\frac{1}{2n}\right)\Gamma\left(\frac{1}{2}+\frac{1}{2n}\right)}{a\left(8an^3\Gamma\left(\frac{3}{2}+\frac{1}{2n}\right)+8cn^3x^{2n}\Gamma\left(\frac{3}{2}+\frac{1}{2n}\right)\right)}+\frac{2n^2xx^n\Gamma\left(\frac{1}{2}+\frac{1}{2n}\right)}{a\left(8an^3\Gamma\left(\frac{3}{2}+\frac{1}{2n}\right)+8cn^3x^{2n}\Gamma\left(\frac{3}{2}+\frac{1}{2n}\right)\right)}+\frac{2nxx^n\Gamma\left(\frac{1}{2}+\frac{1}{2n}\right)}{a\left(8an^3\Gamma\left(\frac{3}{2}+\frac{1}{2n}\right)+8cn^3x^{2n}\Gamma\left(\frac{3}{2}+\frac{1}{2n}\right)\right)}-\frac{xx^n\Phi\left(\frac{cx^{2n}e^{i\pi}}{a},1,\frac{1}{2}+\frac{1}{2n}\right)\Gamma\left(\frac{1}{2}+\frac{1}{2n}\right)}{a\left(8an^3\Gamma\left(\frac{3}{2}+\frac{1}{2n}\right)+8cn^3x^{2n}\Gamma\left(\frac{3}{2}+\frac{1}{2n}\right)\right)}+\frac{cn^2xx^{3n}\Phi\left(\frac{cx^{2n}e^{i\pi}}{a},1,\frac{1}{2}+\frac{1}{2n}\right)\Gamma\left(\frac{1}{2}+\frac{1}{2n}\right)}{a^2\cdot\left(8an^3\Gamma\left(\frac{3}{2}+\frac{1}{2n}\right)+8cn^3x^{2n}\Gamma\left(\frac{3}{2}+\frac{1}{2n}\right)\right)}-\frac{cxx^{3n}\Phi\left(\frac{cx^{2n}e^{i\pi}}{a},1,\frac{1}{2}+\frac{1}{2n}\right)\Gamma\left(\frac{1}{2}+\frac{1}{2n}\right)}{a^2\cdot\left(8an^3\Gamma\left(\frac{3}{2}+\frac{1}{2n}\right)+8cn^3x^{2n}\Gamma\left(\frac{3}{2}+\frac{1}{2n}\right)\right)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+e*x**n)/(a+c*x**(2*n))**2,x)
```

```
[Out] d*(2*n*x*lerchphi(c*x**(2*n)*exp_polar(I*pi)/a, 1, 1/(2*n))*gamma(1/(2*n))/
(a*(8*a*n**3*gamma(1 + 1/(2*n)) + 8*c*n**3*x**(2*n)*gamma(1 + 1/(2*n)))) +
2*n*x*gamma(1/(2*n))/(a*(8*a*n**3*gamma(1 + 1/(2*n)) + 8*c*n**3*x**(2*n)*ga
mma(1 + 1/(2*n)))) - x*lerchphi(c*x**(2*n)*exp_polar(I*pi)/a, 1, 1/(2*n))*g
amma(1/(2*n))/(a*(8*a*n**3*gamma(1 + 1/(2*n)) + 8*c*n**3*x**(2*n)*gamma(1 +
 1/(2*n)))) + 2*c*n*x*x**(2*n)*lerchphi(c*x**(2*n)*exp_polar(I*pi)/a, 1, 1/
(2*n))*gamma(1/(2*n))/(a**2*(8*a*n**3*gamma(1 + 1/(2*n)) + 8*c*n**3*x**(2*n
)*gamma(1 + 1/(2*n)))) - c*x*x**(2*n)*lerchphi(c*x**(2*n)*exp_polar(I*pi)/a
, 1, 1/(2*n))*gamma(1/(2*n))/(a**2*(8*a*n**3*gamma(1 + 1/(2*n)) + 8*c*n**3*
x**(2*n)*gamma(1 + 1/(2*n))))) + e*(n**2*x*x**n*lerchphi(c*x**(2*n)*exp_pol
ar(I*pi)/a, 1, 1/2 + 1/(2*n))*gamma(1/2 + 1/(2*n))/(a*(8*a*n**3*gamma(3/2 +
 1/(2*n)) + 8*c*n**3*x**(2*n)*gamma(3/2 + 1/(2*n)))) + 2*n**2*x*x**n*gamma(
1/2 + 1/(2*n))/(a*(8*a*n**3*gamma(3/2 + 1/(2*n)) + 8*c*n**3*x**(2*n)*gamma(
3/2 + 1/(2*n)))) + 2*n*x*x**n*gamma(1/2 + 1/(2*n))/(a*(8*a*n**3*gamma(3/2 +
```

```
 1/(2*n)) + 8*c*n**3*x**(2*n)*gamma(3/2 + 1/(2*n)))) - x*x**n*lerchphi(c*x*
*(2*n)*exp_polar(I*pi)/a, 1, 1/2 + 1/(2*n))*gamma(1/2 + 1/(2*n))/(a*(8*a*n*
*3*gamma(3/2 + 1/(2*n)) + 8*c*n**3*x**(2*n)*gamma(3/2 + 1/(2*n)))) + c*n**2
*x*x**(3*n)*lerchphi(c*x**(2*n)*exp_polar(I*pi)/a, 1, 1/2 + 1/(2*n))*gamma(
1/2 + 1/(2*n))/(a**2*(8*a*n**3*gamma(3/2 + 1/(2*n)) + 8*c*n**3*x**(2*n)*gam
ma(3/2 + 1/(2*n)))) - c*x*x**(3*n)*lerchphi(c*x**(2*n)*exp_polar(I*pi)/a, 1
, 1/2 + 1/(2*n))*gamma(1/2 + 1/(2*n))/(a**2*(8*a*n**3*gamma(3/2 + 1/(2*n))
+ 8*c*n**3*x**(2*n)*gamma(3/2 + 1/(2*n)))))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+e*x^n)/(a+c*x^(2*n))^2,x, algorithm="giac")
[Out] integrate((x^n*e + d)/(c*x^(2*n) + a)^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{d+e\,x^n}{{\left(a+c\,x^{2\,n}\right)}^2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x^n)/(a + c*x^(2*n))^2,x)
[Out] int((d + e*x^n)/(a + c*x^(2*n))^2, x)
```

3.51 $\int \frac{1}{(d+ex^n)(a+cx^{2n})^2}\,dx$

Optimal. Leaf size=333

$$\frac{cx(d-ex^n)}{2a\,(cd^2+ae^2)\,n\,(a+cx^{2n})}+\frac{cde^2x\,{}_2F_1\left(1,\frac{1}{2n};\frac{1}{2}\left(2+\frac{1}{n}\right);-\frac{cx^{2n}}{a}\right)}{a\,(cd^2+ae^2)^2}-\frac{cd(1-2n)x\,{}_2F_1\left(1,\frac{1}{2n};\frac{1}{2}\left(2+\frac{1}{n}\right);-\frac{cx^{2n}}{a}\right)}{2a^2\,(cd^2+ae^2)\,n}+$$

```
[Out] 1/2*c*x*(d-e*x^n)/a/(a*e^2+c*d^2)/n/(a+c*x^(2*n))+c*d*e^2*x*hypergeom([1, 1
/2/n],[1+1/2/n],-c*x^(2*n)/a)/a/(a*e^2+c*d^2)^2-1/2*c*d*(1-2*n)*x*hypergeom
([1, 1/2/n],[1+1/2/n],-c*x^(2*n)/a)/a^2/(a*e^2+c*d^2)/n+e^4*x*hypergeom([1,
 1/n],[1+1/n],-e*x^n/d)/d/(a*e^2+c*d^2)^2-c*e^3*x^(1+n)*hypergeom([1, 1/2*(
1+n)/n],[3/2+1/2/n],-c*x^(2*n)/a)/a/(a*e^2+c*d^2)^2/(1+n)+1/2*c*e*(1-n)*x^(
1+n)*hypergeom([1, 1/2*(1+n)/n],[3/2+1/2/n],-c*x^(2*n)/a)/a^2/(a*e^2+c*d^2)
/n/(1+n)
```

Rubi [A]
time = 0.16, antiderivative size = 333, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used = {1451, 251, 1445, 1432, 371}

$$\frac{ce(1-n)x^{n+1}\,{}_2F_1\left(1,\frac{n+1}{2n};\frac{1}{2}\left(3+\frac{1}{n}\right);-\frac{cx^{2n}}{a}\right)}{2a^2n(n+1)\,(ae^2+cd^2)}-\frac{cd(1-2n)x\,{}_2F_1\left(1,\frac{1}{2n};\frac{1}{2}\left(2+\frac{1}{n}\right);-\frac{cx^{2n}}{a}\right)}{2a^2n\,(ae^2+cd^2)}+\frac{cde^2x\,{}_2F_1\left(1,\frac{1}{2n};\frac{1}{2}\left(2+\frac{1}{n}\right);-\frac{cx^{2n}}{a}\right)}{a\,(ae^2+cd^2)^2}+\frac{cx(d-ex^n)}{2an\,(ae^2+cd^2)\,(a+cx^{2n})}+\frac{e^4x\,{}_2F_1\left(1,\frac{1}{n};1+\frac{1}{n};-\frac{ex^n}{d}\right)}{d\,(ae^2+cd^2)^2}-\frac{ce^3x^{n+1}\,{}_2F_1\left(1,\frac{n+1}{2n};\frac{1}{2}\left(3+\frac{1}{n}\right);-\frac{cx^{2n}}{a}\right)}{a(n+1)\,(ae^2+cd^2)^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/((d + e*x^n)*(a + c*x^(2*n))^2),x]
```

```
[Out] (c*x*(d - e*x^n))/(2*a*(c*d^2 + a*e^2)*n*(a + c*x^(2*n))) + (c*d*e^2*x*Hype
rgeometric2F1[1, 1/(2*n), (2 + n^(-1))/2, -((c*x^(2*n))/a)])/(a*(c*d^2 + a*
e^2)^2) - (c*d*(1 - 2*n)*x*Hypergeometric2F1[1, 1/(2*n), (2 + n^(-1))/2, -(
(c*x^(2*n))/a)])/(2*a^2*(c*d^2 + a*e^2)*n) + (e^4*x*Hypergeometric2F1[1, n^
(-1), 1 + n^(-1), -((e*x^n)/d)])/(d*(c*d^2 + a*e^2)^2) - (c*e^3*x^(1 + n)*H
ypergeometric2F1[1, (1 + n)/(2*n), (3 + n^(-1))/2, -((c*x^(2*n))/a)])/(a*(c
*d^2 + a*e^2)^2*(1 + n)) + (c*e*(1 - n)*x^(1 + n)*Hypergeometric2F1[1, (1 +
 n)/(2*n), (3 + n^(-1))/2, -((c*x^(2*n))/a)])/(2*a^2*(c*d^2 + a*e^2)*n*(1 +
 n))
```

Rule 251

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, (-b)*(x^n/a)], x] /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p
, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
```

```
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 1432

```
Int[((d_) + (e_.)*(x_)^(n_))/((a_) + (c_.)*(x_)^(n2_)), x_Symbol] :> Dist[d
, Int[1/(a + c*x^(2*n)), x], x] + Dist[e, Int[x^n/(a + c*x^(2*n)), x], x] /
; FreeQ[{a, c, d, e, n}, x] && EqQ[n2, 2*n] && NeQ[c*d^2 + a*e^2, 0] && (Po
sQ[a*c] ||  !IntegerQ[n])
```

Rule 1445

```
Int[((d_) + (e_.)*(x_)^(n_))*((a_) + (c_.)*(x_)^(n2_))^(p_), x_Symbol] :> S
imp[(-x)*(d + e*x^n)*((a + c*x^(2*n))^(p + 1)/(2*a*n*(p + 1))), x] + Dist[1
/(2*a*n*(p + 1)), Int[(d*(2*n*p + 2*n + 1) + e*(2*n*p + 3*n + 1)*x^n)*(a +
c*x^(2*n))^(p + 1), x], x] /; FreeQ[{a, c, d, e, n}, x] && EqQ[n2, 2*n] &&
ILtQ[p, -1]
```

Rule 1451

```
Int[((d_) + (e_.)*(x_)^(n_))^(q_)*((a_) + (c_.)*(x_)^(n2_))^(p_), x_Symbol]
 :> Int[ExpandIntegrand[(d + e*x^n)^q*(a + c*x^(2*n))^p, x], x] /; FreeQ[{a
, c, d, e, n, p, q}, x] && EqQ[n2, 2*n] && NeQ[c*d^2 + a*e^2, 0] && ((Integ
ersQ[p, q] &&  !IntegerQ[n]) || IGtQ[p, 0] || (IGtQ[q, 0] &&  !IntegerQ[n])
)
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{(d+ex^n)\left(a+cx^{2n}\right)^2}\,dx &= \int \left(\frac{e^4}{\left(cd^2+ae^2\right)^2(d+ex^n)} - \frac{c(-d+ex^n)}{\left(cd^2+ae^2\right)\left(a+cx^{2n}\right)^2} - \frac{ce^2(-d+ex^n)}{\left(cd^2+ae^2\right)^2(a+cx \cdots}\right.\\
&= -\frac{\left(ce^2\right)\int \frac{-d+ex^n}{a+cx^{2n}}\,dx}{\left(cd^2+ae^2\right)^2} + \frac{e^4\int \frac{1}{d+ex^n}\,dx}{\left(cd^2+ae^2\right)^2} - \frac{c\int \frac{-d+ex^n}{\left(a+cx^{2n}\right)^2}\,dx}{cd^2+ae^2}\\
&= \frac{cx(d-ex^n)}{2a\left(cd^2+ae^2\right)n\left(a+cx^{2n}\right)} + \frac{e^4x\,{}_2F_1\left(1,\frac{1}{n};1+\frac{1}{n};-\frac{ex^n}{d}\right)}{d\left(cd^2+ae^2\right)^2} + \frac{\left(cde^2\right)\int \frac{1}{a+cx^{2n}}\,dx \cdots}{\left(cd^2+ae^2\right)^2}\\
&= \frac{cx(d-ex^n)}{2a\left(cd^2+ae^2\right)n\left(a+cx^{2n}\right)} + \frac{cde^2x\,{}_2F_1\left(1,\frac{1}{2n};\frac{1}{2}\left(2+\frac{1}{n}\right);-\frac{cx^{2n}}{a}\right)}{a\left(cd^2+ae^2\right)^2} + \frac{e^4x\,{}_2F_1\left(\cdots\right.}{d\left(\cdots\right.}\\
&= \frac{cx(d-ex^n)}{2a\left(cd^2+ae^2\right)n\left(a+cx^{2n}\right)} + \frac{cde^2x\,{}_2F_1\left(1,\frac{1}{2n};\frac{1}{2}\left(2+\frac{1}{n}\right);-\frac{cx^{2n}}{a}\right)}{a\left(cd^2+ae^2\right)^2} - \frac{cd(1-2 \cdots}{\cdots}
\end{aligned}$$

Mathematica [A]
time = 0.35, size = 245, normalized size = 0.74

$$\frac{x\left(cd^2(1+n)\left(cd^2(-1+2n)+ae^2(-1+4n)\right)(a+cx^{2n})\ {}_2F_1\left(1,\frac{1}{2n};1+\frac{1}{2n};-\frac{cx^{2n}}{a}\right)+2a^2e^4n(1+n)(a+cx^{2n})\ {}_2F_1\left(1,\frac{1}{n};1+\frac{1}{n};-\frac{ex^n}{d}\right)+cd\left(a(cd^2+ae^2)(1+n)(d-ex^n)-e(cd^2(-1+n)+ae^2(-1+3n))x^n(a+cx^{2n})\ {}_2F_1\left(1,\frac{1+n}{2n};\frac{1}{2}\left(3+\frac{1}{n}\right);-\frac{cx^{2n}}{a}\right)\right)\right)}{2a^2d\left(cd^2+ae^2\right)^2n(1+n)(a+cx^{2n})}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((d + e*x^n)*(a + c*x^(2*n))^2),x]
```

```
[Out] (x*(c*d^2*(1 + n)*(c*d^2*(-1 + 2*n) + a*e^2*(-1 + 4*n))*(a + c*x^(2*n))*Hyp
ergeometric2F1[1, 1/(2*n), 1 + 1/(2*n), -((c*x^(2*n))/a)] + 2*a^2*e^4*n*(1
+ n)*(a + c*x^(2*n))*Hypergeometric2F1[1, n^(-1), 1 + n^(-1), -((e*x^n)/d)]
 + c*d*(a*(c*d^2 + a*e^2)*(1 + n)*(d - e*x^n) - e*(c*d^2*(-1 + n) + a*e^2*(
-1 + 3*n))*x^n*(a + c*x^(2*n))*Hypergeometric2F1[1, (1 + n)/(2*n), (3 + n^(
-1))/2, -((c*x^(2*n))/a)])))/(2*a^2*d*(c*d^2 + a*e^2)^2*n*(1 + n)*(a + c*x^
(2*n)))
```

Maple [F]
time = 0.02, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(d+e\,x^{n}\right)\left(a+c\,x^{2 n}\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(d+e*x^n)/(a+c*x^(2*n))^2,x)
```

```
[Out] int(1/(d+e*x^n)/(a+c*x^(2*n))^2,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d+e*x^n)/(a+c*x^(2*n))^2,x, algorithm="maxima")
```

```
[Out] e^4*integrate(1/(c^2*d^5 + 2*a*c*d^3*e^2 + a^2*d*e^4 + (c^2*d^4*e + 2*a*c*d
^2*e^3 + a^2*e^5)*x^n), x) + 1/2*(c*d*x - c*x*e^(n*log(x) + 1))/(a^2*c*d^2*
n + a^3*n*e^2 + (a*c^2*d^2*n + a^2*c*n*e^2)*x^(2*n)) - integrate(-1/2*(c^2*
d^3*(2*n - 1) + a*c*d*(4*n - 1)*e^2 - (c^2*d^2*(n - 1)*e + a*c*(3*n - 1)*e^
3)*x^n)/(a^2*c^2*d^4*n + 2*a^3*c*d^2*n*e^2 + a^4*n*e^4 + (a*c^3*d^4*n + 2*a
^2*c^2*d^2*n*e^2 + a^3*c*n*e^4)*x^(2*n)), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d+e*x^n)/(a+c*x^(2*n))^2,x, algorithm="fricas")
```

```
[Out] integral(1/(a^2*x^n*e + a^2*d + (c^2*x^n*e + c^2*d)*x^(4*n) + 2*(a*c*x^n*e
+ a*c*d)*x^(2*n)), x)
```

Sympy [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: HeuristicGCDFailed

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d+e*x**n)/(a+c*x**(2*n))**2,x)
```

```
[Out] Exception raised: HeuristicGCDFailed >> no luck
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d+e*x^n)/(a+c*x^(2*n))^2,x, algorithm="giac")
```

```
[Out] integrate(1/((c*x^(2*n) + a)^2*(x^n*e + d)), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{\left(a+c\,x^{2\,n}\right)^2\,\left(d+e\,x^n\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + c*x^(2*n))^2*(d + e*x^n)),x)
```

```
[Out] int(1/((a + c*x^(2*n))^2*(d + e*x^n)), x)
```

3.52 $\int \frac{1}{(d+ex^n)^2 (a+cx^{2n})^2} \, dx$

Optimal. Leaf size=410

$$\frac{cx(cd^2 - ae^2 - 2cdex^n)}{2a\,(cd^2 + ae^2)^2\, n\,(a + cx^{2n})} + \frac{ce^2(3cd^2 - ae^2)\, x\, {}_2F_1\left(1, \frac{1}{2n}; \frac{1}{2}\left(2 + \frac{1}{n}\right); -\frac{cx^{2n}}{a}\right)}{a\,(cd^2 + ae^2)^3} - \frac{c(cd^2 - ae^2)\,(1 - 2n)x\, {}_2F_1\Big(1,}{2a^2\,(cd^2 + ae^2}$$

```
[Out] 1/2*c*x*(c*d^2-a*e^2-2*c*d*e*x^n)/a/(a*e^2+c*d^2)^2/n/(a+c*x^(2*n))+c*e^2*(
-a*e^2+3*c*d^2)*x*hypergeom([1, 1/2/n],[1+1/2/n],-c*x^(2*n)/a)/a/(a*e^2+c*d
^2)^3-1/2*c*(-a*e^2+c*d^2)*(1-2*n)*x*hypergeom([1, 1/2/n],[1+1/2/n],-c*x^(2
*n)/a)/a^2/(a*e^2+c*d^2)^2/n+4*c*e^4*x*hypergeom([1, 1/n],[1+1/n],-e*x^n/d)
/(a*e^2+c*d^2)^3-4*c^2*d*e^3*x^(1+n)*hypergeom([1, 1/2*(1+n)/n],[3/2+1/2/n]
,-c*x^(2*n)/a)/a/(a*e^2+c*d^2)^3/(1+n)+c^2*d*e*(1-n)*x^(1+n)*hypergeom([1,
1/2*(1+n)/n],[3/2+1/2/n],-c*x^(2*n)/a)/a^2/(a*e^2+c*d^2)^2/n/(1+n)+e^4*x*hy
pergeom([2, 1/n],[1+1/n],-e*x^n/d)/d^2/(a*e^2+c*d^2)^2
```

Rubi [A]
time = 0.26, antiderivative size = 410, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used = {1451, 251, 1445, 1432, 371}

$$\frac{c^2de(1-n)x^{n+1}\, {}_2F_1\left(1, \frac{n+1}{2n}; \frac{1}{2}\left(3 + \frac{1}{n}\right); -\frac{cx^{2n}}{a}\right)}{a^2n(n+1)\,(ae^2 + cd^2)^2} - \frac{c(1-2n)x(cd^2 - ae^2)\, {}_2F_1\left(1, \frac{1}{2n}; \frac{1}{2}\left(2 + \frac{1}{n}\right); -\frac{cx^{2n}}{a}\right)}{2a^2n\,(ae^2 + cd^2)^2} - \frac{4c^2de^3x^{n+1}\, {}_2F_1\left(1, \frac{n+1}{2n}; \frac{1}{2}\left(3 + \frac{1}{n}\right); -\frac{cx^{2n}}{a}\right)}{a(n+1)\,(ae^2 + cd^2)^3} + \frac{ce^2x(3cd^2 - ae^2)\, {}_2F_1\left(1, \frac{1}{2n}; \frac{1}{2}\left(2 + \frac{1}{n}\right); -\frac{cx^{2n}}{a}\right)}{a\,(ae^2 + cd^2)^3} + \frac{cx(-ae^2 + cd^2 - 2cdex^n)}{2an\,(ae^2 + cd^2)^2\,(a + cx^{2n})} + \frac{4ce^4x\, {}_2F_1\left(1, \frac{1}{n}; 1 + \frac{1}{n}; -\frac{ex^n}{d}\right)}{(ae^2 + cd^2)^3} + \frac{e^4x\, {}_2F_1\left(2, \frac{1}{n}; 1 + \frac{1}{n}; -\frac{ex^n}{d}\right)}{d^2\,(ae^2 + cd^2)^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/((d + e*x^n)^2*(a + c*x^(2*n))^2),x]
```

```
[Out] (c*x*(c*d^2 - a*e^2 - 2*c*d*e*x^n))/(2*a*(c*d^2 + a*e^2)^2*n*(a + c*x^(2*n)
)) + (c*e^2*(3*c*d^2 - a*e^2)*x*Hypergeometric2F1[1, 1/(2*n), (2 + n^(-1))/
2, -((c*x^(2*n))/a)])/(a*(c*d^2 + a*e^2)^3) - (c*(c*d^2 - a*e^2)*(1 - 2*n)*
x*Hypergeometric2F1[1, 1/(2*n), (2 + n^(-1))/2, -((c*x^(2*n))/a)])/(2*a^2*(
c*d^2 + a*e^2)^2*n) + (4*c*e^4*x*Hypergeometric2F1[1, n^(-1), 1 + n^(-1), -
((e*x^n)/d)])/(c*d^2 + a*e^2)^3 - (4*c^2*d*e^3*x^(1 + n)*Hypergeometric2F1[
1, (1 + n)/(2*n), (3 + n^(-1))/2, -((c*x^(2*n))/a)])/(a*(c*d^2 + a*e^2)^3*(
1 + n)) + (c^2*d*e*(1 - n)*x^(1 + n)*Hypergeometric2F1[1, (1 + n)/(2*n), (3
 + n^(-1))/2, -((c*x^(2*n))/a)])/(a^2*(c*d^2 + a*e^2)^2*n*(1 + n)) + (e^4*x
*Hypergeometric2F1[2, n^(-1), 1 + n^(-1), -((e*x^n)/d)])/(d^2*(c*d^2 + a*e^
2)^2)
```

Rule 251

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, (-b)*(x^n/a)], x] /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p
, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 1432

```
Int[((d_) + (e_.)*(x_)^(n_))/((a_) + (c_.)*(x_)^(n2_)), x_Symbol] :> Dist[d
, Int[1/(a + c*x^(2*n)), x], x] + Dist[e, Int[x^n/(a + c*x^(2*n)), x], x] /
; FreeQ[{a, c, d, e, n}, x] && EqQ[n2, 2*n] && NeQ[c*d^2 + a*e^2, 0] && (Po
sQ[a*c] ||  !IntegerQ[n])
```

Rule 1445

```
Int[((d_) + (e_.)*(x_)^(n_))*((a_) + (c_.)*(x_)^(n2_))^(p_), x_Symbol] :> S
imp[(-x)*(d + e*x^n)*((a + c*x^(2*n))^(p + 1)/(2*a*n*(p + 1))), x] + Dist[1
/(2*a*n*(p + 1)), Int[(d*(2*n*p + 2*n + 1) + e*(2*n*p + 3*n + 1)*x^n)*(a +
c*x^(2*n))^(p + 1), x], x] /; FreeQ[{a, c, d, e, n}, x] && EqQ[n2, 2*n] &&
ILtQ[p, -1]
```

Rule 1451

```
Int[((d_) + (e_.)*(x_)^(n_))^(q_)*((a_) + (c_.)*(x_)^(n2_))^(p_), x_Symbol]
 :> Int[ExpandIntegrand[(d + e*x^n)^q*(a + c*x^(2*n))^p, x], x] /; FreeQ[{a
, c, d, e, n, p, q}, x] && EqQ[n2, 2*n] && NeQ[c*d^2 + a*e^2, 0] && ((Integ
ersQ[p, q] &&  !IntegerQ[n]) || IGtQ[p, 0] || (IGtQ[q, 0] &&  !IntegerQ[n])
)
```

Rubi steps

$$\int \frac{1}{(d+ex^n)^2 (a+cx^{2n})^2} \, dx = \int \left(\frac{e^4}{(cd^2+ae^2)^2 (d+ex^n)^2} + \frac{4cde^4}{(cd^2+ae^2)^3 (d+ex^n)} - \frac{c(-cd^2+ae^2+2cde}{(cd^2+ae^2)^2 (a+cx^{\cdots}}\right.$$

$$= -\frac{(ce^2)\int \frac{-3cd^2+ae^2+4cdex^n}{a+cx^{2n}}\,dx}{(cd^2+ae^2)^3} + \frac{(4cde^4)\int \frac{1}{d+ex^n}\,dx}{(cd^2+ae^2)^3} - \frac{c\int \frac{-cd^2+ae^2+2cdex^n}{(a+cx^{2n})^2}\,dx}{(cd^2+ae^2)^2} + \cdots$$

$$= \frac{cx(cd^2-ae^2-2cdex^n)}{2a(cd^2+ae^2)^2 n(a+cx^{2n})} + \frac{4ce^4x\,{}_2F_1\left(1,\frac{1}{n};1+\frac{1}{n};-\frac{ex^n}{d}\right)}{(cd^2+ae^2)^3} + \frac{e^4x\,{}_2F_1\left(2,\frac{1}{n};1\cdots\right.}{d^2(cd^2+\cdots}$$

$$= \frac{cx(cd^2-ae^2-2cdex^n)}{2a(cd^2+ae^2)^2 n(a+cx^{2n})} + \frac{ce^2(3cd^2-ae^2)\,x\,{}_2F_1\left(1,\frac{1}{2n};\frac{1}{2}\left(2+\frac{1}{n}\right);-\frac{cx^{2n}}{a}\right)}{a(cd^2+ae^2)^3}\cdots$$

$$= \frac{cx(cd^2-ae^2-2cdex^n)}{2a(cd^2+ae^2)^2 n(a+cx^{2n})} + \frac{ce^2(3cd^2-ae^2)\,x\,{}_2F_1\left(1,\frac{1}{2n};\frac{1}{2}\left(2+\frac{1}{n}\right);-\frac{cx^{2n}}{a}\right)}{a(cd^2+ae^2)^3}\cdots$$

Mathematica [A]
time = 0.67, size = 495, normalized size = 1.21

$$\frac{x\left(\frac{2cde^4}{dn+enx^n}+\frac{2ae^6}{d^2n+denx^n}-\frac{ace^4}{an+cnx^{2n}}-\frac{2c^2de^3x^n}{an+cnx^{2n}}+\frac{c^3d^4}{a^2n+acnx^{2n}}-\frac{2c^3d^3ex^n}{a^2n+acnx^{2n}}+\frac{c(a^2e^4(1-4n)+6acd^2e^2n+c^2d^4(-1+2n))\,{}_2F_1\left(1,\frac{1}{2n};1+\frac{1}{2n};-\frac{cx^{2n}}{a}\right)}{a^2n}+\frac{2e^4(ae^2(-1+n)+cd^2(-1+5n))\,{}_2F_1\left(1,\frac{1}{n};1+\frac{1}{n};-\frac{ex^n}{d}\right)}{d^2n}-\frac{2c^3d^3ex^n\,{}_2F_1\left(1,\frac{1+n}{2n};\frac{1}{2}\left(3+\frac{1}{n}\right);-\frac{cx^{2n}}{a}\right)}{a^2(1+n)}-\frac{10c^2de^3x^n\,{}_2F_1\left(1,\frac{1+n}{2n};\frac{1}{2}\left(3+\frac{1}{n}\right);-\frac{cx^{2n}}{a}\right)}{a(1+n)}+\frac{2c^3d^3ex^n\,{}_2F_1\left(1,\frac{1+n}{2n};\frac{1}{2}\left(3+\frac{1}{n}\right);-\frac{cx^{2n}}{a}\right)}{a^2n(1+n)}+\frac{2c^2de^3x^n\,{}_2F_1\left(1,\frac{1+n}{2n};\frac{1}{2}\left(3+\frac{1}{n}\right);-\frac{cx^{2n}}{a}\right)}{an(1+n)}\right)}{2(cd^2+ae^2)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((d + e*x^n)^2*(a + c*x^(2*n))^2),x]
```

```
[Out] (x*((2*c*d*e^4)/(d*n + e*n*x^n) + (2*a*e^6)/(d^2*n + d*e*n*x^n) - (a*c*e^4)
/(a*n + c*n*x^(2*n)) - (2*c^2*d*e^3*x^n)/(a*n + c*n*x^(2*n)) + (c^3*d^4)/(a
^2*n + a*c*n*x^(2*n)) - (2*c^3*d^3*e*x^n)/(a^2*n + a*c*n*x^(2*n)) + (c*(a^2
*e^4*(1 - 4*n) + 6*a*c*d^2*e^2*n + c^2*d^4*(-1 + 2*n))*Hypergeometric2F1[1,
 1/(2*n), 1 + 1/(2*n), -((c*x^(2*n))/a)])/(a^2*n) + (2*e^4*(a*e^2*(-1 + n)
+ c*d^2*(-1 + 5*n))*Hypergeometric2F1[1, n^(-1), 1 + n^(-1), -((e*x^n)/d)])
/(d^2*n) - (2*c^3*d^3*e*x^n*Hypergeometric2F1[1, (1 + n)/(2*n), (3 + n^(-1)
)/2, -((c*x^(2*n))/a)])/(a^2*(1 + n)) - (10*c^2*d*e^3*x^n*Hypergeometric2F1
[1, (1 + n)/(2*n), (3 + n^(-1))/2, -((c*x^(2*n))/a)])/(a*(1 + n)) + (2*c^3*
d^3*e*x^n*Hypergeometric2F1[1, (1 + n)/(2*n), (3 + n^(-1))/2, -((c*x^(2*n))
/a)])/(a^2*n*(1 + n)) + (2*c^2*d*e^3*x^n*Hypergeometric2F1[1, (1 + n)/(2*n)
, (3 + n^(-1))/2, -((c*x^(2*n))/a)])/(a*n*(1 + n))))/(2*(c*d^2 + a*e^2)^3)
```

Maple [F]
time = 0.02, size = 0, normalized size = 0.00

$$\int \frac{1}{(d+e\,x^n)^2 (a+c\,x^{2n})^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(d+e*x^n)^2/(a+c*x^(2*n))^2,x)
```

```
[Out] int(1/(d+e*x^n)^2/(a+c*x^(2*n))^2,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d+e*x^n)^2/(a+c*x^(2*n))^2,x, algorithm="maxima")
```

```
[Out] (c*d^2*(5*n - 1)*e^4 + a*(n - 1)*e^6)*integrate(1/(c^3*d^8*n + 3*a*c^2*d^6*
n*e^2 + 3*a^2*c*d^4*n*e^4 + a^3*d^2*n*e^6 + (c^3*d^7*n*e + 3*a*c^2*d^5*n*e^
3 + 3*a^2*c*d^3*n*e^5 + a^3*d*n*e^7)*x^n), x) - 1/2*(2*(c^2*d^2*e^2 - a*c*e
^4)*x*x^(2*n) + (c^2*d^3*e + a*c*d*e^3)*x*x^n - (c^2*d^4 - a*c*d^2*e^2 + 2*
a^2*e^4)*x)/(a^2*c^2*d^6*n + 2*a^3*c*d^4*n*e^2 + a^4*d^2*n*e^4 + (a*c^3*d^5
*n*e + 2*a^2*c^2*d^3*n*e^3 + a^3*c*d*n*e^5)*x^(3*n) + (a*c^3*d^6*n + 2*a^2*
c^2*d^4*n*e^2 + a^3*c*d^2*n*e^4)*x^(2*n) + (a^2*c^2*d^5*n*e + 2*a^3*c*d^3*n
*e^3 + a^4*d*n*e^5)*x^n) - integrate(-1/2*(c^3*d^4*(2*n - 1) + 6*a*c^2*d^2*
n*e^2 - a^2*c*(4*n - 1)*e^4 - 2*(c^3*d^3*(n - 1)*e + a*c^2*d*(5*n - 1)*e^3)
*x^n)/(a^2*c^3*d^6*n + 3*a^3*c^2*d^4*n*e^2 + 3*a^4*c*d^2*n*e^4 + a^5*n*e^6
+ (a*c^4*d^6*n + 3*a^2*c^3*d^4*n*e^2 + 3*a^3*c^2*d^2*n*e^4 + a^4*c*n*e^6)*x
^(2*n)), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d+e*x^n)^2/(a+c*x^(2*n))^2,x, algorithm="fricas")
```

```
[Out] integral(1/(2*a^2*d*x^n*e + a^2*d^2 + a^2*x^(2*n)*e^2 + (2*c^2*d*x^n*e + c^
2*d^2 + c^2*x^(2*n)*e^2)*x^(4*n) + 2*(2*a*c*d*x^n*e + a*c*d^2 + a*c*x^(2*n)
*e^2)*x^(2*n)), x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d+e*x**n)**2/(a+c*x**(2*n))**2,x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d+e*x^n)^2/(a+c*x^(2*n))^2,x, algorithm="giac")
[Out] integrate(1/((c*x^(2*n) + a)^2*(x^n*e + d)^2), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{{\left(a+c\,x^{2\,n}\right)}^2\,{\left(d+e\,x^n\right)}^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + c*x^(2*n))^2*(d + e*x^n)^2),x)
[Out] int(1/((a + c*x^(2*n))^2*(d + e*x^n)^2), x)
```

3.53 $\int \frac{(d+ex^n)^3}{(a+cx^{2n})^3}\,dx$

Optimal. Leaf size=424

$$\frac{x\left(d\left(cd^2-3ae^2\right)+e\left(3cd^2-ae^2\right)x^n\right)}{4acn\left(a+cx^{2n}\right)^2}+\frac{e^2x\left(3d+ex^n\right)}{2acn\left(a+cx^{2n}\right)}-\frac{x\left(d\left(cd^2-3ae^2\right)(1-4n)+e\left(3cd^2-ae^2\right)(1-3n)x}{8a^2cn^2\left(a+cx^{2n}\right)}$$

```
[Out] 1/4*x*(d*(-3*a*e^2+c*d^2)+e*(-a*e^2+3*c*d^2)*x^n)/a/c/n/(a+c*x^(2*n))^2+1/2
*e^2*x*(3*d+e*x^n)/a/c/n/(a+c*x^(2*n))-1/8*x*(d*(-3*a*e^2+c*d^2)*(1-4*n)+e*
(-a*e^2+3*c*d^2)*(1-3*n)*x^n)/a^2/c/n^2/(a+c*x^(2*n))+1/8*d*(-3*a*e^2+c*d^2
)*(1-4*n)*(1-2*n)*x*hypergeom([1, 1/2/n],[1+1/2/n],-c*x^(2*n)/a)/a^3/c/n^2-
3/2*d*e^2*(1-2*n)*x*hypergeom([1, 1/2/n],[1+1/2/n],-c*x^(2*n)/a)/a^2/c/n+1/
8*e*(-a*e^2+3*c*d^2)*(1-3*n)*(1-n)*x^(1+n)*hypergeom([1, 1/2*(1+n)/n],[3/2+
1/2/n],-c*x^(2*n)/a)/a^3/c/n^2/(1+n)-1/2*e^3*(1-n)*x^(1+n)*hypergeom([1, 1/
2*(1+n)/n],[3/2+1/2/n],-c*x^(2*n)/a)/a^2/c/n/(1+n)
```

Rubi [A]
time = 0.27, antiderivative size = 424, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$,
Rules used = {1451, 1445, 1432, 251, 371}

$$\frac{e(1-3n)(1-n)x^{n+1}\left(3cd^2-ae^2\right)\,{}_2F_1\left(1,\frac{n+1}{2n};\frac{1}{2}\left(3+\frac{1}{n}\right);-\frac{cx^{2n}}{a}\right)}{8a^3cn^2(n+1)}+\frac{d(1-4n)(1-2n)x\left(cd^2-3ae^2\right)\,{}_2F_1\left(1,\frac{1}{2n};\frac{1}{2}\left(2+\frac{1}{n}\right);-\frac{cx^{2n}}{a}\right)}{8a^3cn^2}-\frac{x\left(e(1-3n)x^n\left(3cd^2-ae^2\right)+d(1-4n)\left(cd^2-3ae^2\right)\right)}{8a^2cn^2\left(a+cx^{2n}\right)}-\frac{3de^2(1-2n)x\,{}_2F_1\left(1,\frac{1}{2n};\frac{1}{2}\left(2+\frac{1}{n}\right);-\frac{cx^{2n}}{a}\right)}{2a^2cn}-\frac{e^3(1-n)x^{n+1}\,{}_2F_1\left(1,\frac{n+1}{2n};\frac{1}{2}\left(3+\frac{1}{n}\right);-\frac{cx^{2n}}{a}\right)}{2a^2cn(n+1)}+\frac{x\left(ex^n\left(3cd^2-ae^2\right)+d\left(cd^2-3ae^2\right)\right)}{4acn\left(a+cx^{2n}\right)^2}+\frac{e^2x\left(3d+ex^n\right)}{2acn\left(a+cx^{2n}\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x^n)^3/(a + c*x^(2*n))^3,x]
```

```
[Out] (x*(d*(c*d^2 - 3*a*e^2) + e*(3*c*d^2 - a*e^2)*x^n))/(4*a*c*n*(a + c*x^(2*n)
)^2) + (e^2*x*(3*d + e*x^n))/(2*a*c*n*(a + c*x^(2*n))) - (x*(d*(c*d^2 - 3*a
*e^2)*(1 - 4*n) + e*(3*c*d^2 - a*e^2)*(1 - 3*n)*x^n))/(8*a^2*c*n^2*(a + c*x
^(2*n))) + (d*(c*d^2 - 3*a*e^2)*(1 - 4*n)*(1 - 2*n)*x*Hypergeometric2F1[1,
1/(2*n), (2 + n^(-1))/2, -((c*x^(2*n))/a)])/(8*a^3*c*n^2) - (3*d*e^2*(1 - 2
*n)*x*Hypergeometric2F1[1, 1/(2*n), (2 + n^(-1))/2, -((c*x^(2*n))/a)])/(2*a
^2*c*n) + (e*(3*c*d^2 - a*e^2)*(1 - 3*n)*(1 - n)*x^(1 + n)*Hypergeometric2F
1[1, (1 + n)/(2*n), (3 + n^(-1))/2, -((c*x^(2*n))/a)])/(8*a^3*c*n^2*(1 + n)
) - (e^3*(1 - n)*x^(1 + n)*Hypergeometric2F1[1, (1 + n)/(2*n), (3 + n^(-1))
/2, -((c*x^(2*n))/a)])/(2*a^2*c*n*(1 + n))
```

Rule 251

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, (-b)*(x^n/a)], x] /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p
, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 1432

```
Int[((d_) + (e_.)*(x_)^(n_))/((a_) + (c_.)*(x_)^(n2_)), x_Symbol] :> Dist[d
, Int[1/(a + c*x^(2*n)), x], x] + Dist[e, Int[x^n/(a + c*x^(2*n)), x], x] /
; FreeQ[{a, c, d, e, n}, x] && EqQ[n2, 2*n] && NeQ[c*d^2 + a*e^2, 0] && (Po
sQ[a*c] || !IntegerQ[n])
```

Rule 1445

```
Int[((d_) + (e_.)*(x_)^(n_))*((a_) + (c_.)*(x_)^(n2_))^(p_), x_Symbol] :> S
imp[(-x)*(d + e*x^n)*((a + c*x^(2*n))^(p + 1)/(2*a*n*(p + 1))), x] + Dist[1
/(2*a*n*(p + 1)), Int[(d*(2*n*p + 2*n + 1) + e*(2*n*p + 3*n + 1)*x^n)*(a +
c*x^(2*n))^(p + 1), x], x] /; FreeQ[{a, c, d, e, n}, x] && EqQ[n2, 2*n] &&
ILtQ[p, -1]
```

Rule 1451

```
Int[((d_) + (e_.)*(x_)^(n_))^(q_)*((a_) + (c_.)*(x_)^(n2_))^(p_), x_Symbol]
 :> Int[ExpandIntegrand[(d + e*x^n)^q*(a + c*x^(2*n))^p, x], x] /; FreeQ[{a
, c, d, e, n, p, q}, x] && EqQ[n2, 2*n] && NeQ[c*d^2 + a*e^2, 0] && ((Integ
ersQ[p, q] && !IntegerQ[n]) || IGtQ[p, 0] || (IGtQ[q, 0] && !IntegerQ[n])
)
```

Rubi steps

$$
\begin{aligned}
\int \frac{(d+ex^n)^3}{(a+cx^{2n})^3}\,dx &= \int \left(\frac{cd^3-3ade^2+(3cd^2e-ae^3)\,x^n}{c\,(a+cx^{2n})^3}+\frac{e^2(3d+ex^n)}{c\,(a+cx^{2n})^2}\right)dx\\
&= \frac{\int \frac{cd^3-3ade^2+(3cd^2e-ae^3)x^n}{(a+cx^{2n})^3}\,dx}{c}+\frac{e^2\int \frac{3d+ex^n}{(a+cx^{2n})^2}\,dx}{c}\\
&= \frac{x(d(cd^2-3ae^2)+e(3cd^2-ae^2)\,x^n)}{4acn\,(a+cx^{2n})^2}+\frac{e^2x(3d+ex^n)}{2acn\,(a+cx^{2n})}-\frac{\int \frac{(cd^3-3ade^2)(1-4n)+(3cd^2e-ae^3)}{(a+cx^{2n})^2}}{4acn}\\
&= \frac{x(d(cd^2-3ae^2)+e(3cd^2-ae^2)\,x^n)}{4acn\,(a+cx^{2n})^2}+\frac{e^2x(3d+ex^n)}{2acn\,(a+cx^{2n})}-\frac{x(d(cd^2-3ae^2)\,(1-4n)+}{8a^2cn^2\,(a}\\
&= \frac{x(d(cd^2-3ae^2)+e(3cd^2-ae^2)\,x^n)}{4acn\,(a+cx^{2n})^2}+\frac{e^2x(3d+ex^n)}{2acn\,(a+cx^{2n})}-\frac{x(d(cd^2-3ae^2)\,(1-4n)+}{8a^2cn^2\,(a}\\
&= \frac{x(d(cd^2-3ae^2)+e(3cd^2-ae^2)\,x^n)}{4acn\,(a+cx^{2n})^2}+\frac{e^2x(3d+ex^n)}{2acn\,(a+cx^{2n})}-\frac{x(d(cd^2-3ae^2)\,(1-4n)+}{8a^2cn^2\,(a}
\end{aligned}
$$

Mathematica [A]
time = 1.01, size = 252, normalized size = 0.59

$$
\frac{x\left(\frac{a\left(-a^2e^2(d(-3+6n)+e(-1+n)x^n)+c^2d^2x^{2n}(d(-1+4n)+3e(-1+3n)x^n)+ac(d^3(-1+6n)+3d^2e(-1+5n)x^n+3de^2x^{2n}+e^3(1+n)x^{3n})\right)}{(a+cx^{2n})^2}+d(-1+2n)\,(3ae^2+cd^2(-1+4n))\,{}_2F_1\left(1,\frac{1}{2n};1+\frac{1}{2n};-\frac{cx^{2n}}{a}\right)+\frac{e(-1+n)(ae^2(1+n)+3cd^2(-1+3n))x^n\,{}_2F_1\left(1,\frac{1+n}{2n};\frac{1}{2}(3+\frac{1}{n});-\frac{cx^{2n}}{a}\right)}{1+n}\right)}{8a^3cn^2}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x^n)^3/(a + c*x^(2*n))^3,x]
```

```
[Out] (x*((a*(-(a^2*e^2*(d*(-3 + 6*n) + e*(-1 + n)*x^n)) + c^2*d^2*x^(2*n)*(d*(-1
 + 4*n) + 3*e*(-1 + 3*n)*x^n) + a*c*(d^3*(-1 + 6*n) + 3*d^2*e*(-1 + 5*n)*x^
n + 3*d*e^2*x^(2*n) + e^3*(1 + n)*x^(3*n))))/(a + c*x^(2*n))^2 + d*(-1 + 2*
n)*(3*a*e^2 + c*d^2*(-1 + 4*n))*Hypergeometric2F1[1, 1/(2*n), 1 + 1/(2*n),
-((c*x^(2*n))/a)] + (e*(-1 + n)*(a*e^2*(1 + n) + 3*c*d^2*(-1 + 3*n))*x^n*Hy
pergeometric2F1[1, (1 + n)/(2*n), (3 + n^(-1))/2, -((c*x^(2*n))/a)])/(1 + n
)))/(8*a^3*c*n^2)
```

Maple [F]
time = 0.02, size = 0, normalized size = 0.00

$$\int \frac{(d+e\,x^n)^3}{(a+c\,x^{2n})^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d+e*x^n)^3/(a+c*x^(2*n))^3,x)
```

```
[Out] int((d+e*x^n)^3/(a+c*x^(2*n))^3,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+e*x^n)^3/(a+c*x^(2*n))^3,x, algorithm="maxima")
```

```
[Out] 1/8*((3*c^2*d^2*(3*n - 1)*e + a*c*(n + 1)*e^3)*x*x^(3*n) + (c^2*d^3*(4*n -
1) + 3*a*c*d*e^2)*x*x^(2*n) + (3*a*c*d^2*(5*n - 1)*e - a^2*(n - 1)*e^3)*x*x
^n + (a*c*d^3*(6*n - 1) - 3*a^2*d*(2*n - 1)*e^2)*x)/(a^2*c^3*n^2*x^(4*n) +
2*a^3*c^2*n^2*x^(2*n) + a^4*c*n^2) + integrate(1/8*((8*n^2 - 6*n + 1)*c*d^3
 + 3*a*d*(2*n - 1)*e^2 + (3*(3*n^2 - 4*n + 1)*c*d^2*e + (n^2 - 1)*a*e^3)*x^
n)/(a^2*c^2*n^2*x^(2*n) + a^3*c*n^2), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+e*x^n)^3/(a+c*x^(2*n))^3,x, algorithm="fricas")
```

```
[Out] integral((3*d^2*x^n*e + d^3 + 3*d*x^(2*n)*e^2 + x^(3*n)*e^3)/(c^3*x^(6*n) +
 3*a*c^2*x^(4*n) + 3*a^2*c*x^(2*n) + a^3), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(d + ex^n)^3}{(a + cx^{2n})^3}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+e*x**n)**3/(a+c*x**(2*n))**3,x)
```

```
[Out] Integral((d + e*x**n)**3/(a + c*x**(2*n))**3, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+e*x^n)^3/(a+c*x^(2*n))^3,x, algorithm="giac")
```

```
[Out] integrate((x^n*e + d)^3/(c*x^(2*n) + a)^3, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(d+e\,x^n)^3}{(a+c\,x^{2\,n})^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x^n)^3/(a + c*x^(2*n))^3,x)
[Out] int((d + e*x^n)^3/(a + c*x^(2*n))^3, x)
```

3.54 $\int \frac{(d+ex^n)^2}{(a+cx^{2n})^3}\, dx$

Optimal. Leaf size=272

$$\frac{x(cd^2-ae^2+2cdex^n)}{4acn\,(a+cx^{2n})^2}-\frac{x((cd^2-ae^2)\,(1-4n)+2cde(1-3n)x^n)}{8a^2cn^2\,(a+cx^{2n})}+\frac{(cd^2-ae^2)\,(1-4n)(1-2n)x\,{}_2F_1\Big(1,\frac{1}{2n}}{8a^3cn^2}$$

```
[Out] 1/4*x*(c*d^2-a*e^2+2*c*d*e*x^n)/a/c/n/(a+c*x^(2*n))^2-1/8*x*((-a*e^2+c*d^2)
*(1-4*n)+2*c*d*e*(1-3*n)*x^n)/a^2/c/n^2/(a+c*x^(2*n))+1/8*(-a*e^2+c*d^2)*(1
-4*n)*(1-2*n)*x*hypergeom([1, 1/2/n],[1+1/2/n],-c*x^(2*n)/a)/a^3/c/n^2+1/4*
d*e*(1-3*n)*(1-n)*x^(1+n)*hypergeom([1, 1/2*(1+n)/n],[3/2+1/2/n],-c*x^(2*n)
/a)/a^3/n^2/(1+n)+e^2*x*hypergeom([2, 1/2/n],[1+1/2/n],-c*x^(2*n)/a)/a^2/c
```

Rubi [A]
time = 0.17, antiderivative size = 272, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used = {1451, 1445, 1432, 251, 371}

$$\frac{(1-4n)(1-2n)x(cd^2-ae^2)\;{}_2F_1\left(1,\frac{1}{2n};\frac{1}{2}\left(2+\frac{1}{n}\right);-\frac{cx^{2n}}{a}\right)}{8a^3cn^2}+\frac{de(1-3n)(1-n)x^{n+1}\,{}_2F_1\left(1,\frac{n+1}{2n};\frac{1}{2}\left(3+\frac{1}{n}\right);-\frac{cx^{2n}}{a}\right)}{4a^3n^2(n+1)}-\frac{x((1-4n)\,(cd^2-ae^2)+2cde(1-3n)x^n)}{8a^2cn^2\,(a+cx^{2n})}+\frac{e^2x\,{}_2F_1\left(2,\frac{1}{2n};\frac{1}{2}\left(2+\frac{1}{n}\right);-\frac{cx^{2n}}{a}\right)}{a^2c}+\frac{x(-ae^2+cd^2+2cdex^n)}{4acn\,(a+cx^{2n})^2}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x^n)^2/(a + c*x^(2*n))^3,x]
```

```
[Out] (x*(c*d^2 - a*e^2 + 2*c*d*e*x^n))/(4*a*c*n*(a + c*x^(2*n))^2) - (x*((c*d^2
- a*e^2)*(1 - 4*n) + 2*c*d*e*(1 - 3*n)*x^n))/(8*a^2*c*n^2*(a + c*x^(2*n)))
+ ((c*d^2 - a*e^2)*(1 - 4*n)*(1 - 2*n)*x*Hypergeometric2F1[1, 1/(2*n), (2 +
 n^(-1))/2, -((c*x^(2*n))/a)])/(8*a^3*c*n^2) + (d*e*(1 - 3*n)*(1 - n)*x^(1
+ n)*Hypergeometric2F1[1, (1 + n)/(2*n), (3 + n^(-1))/2, -((c*x^(2*n))/a)])
/(4*a^3*n^2*(1 + n)) + (e^2*x*Hypergeometric2F1[2, 1/(2*n), (2 + n^(-1))/2,
 -((c*x^(2*n))/a)])/(a^2*c)
```

Rule 251

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, (-b)*(x^n/a)], x] /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p
, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 1432

```
Int[((d_) + (e_.)*(x_)^(n_))/((a_) + (c_.)*(x_)^(n2_)), x_Symbol] :> Dist[d
, Int[1/(a + c*x^(2*n)), x], x] + Dist[e, Int[x^n/(a + c*x^(2*n)), x], x] /
; FreeQ[{a, c, d, e, n}, x] && EqQ[n2, 2*n] && NeQ[c*d^2 + a*e^2, 0] && (Po
sQ[a*c] ||  !IntegerQ[n])
```

Rule 1445

```
Int[((d_) + (e_.)*(x_)^(n_))*((a_) + (c_.)*(x_)^(n2_))^(p_), x_Symbol] :> S
imp[(-x)*(d + e*x^n)*((a + c*x^(2*n))^(p + 1)/(2*a*n*(p + 1))), x] + Dist[1
/(2*a*n*(p + 1)), Int[(d*(2*n*p + 2*n + 1) + e*(2*n*p + 3*n + 1)*x^n)*(a +
c*x^(2*n))^(p + 1), x], x] /; FreeQ[{a, c, d, e, n}, x] && EqQ[n2, 2*n] &&
ILtQ[p, -1]
```

Rule 1451

```
Int[((d_) + (e_.)*(x_)^(n_))^(q_)*((a_) + (c_.)*(x_)^(n2_))^(p_), x_Symbol]
 :> Int[ExpandIntegrand[(d + e*x^n)^q*(a + c*x^(2*n))^p, x], x] /; FreeQ[{a
, c, d, e, n, p, q}, x] && EqQ[n2, 2*n] && NeQ[c*d^2 + a*e^2, 0] && ((Integ
ersQ[p, q] &&  !IntegerQ[n]) || IGtQ[p, 0] || (IGtQ[q, 0] &&  !IntegerQ[n])
)
```

Rubi steps

$$\begin{aligned}
\int \frac{(d+ex^n)^2}{(a+cx^{2n})^3}\,dx &= \int \left(\frac{cd^2-ae^2+2cdex^n}{c\,(a+cx^{2n})^3}+\frac{e^2}{c\,(a+cx^{2n})^2}\right)dx \\
&= \frac{\int \frac{cd^2-ae^2+2cdex^n}{(a+cx^{2n})^3}\,dx}{c}+\frac{e^2\int \frac{1}{(a+cx^{2n})^2}\,dx}{c} \\
&= \frac{x(cd^2-ae^2+2cdex^n)}{4acn\,(a+cx^{2n})^2}+\frac{e^2x\,{}_2F_1\left(2,\frac{1}{2n};\frac{1}{2}\left(2+\frac{1}{n}\right);-\frac{cx^{2n}}{a}\right)}{a^2c}-\frac{\int \frac{(cd^2-ae^2)(1-4n)+2cde(1-3n}{(a+cx^{2n})^2}}{4acn} \\
&= \frac{x(cd^2-ae^2+2cdex^n)}{4acn\,(a+cx^{2n})^2}-\frac{x((cd^2-ae^2)(1-4n)+2cde(1-3n)x^n)}{8a^2cn^2\,(a+cx^{2n})}+\frac{e^2x\,{}_2F_1\left(2,\frac{1}{2n};\frac{1}{2}\right.}{a} \\
&= \frac{x(cd^2-ae^2+2cdex^n)}{4acn\,(a+cx^{2n})^2}-\frac{x((cd^2-ae^2)(1-4n)+2cde(1-3n)x^n)}{8a^2cn^2\,(a+cx^{2n})}+\frac{e^2x\,{}_2F_1\left(2,\frac{1}{2n};\frac{1}{2}\right.}{a} \\
&= \frac{x(cd^2-ae^2+2cdex^n)}{4acn\,(a+cx^{2n})^2}-\frac{x((cd^2-ae^2)(1-4n)+2cde(1-3n)x^n)}{8a^2cn^2\,(a+cx^{2n})}+\frac{(cd^2-ae^2)(1-}{}
\end{aligned}$$

Mathematica [A]

time = 0.70, size = 212, normalized size = 0.78

$$\frac{x\left(\frac{a(a^2e^2(1-2n)+c^2dx^{2n}(d(-1+4n)+2e(-1+3n)x^n)+ac(d^2(-1+6n)+2de(-1+5n)x^n+e^2x^{2n}))}{c(a+cx^{2n})^2}+\frac{(-1+2n)(ae^2+cd^2(-1+4n))\,{}_2F_1\left(1,\frac{1}{2n};1+\frac{1}{2n};-\frac{cx^{2n}}{a}\right)}{c}+\frac{2de(1-4n+3n^2)x^n\,{}_2F_1\left(1,\frac{1+n}{2n};\frac{1}{2}\left(3+\frac{1}{n}\right);-\frac{cx^{2n}}{a}\right)}{1+n}\right)}{8a^3n^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x^n)^2/(a + c*x^(2*n))^3,x]
```

```
[Out] (x*((a*(a^2*e^2*(1 - 2*n) + c^2*d*x^(2*n)*(d*(-1 + 4*n) + 2*e*(-1 + 3*n)*x^
n) + a*c*(d^2*(-1 + 6*n) + 2*d*e*(-1 + 5*n)*x^n + e^2*x^(2*n))))/(c*(a + c*
x^(2*n))^2) + ((-1 + 2*n)*(a*e^2 + c*d^2*(-1 + 4*n))*Hypergeometric2F1[1, 1
/(2*n), 1 + 1/(2*n), -((c*x^(2*n))/a)])/c + (2*d*e*(1 - 4*n + 3*n^2)*x^n*Hy
pergeometric2F1[1, (1 + n)/(2*n), (3 + n^(-1))/2, -((c*x^(2*n))/a)])/(1 + n
)))/(8*a^3*n^2)
```

Maple [F]
time = 0.02, size = 0, normalized size = 0.00

$$\int \frac{(d+e\,x^n)^2}{(a+c\,x^{2n})^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d+e*x^n)^2/(a+c*x^(2*n))^3,x)
```

```
[Out] int((d+e*x^n)^2/(a+c*x^(2*n))^3,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+e*x^n)^2/(a+c*x^(2*n))^3,x, algorithm="maxima")
```

```
[Out] 1/8*(2*c^2*d*(3*n - 1)*x*e^(3*n*log(x) + 1) + 2*a*c*d*(5*n - 1)*x*e^(n*log(
x) + 1) + (c^2*d^2*(4*n - 1) + a*c*e^2)*x*x^(2*n) + (a*c*d^2*(6*n - 1) - a^
2*(2*n - 1)*e^2)*x)/(a^2*c^3*n^2*x^(4*n) + 2*a^3*c^2*n^2*x^(2*n) + a^4*c*n^
2) + integrate(1/8*((8*n^2 - 6*n + 1)*c*d^2 + 2*(3*n^2 - 4*n + 1)*c*d*e^(n*
log(x) + 1) + a*(2*n - 1)*e^2)/(a^2*c^2*n^2*x^(2*n) + a^3*c*n^2), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+e*x^n)^2/(a+c*x^(2*n))^3,x, algorithm="fricas")
```

```
[Out] integral((2*d*x^n*e + d^2 + x^(2*n)*e^2)/(c^3*x^(6*n) + 3*a*c^2*x^(4*n) + 3
*a^2*c*x^(2*n) + a^3), x)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(d+ex^n)^2}{(a+cx^{2n})^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+e*x**n)**2/(a+c*x**(2*n))**3,x)
```

```
[Out] Integral((d + e*x**n)**2/(a + c*x**(2*n))**3, x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+e*x^n)^2/(a+c*x^(2*n))^3,x, algorithm="giac")
```

```
[Out] integrate((x^n*e + d)^2/(c*x^(2*n) + a)^3, x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{{\left(d+e\,x^n\right)}^2}{{\left(a+c\,x^{2\,n}\right)}^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x^n)^2/(a + c*x^(2*n))^3,x)
```

```
[Out] int((d + e*x^n)^2/(a + c*x^(2*n))^3, x)
```

3.55 $\int \frac{d+ex^n}{(a+cx^{2n})^3} \, dx$

Optimal. Leaf size=184

$$\frac{x(d+ex^n)}{4an\left(a+cx^{2n}\right)^2}-\frac{x(d(1-4n)+e(1-3n)x^n)}{8a^2n^2\left(a+cx^{2n}\right)}+\frac{d(1-4n)(1-2n)x\,{}_2F_1\left(1,\frac{1}{2n};\frac{1}{2}\left(2+\frac{1}{n}\right);-\frac{cx^{2n}}{a}\right)}{8a^3n^2}+\frac{e(1-3n)}{}$$

```
[Out] 1/4*x*(d+e*x^n)/a/n/(a+c*x^(2*n))^2-1/8*x*(d*(1-4*n)+e*(1-3*n)*x^n)/a^2/n^2
/(a+c*x^(2*n))+1/8*d*(1-4*n)*(1-2*n)*x*hypergeom([1, 1/2/n],[1+1/2/n],-c*x^
(2*n)/a)/a^3/n^2+1/8*e*(1-3*n)*(1-n)*x^(1+n)*hypergeom([1, 1/2*(1+n)/n],[3/
2+1/2/n],-c*x^(2*n)/a)/a^3/n^2/(1+n)
```

Rubi [A]
time = 0.07, antiderivative size = 184, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.210$,
Rules used = {1445, 1432, 251, 371}

$$\frac{d(1-4n)(1-2n)x\,{}_2F_1\left(1,\frac{1}{2n};\frac{1}{2}\left(2+\frac{1}{n}\right);-\frac{cx^{2n}}{a}\right)}{8a^3n^2}+\frac{e(1-3n)(1-n)x^{n+1}\,{}_2F_1\left(1,\frac{n+1}{2n};\frac{1}{2}\left(3+\frac{1}{n}\right);-\frac{cx^{2n}}{a}\right)}{8a^3n^2(n+1)}-\frac{x(d(1-4n)+e(1-3n)x^n)}{8a^2n^2\left(a+cx^{2n}\right)}+\frac{x(d+ex^n)}{4an\left(a+cx^{2n}\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x^n)/(a + c*x^(2*n))^3,x]
```

```
[Out] (x*(d + e*x^n))/(4*a*n*(a + c*x^(2*n))^2) - (x*(d*(1 - 4*n) + e*(1 - 3*n)*x
^n))/(8*a^2*n^2*(a + c*x^(2*n))) + (d*(1 - 4*n)*(1 - 2*n)*x*Hypergeometric2
F1[1, 1/(2*n), (2 + n^(-1))/2, -((c*x^(2*n))/a)])/(8*a^3*n^2) + (e*(1 - 3*n
)*(1 - n)*x^(1 + n)*Hypergeometric2F1[1, (1 + n)/(2*n), (3 + n^(-1))/2, -((
c*x^(2*n))/a)])/(8*a^3*n^2*(1 + n))
```

Rule 251

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, (-b)*(x^n/a)], x] /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p
, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 1432

```
Int[((d_) + (e_.)*(x_)^(n_))/((a_) + (c_.)*(x_)^(n2_)), x_Symbol] :> Dist[d
, Int[1/(a + c*x^(2*n)), x], x] + Dist[e, Int[x^n/(a + c*x^(2*n)), x], x] /
; FreeQ[{a, c, d, e, n}, x] && EqQ[n2, 2*n] && NeQ[c*d^2 + a*e^2, 0] && (Po
sQ[a*c] ||  !IntegerQ[n])
```

Rule 1445

```
Int[((d_) + (e_.)*(x_)^(n_))*((a_) + (c_.)*(x_)^(n2_))^(p_), x_Symbol] :> S
imp[(-x)*(d + e*x^n)*((a + c*x^(2*n))^(p + 1)/(2*a*n*(p + 1))), x] + Dist[1
/(2*a*n*(p + 1)), Int[(d*(2*n*p + 2*n + 1) + e*(2*n*p + 3*n + 1)*x^n)*(a +
c*x^(2*n))^(p + 1), x], x] /; FreeQ[{a, c, d, e, n}, x] && EqQ[n2, 2*n] &&
ILtQ[p, -1]
```

Rubi steps

$$\int \frac{d+ex^n}{(a+cx^{2n})^3}\,dx = \frac{x(d+ex^n)}{4an\,(a+cx^{2n})^2} - \frac{\int \frac{d(1-4n)+e(1-3n)x^n}{(a+cx^{2n})^2}\,dx}{4an}$$

$$= \frac{x(d+ex^n)}{4an\,(a+cx^{2n})^2} - \frac{x(d(1-4n)+e(1-3n)x^n)}{8a^2n^2\,(a+cx^{2n})} + \frac{\int \frac{d(1-4n)(1-2n)+e(1-3n)(1-n)x^n}{a+cx^{2n}}\,dx}{8a^2n^2}$$

$$= \frac{x(d+ex^n)}{4an\,(a+cx^{2n})^2} - \frac{x(d(1-4n)+e(1-3n)x^n)}{8a^2n^2\,(a+cx^{2n})} + \frac{(d(1-4n)(1-2n))\int \frac{1}{a+cx^{2n}}\,dx}{8a^2n^2} + \frac{(\cdots}{\cdots}$$

$$= \frac{x(d+ex^n)}{4an\,(a+cx^{2n})^2} - \frac{x(d(1-4n)+e(1-3n)x^n)}{8a^2n^2\,(a+cx^{2n})} + \frac{d(1-4n)(1-2n)x\,{}_2F_1\left(1,\frac{1}{2n};\frac{1}{2}(2+\cdots\right.}{8a^3n^2}$$

Mathematica [A]
time = 0.23, size = 164, normalized size = 0.89

$$\frac{x\left(\frac{a(cx^{2n}(d(-1+4n)+e(-1+3n)x^n)+a(d(-1+6n)+e(-1+5n)x^n))}{(a+cx^{2n})^2}+d(1-6n+8n^2)\,{}_2F_1\left(1,\frac{1}{2n};1+\frac{1}{2n};-\frac{cx^{2n}}{a}\right)+\frac{e(1-4n+3n^2)x^n\,{}_2F_1\left(1,\frac{1+n}{2n};\frac{1}{2}\left(3+\frac{1}{n}\right);-\frac{cx^{2n}}{a}\right)}{1+n}\right)}{8a^3n^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x^n)/(a + c*x^(2*n))^3,x]
```

```
[Out] (x*((a*(c*x^(2*n)*(d*(-1 + 4*n) + e*(-1 + 3*n)*x^n) + a*(d*(-1 + 6*n) + e*(
-1 + 5*n)*x^n)))/(a + c*x^(2*n))^2 + d*(1 - 6*n + 8*n^2)*Hypergeometric2F1[
1, 1/(2*n), 1 + 1/(2*n), -((c*x^(2*n))/a)] + (e*(1 - 4*n + 3*n^2)*x^n*Hyper
geometric2F1[1, (1 + n)/(2*n), (3 + n^(-1))/2, -((c*x^(2*n))/a)])/(1 + n)))
/(8*a^3*n^2)
```

Maple [F]
time = 0.02, size = 0, normalized size = 0.00

$$\int \frac{d+e\,x^{n}}{\left(a+c\,x^{2n}\right)^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d+e*x^n)/(a+c*x^(2*n))^3,x)
```

```
[Out] int((d+e*x^n)/(a+c*x^(2*n))^3,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+e*x^n)/(a+c*x^(2*n))^3,x, algorithm="maxima")
```

```
[Out] 1/8*(c*d*(4*n - 1)*x*x^(2*n) + a*d*(6*n - 1)*x + c*(3*n - 1)*x*e^(3*n*log(x
) + 1) + a*(5*n - 1)*x*e^(n*log(x) + 1))/(a^2*c^2*n^2*x^(4*n) + 2*a^3*c*n^2
*x^(2*n) + a^4*n^2) + integrate(1/8*((8*n^2 - 6*n + 1)*d + (3*n^2 - 4*n + 1
)*e^(n*log(x) + 1))/(a^2*c*n^2*x^(2*n) + a^3*n^2), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+e*x^n)/(a+c*x^(2*n))^3,x, algorithm="fricas")
```

```
[Out] integral((x^n*e + d)/(c^3*x^(6*n) + 3*a*c^2*x^(4*n) + 3*a^2*c*x^(2*n) + a^3
), x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+e*x**n)/(a+c*x**(2*n))**3,x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+e*x^n)/(a+c*x^(2*n))^3,x, algorithm="giac")
[Out] integrate((x^n*e + d)/(c*x^(2*n) + a)^3, x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{d+e\,x^n}{{\left(a+c\,x^{2\,n}\right)}^3} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x^n)/(a + c*x^(2*n))^3,x)
[Out] int((d + e*x^n)/(a + c*x^(2*n))^3, x)
```

3.56 $\int \frac{1}{(d+ex^n)(a+cx^{2n})^3}\,dx$

Optimal. Leaf size=582

$$\frac{cx(d-ex^n)}{4a\left(cd^2+ae^2\right)n\left(a+cx^{2n}\right)^2}+\frac{ce^2x(d-ex^n)}{2a\left(cd^2+ae^2\right)^2n\left(a+cx^{2n}\right)}-\frac{cx(d(1-4n)-e(1-3n)x^n)}{8a^2\left(cd^2+ae^2\right)n^2\left(a+cx^{2n}\right)}+\frac{cde^4x\,{}_2F_1\left(1,\frac{1}{2n};\frac{1}{2}\right.}{a\left(cd^2+\right.}$$

[Out] 1/4*c*x*(d-e*x^n)/a/(a*e^2+c*d^2)/n/(a+c*x^(2*n))^2+1/2*c*e^2*x*(d-e*x^n)/a/(a*e^2+c*d^2)^2/n/(a+c*x^(2*n))-1/8*c*x*(d*(1-4*n)-e*(1-3*n)*x^n)/a^2/(a*e^2+c*d^2)/n^2/(a+c*x^(2*n))+c*d*e^4*x*hypergeom([1, 1/2/n],[1+1/2/n],-c*x^(2*n)/a)/a/(a*e^2+c*d^2)^3+1/8*c*d*(1-4*n)*(1-2*n)*x*hypergeom([1, 1/2/n],[1+1/2/n],-c*x^(2*n)/a)/a^3/(a*e^2+c*d^2)/n^2-1/2*c*d*e^2*(1-2*n)*x*hypergeom([1, 1/2/n],[1+1/2/n],-c*x^(2*n)/a)/a^2/(a*e^2+c*d^2)^2/n+e^6*x*hypergeom([1, 1/n],[1+1/n],-e*x^n/d)/d/(a*e^2+c*d^2)^3-c*e^5*x^(1+n)*hypergeom([1, 1/2*(1+n)/n],[3/2+1/2/n],-c*x^(2*n)/a)/a/(a*e^2+c*d^2)^3/(1+n)-1/8*c*e*(1-3*n)*(1-n)*x^(1+n)*hypergeom([1, 1/2*(1+n)/n],[3/2+1/2/n],-c*x^(2*n)/a)/a^3/(a*e^2+c*d^2)/n^2/(1+n)+1/2*c*e^3*(1-n)*x^(1+n)*hypergeom([1, 1/2*(1+n)/n],[3/2+1/2/n],-c*x^(2*n)/a)/a^2/(a*e^2+c*d^2)^2/n/(1+n)

Rubi [A]
time = 0.29, antiderivative size = 582, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used = {1451, 251, 1445, 1432, 371}

$$-\frac{ce(1-3n)(1-n)x^{n+1}\,{}_2F_1\left(1,\frac{n+1}{2n};\frac{1}{2}\left(3+\frac{1}{n}\right);-\frac{cx^{2n}}{a}\right)}{8a^3n^2(n+1)\left(ae^2+cd^2\right)}+\frac{cd(1-4n)(1-2n)x\,{}_2F_1\left(1,\frac{1}{2n};\frac{1}{2}\left(2+\frac{1}{n}\right);-\frac{cx^{2n}}{a}\right)}{8a^3n^2\left(ae^2+cd^2\right)}-\frac{cde^2(1-2n)x\,{}_2F_1\left(1,\frac{1}{2n};\frac{1}{2}\left(2+\frac{1}{n}\right);-\frac{cx^{2n}}{a}\right)}{2a^2n\left(ae^2+cd^2\right)^2}-\frac{cx(d(1-4n)-e(1-3n)x^n)}{8a^2n^2\left(ae^2+cd^2\right)\left(a+cx^{2n}\right)}+\frac{ce^3(1-n)x^{n+1}\,{}_2F_1\left(1,\frac{n+1}{2n};\frac{1}{2}\left(3+\frac{1}{n}\right);-\frac{cx^{2n}}{a}\right)}{2a^2n(n+1)\left(ae^2+cd^2\right)^2}+\frac{ce^2x(d-ex^n)}{2an\left(ae^2+cd^2\right)^2\left(a+cx^{2n}\right)}+\frac{cx(d-ex^n)}{4an\left(ae^2+cd^2\right)\left(a+cx^{2n}\right)^2}+\frac{e^6x\,{}_2F_1\left(1,\frac{1}{n};1+\frac{1}{n};-\frac{ex^n}{d}\right)}{d\left(ae^2+cd^2\right)^3}-\frac{ce^5x^{n+1}\,{}_2F_1\left(1,\frac{n+1}{2n};\frac{1}{2}\left(3+\frac{1}{n}\right);-\frac{cx^{2n}}{a}\right)}{a(n+1)\left(ae^2+cd^2\right)^3}+\frac{cde^4x\,{}_2F_1\left(1,\frac{1}{2n};\frac{1}{2}\left(2+\frac{1}{n}\right);-\frac{cx^{2n}}{a}\right)}{a\left(ae^2+cd^2\right)^3}$$

Antiderivative was successfully verified.

[In] Int[1/((d + e*x^n)*(a + c*x^(2*n))^3),x]

[Out] (c*x*(d - e*x^n))/(4*a*(c*d^2 + a*e^2)*n*(a + c*x^(2*n))^2) + (c*e^2*x*(d - e*x^n))/(2*a*(c*d^2 + a*e^2)^2*n*(a + c*x^(2*n))) - (c*x*(d*(1 - 4*n) - e*(1 - 3*n)*x^n))/(8*a^2*(c*d^2 + a*e^2)*n^2*(a + c*x^(2*n))) + (c*d*e^4*x*Hypergeometric2F1[1, 1/(2*n), (2 + n^(-1))/2, -((c*x^(2*n))/a)])/(a*(c*d^2 + a*e^2)^3) + (c*d*(1 - 4*n)*(1 - 2*n)*x*Hypergeometric2F1[1, 1/(2*n), (2 + n^(-1))/2, -((c*x^(2*n))/a)])/(8*a^3*(c*d^2 + a*e^2)*n^2) - (c*d*e^2*(1 - 2*n)*x*Hypergeometric2F1[1, 1/(2*n), (2 + n^(-1))/2, -((c*x^(2*n))/a)])/(2*a^2*(c*d^2 + a*e^2)^2*n) + (e^6*x*Hypergeometric2F1[1, n^(-1), 1 + n^(-1), -((e*x^n)/d)])/(d*(c*d^2 + a*e^2)^3) - (c*e^5*x^(1 + n)*Hypergeometric2F1[1, (1 + n)/(2*n), (3 + n^(-1))/2, -((c*x^(2*n))/a)])/(a*(c*d^2 + a*e^2)^3*(1 + n)) - (c*e*(1 - 3*n)*(1 - n)*x^(1 + n)*Hypergeometric2F1[1, (1 + n)/(2*n), (3 + n^(-1))/2, -((c*x^(2*n))/a)])/(8*a^3*(c*d^2 + a*e^2)*n^2*(1 + n)) + (c*e^3*(1 - n)*x^(1 + n)*Hypergeometric2F1[1, (1 + n)/(2*n), (3 + n^(-1))/2, -((c*x^(2*n))/a)])/(2*a^2*(c*d^2 + a*e^2)^2*n*(1 + n))

Rule 251

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, (-b)*(x^n/a)], x] /; FreeQ[{a, b, n, p}, x] && !IGtQ[p
, 0] && !IntegerQ[1/n] && !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 1432

```
Int[((d_) + (e_.)*(x_)^(n_))/((a_) + (c_.)*(x_)^(n2_)), x_Symbol] :> Dist[d
, Int[1/(a + c*x^(2*n)), x], x] + Dist[e, Int[x^n/(a + c*x^(2*n)), x], x] /
; FreeQ[{a, c, d, e, n}, x] && EqQ[n2, 2*n] && NeQ[c*d^2 + a*e^2, 0] && (Po
sQ[a*c] || !IntegerQ[n])
```

Rule 1445

```
Int[((d_) + (e_.)*(x_)^(n_))*((a_) + (c_.)*(x_)^(n2_))^(p_), x_Symbol] :> S
imp[(-x)*(d + e*x^n)*((a + c*x^(2*n))^(p + 1)/(2*a*n*(p + 1))), x] + Dist[1
/(2*a*n*(p + 1)), Int[(d*(2*n*p + 2*n + 1) + e*(2*n*p + 3*n + 1)*x^n)*(a +
c*x^(2*n))^(p + 1), x], x] /; FreeQ[{a, c, d, e, n}, x] && EqQ[n2, 2*n] &&
ILtQ[p, -1]
```

Rule 1451

```
Int[((d_) + (e_.)*(x_)^(n_))^(q_)*((a_) + (c_.)*(x_)^(n2_))^(p_), x_Symbol]
 :> Int[ExpandIntegrand[(d + e*x^n)^q*(a + c*x^(2*n))^p, x], x] /; FreeQ[{a
, c, d, e, n, p, q}, x] && EqQ[n2, 2*n] && NeQ[c*d^2 + a*e^2, 0] && ((Integ
ersQ[p, q] && !IntegerQ[n]) || IGtQ[p, 0] || (IGtQ[q, 0] && !IntegerQ[n])
)
```

Rubi steps

$$\int \frac{1}{(d+ex^n)(a+cx^{2n})^3}\,dx = \int \left(\frac{e^6}{(cd^2+ae^2)^3(d+ex^n)} - \frac{c(-d+ex^n)}{(cd^2+ae^2)(a+cx^{2n})^3} - \frac{ce^2(-d+ex^n)}{(cd^2+ae^2)^2(a+cx^{2n}}\right.$$

$$= -\frac{(ce^4)\int \frac{-d+ex^n}{a+cx^{2n}}\,dx}{(cd^2+ae^2)^3} + \frac{e^6\int \frac{1}{d+ex^n}\,dx}{(cd^2+ae^2)^3} - \frac{(ce^2)\int \frac{-d+ex^n}{(a+cx^{2n})^2}\,dx}{(cd^2+ae^2)^2} - \frac{c\int \frac{-d+ex^n}{(a+cx^{2n})^3}\,dx}{cd^2+ae^2}$$

$$= \frac{cx(d-ex^n)}{4a(cd^2+ae^2)n(a+cx^{2n})^2} + \frac{ce^2x(d-ex^n)}{2a(cd^2+ae^2)^2n(a+cx^{2n})} + \frac{e^6x\,{}_2F_1\left(1,\frac{1}{n};1+\right.}{d(cd^2+a}$$

$$= \frac{cx(d-ex^n)}{4a(cd^2+ae^2)n(a+cx^{2n})^2} + \frac{ce^2x(d-ex^n)}{2a(cd^2+ae^2)^2n(a+cx^{2n})} - \frac{cx(d(1-4n)-}{8a^2(cd^2+ae^2)}$$

$$= \frac{cx(d-ex^n)}{4a(cd^2+ae^2)n(a+cx^{2n})^2} + \frac{ce^2x(d-ex^n)}{2a(cd^2+ae^2)^2n(a+cx^{2n})} - \frac{cx(d(1-4n)-}{8a^2(cd^2+ae^2)}$$

$$= \frac{cx(d-ex^n)}{4a(cd^2+ae^2)n(a+cx^{2n})^2} + \frac{ce^2x(d-ex^n)}{2a(cd^2+ae^2)^2n(a+cx^{2n})} - \frac{cx(d(1-4n)-}{8a^2(cd^2+ae^2)}$$

Mathematica [A]
time = 0.85, size = 1031, normalized size = 1.77

Antiderivative was successfully verified.

```
[In] Integrate[1/((d + e*x^n)*(a + c*x^(2*n))^3),x]
```

```
[Out] (x*((2*c*(c*d^2 + a*e^2)^2*(d - e*x^n))/(a*n*(a + c*x^(2*n))^2) + (c*(c*d^2
 + a*e^2)*(c*d^2*(d*(-1 + 4*n) - e*(-1 + 3*n)*x^n) + a*e^2*(d*(-1 + 8*n) -
e*(-1 + 7*n)*x^n)))/(a^2*n^2*(a + c*x^(2*n))) + (8*c^3*d^5*Hypergeometric2F
1[1, 1/(2*n), 1 + 1/(2*n), -((c*x^(2*n))/a)])/a^3 + (24*c^2*d^3*e^2*Hyperge
ometric2F1[1, 1/(2*n), 1 + 1/(2*n), -((c*x^(2*n))/a)])/a^2 + (24*c*d*e^4*Hy
pergeometric2F1[1, 1/(2*n), 1 + 1/(2*n), -((c*x^(2*n))/a)])/a + (c^3*d^5*Hy
pergeometric2F1[1, 1/(2*n), 1 + 1/(2*n), -((c*x^(2*n))/a)])/(a^3*n^2) + (2*
c^2*d^3*e^2*Hypergeometric2F1[1, 1/(2*n), 1 + 1/(2*n), -((c*x^(2*n))/a)])/(
a^2*n^2) + (c*d*e^4*Hypergeometric2F1[1, 1/(2*n), 1 + 1/(2*n), -((c*x^(2*n)
)/a)])/(a*n^2) - (6*c^3*d^5*Hypergeometric2F1[1, 1/(2*n), 1 + 1/(2*n), -((c
*x^(2*n))/a)])/(a^3*n) - (16*c^2*d^3*e^2*Hypergeometric2F1[1, 1/(2*n), 1 +
1/(2*n), -((c*x^(2*n))/a)])/(a^2*n) - (10*c*d*e^4*Hypergeometric2F1[1, 1/(2
*n), 1 + 1/(2*n), -((c*x^(2*n))/a)])/(a*n) + (8*e^6*Hypergeometric2F1[1, n^
(-1), 1 + n^(-1), -((e*x^n)/d)])/d - (3*c^3*d^4*e*x^n*Hypergeometric2F1[1,
(1 + n)/(2*n), (3 + n^(-1))/2, -((c*x^(2*n))/a)])/(a^3*(1 + n)) - (10*c^2*d
^2*e^3*x^n*Hypergeometric2F1[1, (1 + n)/(2*n), (3 + n^(-1))/2, -((c*x^(2*n)
```

```
)/a)])/(a^2*(1 + n)) - (15*c*e^5*x^n*Hypergeometric2F1[1, (1 + n)/(2*n), (3
 + n^(-1))/2, -((c*x^(2*n))/a)])/(a*(1 + n)) - (c^3*d^4*e*x^n*Hypergeometri
c2F1[1, (1 + n)/(2*n), (3 + n^(-1))/2, -((c*x^(2*n))/a)])/(a^3*n^2*(1 + n))
 - (2*c^2*d^2*e^3*x^n*Hypergeometric2F1[1, (1 + n)/(2*n), (3 + n^(-1))/2, -
((c*x^(2*n))/a)])/(a^2*n^2*(1 + n)) - (c*e^5*x^n*Hypergeometric2F1[1, (1 +
n)/(2*n), (3 + n^(-1))/2, -((c*x^(2*n))/a)])/(a*n^2*(1 + n)) + (4*c^3*d^4*e
*x^n*Hypergeometric2F1[1, (1 + n)/(2*n), (3 + n^(-1))/2, -((c*x^(2*n))/a)])
/(a^3*n*(1 + n)) + (12*c^2*d^2*e^3*x^n*Hypergeometric2F1[1, (1 + n)/(2*n),
(3 + n^(-1))/2, -((c*x^(2*n))/a)])/(a^2*n*(1 + n)) + (8*c*e^5*x^n*Hypergeom
etric2F1[1, (1 + n)/(2*n), (3 + n^(-1))/2, -((c*x^(2*n))/a)])/(a*n*(1 + n))
))/(8*(c*d^2 + a*e^2)^3)
```

Maple [F]
time = 0.04, size = 0, normalized size = 0.00

$$\int \frac{1}{(d+e\,x^{n})\,(a+c\,x^{2n})^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(d+e*x^n)/(a+c*x^(2*n))^3,x)
```

```
[Out] int(1/(d+e*x^n)/(a+c*x^(2*n))^3,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d+e*x^n)/(a+c*x^(2*n))^3,x, algorithm="maxima")
```

```
[Out] e^6*integrate(1/(c^3*d^7 + 3*a*c^2*d^5*e^2 + 3*a^2*c*d^3*e^4 + a^3*d*e^6 +
(c^3*d^6*e + 3*a*c^2*d^4*e^3 + 3*a^2*c*d^2*e^5 + a^3*e^7)*x^n), x) - 1/8*((
c^3*d^2*(3*n - 1)*e + a*c^2*(7*n - 1)*e^3)*x*x^(3*n) - (c^3*d^3*(4*n - 1) +
 a*c^2*d*(8*n - 1)*e^2)*x*x^(2*n) + (a*c^2*d^2*(5*n - 1)*e + a^2*c*(9*n - 1
)*e^3)*x*x^n - (a*c^2*d^3*(6*n - 1) + a^2*c*d*(10*n - 1)*e^2)*x)/(a^4*c^2*d
^4*n^2 + 2*a^5*c*d^2*n^2*e^2 + a^6*n^2*e^4 + (a^2*c^4*d^4*n^2 + 2*a^3*c^3*d
^2*n^2*e^2 + a^4*c^2*n^2*e^4)*x^(4*n) + 2*(a^3*c^3*d^4*n^2 + 2*a^4*c^2*d^2*
n^2*e^2 + a^5*c*n^2*e^4)*x^(2*n)) - integrate(-1/8*((8*n^2 - 6*n + 1)*c^3*d
^5 + 2*(12*n^2 - 8*n + 1)*a*c^2*d^3*e^2 + (24*n^2 - 10*n + 1)*a^2*c*d*e^4 -
 ((3*n^2 - 4*n + 1)*c^3*d^4*e + 2*(5*n^2 - 6*n + 1)*a*c^2*d^2*e^3 + (15*n^2
 - 8*n + 1)*a^2*c*e^5)*x^n)/(a^3*c^3*d^6*n^2 + 3*a^4*c^2*d^4*n^2*e^2 + 3*a^
5*c*d^2*n^2*e^4 + a^6*n^2*e^6 + (a^2*c^4*d^6*n^2 + 3*a^3*c^3*d^4*n^2*e^2 +
3*a^4*c^2*d^2*n^2*e^4 + a^5*c*n^2*e^6)*x^(2*n)), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d+e*x^n)/(a+c*x^(2*n))^3,x, algorithm="fricas")
```

```
[Out] integral(1/(a^3*x^n*e + a^3*d + (c^3*x^n*e + c^3*d)*x^(6*n) + 3*(a*c^2*x^n*
e + a*c^2*d)*x^(4*n) + 3*(a^2*c*x^n*e + a^2*c*d)*x^(2*n)), x)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d+e*x**n)/(a+c*x**(2*n))**3,x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d+e*x^n)/(a+c*x^(2*n))^3,x, algorithm="giac")
```

```
[Out] integrate(1/((c*x^(2*n) + a)^3*(x^n*e + d)), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{{\left(a+c\,x^{2\,n}\right)}^3\,\left(d+e\,x^n\right)} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + c*x^(2*n))^3*(d + e*x^n)),x)
```

```
[Out] int(1/((a + c*x^(2*n))^3*(d + e*x^n)), x)
```

3.57 $\int \frac{1}{(d+ex^n)^2 (a+cx^{2n})^3} \, dx$

Optimal. Leaf size=701

$$\frac{cx(cd^2-ae^2-2cdex^n)}{4a\left(cd^2+ae^2\right)^2n\left(a+cx^{2n}\right)^2}+\frac{ce^2x(3cd^2-ae^2-4cdex^n)}{2a\left(cd^2+ae^2\right)^3n\left(a+cx^{2n}\right)}-\frac{cx\left(\left(cd^2-ae^2\right)(1-4n)-2cde(1-3n)x^n\right)}{8a^2\left(cd^2+ae^2\right)^2n^2\left(a+cx^{2n}\right)}+\frac{c}{-}$$

```
[Out] 1/4*c*x*(c*d^2-a*e^2-2*c*d*e*x^n)/a/(a*e^2+c*d^2)^2/n/(a+c*x^(2*n))^2+1/2*c
*e^2*x*(3*c*d^2-a*e^2-4*c*d*e*x^n)/a/(a*e^2+c*d^2)^3/n/(a+c*x^(2*n))-1/8*c*
x*((-a*e^2+c*d^2)*(1-4*n)-2*c*d*e*(1-3*n)*x^n)/a^2/(a*e^2+c*d^2)^2/n^2/(a+c
*x^(2*n))+c*e^4*(-a*e^2+5*c*d^2)*x*hypergeom([1, 1/2/n],[1+1/2/n],-c*x^(2*n
)/a)/a/(a*e^2+c*d^2)^4+1/8*c*(-a*e^2+c*d^2)*(1-4*n)*(1-2*n)*x*hypergeom([1,
 1/2/n],[1+1/2/n],-c*x^(2*n)/a)/a^3/(a*e^2+c*d^2)^2/n^2-1/2*c*e^2*(-a*e^2+3
*c*d^2)*(1-2*n)*x*hypergeom([1, 1/2/n],[1+1/2/n],-c*x^(2*n)/a)/a^2/(a*e^2+c
*d^2)^3/n+6*c*e^6*x*hypergeom([1, 1/n],[1+1/n],-e*x^n/d)/(a*e^2+c*d^2)^4-6*
c^2*d*e^5*x^(1+n)*hypergeom([1, 1/2*(1+n)/n],[3/2+1/2/n],-c*x^(2*n)/a)/a/(a
*e^2+c*d^2)^4/(1+n)-1/4*c^2*d*e*(1-3*n)*(1-n)*x^(1+n)*hypergeom([1, 1/2*(1+
n)/n],[3/2+1/2/n],-c*x^(2*n)/a)/a^3/(a*e^2+c*d^2)^2/n^2/(1+n)+2*c^2*d*e^3*(
1-n)*x^(1+n)*hypergeom([1, 1/2*(1+n)/n],[3/2+1/2/n],-c*x^(2*n)/a)/a^2/(a*e^
2+c*d^2)^3/n/(1+n)+e^6*x*hypergeom([2, 1/n],[1+1/n],-e*x^n/d)/d^2/(a*e^2+c*
d^2)^3
```

Rubi [A]
time = 0.48, antiderivative size = 701, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used = {1451, 251, 1445, 1432, 371}

$$-\frac{c^2de(1-3n)(1-n)x^{n+1}\,{}_2F_1\left(1,\frac{n+1}{2n};\frac{1}{2}\left(3+\frac{1}{n}\right);-\frac{cx^{2n}}{a}\right)}{4a^3n^2(n+1)\left(ae^2+cd^2\right)^2}+\frac{c(1-4n)(1-2n)x(cd^2-ae^2)\,{}_2F_1\left(1,\frac{1}{2n};\frac{1}{2}\left(2+\frac{1}{n}\right);-\frac{cx^{2n}}{a}\right)}{8a^3n^2\left(ae^2+cd^2\right)^2}+\frac{2c^2de^3(1-n)x^{n+1}\,{}_2F_1\left(1,\frac{n+1}{2n};\frac{1}{2}\left(3+\frac{1}{n}\right);-\frac{cx^{2n}}{a}\right)}{a^2n(n+1)\left(ae^2+cd^2\right)^3}-\frac{ce^2(1-2n)x(3cd^2-ae^2)\,{}_2F_1\left(1,\frac{1}{2n};\frac{1}{2}\left(2+\frac{1}{n}\right);-\frac{cx^{2n}}{a}\right)}{2a^2n\left(ae^2+cd^2\right)^3}-\frac{cx\left((1-4n)\left(cd^2-ae^2\right)-2cde(1-3n)x^n\right)}{8a^2n^2\left(ae^2+cd^2\right)^2\left(a+cx^{2n}\right)}-\frac{6c^2de^5x^{n+1}\,{}_2F_1\left(1,\frac{n+1}{2n};\frac{1}{2}\left(3+\frac{1}{n}\right);-\frac{cx^{2n}}{a}\right)}{a(n+1)\left(ae^2+cd^2\right)^4}+\frac{ce^2x(-ae^2+3cd^2-4cdex^n)}{2an\left(ae^2+cd^2\right)^3\left(a+cx^{2n}\right)}+\frac{cx(-ae^2+cd^2-2cdex^n)}{4an\left(ae^2+cd^2\right)^2\left(a+cx^{2n}\right)^2}+\frac{6ce^6x\,{}_2F_1\left(1,\frac{1}{n};1+\frac{1}{n};-\frac{ex^n}{d}\right)}{\left(ae^2+cd^2\right)^4}+\frac{e^6x\,{}_2F_1\left(2,\frac{1}{n};1+\frac{1}{n};-\frac{ex^n}{d}\right)}{d^2\left(ae^2+cd^2\right)^3}+\frac{ce^4x(5cd^2-ae^2)\,{}_2F_1\left(1,\frac{1}{2n};\frac{1}{2}\left(2+\frac{1}{n}\right);-\frac{cx^{2n}}{a}\right)}{a\left(ae^2+cd^2\right)^4}$$

Antiderivative was successfully verified.

```
[In] Int[1/((d + e*x^n)^2*(a + c*x^(2*n))^3),x]
```

```
[Out] (c*x*(c*d^2 - a*e^2 - 2*c*d*e*x^n))/(4*a*(c*d^2 + a*e^2)^2*n*(a + c*x^(2*n)
)^2) + (c*e^2*x*(3*c*d^2 - a*e^2 - 4*c*d*e*x^n))/(2*a*(c*d^2 + a*e^2)^3*n*(
a + c*x^(2*n))) - (c*x*((c*d^2 - a*e^2)*(1 - 4*n) - 2*c*d*e*(1 - 3*n)*x^n))
/(8*a^2*(c*d^2 + a*e^2)^2*n^2*(a + c*x^(2*n))) + (c*e^4*(5*c*d^2 - a*e^2)*x
*Hypergeometric2F1[1, 1/(2*n), (2 + n^(-1))/2, -((c*x^(2*n))/a)])/(a*(c*d^2
 + a*e^2)^4) + (c*(c*d^2 - a*e^2)*(1 - 4*n)*(1 - 2*n)*x*Hypergeometric2F1[1
, 1/(2*n), (2 + n^(-1))/2, -((c*x^(2*n))/a)])/(8*a^3*(c*d^2 + a*e^2)^2*n^2)
 - (c*e^2*(3*c*d^2 - a*e^2)*(1 - 2*n)*x*Hypergeometric2F1[1, 1/(2*n), (2 +
n^(-1))/2, -((c*x^(2*n))/a)])/(2*a^2*(c*d^2 + a*e^2)^3*n) + (6*c*e^6*x*Hype
rgeometric2F1[1, n^(-1), 1 + n^(-1), -((e*x^n)/d)])/(c*d^2 + a*e^2)^4 - (6*
c^2*d*e^5*x^(1 + n)*Hypergeometric2F1[1, (1 + n)/(2*n), (3 + n^(-1))/2, -((
c*x^(2*n))/a)])/(a*(c*d^2 + a*e^2)^4*(1 + n)) - (c^2*d*e*(1 - 3*n)*(1 - n)*
```

```
x^(1 + n)*Hypergeometric2F1[1, (1 + n)/(2*n), (3 + n^(-1))/2, -((c*x^(2*n))
/a)])/(4*a^3*(c*d^2 + a*e^2)^2*n^2*(1 + n)) + (2*c^2*d*e^3*(1 - n)*x^(1 + n
)*Hypergeometric2F1[1, (1 + n)/(2*n), (3 + n^(-1))/2, -((c*x^(2*n))/a)])/(a
^2*(c*d^2 + a*e^2)^3*n*(1 + n)) + (e^6*x*Hypergeometric2F1[2, n^(-1), 1 + n
^(-1), -((e*x^n)/d)])/(d^2*(c*d^2 + a*e^2)^3)
```

Rule 251

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, (-b)*(x^n/a)], x] /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p
, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 1432

```
Int[((d_) + (e_.)*(x_)^(n_))/((a_) + (c_.)*(x_)^(n2_)), x_Symbol] :> Dist[d
, Int[1/(a + c*x^(2*n)), x], x] + Dist[e, Int[x^n/(a + c*x^(2*n)), x], x] /
; FreeQ[{a, c, d, e, n}, x] && EqQ[n2, 2*n] && NeQ[c*d^2 + a*e^2, 0] && (Po
sQ[a*c] ||  !IntegerQ[n])
```

Rule 1445

```
Int[((d_) + (e_.)*(x_)^(n_))*((a_) + (c_.)*(x_)^(n2_))^(p_), x_Symbol] :> S
imp[(-x)*(d + e*x^n)*((a + c*x^(2*n))^(p + 1)/(2*a*n*(p + 1))), x] + Dist[1
/(2*a*n*(p + 1)), Int[(d*(2*n*p + 2*n + 1) + e*(2*n*p + 3*n + 1)*x^n)*(a +
c*x^(2*n))^(p + 1), x], x] /; FreeQ[{a, c, d, e, n}, x] && EqQ[n2, 2*n] &&
ILtQ[p, -1]
```

Rule 1451

```
Int[((d_) + (e_.)*(x_)^(n_))^(q_)*((a_) + (c_.)*(x_)^(n2_))^(p_), x_Symbol]
 :> Int[ExpandIntegrand[(d + e*x^n)^q*(a + c*x^(2*n))^p, x], x] /; FreeQ[{a
, c, d, e, n, p, q}, x] && EqQ[n2, 2*n] && NeQ[c*d^2 + a*e^2, 0] && ((Integ
ersQ[p, q] &&  !IntegerQ[n]) || IGtQ[p, 0] || (IGtQ[q, 0] &&  !IntegerQ[n])
)
```

Rubi steps

$$\int \frac{1}{(d+ex^n)^2 (a+cx^{2n})^3}\,dx = \int \left(\frac{e^6}{(cd^2+ae^2)^3 (d+ex^n)^2} + \frac{6cde^6}{(cd^2+ae^2)^4 (d+ex^n)} - \frac{c(-cd^2+ae^2+2cd}{(cd^2+ae^2)^2 (a+c}\right.$$

$$= -\frac{(ce^4)\int \frac{-5cd^2+ae^2+6cdex^n}{a+cx^{2n}}\,dx}{(cd^2+ae^2)^4} + \frac{(6cde^6)\int \frac{1}{d+ex^n}\,dx}{(cd^2+ae^2)^4} - \frac{(ce^2)\int \frac{-3cd^2+ae^2+4cdex^n}{(a+cx^{2n})^2}}{(cd^2+ae^2)^3}$$

$$= \frac{cx(cd^2-ae^2-2cdex^n)}{4a(cd^2+ae^2)^2 n (a+cx^{2n})^2} + \frac{ce^2x(3cd^2-ae^2-4cdex^n)}{2a(cd^2+ae^2)^3 n (a+cx^{2n})} + \frac{6ce^6x\,{}_2F_1(1,}{(cd^2}$$

$$= \frac{cx(cd^2-ae^2-2cdex^n)}{4a(cd^2+ae^2)^2 n (a+cx^{2n})^2} + \frac{ce^2x(3cd^2-ae^2-4cdex^n)}{2a(cd^2+ae^2)^3 n (a+cx^{2n})} - \frac{cx((cd^2-ae}{8a^2(c}$$

$$= \frac{cx(cd^2-ae^2-2cdex^n)}{4a(cd^2+ae^2)^2 n (a+cx^{2n})^2} + \frac{ce^2x(3cd^2-ae^2-4cdex^n)}{2a(cd^2+ae^2)^3 n (a+cx^{2n})} - \frac{cx((cd^2-ae}{8a^2(c}$$

$$= \frac{cx(cd^2-ae^2-2cdex^n)}{4a(cd^2+ae^2)^2 n (a+cx^{2n})^2} + \frac{ce^2x(3cd^2-ae^2-4cdex^n)}{2a(cd^2+ae^2)^3 n (a+cx^{2n})} - \frac{cx((cd^2-ae}{8a^2(c}$$

Mathematica [A]
time = 1.55, size = 1241, normalized size = 1.77

Antiderivative was successfully verified.

```
[In] Integrate[1/((d + e*x^n)^2*(a + c*x^(2*n))^3),x]
```

```
[Out] (x*((8*c*d^2*e^6 + 8*a*e^8)/(d^2*n + d*e*n*x^n) + (2*c*(c*d^2 + a*e^2)^2*(-
(a*e^2) + c*d*(d - 2*e*x^n)))/(a*n*(a + c*x^(2*n))^2) + (c*(c*d^2 + a*e^2)*
(a^2*e^4*(1 - 8*n) + c^2*d^3*(d*(-1 + 4*n) - 2*e*(-1 + 3*n)*x^n) + 2*a*c*d*
e^2*(6*d*n - e*(-1 + 11*n)*x^n)))/(a^2*n^2*(a + c*x^(2*n))) + (8*c^4*d^6*Hy
pergeometric2F1[1, 1/(2*n), 1 + 1/(2*n), -((c*x^(2*n))/a)])/a^3 + (32*c^3*d
^4*e^2*Hypergeometric2F1[1, 1/(2*n), 1 + 1/(2*n), -((c*x^(2*n))/a)])/a^2 +
(48*c^2*d^2*e^4*Hypergeometric2F1[1, 1/(2*n), 1 + 1/(2*n), -((c*x^(2*n))/a)
])/a - 24*c*e^6*Hypergeometric2F1[1, 1/(2*n), 1 + 1/(2*n), -((c*x^(2*n))/a)
] + (c^4*d^6*Hypergeometric2F1[1, 1/(2*n), 1 + 1/(2*n), -((c*x^(2*n))/a)])/
(a^3*n^2) + (c^3*d^4*e^2*Hypergeometric2F1[1, 1/(2*n), 1 + 1/(2*n), -((c*x^
(2*n))/a)])/(a^2*n^2) - (c^2*d^2*e^4*Hypergeometric2F1[1, 1/(2*n), 1 + 1/(2
*n), -((c*x^(2*n))/a)])/(a*n^2) - (c*e^6*Hypergeometric2F1[1, 1/(2*n), 1 +
1/(2*n), -((c*x^(2*n))/a)])/n^2 - (6*c^4*d^6*Hypergeometric2F1[1, 1/(2*n),
1 + 1/(2*n), -((c*x^(2*n))/a)])/(a^3*n) - (18*c^3*d^4*e^2*Hypergeometric2F1
[1, 1/(2*n), 1 + 1/(2*n), -((c*x^(2*n))/a)])/(a^2*n) - (2*c^2*d^2*e^4*Hyper
geometric2F1[1, 1/(2*n), 1 + 1/(2*n), -((c*x^(2*n))/a)])/(a*n) + (10*c*e^6*
```

```
Hypergeometric2F1[1, 1/(2*n), 1 + 1/(2*n), -((c*x^(2*n))/a)])/n + (8*e^6*(a
*e^2*(-1 + n) + c*d^2*(-1 + 7*n))*Hypergeometric2F1[1, n^(-1), 1 + n^(-1),
-((e*x^n)/d)])/(d^2*n) - (6*c^4*d^5*e*x^n*Hypergeometric2F1[1, (1 + n)/(2*n
), (3 + n^(-1))/2, -((c*x^(2*n))/a)])/(a^3*(1 + n)) - (28*c^3*d^3*e^3*x^n*H
ypergeometric2F1[1, (1 + n)/(2*n), (3 + n^(-1))/2, -((c*x^(2*n))/a)])/(a^2*
(1 + n)) - (70*c^2*d*e^5*x^n*Hypergeometric2F1[1, (1 + n)/(2*n), (3 + n^(-1
))/2, -((c*x^(2*n))/a)])/(a*(1 + n)) - (2*c^4*d^5*e*x^n*Hypergeometric2F1[1
, (1 + n)/(2*n), (3 + n^(-1))/2, -((c*x^(2*n))/a)])/(a^3*n^2*(1 + n)) - (4*
c^3*d^3*e^3*x^n*Hypergeometric2F1[1, (1 + n)/(2*n), (3 + n^(-1))/2, -((c*x^
(2*n))/a)])/(a^2*n^2*(1 + n)) - (2*c^2*d*e^5*x^n*Hypergeometric2F1[1, (1 +
n)/(2*n), (3 + n^(-1))/2, -((c*x^(2*n))/a)])/(a*n^2*(1 + n)) + (8*c^4*d^5*e
*x^n*Hypergeometric2F1[1, (1 + n)/(2*n), (3 + n^(-1))/2, -((c*x^(2*n))/a)])
/(a^3*n*(1 + n)) + (32*c^3*d^3*e^3*x^n*Hypergeometric2F1[1, (1 + n)/(2*n),
(3 + n^(-1))/2, -((c*x^(2*n))/a)])/(a^2*n*(1 + n)) + (24*c^2*d*e^5*x^n*Hype
rgeometric2F1[1, (1 + n)/(2*n), (3 + n^(-1))/2, -((c*x^(2*n))/a)])/(a*n*(1
+ n))))/(8*(c*d^2 + a*e^2)^4)
```

Maple [F]
time = 0.04, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(d+e\,x^{n}\right)^{2}\left(a+c\,x^{2n}\right)^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(d+e*x^n)^2/(a+c*x^(2*n))^3,x)
```

```
[Out] int(1/(d+e*x^n)^2/(a+c*x^(2*n))^3,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d+e*x^n)^2/(a+c*x^(2*n))^3,x, algorithm="maxima")
```

```
[Out] (c*d^2*(7*n - 1)*e^6 + a*(n - 1)*e^8)*integrate(1/(c^4*d^10*n + 4*a*c^3*d^8
*n*e^2 + 6*a^2*c^2*d^6*n*e^4 + 4*a^3*c*d^4*n*e^6 + a^4*d^2*n*e^8 + (c^4*d^9
*n*e + 4*a*c^3*d^7*n*e^3 + 6*a^2*c^2*d^5*n*e^5 + 4*a^3*c*d^3*n*e^7 + a^4*d*
n*e^9)*x^n), x) - 1/8*(2*(c^4*d^4*(3*n - 1)*e^2 + a*c^3*d^2*(11*n - 1)*e^4
- 4*a^2*c^2*n*e^6)*x*x^(4*n) + (c^4*d^5*(2*n - 1)*e + 2*a*c^3*d^3*(5*n - 1)
*e^3 + a^2*c^2*d*(8*n - 1)*e^5)*x*x^(3*n) - (c^4*d^6*(4*n - 1) + 2*a*c^3*d^
4*(n + 1)*e^2 - a^2*c^2*d^2*(34*n - 3)*e^4 + 16*a^3*c*n*e^6)*x*x^(2*n) + (a
*c^3*d^5*(4*n - 1)*e + 2*a^2*c^2*d^3*(7*n - 1)*e^3 + a^3*c*d*(10*n - 1)*e^5
)*x*x^n - (a*c^3*d^6*(6*n - 1) + 12*a^2*c^2*d^4*n*e^2 - a^3*c*d^2*(10*n - 1
)*e^4 + 8*a^4*n*e^6)*x)/(a^4*c^3*d^8*n^2 + 3*a^5*c^2*d^6*n^2*e^2 + 3*a^6*c*
```

```
d^4*n^2*e^4 + a^7*d^2*n^2*e^6 + (a^2*c^5*d^7*n^2*e + 3*a^3*c^4*d^5*n^2*e^3
+ 3*a^4*c^3*d^3*n^2*e^5 + a^5*c^2*d*n^2*e^7)*x^(5*n) + (a^2*c^5*d^8*n^2 + 3
*a^3*c^4*d^6*n^2*e^2 + 3*a^4*c^3*d^4*n^2*e^4 + a^5*c^2*d^2*n^2*e^6)*x^(4*n)
 + 2*(a^3*c^4*d^7*n^2*e + 3*a^4*c^3*d^5*n^2*e^3 + 3*a^5*c^2*d^3*n^2*e^5 + a
^6*c*d*n^2*e^7)*x^(3*n) + 2*(a^3*c^4*d^8*n^2 + 3*a^4*c^3*d^6*n^2*e^2 + 3*a^
5*c^2*d^4*n^2*e^4 + a^6*c*d^2*n^2*e^6)*x^(2*n) + (a^4*c^3*d^7*n^2*e + 3*a^5
*c^2*d^5*n^2*e^3 + 3*a^6*c*d^3*n^2*e^5 + a^7*d*n^2*e^7)*x^n) - integrate(-1
/8*((8*n^2 - 6*n + 1)*c^4*d^6 + (32*n^2 - 18*n + 1)*a*c^3*d^4*e^2 + (48*n^2
 - 2*n - 1)*a^2*c^2*d^2*e^4 - (24*n^2 - 10*n + 1)*a^3*c*e^6 - 2*((3*n^2 - 4
*n + 1)*c^4*d^5*e + 2*(7*n^2 - 8*n + 1)*a*c^3*d^3*e^3 + (35*n^2 - 12*n + 1)
*a^2*c^2*d*e^5)*x^n)/(a^3*c^4*d^8*n^2 + 4*a^4*c^3*d^6*n^2*e^2 + 6*a^5*c^2*d
^4*n^2*e^4 + 4*a^6*c*d^2*n^2*e^6 + a^7*n^2*e^8 + (a^2*c^5*d^8*n^2 + 4*a^3*c
^4*d^6*n^2*e^2 + 6*a^4*c^3*d^4*n^2*e^4 + 4*a^5*c^2*d^2*n^2*e^6 + a^6*c*n^2*
e^8)*x^(2*n)), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d+e*x^n)^2/(a+c*x^(2*n))^3,x, algorithm="fricas")
```

```
[Out] integral(1/(2*a^3*d*x^n*e + a^3*d^2 + a^3*x^(2*n)*e^2 + (2*c^3*d*x^n*e + c^
3*d^2 + c^3*x^(2*n)*e^2)*x^(6*n) + 3*(2*a*c^2*d*x^n*e + a*c^2*d^2 + a*c^2*x
^(2*n)*e^2)*x^(4*n) + 3*(2*a^2*c*d*x^n*e + a^2*c*d^2 + a^2*c*x^(2*n)*e^2)*x
^(2*n)), x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d+e*x**n)**2/(a+c*x**(2*n))**3,x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d+e*x^n)^2/(a+c*x^(2*n))^3,x, algorithm="giac")
```

```
[Out] integrate(1/((c*x^(2*n) + a)^3*(x^n*e + d)^2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{(a+c\,x^{2\,n})^3\,(d+e\,x^n)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + c*x^(2*n))^3*(d + e*x^n)^2),x)
[Out] int(1/((a + c*x^(2*n))^3*(d + e*x^n)^2), x)
```

3.58 $\int \frac{1}{(d+ex^n)\sqrt{a+cx^{2n}}}\,dx$

Optimal. Leaf size=171

$$\frac{x\sqrt{1+\frac{cx^{2n}}{a}}\,F_1\left(\frac{1}{2n};\frac{1}{2},1;\frac{1}{2}\left(2+\frac{1}{n}\right);-\frac{cx^{2n}}{a},\frac{e^2x^{2n}}{d^2}\right)}{d\sqrt{a+cx^{2n}}}-\frac{ex^{1+n}\sqrt{1+\frac{cx^{2n}}{a}}\,F_1\left(\frac{1+n}{2n};\frac{1}{2},1;\frac{1}{2}\left(3+\frac{1}{n}\right);-\frac{cx^{2n}}{a},\frac{e^2x^{2n}}{d^2}\right)}{d^2(1+n)\sqrt{a+cx^{2n}}}$$

```
[Out] x*AppellF1(1/2/n,1,1/2,1+1/2/n,e^2*x^(2*n)/d^2,-c*x^(2*n)/a)*(1+c*x^(2*n)/a
)^(1/2)/d/(a+c*x^(2*n))^(1/2)-e*x^(1+n)*AppellF1(1/2*(1+n)/n,1,1/2,3/2+1/2/
n,e^2*x^(2*n)/d^2,-c*x^(2*n)/a)*(1+c*x^(2*n)/a)^(1/2)/d^2/(1+n)/(a+c*x^(2*n
))^(1/2)
```

Rubi [A]
time = 0.12, antiderivative size = 171, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {1452, 441, 440, 525, 524}

$$\frac{x\sqrt{\frac{cx^{2n}}{a}+1}\,F_1\left(\frac{1}{2n};\frac{1}{2},1;\frac{1}{2}\left(2+\frac{1}{n}\right);-\frac{cx^{2n}}{a},\frac{e^2x^{2n}}{d^2}\right)}{d\sqrt{a+cx^{2n}}}-\frac{ex^{n+1}\sqrt{\frac{cx^{2n}}{a}+1}\,F_1\left(\frac{n+1}{2n};\frac{1}{2},1;\frac{1}{2}\left(3+\frac{1}{n}\right);-\frac{cx^{2n}}{a},\frac{e^2x^{2n}}{d^2}\right)}{d^2(n+1)\sqrt{a+cx^{2n}}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((d + e*x^n)*Sqrt[a + c*x^(2*n)]),x]
```

```
[Out] (x*Sqrt[1 + (c*x^(2*n))/a]*AppellF1[1/(2*n), 1/2, 1, (2 + n^(-1))/2, -((c*x
^(2*n))/a), (e^2*x^(2*n))/d^2])/(d*Sqrt[a + c*x^(2*n)]) - (e*x^(1 + n)*Sqrt
[1 + (c*x^(2*n))/a]*AppellF1[(1 + n)/(2*n), 1/2, 1, (3 + n^(-1))/2, -((c*x^
(2*n))/a), (e^2*x^(2*n))/d^2])/(d^2*(1 + n)*Sqrt[a + c*x^(2*n)])
```

Rule 440

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, (-b)*(x^n/a), (-d)*(x^n/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 441

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Dist[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]),
 Int[(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q}
, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 524

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 525

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^
n/a))^FracPart[p]), Int[(e*x)^m*(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 1452

```
Int[((d_) + (e_.)*(x_)^(n_))^(q_)*((a_) + (c_.)*(x_)^(n2_))^(p_), x_Symbol]
 :> Int[ExpandIntegrand[(a + c*x^(2*n))^p, (d/(d^2 - e^2*x^(2*n)) - e*(x^n/
(d^2 - e^2*x^(2*n))))^(-q), x], x] /; FreeQ[{a, c, d, e, n, p}, x] && EqQ[n
2, 2*n] && NeQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && ILtQ[q, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{(d+ex^n)\sqrt{a+cx^{2n}}}\,dx &= \int \left(\frac{d}{\sqrt{a+cx^{2n}}\,(d^2-e^2x^{2n})}+\frac{ex^n}{\sqrt{a+cx^{2n}}\,(-d^2+e^2x^{2n})}\right)dx\\
&= d\int \frac{1}{\sqrt{a+cx^{2n}}\,(d^2-e^2x^{2n})}\,dx+e\int \frac{x^n}{\sqrt{a+cx^{2n}}\,(-d^2+e^2x^{2n})}\,dx\\
&= \frac{\left(d\sqrt{1+\frac{cx^{2n}}{a}}\right)\int \frac{1}{\sqrt{1+\frac{cx^{2n}}{a}}\,(d^2-e^2x^{2n})}\,dx}{\sqrt{a+cx^{2n}}}+\frac{\left(e\sqrt{1+\frac{cx^{2n}}{a}}\right)\int \frac{\cdots}{\sqrt{1+\frac{cx^{2n}}{a}}\cdots}}{\sqrt{a+cx^{2n}}}\\
&= \frac{x\sqrt{1+\frac{cx^{2n}}{a}}\,F_1\left(\frac{1}{2n};\frac{1}{2},1;\frac{1}{2}\left(2+\frac{1}{n}\right);-\frac{cx^{2n}}{a},\frac{e^2x^{2n}}{d^2}\right)}{d\sqrt{a+cx^{2n}}}-\frac{ex^{1+n}\sqrt{1+\frac{cx^{2n}}{a}}\,F_1\left(\frac{1+n}{2n}\cdots\right.}{d^2(1+n)\cdots}\end{aligned}$$

Mathematica [F]
time = 0.16, size = 0, normalized size = 0.00

$$\int \frac{1}{(d+ex^n)\sqrt{a+cx^{2n}}}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[1/((d + e*x^n)*Sqrt[a + c*x^(2*n)]),x]
```

```
[Out] Integrate[1/((d + e*x^n)*Sqrt[a + c*x^(2*n)]), x]
```

Maple [F]
time = 0.07, size = 0, normalized size = 0.00

$$\int \frac{1}{(d+e\,x^n)\,\sqrt{a+c\,x^{2n}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(d+e*x^n)/(a+c*x^(2*n))^(1/2),x)
```

```
[Out] int(1/(d+e*x^n)/(a+c*x^(2*n))^(1/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d+e*x^n)/(a+c*x^(2*n))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(c*x^(2*n) + a)*(x^n*e + d)), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d+e*x^n)/(a+c*x^(2*n))^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(c*x^(2*n) + a)/(a*x^n*e + a*d + (c*x^n*e + c*d)*x^(2*n)), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{a+cx^{2n}}\,(d+ex^n)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d+e*x**n)/(a+c*x**(2*n))**(1/2),x)
```

```
[Out] Integral(1/(sqrt(a + c*x**(2*n))*(d + e*x**n)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d+e*x^n)/(a+c*x^(2*n))^(1/2),x, algorithm="giac")
[Out] integrate(1/(sqrt(c*x^(2*n) + a)*(x^n*e + d)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\sqrt{a+c\,x^{2\,n}}\,\left(d+e\,x^n\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + c*x^(2*n))^(1/2)*(d + e*x^n)),x)
[Out] int(1/((a + c*x^(2*n))^(1/2)*(d + e*x^n)), x)
```

3.59 $\int (d+ex^n)^q \left(a+cx^{2n}\right)^p \, dx$

Optimal. Leaf size=24

$$\text{Int}\left((d+ex^n)^q \left(a+cx^{2n}\right)^p, x\right)$$

```
[Out] Unintegrable((d+e*x^n)^q*(a+c*x^(2*n))^p,x)
```

Rubi [A]
time = 0.01, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int (d+ex^n)^q \left(a+cx^{2n}\right)^p \, dx$$

Verification is not applicable to the result.

```
[In] Int[(d + e*x^n)^q*(a + c*x^(2*n))^p,x]
[Out] Defer[Int][(d + e*x^n)^q*(a + c*x^(2*n))^p, x]
```

Rubi steps

$$\int (d+ex^n)^q \left(a+cx^{2n}\right)^p \, dx = \int (d+ex^n)^q \left(a+cx^{2n}\right)^p \, dx$$

Mathematica [A]
time = 0.14, size = 0, normalized size = 0.00

$$\int (d+ex^n)^q \left(a+cx^{2n}\right)^p \, dx$$

Verification is not applicable to the result.

```
[In] Integrate[(d + e*x^n)^q*(a + c*x^(2*n))^p,x]
[Out] Integrate[(d + e*x^n)^q*(a + c*x^(2*n))^p, x]
```

Maple [A]
time = 0.08, size = 0, normalized size = 0.00

$$\int (d+e\,x^n)^q \left(a+c\,x^{2n}\right)^p \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d+e*x^n)^q*(a+c*x^(2*n))^p,x)
```

```
[Out] int((d+e*x^n)^q*(a+c*x^(2*n))^p,x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+e*x^n)^q*(a+c*x^(2*n))^p,x, algorithm="maxima")
[Out] integrate((c*x^(2*n) + a)^p*(x^n*e + d)^q, x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+e*x^n)^q*(a+c*x^(2*n))^p,x, algorithm="fricas")
[Out] integral((c*x^(2*n) + a)^p*(x^n*e + d)^q, x)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+e*x**n)**q*(a+c*x**(2*n))**p,x)
[Out] Timed out
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+e*x^n)^q*(a+c*x^(2*n))^p,x, algorithm="giac")
[Out] integrate((c*x^(2*n) + a)^p*(x^n*e + d)^q, x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.04

$$\int (a + c\,x^{2\,n})^p\,(d + e\,x^n)^q\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + c*x^(2*n))^p*(d + e*x^n)^q,x)
[Out] int((a + c*x^(2*n))^p*(d + e*x^n)^q, x)
```

3.60 $\int (d+ex^n)^3 (a+cx^{2n})^p \, dx$

Optimal. Leaf size=299

$$\frac{3de^2x^{1+2n}(a+cx^{2n})^p\left(1+\frac{cx^{2n}}{a}\right)^{-p}{}_2F_1\left(\frac{1}{2}\left(2+\frac{1}{n}\right),-p;\frac{1}{2}\left(4+\frac{1}{n}\right);-\frac{cx^{2n}}{a}\right)}{1+2n}+\frac{e^3x^{1+3n}(a+cx^{2n})^p\left(1+\frac{cx^{2n}}{a}\right)^{-}$$

```
[Out] 3*d*e^2*x^(1+2*n)*(a+c*x^(2*n))^p*hypergeom([-p, 1+1/2/n],[2+1/2/n],-c*x^(2
*n)/a)/(1+2*n)/((1+c*x^(2*n)/a)^p)+e^3*x^(1+3*n)*(a+c*x^(2*n))^p*hypergeom(
[-p, 3/2+1/2/n],[5/2+1/2/n],-c*x^(2*n)/a)/(1+3*n)/((1+c*x^(2*n)/a)^p)+d^3*x
*(a+c*x^(2*n))^p*hypergeom([-p, 1/2/n],[1+1/2/n],-c*x^(2*n)/a)/((1+c*x^(2*n
)/a)^p)+3*d^2*e*x^(1+n)*(a+c*x^(2*n))^p*hypergeom([-p, 1/2*(1+n)/n],[3/2+1/
2/n],-c*x^(2*n)/a)/(1+n)/((1+c*x^(2*n)/a)^p)
```

Rubi [A]

time = 0.11, antiderivative size = 299, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used = {1451, 252, 251, 372, 371}

$$d^3x(a+cx^{2n})^p\left(\frac{cx^{2n}}{a}+1\right)^{-p}{}_2F_1\left(\frac{1}{2n},-p;\frac{1}{2}\left(2+\frac{1}{n}\right);-\frac{cx^{2n}}{a}\right)+\frac{3d^2ex^{n+1}(a+cx^{2n})^p\left(\frac{cx^{2n}}{a}+1\right)^{-p}{}_2F_1\left(\frac{n+1}{2n},-p;\frac{1}{2}\left(3+\frac{1}{n}\right);-\frac{cx^{2n}}{a}\right)}{n+1}+\frac{3de^2x^{2n+1}(a+cx^{2n})^p\left(\frac{cx^{2n}}{a}+1\right)^{-p}{}_2F_1\left(\frac{1}{2}\left(2+\frac{1}{n}\right),-p;\frac{1}{2}\left(4+\frac{1}{n}\right);-\frac{cx^{2n}}{a}\right)}{2n+1}+\frac{e^3x^{3n+1}(a+cx^{2n})^p\left(\frac{cx^{2n}}{a}+1\right)^{-p}{}_2F_1\left(\frac{1}{2}\left(3+\frac{1}{n}\right),-p;\frac{1}{2}\left(5+\frac{1}{n}\right);-\frac{cx^{2n}}{a}\right)}{3n+1}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x^n)^3*(a + c*x^(2*n))^p,x]
```

```
[Out] (3*d*e^2*x^(1 + 2*n)*(a + c*x^(2*n))^p*Hypergeometric2F1[(2 + n^(-1))/2, -p
, (4 + n^(-1))/2, -((c*x^(2*n))/a)])/((1 + 2*n)*(1 + (c*x^(2*n))/a)^p) + (e
^3*x^(1 + 3*n)*(a + c*x^(2*n))^p*Hypergeometric2F1[(3 + n^(-1))/2, -p, (5 +
 n^(-1))/2, -((c*x^(2*n))/a)])/((1 + 3*n)*(1 + (c*x^(2*n))/a)^p) + (d^3*x*(
a + c*x^(2*n))^p*Hypergeometric2F1[1/(2*n), -p, (2 + n^(-1))/2, -((c*x^(2*n
))/a)])/(1 + (c*x^(2*n))/a)^p + (3*d^2*e*x^(1 + n)*(a + c*x^(2*n))^p*Hyperg
eometric2F1[(1 + n)/(2*n), -p, (3 + n^(-1))/2, -((c*x^(2*n))/a)])/((1 + n)*
(1 + (c*x^(2*n))/a)^p)
```

Rule 251

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, (-b)*(x^n/a)], x] /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p
, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

Rule 252

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^IntPart[p]*((a + b*x
^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]), Int[(1 + b*(x^n/a))^p, x], x]
 /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Sim
```

```
plify[1/n + p], 0] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 372

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^I
ntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]), Int[(c*x)^
m*(1 + b*(x^n/a))^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0]
 &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

Rule 1451

```
Int[((d_) + (e_.)*(x_)^(n_))^(q_)*((a_) + (c_.)*(x_)^(n2_))^(p_), x_Symbol]
 :> Int[ExpandIntegrand[(d + e*x^n)^q*(a + c*x^(2*n))^p, x], x] /; FreeQ[{a
, c, d, e, n, p, q}, x] && EqQ[n2, 2*n] && NeQ[c*d^2 + a*e^2, 0] && ((Integ
ersQ[p, q] &&  !IntegerQ[n]) || IGtQ[p, 0] || (IGtQ[q, 0] &&  !IntegerQ[n])
)
```

Rubi steps

$$\int (d+ex^n)^3 \left(a+cx^{2n}\right)^p \, dx = \int \left(d^3\left(a+cx^{2n}\right)^p + 3d^2ex^n\left(a+cx^{2n}\right)^p + 3de^2x^{2n}\left(a+cx^{2n}\right)^p + e^3x^{3n}\left(a+cx\right.\right.$$

$$= d^3\int \left(a+cx^{2n}\right)^p \, dx + \left(3d^2e\right)\int x^n\left(a+cx^{2n}\right)^p \, dx + \left(3de^2\right)\int x^{2n}\left(a+cx^{2n}\right.$$

$$= \left(d^3\left(a+cx^{2n}\right)^p\left(1+\frac{cx^{2n}}{a}\right)^{-p}\right)\int \left(1+\frac{cx^{2n}}{a}\right)^p dx + \left(3d^2e\left(a+cx^{2n}\right)^p\left(1\right.\right.$$

$$= \frac{3de^2x^{1+2n}\left(a+cx^{2n}\right)^p\left(1+\frac{cx^{2n}}{a}\right)^{-p} {}_2F_1\left(\frac{1}{2}\left(2+\frac{1}{n}\right),-p;\frac{1}{2}\left(4+\frac{1}{n}\right);-\frac{cx^{2n}}{a}\right)}{1+2n} +$$

Mathematica [A]
time = 0.24, size = 213, normalized size = 0.71

$$x\left(a+cx^{2n}\right)^p\left(1+\frac{cx^{2n}}{a}\right)^{-p}\left(\frac{3de^2x^{2n}\,{}_2F_1\left(1+\frac{1}{2n},-p;2+\frac{1}{2n};-\frac{cx^{2n}}{a}\right)}{1+2n}+\frac{e^3x^{3n}\,{}_2F_1\left(\frac{1}{2}\left(3+\frac{1}{n}\right),-p;\frac{1}{2}\left(5+\frac{1}{n}\right);-\frac{cx^{2n}}{a}\right)}{1+3n}+d^2\left(d\,{}_2F_1\left(\frac{1}{2n},-p;1+\frac{1}{2n};-\frac{cx^{2n}}{a}\right)+\frac{3ex^n\,{}_2F_1\left(\frac{1+n}{2n},-p;\frac{1}{2}\left(3+\frac{1}{n}\right);-\frac{cx^{2n}}{a}\right)}{1+n}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x^n)^3*(a + c*x^(2*n))^p,x]
```

```
[Out] (x*(a + c*x^(2*n))^p*((3*d*e^2*x^(2*n)*Hypergeometric2F1[1 + 1/(2*n), -p, 2
 + 1/(2*n), -((c*x^(2*n))/a)])/(1 + 2*n) + (e^3*x^(3*n)*Hypergeometric2F1[(
3 + n^(-1))/2, -p, (5 + n^(-1))/2, -((c*x^(2*n))/a)])/(1 + 3*n) + d^2*(d*Hy
pergeometric2F1[1/(2*n), -p, 1 + 1/(2*n), -((c*x^(2*n))/a)] + (3*e*x^n*Hype
rgeometric2F1[(1 + n)/(2*n), -p, (3 + n^(-1))/2, -((c*x^(2*n))/a)])/(1 + n)
)))/(1 + (c*x^(2*n))/a)^p
```

Maple [F]
time = 0.05, size = 0, normalized size = 0.00

$$\int \left(d+e\,x^{n}\right)^{3} \left(a+c\,x^{2 n}\right)^{p}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d+e*x^n)^3*(a+c*x^(2*n))^p,x)
```

```
[Out] int((d+e*x^n)^3*(a+c*x^(2*n))^p,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+e*x^n)^3*(a+c*x^(2*n))^p,x, algorithm="maxima")
```

```
[Out] integrate((x^n*e + d)^3*(c*x^(2*n) + a)^p, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+e*x^n)^3*(a+c*x^(2*n))^p,x, algorithm="fricas")
```

```
[Out] integral((3*d^2*x^n*e + d^3 + 3*d*x^(2*n)*e^2 + x^(3*n)*e^3)*(c*x^(2*n) + a
)^p, x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+e*x**n)**3*(a+c*x**(2*n))**p,x)
```

```
[Out] Timed out
```

Giac [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+e*x^n)^3*(a+c*x^(2*n))^p,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Unable to divide, perhaps due to roun
ding error%%%{-16,[1,0,6,3,2,4,4,1]%%%}+%%%{-64,[1,0,6,3,2,3,4,1]%%%}+%%%{-
96,[1,
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \left(a+c\,x^{2\,n}\right)^p\,\left(d+e\,x^n\right)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + c*x^(2*n))^p*(d + e*x^n)^3,x)
```

```
[Out] int((a + c*x^(2*n))^p*(d + e*x^n)^3, x)
```

3.61 $\int (d+ex^n)^2 (a+cx^{2n})^p \, dx$

Optimal. Leaf size=217

$$\frac{e^2x^{1+2n}(a+cx^{2n})^p\left(1+\frac{cx^{2n}}{a}\right)^{-p}{}_2F_1\left(\frac{1}{2}\left(2+\frac{1}{n}\right),-p;\frac{1}{2}\left(4+\frac{1}{n}\right);-\frac{cx^{2n}}{a}\right)}{1+2n}+d^2x(a+cx^{2n})^p\left(1+\frac{cx^{2n}}{a}\right)^{-p}{}_2F_1$$

```
[Out] e^2*x^(1+2*n)*(a+c*x^(2*n))^p*hypergeom([-p, 1+1/2/n],[2+1/2/n],-c*x^(2*n)/
a)/(1+2*n)/((1+c*x^(2*n)/a)^p)+d^2*x*(a+c*x^(2*n))^p*hypergeom([-p, 1/2/n],
[1+1/2/n],-c*x^(2*n)/a)/((1+c*x^(2*n)/a)^p)+2*d*e*x^(1+n)*(a+c*x^(2*n))^p*h
ypergeom([-p, 1/2*(1+n)/n],[3/2+1/2/n],-c*x^(2*n)/a)/(1+n)/((1+c*x^(2*n)/a)
^p)
```

Rubi [A]
time = 0.07, antiderivative size = 217, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used = {1451, 252, 251, 372, 371}

$$d^2x(a+cx^{2n})^p\left(\frac{cx^{2n}}{a}+1\right)^{-p}{}_2F_1\left(\frac{1}{2n},-p;\frac{1}{2}\left(2+\frac{1}{n}\right);-\frac{cx^{2n}}{a}\right)+\frac{2dex^{n+1}(a+cx^{2n})^p\left(\frac{cx^{2n}}{a}+1\right)^{-p}{}_2F_1\left(\frac{n+1}{2n},-p;\frac{1}{2}\left(3+\frac{1}{n}\right);-\frac{cx^{2n}}{a}\right)}{n+1}+\frac{e^2x^{2n+1}(a+cx^{2n})^p\left(\frac{cx^{2n}}{a}+1\right)^{-p}{}_2F_1\left(\frac{1}{2}\left(2+\frac{1}{n}\right),-p;\frac{1}{2}\left(4+\frac{1}{n}\right);-\frac{cx^{2n}}{a}\right)}{2n+1}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x^n)^2*(a + c*x^(2*n))^p,x]
```

```
[Out] (e^2*x^(1 + 2*n)*(a + c*x^(2*n))^p*Hypergeometric2F1[(2 + n^(-1))/2, -p, (4
 + n^(-1))/2, -((c*x^(2*n))/a)])/((1 + 2*n)*(1 + (c*x^(2*n))/a)^p) + (d^2*x
*(a + c*x^(2*n))^p*Hypergeometric2F1[1/(2*n), -p, (2 + n^(-1))/2, -((c*x^(2
*n))/a)])/(1 + (c*x^(2*n))/a)^p + (2*d*e*x^(1 + n)*(a + c*x^(2*n))^p*Hyperg
eometric2F1[(1 + n)/(2*n), -p, (3 + n^(-1))/2, -((c*x^(2*n))/a)])/((1 + n)*
(1 + (c*x^(2*n))/a)^p)
```

Rule 251

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, (-b)*(x^n/a)], x] /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p
, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

Rule 252

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^IntPart[p]*((a + b*x
^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]), Int[(1 + b*(x^n/a))^p, x], x]
 /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Sim
plify[1/n + p], 0] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 372

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^I
ntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]), Int[(c*x)^
m*(1 + b*(x^n/a))^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0]
 && !(ILtQ[p, 0] || GtQ[a, 0])
```

Rule 1451

```
Int[((d_) + (e_.)*(x_)^(n_))^(q_)*((a_) + (c_.)*(x_)^(n2_))^(p_), x_Symbol]
 :> Int[ExpandIntegrand[(d + e*x^n)^q*(a + c*x^(2*n))^p, x], x] /; FreeQ[{a
, c, d, e, n, p, q}, x] && EqQ[n2, 2*n] && NeQ[c*d^2 + a*e^2, 0] && ((Integ
ersQ[p, q] && !IntegerQ[n]) || IGtQ[p, 0] || (IGtQ[q, 0] && !IntegerQ[n])
)
```

Rubi steps

$$\begin{aligned}
\int (d+ex^n)^2 \left(a+cx^{2n}\right)^p \, dx &= \int \left(d^2\left(a+cx^{2n}\right)^p + 2dex^n\left(a+cx^{2n}\right)^p + e^2x^{2n}\left(a+cx^{2n}\right)^p\right) dx \\
&= d^2 \int \left(a+cx^{2n}\right)^p \, dx + (2de) \int x^n\left(a+cx^{2n}\right)^p \, dx + e^2 \int x^{2n}\left(a+cx^{2n}\right)^p \, dx \\
&= \left(d^2\left(a+cx^{2n}\right)^p \left(1+\frac{cx^{2n}}{a}\right)^{-p}\right) \int \left(1+\frac{cx^{2n}}{a}\right)^p dx + \left(2de\left(a+cx^{2n}\right)^p \left(1 \right.\right. \\
&= \frac{e^2x^{1+2n}\left(a+cx^{2n}\right)^p \left(1+\frac{cx^{2n}}{a}\right)^{-p} {}_2F_1\left(\frac{1}{2}\left(2+\frac{1}{n}\right), -p; \frac{1}{2}\left(4+\frac{1}{n}\right); -\frac{cx^{2n}}{a}\right)}{1+2n} + d^2x
\end{aligned}$$

Mathematica [A]
time = 0.15, size = 171, normalized size = 0.79

$$\frac{x\left(a+cx^{2n}\right)^p \left(1+\frac{cx^{2n}}{a}\right)^{-p} \left(e^2(1+n)x^{2n} {}_2F_1\left(1+\frac{1}{2n}, -p; 2+\frac{1}{2n}; -\frac{cx^{2n}}{a}\right) + d(1+2n)\left(d(1+n) {}_2F_1\left(\frac{1}{2n}, -p; 1+\frac{1}{2n}; -\frac{cx^{2n}}{a}\right) + 2ex^n {}_2F_1\left(\frac{1+n}{2n}, -p; \frac{1}{2}\left(3+\frac{1}{n}\right); -\frac{cx^{2n}}{a}\right)\right)\right)}{(1+n)(1+2n)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x^n)^2*(a + c*x^(2*n))^p,x]
```

```
[Out] (x*(a + c*x^(2*n))^p*(e^2*(1 + n)*x^(2*n)*Hypergeometric2F1[1 + 1/(2*n), -p
, 2 + 1/(2*n), -((c*x^(2*n))/a)] + d*(1 + 2*n)*(d*(1 + n)*Hypergeometric2F1
```

```
[1/(2*n), -p, 1 + 1/(2*n), -((c*x^(2*n))/a)] + 2*e*x^n*Hypergeometric2F1[(1
 + n)/(2*n), -p, (3 + n^(-1))/2, -((c*x^(2*n))/a)])))/((1 + n)*(1 + 2*n)*(1
 + (c*x^(2*n))/a)^p)
```

Maple [F]
time = 0.05, size = 0, normalized size = 0.00

$$\int (d + e\,x^{n})^{2} \left(a + c\,x^{2n}\right)^{p}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d+e*x^n)^2*(a+c*x^(2*n))^p,x)
```

```
[Out] int((d+e*x^n)^2*(a+c*x^(2*n))^p,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+e*x^n)^2*(a+c*x^(2*n))^p,x, algorithm="maxima")
```

```
[Out] integrate((x^n*e + d)^2*(c*x^(2*n) + a)^p, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+e*x^n)^2*(a+c*x^(2*n))^p,x, algorithm="fricas")
```

```
[Out] integral((2*d*x^n*e + d^2 + x^(2*n)*e^2)*(c*x^(2*n) + a)^p, x)
```

Sympy [C] Result contains complex when optimal does not.
time = 214.77, size = 177, normalized size = 0.82

$$\frac{a^{p} d^{2} x \Gamma\left(\frac{1}{2n}\right) {}_{2}F_{1}\left(\begin{matrix} \frac{1}{2n}, -p \\ 1 + \frac{1}{2n} \end{matrix}\middle| {\frac{c x^{2n} e^{i\pi}}{a}} \right)}{2n\Gamma\left(1 + \frac{1}{2n}\right)} + \frac{a^{p} d e x x^{n} \Gamma\left(\frac{1}{2} + \frac{1}{2n}\right) {}_{2}F_{1}\left(\begin{matrix} -p, \frac{1}{2} + \frac{1}{2n} \\ \frac{3}{2} + \frac{1}{2n} \end{matrix}\middle| {\frac{c x^{2n} e^{i\pi}}{a}} \right)}{n\Gamma\left(\frac{3}{2} + \frac{1}{2n}\right)} + \frac{a^{p} e^{2} x x^{2n} \Gamma\left(1 + \frac{1}{2n}\right) {}_{2}F_{1}\left(\begin{matrix} -p, 1 + \frac{1}{2n} \\ 2 + \frac{1}{2n} \end{matrix}\middle| {\frac{c x^{2n} e^{i\pi}}{a}} \right)}{2n\Gamma\left(2 + \frac{1}{2n}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+e*x**n)**2*(a+c*x**(2*n))**p,x)
```

```
[Out] a**p*d**2*x*gamma(1/(2*n))*hyper((1/(2*n), -p), (1 + 1/(2*n),), c*x**(2*n)*
exp_polar(I*pi)/a)/(2*n*gamma(1 + 1/(2*n))) + a**p*d*e*x*x**n*gamma(1/2 + 1
```

```
/(2*n))*hyper((-p, 1/2 + 1/(2*n)), (3/2 + 1/(2*n),), c*x**(2*n)*exp_polar(I
*pi)/a)/(n*gamma(3/2 + 1/(2*n))) + a**p*e**2*x*x**(2*n)*gamma(1 + 1/(2*n))*
hyper((-p, 1 + 1/(2*n)), (2 + 1/(2*n),), c*x**(2*n)*exp_polar(I*pi)/a)/(2*n
*gamma(2 + 1/(2*n)))
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+e*x^n)^2*(a+c*x^(2*n))^p,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Unable to divide, perhaps due to roun
ding error%%%{4,[0,0,3,2,0,2,3,1]%%%}+%%%{8,[0,0,3,2,0,1,3,1]%%%}+%%%{4,[0,
0,3,2,
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int (a + c\,x^{2\,n})^p\,(d + e\,x^n)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + c*x^(2*n))^p*(d + e*x^n)^2,x)
```

```
[Out] int((a + c*x^(2*n))^p*(d + e*x^n)^2, x)
```

3.62 $\int (d+ex^n)\left(a+cx^{2n}\right)^p\,dx$

Optimal. Leaf size=135

$$dx\left(a+cx^{2n}\right)^p\left(1+\frac{cx^{2n}}{a}\right)^{-p}{}_2F_1\left(\frac{1}{2n},-p;\frac{1}{2}\left(2+\frac{1}{n}\right);-\frac{cx^{2n}}{a}\right)+\frac{ex^{1+n}\left(a+cx^{2n}\right)^p\left(1+\frac{cx^{2n}}{a}\right)^{-p}{}_2F_1\left(\frac{1+n}{2n},\right.}{1+n}$$

```
[Out] d*x*(a+c*x^(2*n))^p*hypergeom([-p, 1/2/n],[1+1/2/n],-c*x^(2*n)/a)/((1+c*x^(
2*n)/a)^p)+e*x^(1+n)*(a+c*x^(2*n))^p*hypergeom([-p, 1/2*(1+n)/n],[3/2+1/2/n
],-c*x^(2*n)/a)/(1+n)/((1+c*x^(2*n)/a)^p)
```

Rubi [A]
time = 0.04, antiderivative size = 135, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.263$, Rules used = {1447, 252, 251, 372, 371}

$$dx\left(a+cx^{2n}\right)^p\left(\frac{cx^{2n}}{a}+1\right)^{-p}{}_2F_1\left(\frac{1}{2n},-p;\frac{1}{2}\left(2+\frac{1}{n}\right);-\frac{cx^{2n}}{a}\right)+\frac{ex^{n+1}\left(a+cx^{2n}\right)^p\left(\frac{cx^{2n}}{a}+1\right)^{-p}{}_2F_1\left(\frac{n+1}{2n},-p;\frac{1}{2}\left(3+\frac{1}{n}\right);-\frac{cx^{2n}}{a}\right)}{n+1}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x^n)*(a + c*x^(2*n))^p,x]
```

```
[Out] (d*x*(a + c*x^(2*n))^p*Hypergeometric2F1[1/(2*n), -p, (2 + n^(-1))/2, -((c*
x^(2*n))/a)])/(1 + (c*x^(2*n))/a)^p + (e*x^(1 + n)*(a + c*x^(2*n))^p*Hyperg
eometric2F1[(1 + n)/(2*n), -p, (3 + n^(-1))/2, -((c*x^(2*n))/a)])/((1 + n)*
(1 + (c*x^(2*n))/a)^p)
```

Rule 251

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, (-b)*(x^n/a)], x] /; FreeQ[{a, b, n, p}, x] && !IGtQ[p
, 0] && !IntegerQ[1/n] && !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

Rule 252

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^IntPart[p]*((a + b*x
^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]), Int[(1 + b*(x^n/a))^p, x], x]
 /; FreeQ[{a, b, n, p}, x] && !IGtQ[p, 0] && !IntegerQ[1/n] && !ILtQ[Sim
plify[1/n + p], 0] && !(IntegerQ[p] || GtQ[a, 0])
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILt
```

```
Q[p, 0] || GtQ[a, 0])
```

Rule 372

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^I
ntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]), Int[(c*x)^
m*(1 + b*(x^n/a))^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0]
 && !(ILtQ[p, 0] || GtQ[a, 0])
```

Rule 1447

```
Int[((d_) + (e_.)*(x_)^(n_))*((a_) + (c_.)*(x_)^(n2_))^(p_), x_Symbol] :> I
nt[ExpandIntegrand[(d + e*x^n)*(a + c*x^(2*n))^p, x], x] /; FreeQ[{a, c, d,
 e, n}, x] && EqQ[n2, 2*n]
```

Rubi steps

$$\begin{aligned}\int (d+ex^n)\left(a+cx^{2n}\right)^p\,dx &= \int \left(d\left(a+cx^{2n}\right)^p+ex^n\left(a+cx^{2n}\right)^p\right)\,dx\\ &= d\int \left(a+cx^{2n}\right)^p\,dx+e\int x^n\left(a+cx^{2n}\right)^p\,dx\\ &= \left(d\left(a+cx^{2n}\right)^p\left(1+\frac{cx^{2n}}{a}\right)^{-p}\right)\int \left(1+\frac{cx^{2n}}{a}\right)^p\,dx+\left(e\left(a+cx^{2n}\right)^p\left(1+\frac{cx}{a}\right.\right.\\ &= dx\left(a+cx^{2n}\right)^p\left(1+\frac{cx^{2n}}{a}\right)^{-p}{}_2F_1\left(\frac{1}{2n},-p;\frac{1}{2}\left(2+\frac{1}{n}\right);-\frac{cx^{2n}}{a}\right)+\frac{ex^{1+n}(a+}{}\end{aligned}$$

Mathematica [A]
time = 0.07, size = 110, normalized size = 0.81

$$\frac{x\left(a+cx^{2n}\right)^p\left(1+\frac{cx^{2n}}{a}\right)^{-p}\left(d(1+n)\,{}_2F_1\left(\frac{1}{2n},-p;1+\frac{1}{2n};-\frac{cx^{2n}}{a}\right)+ex^n\,{}_2F_1\left(\frac{1+n}{2n},-p;\frac{1}{2}\left(3+\frac{1}{n}\right);-\frac{cx^{2n}}{a}\right)\right)}{1+n}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x^n)*(a + c*x^(2*n))^p,x]
[Out] (x*(a + c*x^(2*n))^p*(d*(1 + n)*Hypergeometric2F1[1/(2*n), -p, 1 + 1/(2*n),
 -((c*x^(2*n))/a)] + e*x^n*Hypergeometric2F1[(1 + n)/(2*n), -p, (3 + n^(-1)
)/2, -((c*x^(2*n))/a)]))/((1 + n)*(1 + (c*x^(2*n))/a)^p)
```

Maple [F]
time = 0.05, size = 0, normalized size = 0.00

$$\int (d+e\,x^n)\left(a+c\,x^{2n}\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d+e*x^n)*(a+c*x^(2*n))^p,x)
```

```
[Out] int((d+e*x^n)*(a+c*x^(2*n))^p,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+e*x^n)*(a+c*x^(2*n))^p,x, algorithm="maxima")
```

```
[Out] integrate((x^n*e + d)*(c*x^(2*n) + a)^p, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+e*x^n)*(a+c*x^(2*n))^p,x, algorithm="fricas")
```

```
[Out] integral((x^n*e + d)*(c*x^(2*n) + a)^p, x)
```

Sympy [C] Result contains complex when optimal does not.
time = 129.60, size = 114, normalized size = 0.84

$$\frac{a^p dx\Gamma\left(\frac{1}{2n}\right) {}_2F_1\left(\begin{matrix} \frac{1}{2n}, -p \\ 1+\frac{1}{2n} \end{matrix}\middle| \frac{cx^{2n}e^{i\pi}}{a}\right)}{2n\Gamma\left(1+\frac{1}{2n}\right)} + \frac{a^p exx^n\Gamma\left(\frac{1}{2}+\frac{1}{2n}\right) {}_2F_1\left(\begin{matrix} -p, \frac{1}{2}+\frac{1}{2n} \\ \frac{3}{2}+\frac{1}{2n} \end{matrix}\middle| \frac{cx^{2n}e^{i\pi}}{a}\right)}{2n\Gamma\left(\frac{3}{2}+\frac{1}{2n}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+e*x**n)*(a+c*x**(2*n))**p,x)
```

```
[Out] a**p*d*x*gamma(1/(2*n))*hyper((1/(2*n), -p), (1 + 1/(2*n),), c*x**(2*n)*exp
_polar(I*pi)/a)/(2*n*gamma(1 + 1/(2*n))) + a**p*e*x*x**n*gamma(1/2 + 1/(2*n
))*hyper((-p, 1/2 + 1/(2*n)), (3/2 + 1/(2*n),), c*x**(2*n)*exp_polar(I*pi)/
a)/(2*n*gamma(3/2 + 1/(2*n)))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+e*x^n)*(a+c*x^(2*n))^p,x, algorithm="giac")
```

```
[Out] integrate((x^n*e + d)*(c*x^(2*n) + a)^p, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \left(a+c\,x^{2\,n}\right)^p\,\left(d+e\,x^n\right) \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + c*x^(2*n))^p*(d + e*x^n),x)
```

```
[Out] int((a + c*x^(2*n))^p*(d + e*x^n), x)
```

3.63 $\int \frac{(a+cx^{2n})^p}{d+ex^n} dx$

Optimal. Leaf size=167

$$\frac{x(a+cx^{2n})^p \left(1+\frac{cx^{2n}}{a}\right)^{-p} F_1\left(\frac{1}{2n}; -p, 1; \frac{1}{2}\left(2+\frac{1}{n}\right); -\frac{cx^{2n}}{a}, \frac{e^2x^{2n}}{d^2}\right)}{d} - \frac{ex^{1+n}(a+cx^{2n})^p \left(1+\frac{cx^{2n}}{a}\right)^{-p} F_1\left(\frac{1+n}{2n}; -\right.}{d^2(1+n)}$$

```
[Out] x*(a+c*x^(2*n))^p*AppellF1(1/2/n,1,-p,1+1/2/n,e^2*x^(2*n)/d^2,-c*x^(2*n)/a)
/d/((1+c*x^(2*n)/a)^p)-e*x^(1+n)*(a+c*x^(2*n))^p*AppellF1(1/2*(1+n)/n,1,-p,
3/2+1/2/n,e^2*x^(2*n)/d^2,-c*x^(2*n)/a)/d^2/(1+n)/((1+c*x^(2*n)/a)^p)
```

Rubi [A]
time = 0.10, antiderivative size = 167, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used = {1452, 441, 440, 525, 524}

$$\frac{x(a+cx^{2n})^p \left(\frac{cx^{2n}}{a}+1\right)^{-p} F_1\left(\frac{1}{2n}; -p, 1; \frac{1}{2}\left(2+\frac{1}{n}\right); -\frac{cx^{2n}}{a}, \frac{e^2x^{2n}}{d^2}\right)}{d} - \frac{ex^{n+1}(a+cx^{2n})^p \left(\frac{cx^{2n}}{a}+1\right)^{-p} F_1\left(\frac{n+1}{2n}; -p, 1; \frac{1}{2}\left(3+\frac{1}{n}\right); -\frac{cx^{2n}}{a}, \frac{e^2x^{2n}}{d^2}\right)}{d^2(n+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + c*x^(2*n))^p/(d + e*x^n),x]
```

```
[Out] (x*(a + c*x^(2*n))^p*AppellF1[1/(2*n), -p, 1, (2 + n^(-1))/2, -((c*x^(2*n))
/a), (e^2*x^(2*n))/d^2])/(d*(1 + (c*x^(2*n))/a)^p) - (e*x^(1 + n)*(a + c*x^
(2*n))^p*AppellF1[(1 + n)/(2*n), -p, 1, (3 + n^(-1))/2, -((c*x^(2*n))/a), (
e^2*x^(2*n))/d^2])/(d^2*(1 + n)*(1 + (c*x^(2*n))/a)^p)
```

Rule 440

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, (-b)*(x^n/a), (-d)*(x^n/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 441

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Dist[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]),
 Int[(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q}
, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 524

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
```

```
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 525

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^
n/a))^FracPart[p]), Int[(e*x)^m*(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 1452

```
Int[((d_) + (e_.)*(x_)^(n_))^(q_)*((a_) + (c_.)*(x_)^(n2_))^(p_), x_Symbol]
 :> Int[ExpandIntegrand[(a + c*x^(2*n))^p, (d/(d^2 - e^2*x^(2*n)) - e*(x^n/
(d^2 - e^2*x^(2*n))))^(-q), x], x] /; FreeQ[{a, c, d, e, n, p}, x] && EqQ[n
2, 2*n] && NeQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && ILtQ[q, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{(a+cx^{2n})^p}{d+ex^n}\,dx &= \int \left(\frac{d(a+cx^{2n})^p}{d^2-e^2x^{2n}}+\frac{ex^n(a+cx^{2n})^p}{-d^2+e^2x^{2n}}\right)dx\\ &= d\int \frac{(a+cx^{2n})^p}{d^2-e^2x^{2n}}\,dx+e\int \frac{x^n(a+cx^{2n})^p}{-d^2+e^2x^{2n}}\,dx\\ &= \left(d\left(a+cx^{2n}\right)^p\left(1+\frac{cx^{2n}}{a}\right)^{-p}\right)\int \frac{\left(1+\frac{cx^{2n}}{a}\right)^p}{d^2-e^2x^{2n}}\,dx+\left(e\left(a+cx^{2n}\right)^p\left(1+\frac{cx^{2n}}{a}\right)^{-p}\right)\int\\ &= \frac{x(a+cx^{2n})^p\left(1+\frac{cx^{2n}}{a}\right)^{-p}F_1\left(\frac{1}{2n};-p,1;\frac{1}{2}\left(2+\frac{1}{n}\right);-\frac{cx^{2n}}{a},\frac{e^2x^{2n}}{d^2}\right)}{d}-\frac{ex^{1+n}(a+cx^{2n})^p\left(1\right.}{}\end{aligned}$$

Mathematica [F]
time = 0.09, size = 0, normalized size = 0.00

$$\int \frac{(a+cx^{2n})^p}{d+ex^n}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(a + c*x^(2*n))^p/(d + e*x^n),x]
[Out] Integrate[(a + c*x^(2*n))^p/(d + e*x^n), x]
```

Maple [**F**]
time = 0.09, size = 0, normalized size = 0.00

$$\int \frac{(a + c\,x^{2n})^p}{d + e\,x^n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+c*x^(2*n))^p/(d+e*x^n),x)
[Out] int((a+c*x^(2*n))^p/(d+e*x^n),x)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+c*x^(2*n))^p/(d+e*x^n),x, algorithm="maxima")
[Out] integrate((c*x^(2*n) + a)^p/(x^n*e + d), x)
```

Fricas [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+c*x^(2*n))^p/(d+e*x^n),x, algorithm="fricas")
[Out] integral((c*x^(2*n) + a)^p/(x^n*e + d), x)
```

Sympy [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: HeuristicGCDFailed

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+c*x**(2*n))**p/(d+e*x**n),x)
[Out] Exception raised: HeuristicGCDFailed >> no luck
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+c*x^(2*n))^p/(d+e*x^n),x, algorithm="giac")
```

```
[Out] integrate((c*x^(2*n) + a)^p/(x^n*e + d), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(a + c\,x^{2\,n})^p}{d + e\,x^n} \,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + c*x^(2*n))^p/(d + e*x^n),x)
```

```
[Out] int((a + c*x^(2*n))^p/(d + e*x^n), x)
```

3.64 $\int \frac{(a+cx^{2n})^p}{(d+ex^n)^2} dx$

Optimal. Leaf size=261

$$\frac{e^2x^{1+2n}(a+cx^{2n})^p\left(1+\frac{cx^{2n}}{a}\right)^{-p}F_1\left(\frac{1}{2}\left(2+\frac{1}{n}\right);-p,2;\frac{1}{2}\left(4+\frac{1}{n}\right);-\frac{cx^{2n}}{a},\frac{e^2x^{2n}}{d^2}\right)}{d^4(1+2n)}+\frac{x(a+cx^{2n})^p\left(1+\frac{cx^{2n}}{a}\right)^{-p}F}{}$$

```
[Out] e^2*x^(1+2*n)*(a+c*x^(2*n))^p*AppellF1(1+1/2/n,2,-p,2+1/2/n,e^2*x^(2*n)/d^2
,-c*x^(2*n)/a)/d^4/(1+2*n)/((1+c*x^(2*n)/a)^p)+x*(a+c*x^(2*n))^p*AppellF1(1
/2/n,2,-p,1+1/2/n,e^2*x^(2*n)/d^2,-c*x^(2*n)/a)/d^2/((1+c*x^(2*n)/a)^p)-2*e
*x^(1+n)*(a+c*x^(2*n))^p*AppellF1(1/2*(1+n)/n,2,-p,3/2+1/2/n,e^2*x^(2*n)/d^
2,-c*x^(2*n)/a)/d^3/(1+n)/((1+c*x^(2*n)/a)^p)
```

Rubi [A]

time = 0.17, antiderivative size = 261, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used = {1452, 441, 440, 525, 524}

$$\frac{x(a+cx^{2n})^p\left(\frac{cx^{2n}}{a}+1\right)^{-p}F_1\left(\frac{1}{2n};-p,2;\frac{1}{2}\left(2+\frac{1}{n}\right);-\frac{cx^{2n}}{a},\frac{e^2x^{2n}}{d^2}\right)}{d^2}+\frac{e^2x^{2n+1}(a+cx^{2n})^p\left(\frac{cx^{2n}}{a}+1\right)^{-p}F_1\left(\frac{1}{2}\left(2+\frac{1}{n}\right);-p,2;\frac{1}{2}\left(4+\frac{1}{n}\right);-\frac{cx^{2n}}{a},\frac{e^2x^{2n}}{d^2}\right)}{d^4(2n+1)}-\frac{2ex^{n+1}(a+cx^{2n})^p\left(\frac{cx^{2n}}{a}+1\right)^{-p}F_1\left(\frac{n+1}{2n};-p,2;\frac{1}{2}\left(3+\frac{1}{n}\right);-\frac{cx^{2n}}{a},\frac{e^2x^{2n}}{d^2}\right)}{d^3(n+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + c*x^(2*n))^p/(d + e*x^n)^2,x]
```

```
[Out] (e^2*x^(1 + 2*n)*(a + c*x^(2*n))^p*AppellF1[(2 + n^(-1))/2, -p, 2, (4 + n^(
-1))/2, -((c*x^(2*n))/a), (e^2*x^(2*n))/d^2])/(d^4*(1 + 2*n)*(1 + (c*x^(2*n
))/a)^p) + (x*(a + c*x^(2*n))^p*AppellF1[1/(2*n), -p, 2, (2 + n^(-1))/2, -(
(c*x^(2*n))/a), (e^2*x^(2*n))/d^2])/(d^2*(1 + (c*x^(2*n))/a)^p) - (2*e*x^(1
 + n)*(a + c*x^(2*n))^p*AppellF1[(1 + n)/(2*n), -p, 2, (3 + n^(-1))/2, -((c
*x^(2*n))/a), (e^2*x^(2*n))/d^2])/(d^3*(1 + n)*(1 + (c*x^(2*n))/a)^p)
```

Rule 440

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, (-b)*(x^n/a), (-d)*(x^n/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 441

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Dist[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]),
 Int[(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q}
, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 524

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 525

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^
n/a))^FracPart[p]), Int[(e*x)^m*(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 1452

```
Int[((d_) + (e_.)*(x_)^(n_))^(q_)*((a_) + (c_.)*(x_)^(n2_))^(p_), x_Symbol]
 :> Int[ExpandIntegrand[(a + c*x^(2*n))^p, (d/(d^2 - e^2*x^(2*n)) - e*(x^n/
(d^2 - e^2*x^(2*n))))^(-q), x], x] /; FreeQ[{a, c, d, e, n, p}, x] && EqQ[n
2, 2*n] && NeQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && ILtQ[q, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{(a+cx^{2n})^p}{(d+ex^n)^2}\,dx &= \int \left(\frac{d^2(a+cx^{2n})^p}{(d^2-e^2x^{2n})^2} - \frac{2dex^n(a+cx^{2n})^p}{(-d^2+e^2x^{2n})^2} + \frac{e^2x^{2n}(a+cx^{2n})^p}{(-d^2+e^2x^{2n})^2}\right)dx\\
&= d^2\int \frac{(a+cx^{2n})^p}{(d^2-e^2x^{2n})^2}\,dx - (2de)\int \frac{x^n(a+cx^{2n})^p}{(-d^2+e^2x^{2n})^2}\,dx + e^2\int \frac{x^{2n}(a+cx^{2n})^p}{(-d^2+e^2x^{2n})^2}\,dx\\
&= \left(d^2(a+cx^{2n})^p\left(1+\frac{cx^{2n}}{a}\right)^{-p}\right)\int \frac{\left(1+\frac{cx^{2n}}{a}\right)^p}{(d^2-e^2x^{2n})^2}\,dx - \left(2de(a+cx^{2n})^p\left(1+\frac{cx^{2n}}{a}\right)^{-p}\right.\\
&= \frac{e^2x^{1+2n}(a+cx^{2n})^p\left(1+\frac{cx^{2n}}{a}\right)^{-p}F_1\left(\frac{1}{2}\left(2+\frac{1}{n}\right);-p,2;\frac{1}{2}\left(4+\frac{1}{n}\right);-\frac{cx^{2n}}{a},\frac{e^2x^{2n}}{d^2}\right)}{d^4(1+2n)} + \frac{x(a+c}{}\end{aligned}$$

Mathematica [F]
time = 0.11, size = 0, normalized size = 0.00

$$\int \frac{(a+cx^{2n})^p}{(d+ex^n)^2}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(a + c*x^(2*n))^p/(d + e*x^n)^2,x]
[Out] Integrate[(a + c*x^(2*n))^p/(d + e*x^n)^2, x]
```

Maple [F]
time = 0.07, size = 0, normalized size = 0.00

$$\int \frac{(a+c\,x^{2n})^{p}}{(d+e\,x^{n})^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+c*x^(2*n))^p/(d+e*x^n)^2,x)
[Out] int((a+c*x^(2*n))^p/(d+e*x^n)^2,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+c*x^(2*n))^p/(d+e*x^n)^2,x, algorithm="maxima")
[Out] integrate((c*x^(2*n) + a)^p/(x^n*e + d)^2, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+c*x^(2*n))^p/(d+e*x^n)^2,x, algorithm="fricas")
[Out] integral((c*x^(2*n) + a)^p/(2*d*x^n*e + d^2 + x^(2*n)*e^2), x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+c*x**(2*n))**p/(d+e*x**n)**2,x)
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+c*x^(2*n))^p/(d+e*x^n)^2,x, algorithm="giac")
```

```
[Out] integrate((c*x^(2*n) + a)^p/(x^n*e + d)^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(a + c\,x^{2\,n})^p}{(d + e\,x^n)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + c*x^(2*n))^p/(d + e*x^n)^2,x)
```

```
[Out] int((a + c*x^(2*n))^p/(d + e*x^n)^2, x)
```

3.65 $\int \frac{(a+cx^{2n})^p}{(d+ex^n)^3} \, dx$

Optimal. Leaf size=357

$$\frac{3e^2x^{1+2n}(a+cx^{2n})^p\left(1+\frac{cx^{2n}}{a}\right)^{-p}F_1\left(\frac{1}{2}\left(2+\frac{1}{n}\right);-p,3;\frac{1}{2}\left(4+\frac{1}{n}\right);-\frac{cx^{2n}}{a},\frac{e^2x^{2n}}{d^2}\right)}{d^5(1+2n)}-\frac{e^3x^{1+3n}(a+cx^{2n})^p\left(1+\frac{c}{}\right.}{}$$

```
[Out] 3*e^2*x^(1+2*n)*(a+c*x^(2*n))^p*AppellF1(1+1/2/n,3,-p,2+1/2/n,e^2*x^(2*n)/d
^2,-c*x^(2*n)/a)/d^5/(1+2*n)/((1+c*x^(2*n)/a)^p)-e^3*x^(1+3*n)*(a+c*x^(2*n)
)^p*AppellF1(3/2+1/2/n,3,-p,5/2+1/2/n,e^2*x^(2*n)/d^2,-c*x^(2*n)/a)/d^6/(1+
3*n)/((1+c*x^(2*n)/a)^p)+x*(a+c*x^(2*n))^p*AppellF1(1/2/n,3,-p,1+1/2/n,e^2*
x^(2*n)/d^2,-c*x^(2*n)/a)/d^3/((1+c*x^(2*n)/a)^p)-3*e*x^(1+n)*(a+c*x^(2*n))
^p*AppellF1(1/2*(1+n)/n,3,-p,3/2+1/2/n,e^2*x^(2*n)/d^2,-c*x^(2*n)/a)/d^4/(1
+n)/((1+c*x^(2*n)/a)^p)
```

Rubi [A]
time = 0.23, antiderivative size = 357, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used = {1452, 441, 440, 525, 524}

$$-\frac{e^3x^{3n+1}(a+cx^{2n})^p\left(\frac{cx^{2n}}{a}+1\right)^{-p}F_1\left(\frac{1}{2}\left(3+\frac{1}{n}\right);-p,3;\frac{1}{2}\left(5+\frac{1}{n}\right);-\frac{cx^{2n}}{a},\frac{e^2x^{2n}}{d^2}\right)}{d^6(3n+1)}+\frac{3e^2x^{2n+1}(a+cx^{2n})^p\left(\frac{cx^{2n}}{a}+1\right)^{-p}F_1\left(\frac{1}{2}\left(2+\frac{1}{n}\right);-p,3;\frac{1}{2}\left(4+\frac{1}{n}\right);-\frac{cx^{2n}}{a},\frac{e^2x^{2n}}{d^2}\right)}{d^5(2n+1)}-\frac{3ex^{n+1}(a+cx^{2n})^p\left(\frac{cx^{2n}}{a}+1\right)^{-p}F_1\left(\frac{n+1}{2n};-p,3;\frac{1}{2}\left(3+\frac{1}{n}\right);-\frac{cx^{2n}}{a},\frac{e^2x^{2n}}{d^2}\right)}{d^4(n+1)}+\frac{x(a+cx^{2n})^p\left(\frac{cx^{2n}}{a}+1\right)^{-p}F_1\left(\frac{1}{2n};-p,3;\frac{1}{2}\left(2+\frac{1}{n}\right);-\frac{cx^{2n}}{a},\frac{e^2x^{2n}}{d^2}\right)}{d^3}$$

Antiderivative was successfully verified.

```
[In] Int[(a + c*x^(2*n))^p/(d + e*x^n)^3,x]
```

```
[Out] (3*e^2*x^(1 + 2*n)*(a + c*x^(2*n))^p*AppellF1[(2 + n^(-1))/2, -p, 3, (4 + n
^(-1))/2, -((c*x^(2*n))/a), (e^2*x^(2*n))/d^2])/(d^5*(1 + 2*n)*(1 + (c*x^(2
*n))/a)^p) - (e^3*x^(1 + 3*n)*(a + c*x^(2*n))^p*AppellF1[(3 + n^(-1))/2, -p
, 3, (5 + n^(-1))/2, -((c*x^(2*n))/a), (e^2*x^(2*n))/d^2])/(d^6*(1 + 3*n)*(
1 + (c*x^(2*n))/a)^p) + (x*(a + c*x^(2*n))^p*AppellF1[1/(2*n), -p, 3, (2 +
n^(-1))/2, -((c*x^(2*n))/a), (e^2*x^(2*n))/d^2])/(d^3*(1 + (c*x^(2*n))/a)^p
) - (3*e*x^(1 + n)*(a + c*x^(2*n))^p*AppellF1[(1 + n)/(2*n), -p, 3, (3 + n^
(-1))/2, -((c*x^(2*n))/a), (e^2*x^(2*n))/d^2])/(d^4*(1 + n)*(1 + (c*x^(2*n)
)/a)^p)
```

Rule 440

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, (-b)*(x^n/a), (-d)*(x^n/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 441

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Dist[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]),
```

```
 Int[(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q}
, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 524

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 525

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^
n/a))^FracPart[p]), Int[(e*x)^m*(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 1452

```
Int[((d_) + (e_.)*(x_)^(n_))^(q_)*((a_) + (c_.)*(x_)^(n2_))^(p_), x_Symbol]
 :> Int[ExpandIntegrand[(a + c*x^(2*n))^p, (d/(d^2 - e^2*x^(2*n)) - e*(x^n/
(d^2 - e^2*x^(2*n))))^(-q), x], x] /; FreeQ[{a, c, d, e, n, p}, x] && EqQ[n
2, 2*n] && NeQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && ILtQ[q, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{(a+cx^{2n})^p}{(d+ex^n)^3}\,dx &= \int \left(\frac{d^3(a+cx^{2n})^p}{(d^2-e^2x^{2n})^3}+\frac{3d^2ex^n(a+cx^{2n})^p}{(-d^2+e^2x^{2n})^3}-\frac{3de^2x^{2n}(a+cx^{2n})^p}{(-d^2+e^2x^{2n})^3}+\frac{e^3x^{3n}(a+cx^{2n})^p}{(-d^2+e^2x^{2n})^3}\right)dx\\
&= d^3\int \frac{(a+cx^{2n})^p}{(d^2-e^2x^{2n})^3}\,dx+\left(3d^2e\right)\int \frac{x^n(a+cx^{2n})^p}{(-d^2+e^2x^{2n})^3}\,dx-\left(3de^2\right)\int \frac{x^{2n}(a+cx^{2n})^p}{(-d^2+e^2x^{2n})^3}\,dx+\ldots\\
&= \left(d^3(a+cx^{2n})^p\left(1+\frac{cx^{2n}}{a}\right)^{-p}\right)\int \frac{\left(1+\frac{cx^{2n}}{a}\right)^p}{(d^2-e^2x^{2n})^3}\,dx+\left(3d^2e(a+cx^{2n})^p\left(1+\frac{cx^{2n}}{a}\right)^{-p}\ldots\right.\\
&= \frac{3e^2x^{1+2n}(a+cx^{2n})^p\left(1+\frac{cx^{2n}}{a}\right)^{-p}F_1\left(\frac{1}{2}\left(2+\frac{1}{n}\right);-p,3;\frac{1}{2}\left(4+\frac{1}{n}\right);-\frac{cx^{2n}}{a},\frac{e^2x^{2n}}{d^2}\right)}{d^5(1+2n)}-\frac{e^3x^{1+3\ldots}}{\ldots}\end{aligned}$$

Mathematica [F]

time = 0.26, size = 0, normalized size = 0.00

$$\int \frac{(a+cx^{2n})^p}{(d+ex^n)^3}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(a + c*x^(2*n))^p/(d + e*x^n)^3,x]
```

```
[Out] Integrate[(a + c*x^(2*n))^p/(d + e*x^n)^3, x]
```

Maple [F]
time = 0.07, size = 0, normalized size = 0.00

$$\int \frac{\left(a+c\,x^{2 n}\right)^{p}}{\left(d+e\,x^{n}\right)^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+c*x^(2*n))^p/(d+e*x^n)^3,x)
```

```
[Out] int((a+c*x^(2*n))^p/(d+e*x^n)^3,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+c*x^(2*n))^p/(d+e*x^n)^3,x, algorithm="maxima")
```

```
[Out] integrate((c*x^(2*n) + a)^p/(x^n*e + d)^3, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+c*x^(2*n))^p/(d+e*x^n)^3,x, algorithm="fricas")
```

```
[Out] integral((c*x^(2*n) + a)^p/(3*d^2*x^n*e + d^3 + 3*d*x^(2*n)*e^2 + x^(3*n)*e
^3), x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+c*x**(2*n))**p/(d+e*x**n)**3,x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+c*x^(2*n))^p/(d+e*x^n)^3,x, algorithm="giac")
[Out] integrate((c*x^(2*n) + a)^p/(x^n*e + d)^3, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(a+c\,x^{2\,n})^p}{(d+e\,x^n)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + c*x^(2*n))^p/(d + e*x^n)^3,x)
[Out] int((a + c*x^(2*n))^p/(d + e*x^n)^3, x)
```

3.66 $\int (d+ex^n) (a+bx^n+cx^{2n}) \, dx$

Optimal. Leaf size=62

$$adx+\frac{(bd+ae)x^{1+n}}{1+n}+\frac{(cd+be)x^{1+2n}}{1+2n}+\frac{cex^{1+3n}}{1+3n}$$

```
[Out] a*d*x+(a*e+b*d)*x^(1+n)/(1+n)+(b*e+c*d)*x^(1+2*n)/(1+2*n)+c*e*x^(1+3*n)/(1+
3*n)
```

Rubi [A]
time = 0.03, antiderivative size = 62, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.045$, Rules used = {1421}

$$\frac{x^{n+1}(ae+bd)}{n+1}+adx+\frac{x^{2n+1}(be+cd)}{2n+1}+\frac{cex^{3n+1}}{3n+1}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x^n)*(a + b*x^n + c*x^(2*n)),x]
```

```
[Out] a*d*x + ((b*d + a*e)*x^(1 + n))/(1 + n) + ((c*d + b*e)*x^(1 + 2*n))/(1 + 2*
n) + (c*e*x^(1 + 3*n))/(1 + 3*n)
```

Rule 1421

```
Int[((d_) + (e_.)*(x_)^(n_))^(q_.)*((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2
_)), x_Symbol] :> Int[ExpandIntegrand[(d + e*x^n)^q*(a + b*x^n + c*x^(2*n))
, x], x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c
, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IGtQ[q, 0]
```

Rubi steps

$$\begin{aligned}\int (d+ex^n) \left(a+bx^n+cx^{2n}\right) \, dx &= \int \left(ad+(bd+ae)x^n+(cd+be)x^{2n}+cex^{3n}\right) \, dx\\ &= adx+\frac{(bd+ae)x^{1+n}}{1+n}+\frac{(cd+be)x^{1+2n}}{1+2n}+\frac{cex^{1+3n}}{1+3n}\end{aligned}$$

Mathematica [A]
time = 0.17, size = 57, normalized size = 0.92

$$x\left(ad+\frac{(bd+ae)x^n}{1+n}+\frac{(cd+be)x^{2n}}{1+2n}+\frac{cex^{3n}}{1+3n}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x^n)*(a + b*x^n + c*x^(2*n)),x]
```

```
[Out] x*(a*d + ((b*d + a*e)*x^n)/(1 + n) + ((c*d + b*e)*x^(2*n))/(1 + 2*n) + (c*e
*x^(3*n))/(1 + 3*n))
```

Maple [A]
time = 0.02, size = 60, normalized size = 0.97

method	result	size
risch	$adx+\frac{(ae+bd)x\,x^{n}}{1+n}+\frac{(eb+cd)x\,x^{2n}}{1+2n}+\frac{cex\,x^{3n}}{1+3n}$	60
norman	$adx+\frac{(ae+bd)x\,\mathrm{e}^{n\ln(x)}}{1+n}+\frac{(eb+cd)x\,\mathrm{e}^{2n\ln(x)}}{1+2n}+\frac{cex\,\mathrm{e}^{3n\ln(x)}}{1+3n}$	66

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d+e*x^n)*(a+b*x^n+c*x^(2*n)),x,method=_RETURNVERBOSE)
```

```
[Out] a*d*x+(a*e+b*d)/(1+n)*x*x^n+(b*e+c*d)/(1+2*n)*x*(x^n)^2+c*e/(1+3*n)*x*(x^n)
^3
```

Maxima [A]
time = 0.29, size = 82, normalized size = 1.32

$$adx+\frac{cex^{3\,n+1}}{3\,n+1}+\frac{cdx^{2\,n+1}}{2\,n+1}+\frac{bex^{2\,n+1}}{2\,n+1}+\frac{bdx^{n+1}}{n+1}+\frac{aex^{n+1}}{n+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+e*x^n)*(a+b*x^n+c*x^(2*n)),x, algorithm="maxima")
```

```
[Out] a*d*x + c*e*x^(3*n + 1)/(3*n + 1) + c*d*x^(2*n + 1)/(2*n + 1) + b*e*x^(2*n
+ 1)/(2*n + 1) + b*d*x^(n + 1)/(n + 1) + a*e*x^(n + 1)/(n + 1)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 145 vs. $2(65) = 130$.
time = 0.35, size = 145, normalized size = 2.34

$$\frac{(2\,cn^2+3\,cn+c)xx^{3\,n}e+(6\,adn^3+11\,adn^2+6\,adn+ad)x+((3\,bn^2+4\,bn+b)xe+(3\,cdn^2+4\,cdn+cd)x)x^{2\,n}+((6\,an^2+5\,an+a)xe+(6\,bdn^2+5\,bdn+bd)x)x^{n}}{6\,n^3+11\,n^2+6\,n+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+e*x^n)*(a+b*x^n+c*x^(2*n)),x, algorithm="fricas")
```

```
[Out] ((2*c*n^2 + 3*c*n + c)*x*x^(3*n)*e + (6*a*d*n^3 + 11*a*d*n^2 + 6*a*d*n + a*
d)*x + ((3*b*n^2 + 4*b*n + b)*x*e + (3*c*d*n^2 + 4*c*d*n + c*d)*x)*x^(2*n)
+ ((6*a*n^2 + 5*a*n + a)*x*e + (6*b*d*n^2 + 5*b*d*n + b*d)*x)*x^n)/(6*n^3 +
 11*n^2 + 6*n + 1)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 656 vs. $2(53) = 106$.
time = 0.36, size = 656, normalized size = 10.58

$$\begin{cases} adx + ae\log(x) + bd\log(x) - \frac{be}{x} - \frac{cd}{x} - \frac{ce}{2x^2} & \text{for } n = -1 \\ adx + 2ae\sqrt{x} + 2bd\sqrt{x} + be\log(x) + cd\log(x) - \frac{2ce}{\sqrt{x}} & \text{for } n = -\frac{1}{2} \\ adx + \frac{3aex^{\frac{2}{3}}}{2} + \frac{3bdx^{\frac{2}{3}}}{2} + 3be\sqrt[3]{x} + 3cd\sqrt[3]{x} + ce\log(x) & \text{for } n = -\frac{1}{3} \\ \frac{6adn^3x}{6n^3+11n^2+6n+1} + \frac{11adn^2x}{6n^3+11n^2+6n+1} + \frac{6adnx}{6n^3+11n^2+6n+1} + \frac{adx}{6n^3+11n^2+6n+1} + \frac{6aen^2xx^n}{6n^3+11n^2+6n+1} + \frac{5aenxx^n}{6n^3+11n^2+6n+1} + \frac{aexx^n}{6n^3+11n^2+6n+1} + \frac{6bdn^2xx^n}{6n^3+11n^2+6n+1} + \frac{5bdnxx^n}{6n^3+11n^2+6n+1} + \frac{bdxx^n}{6n^3+11n^2+6n+1} + \frac{3ben^2xx^{2n}}{6n^3+11n^2+6n+1} + \frac{4benxx^{2n}}{6n^3+11n^2+6n+1} + \frac{bexx^{2n}}{6n^3+11n^2+6n+1} + \frac{3cdn^2xx^{2n}}{6n^3+11n^2+6n+1} + \frac{4cdnxx^{2n}}{6n^3+11n^2+6n+1} + \frac{cdxx^{2n}}{6n^3+11n^2+6n+1} + \frac{2cen^2xx^{3n}}{6n^3+11n^2+6n+1} + \frac{3cenxx^{3n}}{6n^3+11n^2+6n+1} + \frac{cexx^{3n}}{6n^3+11n^2+6n+1} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+e*x**n)*(a+b*x**n+c*x**(2*n)),x)
```

```
[Out] Piecewise((a*d*x + a*e*log(x) + b*d*log(x) - b*e/x - c*d/x - c*e/(2*x**2),
Eq(n, -1)), (a*d*x + 2*a*e*sqrt(x) + 2*b*d*sqrt(x) + b*e*log(x) + c*d*log(x
) - 2*c*e/sqrt(x), Eq(n, -1/2)), (a*d*x + 3*a*e*x**(2/3)/2 + 3*b*d*x**(2/3)
/2 + 3*b*e*x**(1/3) + 3*c*d*x**(1/3) + c*e*log(x), Eq(n, -1/3)), (6*a*d*n**
3*x/(6*n**3 + 11*n**2 + 6*n + 1) + 11*a*d*n**2*x/(6*n**3 + 11*n**2 + 6*n +
1) + 6*a*d*n*x/(6*n**3 + 11*n**2 + 6*n + 1) + a*d*x/(6*n**3 + 11*n**2 + 6*n
 + 1) + 6*a*e*n**2*x*x**n/(6*n**3 + 11*n**2 + 6*n + 1) + 5*a*e*n*x*x**n/(6*
n**3 + 11*n**2 + 6*n + 1) + a*e*x*x**n/(6*n**3 + 11*n**2 + 6*n + 1) + 6*b*d
*n**2*x*x**n/(6*n**3 + 11*n**2 + 6*n + 1) + 5*b*d*n*x*x**n/(6*n**3 + 11*n**
2 + 6*n + 1) + b*d*x*x**n/(6*n**3 + 11*n**2 + 6*n + 1) + 3*b*e*n**2*x*x**(2
*n)/(6*n**3 + 11*n**2 + 6*n + 1) + 4*b*e*n*x*x**(2*n)/(6*n**3 + 11*n**2 + 6
*n + 1) + b*e*x*x**(2*n)/(6*n**3 + 11*n**2 + 6*n + 1) + 3*c*d*n**2*x*x**(2*
n)/(6*n**3 + 11*n**2 + 6*n + 1) + 4*c*d*n*x*x**(2*n)/(6*n**3 + 11*n**2 + 6*
n + 1) + c*d*x*x**(2*n)/(6*n**3 + 11*n**2 + 6*n + 1) + 2*c*e*n**2*x*x**(3*n
)/(6*n**3 + 11*n**2 + 6*n + 1) + 3*c*e*n*x*x**(3*n)/(6*n**3 + 11*n**2 + 6*n
 + 1) + c*e*x*x**(3*n)/(6*n**3 + 11*n**2 + 6*n + 1), True))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 207 vs. $2(65) = 130$.
time = 4.17, size = 207, normalized size = 3.34

$$\frac{6\,adn^3x + 3\,cdn^2xx^{2\,n} + 6\,bdn^2xx^n + 2\,cn^2xx^{3\,n}e + 3\,bn^2xx^{2\,n}e + 6\,an^2xx^ne + 11\,adn^2x + 4\,cdnxx^{2\,n} + 5\,bdnxx^n + 3\,cnxx^{3\,n}e + 4\,bnxx^{2\,n}e + 5\,anxx^ne + 6\,adnx + cdxx^{2\,n} + bdxx^n + cxx^{3\,n}e + bxx^{2\,n}e + axx^ne + adx}{6\,n^3 + 11\,n^2 + 6\,n + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+e*x^n)*(a+b*x^n+c*x^(2*n)),x, algorithm="giac")
```

```
[Out] (6*a*d*n^3*x + 3*c*d*n^2*x*x^(2*n) + 6*b*d*n^2*x*x^n + 2*c*n^2*x*x^(3*n)*e
+ 3*b*n^2*x*x^(2*n)*e + 6*a*n^2*x*x^n*e + 11*a*d*n^2*x + 4*c*d*n*x*x^(2*n)
+ 5*b*d*n*x*x^n + 3*c*n*x*x^(3*n)*e + 4*b*n*x*x^(2*n)*e + 5*a*n*x*x^n*e + 6
*a*d*n*x + c*d*x*x^(2*n) + b*d*x*x^n + c*x*x^(3*n)*e + b*x*x^(2*n)*e + a*x*
x^n*e + a*d*x)/(6*n^3 + 11*n^2 + 6*n + 1)
```

Mupad [B]
time = 1.66, size = 59, normalized size = 0.95

$$a\,d\,x + \frac{x\,x^{2\,n}\,\left(b\,e + c\,d\right)}{2\,n + 1} + \frac{x\,x^n\,\left(a\,e + b\,d\right)}{n + 1} + \frac{c\,e\,x\,x^{3\,n}}{3\,n + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x^n)*(a + b*x^n + c*x^(2*n)),x)
```

```
[Out] a*d*x + (x*x^(2*n)*(b*e + c*d))/(2*n + 1) + (x*x^n*(a*e + b*d))/(n + 1) + (
c*e*x*x^(3*n))/(3*n + 1)
```

3.67 $\int (d+ex^n)\left(a+bx^n+cx^{2n}\right)^2\,dx$

Optimal. Leaf size=132

$$a^2dx+\frac{a(2bd+ae)x^{1+n}}{1+n}+\frac{(b^2d+2acd+2abe)\,x^{1+2n}}{1+2n}+\frac{(2bcd+b^2e+2ace)\,x^{1+3n}}{1+3n}+\frac{c(cd+2be)x^{1+4n}}{1+4n}+\frac{c^2ex^{1+}}{1+5n}$$

```
[Out] a^2*d*x+a*(a*e+2*b*d)*x^(1+n)/(1+n)+(2*a*b*e+2*a*c*d+b^2*d)*x^(1+2*n)/(1+2*
n)+(2*a*c*e+b^2*e+2*b*c*d)*x^(1+3*n)/(1+3*n)+c*(2*b*e+c*d)*x^(1+4*n)/(1+4*n
)+c^2*e*x^(1+5*n)/(1+5*n)
```

Rubi [A]
time = 0.07, antiderivative size = 132, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.042$, Rules used = {1446}

$$a^2dx+\frac{x^{2n+1}(2abe+2acd+b^2d)}{2n+1}+\frac{x^{3n+1}(2ace+b^2e+2bcd)}{3n+1}+\frac{ax^{n+1}(ae+2bd)}{n+1}+\frac{cx^{4n+1}(2be+cd)}{4n+1}+\frac{c^2ex^{5n+1}}{5n+1}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x^n)*(a + b*x^n + c*x^(2*n))^2,x]
```

```
[Out] a^2*d*x + (a*(2*b*d + a*e)*x^(1 + n))/(1 + n) + ((b^2*d + 2*a*c*d + 2*a*b*e
)*x^(1 + 2*n))/(1 + 2*n) + ((2*b*c*d + b^2*e + 2*a*c*e)*x^(1 + 3*n))/(1 + 3
*n) + (c*(c*d + 2*b*e)*x^(1 + 4*n))/(1 + 4*n) + (c^2*e*x^(1 + 5*n))/(1 + 5*
n)
```

Rule 1446

```
Int[((d_) + (e_.)*(x_)^(n_))*((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_))^(p
_), x_Symbol] :> Int[ExpandIntegrand[(d + e*x^n)*(a + b*x^n + c*x^(2*n))^p,
 x], x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c,
 0]
```

Rubi steps

$$\int (d+ex^n)\left(a+bx^n+cx^{2n}\right)^2\,dx=\int \left(a^2d+a(2bd+ae)x^n+\left(b^2d+2acd+2abe\right)x^{2n}+\left(2bcd+b^2e+2\right.\right.$$

$$=a^2dx+\frac{a(2bd+ae)x^{1+n}}{1+n}+\frac{(b^2d+2acd+2abe)\,x^{1+2n}}{1+2n}+\frac{(2bcd+b^2e+}{1+}$$

Mathematica [A]
time = 0.75, size = 123, normalized size = 0.93

$$x\left(a^2d+\frac{a(2bd+ae)x^n}{1+n}+\frac{(b^2d+2acd+2abe)\,x^{2n}}{1+2n}+\frac{(2bcd+b^2e+2ace)\,x^{3n}}{1+3n}+\frac{c(cd+2be)x^{4n}}{1+4n}+\frac{c^2ex^{5n}}{1+5n}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x^n)*(a + b*x^n + c*x^(2*n))^2,x]
```

```
[Out] x*(a^2*d + (a*(2*b*d + a*e)*x^n)/(1 + n) + ((b^2*d + 2*a*c*d + 2*a*b*e)*x^(
2*n))/(1 + 2*n) + ((2*b*c*d + b^2*e + 2*a*c*e)*x^(3*n))/(1 + 3*n) + (c*(c*d
 + 2*b*e)*x^(4*n))/(1 + 4*n) + (c^2*e*x^(5*n))/(1 + 5*n))
```

Maple [A]
time = 0.03, size = 128, normalized size = 0.97

method	result
risch	$a^2dx+\frac{(2ace+b^2e+2bcd)x\,x^{3n}}{1+3n}+\frac{(2abe+2acd+b^2d)x\,x^{2n}}{1+2n}+\frac{a(ae+2bd)x\,x^n}{1+n}+\frac{c(2eb+cd)x\,x^{4n}}{1+4n}+\frac{c^2ex\,x^{5n}}{1+5n}$
norman	$a^2dx+\frac{(2ace+b^2e+2bcd)x\,\mathrm{e}^{3n\ln(x)}}{1+3n}+\frac{(2abe+2acd+b^2d)x\,\mathrm{e}^{2n\ln(x)}}{1+2n}+\frac{a(ae+2bd)x\,\mathrm{e}^{n\ln(x)}}{1+n}+\frac{c(2eb+cd)x\,\mathrm{e}^{4n\ln(x)}}{1+4n}+\frac{c^2ex\,\mathrm{e}^{5n}}{1+5}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d+e*x^n)*(a+b*x^n+c*x^(2*n))^2,x,method=_RETURNVERBOSE)
```

```
[Out] a^2*d*x+(2*a*c*e+b^2*e+2*b*c*d)/(1+3*n)*x*(x^n)^3+(2*a*b*e+2*a*c*d+b^2*d)/(
1+2*n)*x*(x^n)^2+a*(a*e+2*b*d)/(1+n)*x*x^n+c*(2*b*e+c*d)/(1+4*n)*x*(x^n)^4+
c^2*e/(1+5*n)*x*(x^n)^5
```

Maxima [A]
time = 0.28, size = 208, normalized size = 1.58

$$a^2dx+\frac{c^2ex^{5n+1}}{5n+1}+\frac{c^2dx^{4n+1}}{4n+1}+\frac{2\,bcex^{4n+1}}{4n+1}+\frac{2\,bcdx^{3n+1}}{3n+1}+\frac{b^2ex^{3n+1}}{3n+1}+\frac{2\,acex^{3n+1}}{3n+1}+\frac{b^2dx^{2n+1}}{2n+1}+\frac{2\,acdx^{2n+1}}{2n+1}+\frac{2\,abex^{2n+1}}{2n+1}+\frac{2\,abdx^{n+1}}{n+1}+\frac{a^2ex^{n+1}}{n+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+e*x^n)*(a+b*x^n+c*x^(2*n))^2,x, algorithm="maxima")
```

```
[Out] a^2*d*x + c^2*e*x^(5*n + 1)/(5*n + 1) + c^2*d*x^(4*n + 1)/(4*n + 1) + 2*b*c
*e*x^(4*n + 1)/(4*n + 1) + 2*b*c*d*x^(3*n + 1)/(3*n + 1) + b^2*e*x^(3*n + 1
)/(3*n + 1) + 2*a*c*e*x^(3*n + 1)/(3*n + 1) + b^2*d*x^(2*n + 1)/(2*n + 1) +
 2*a*c*d*x^(2*n + 1)/(2*n + 1) + 2*a*b*e*x^(2*n + 1)/(2*n + 1) + 2*a*b*d*x^
(n + 1)/(n + 1) + a^2*e*x^(n + 1)/(n + 1)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 504 vs. $2(138) = 276$.
time = 0.37, size = 504, normalized size = 3.82

$$\frac{(24\,c^2n^4+50\,c^2n^3+35\,c^2n^2+10\,c^2n+c^2)xx^{5\,n}e+(120\,a^2dn^5+274\,a^2dn^4+225\,a^2dn^3+85\,a^2dn^2+15\,a^2dn+a^2d)x+(2\,(30\,bcn^4+61\,bcn^3+41\,bcn^2+11\,bcn+bc)xe+(30\,c^2dn^4+61\,c^2dn^3+41\,c^2dn^2+11\,c^2dn+c^2d)x)x^{4\,n}+((40\,(b^2+2\,ac)n^4+78\,(b^2+2\,ac)n^3+49\,(b^2+2\,ac)n^2+b^2+2\,ac+12\,(b^2+2\,ac)n)xe+2\,(40\,bcdn^4+78\,bcdn^3+49\,bcdn^2+12\,bcdn+bcd)x)x^{3\,n}+(2\,(60\,abn^4+107\,abn^3+59\,abn^2+13\,abn+ab)xe+(60\,(b^2+2\,ac)dn^4+107\,(b^2+2\,ac)dn^3+59\,(b^2+2\,ac)dn^2+13\,(b^2+2\,ac)dn+(b^2+2\,ac)d)x)x^{2\,n}+((120\,a^2n^4+154\,a^2n^3+71\,a^2n^2+14\,a^2n+a^2)xe+2\,(120\,abdn^4+154\,abdn^3+71\,abdn^2+14\,abdn+abd)x)x^n}{120\,n^5+274\,n^4+225\,n^3+85\,n^2+15\,n+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+e*x^n)*(a+b*x^n+c*x^(2*n))^2,x, algorithm="fricas")
```

[Out] ((24*c^2*n^4 + 50*c^2*n^3 + 35*c^2*n^2 + 10*c^2*n + c^2)*x*x^(5*n)*e + (120*a^2*d*n^5 + 274*a^2*d*n^4 + 225*a^2*d*n^3 + 85*a^2*d*n^2 + 15*a^2*d*n + a^2*d)*x + (2*(30*b*c*n^4 + 61*b*c*n^3 + 41*b*c*n^2 + 11*b*c*n + b*c)*x*e + (30*c^2*d*n^4 + 61*c^2*d*n^3 + 41*c^2*d*n^2 + 11*c^2*d*n + c^2*d)*x)*x^(4*n) + ((40*(b^2 + 2*a*c)*n^4 + 78*(b^2 + 2*a*c)*n^3 + 49*(b^2 + 2*a*c)*n^2 + b^2 + 2*a*c + 12*(b^2 + 2*a*c)*n)*x*e + 2*(40*b*c*d*n^4 + 78*b*c*d*n^3 + 49*b*c*d*n^2 + 12*b*c*d*n + b*c*d)*x)*x^(3*n) + (2*(60*a*b*n^4 + 107*a*b*n^3 + 59*a*b*n^2 + 13*a*b*n + a*b)*x*e + (60*(b^2 + 2*a*c)*d*n^4 + 107*(b^2 + 2*a*c)*d*n^3 + 59*(b^2 + 2*a*c)*d*n^2 + 13*(b^2 + 2*a*c)*d*n + (b^2 + 2*a*c)*d)*x)*x^(2*n) + ((120*a^2*n^4 + 154*a^2*n^3 + 71*a^2*n^2 + 14*a^2*n + a^2)*x*e + 2*(120*a*b*d*n^4 + 154*a*b*d*n^3 + 71*a*b*d*n^2 + 14*a*b*d*n + a*b*d)*x)*x^n)/(120*n^5 + 274*n^4 + 225*n^3 + 85*n^2 + 15*n + 1)

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 3128 vs. $2(124) = 248$.
time = 1.83, size = 3128, normalized size = 23.70

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d+e*x**n)*(a+b*x**n+c*x**(2*n))**2,x)

[Out] Piecewise((a**2*d*x + a**2*e*log(x) + 2*a*b*d*log(x) - 2*a*b*e/x - 2*a*c*d/x - a*c*e/x**2 - b**2*d/x - b**2*e/(2*x**2) - b*c*d/x**2 - 2*b*c*e/(3*x**3) - c**2*d/(3*x**3) - c**2*e/(4*x**4), Eq(n, -1)), (a**2*d*x + 2*a**2*e*sqrt(x) + 4*a*b*d*sqrt(x) + 2*a*b*e*log(x) + 2*a*c*d*log(x) - 4*a*c*e/sqrt(x) + b**2*d*log(x) - 2*b**2*e/sqrt(x) - 4*b*c*d/sqrt(x) - 2*b*c*e/x - c**2*d/x - 2*c**2*e/(3*x**(3/2)), Eq(n, -1/2)), (a**2*d*x + 3*a**2*e*x**(2/3)/2 + 3*a*b*d*x**(2/3) + 6*a*b*e*x**(1/3) + 6*a*c*d*x**(1/3) + 2*a*c*e*log(x) + 3*b**2*d*x**(1/3) + b**2*e*log(x) + 2*b*c*d*log(x) - 6*b*c*e/x**(1/3) - 3*c**2*d/x**(1/3) - 3*c**2*e/(2*x**(2/3)), Eq(n, -1/3)), (a**2*d*x + 4*a**2*e*x**(3/4)/3 + 8*a*b*d*x**(3/4)/3 + 4*a*b*e*sqrt(x) + 4*a*c*d*sqrt(x) + 8*a*c*e*x**(1/4) + 2*b**2*d*sqrt(x) + 4*b**2*e*x**(1/4) + 8*b*c*d*x**(1/4) + 2*b*c*e*log(x) + c**2*d*log(x) - 4*c**2*e/x**(1/4), Eq(n, -1/4)), (a**2*d*x + 5*a**2*e*x**(4/5)/4 + 5*a*b*d*x**(4/5)/2 + 10*a*b*e*x**(3/5)/3 + 10*a*c*d*x**(3/5)/3 + 5*a*c*e*x**(2/5) + 5*b**2*d*x**(3/5)/3 + 5*b**2*e*x**(2/5)/2 + 5*b*c*d*x**(2/5) + 10*b*c*e*x**(1/5) + 5*c**2*d*x**(1/5) + c**2*e*log(x), Eq(n, -1/5)), (120*a**2*d*n**5*x/(120*n**5 + 274*n**4 + 225*n**3 + 85*n**2 + 15*n + 1) + 274*a**2*d*n**4*x/(120*n**5 + 274*n**4 + 225*n**3 + 85*n**2 + 15*n + 1) + 225*a**2*d*n**3*x/(120*n**5 + 274*n**4 + 225*n**3 + 85*n**2 + 15*n + 1) + 85*a**2*d*n**2*x/(120*n**5 + 274*n**4 + 225*n**3 + 85*n**2 + 15*n + 1) + 15*a**2*d*n*x/(120*n**5 + 274*n**4 + 225*n**3 + 85*n**2 + 15*n + 1) + a**2*d*x/(120*n**5 + 274*n**4 + 225*n**3 + 85*n**2 + 15*n + 1) + 120*a**2*e*n**4*x*x**n/(120*n**5 + 274*n**4 + 225*n**3 + 85*n**2 + 15*n + 1) + 154*a**2*e*n**3*x*x**n/(120*n**5 + 274*n**4 + 225*n**3 + 85*n**2 + 15*n + 1) + 7

```
1*a**2*e*n**2*x*x**n/(120*n**5 + 274*n**4 + 225*n**3 + 85*n**2 + 15*n + 1)
+ 14*a**2*e*n*x*x**n/(120*n**5 + 274*n**4 + 225*n**3 + 85*n**2 + 15*n + 1)
+ a**2*e*x*x**n/(120*n**5 + 274*n**4 + 225*n**3 + 85*n**2 + 15*n + 1) + 240
*a*b*d*n**4*x*x**n/(120*n**5 + 274*n**4 + 225*n**3 + 85*n**2 + 15*n + 1) +
308*a*b*d*n**3*x*x**n/(120*n**5 + 274*n**4 + 225*n**3 + 85*n**2 + 15*n + 1)
 + 142*a*b*d*n**2*x*x**n/(120*n**5 + 274*n**4 + 225*n**3 + 85*n**2 + 15*n +
 1) + 28*a*b*d*n*x*x**n/(120*n**5 + 274*n**4 + 225*n**3 + 85*n**2 + 15*n +
1) + 2*a*b*d*x*x**n/(120*n**5 + 274*n**4 + 225*n**3 + 85*n**2 + 15*n + 1) +
 120*a*b*e*n**4*x*x**(2*n)/(120*n**5 + 274*n**4 + 225*n**3 + 85*n**2 + 15*n
 + 1) + 214*a*b*e*n**3*x*x**(2*n)/(120*n**5 + 274*n**4 + 225*n**3 + 85*n**2
 + 15*n + 1) + 118*a*b*e*n**2*x*x**(2*n)/(120*n**5 + 274*n**4 + 225*n**3 +
85*n**2 + 15*n + 1) + 26*a*b*e*n*x*x**(2*n)/(120*n**5 + 274*n**4 + 225*n**3
 + 85*n**2 + 15*n + 1) + 2*a*b*e*x*x**(2*n)/(120*n**5 + 274*n**4 + 225*n**3
 + 85*n**2 + 15*n + 1) + 120*a*c*d*n**4*x*x**(2*n)/(120*n**5 + 274*n**4 + 2
25*n**3 + 85*n**2 + 15*n + 1) + 214*a*c*d*n**3*x*x**(2*n)/(120*n**5 + 274*n
**4 + 225*n**3 + 85*n**2 + 15*n + 1) + 118*a*c*d*n**2*x*x**(2*n)/(120*n**5
+ 274*n**4 + 225*n**3 + 85*n**2 + 15*n + 1) + 26*a*c*d*n*x*x**(2*n)/(120*n*
*5 + 274*n**4 + 225*n**3 + 85*n**2 + 15*n + 1) + 2*a*c*d*x*x**(2*n)/(120*n*
*5 + 274*n**4 + 225*n**3 + 85*n**2 + 15*n + 1) + 80*a*c*e*n**4*x*x**(3*n)/(
120*n**5 + 274*n**4 + 225*n**3 + 85*n**2 + 15*n + 1) + 156*a*c*e*n**3*x*x**
(3*n)/(120*n**5 + 274*n**4 + 225*n**3 + 85*n**2 + 15*n + 1) + 98*a*c*e*n**2
*x*x**(3*n)/(120*n**5 + 274*n**4 + 225*n**3 + 85*n**2 + 15*n + 1) + 24*a*c*
e*n*x*x**(3*n)/(120*n**5 + 274*n**4 + 225*n**3 + 85*n**2 + 15*n + 1) + 2*a*
c*e*x*x**(3*n)/(120*n**5 + 274*n**4 + 225*n**3 + 85*n**2 + 15*n + 1) + 60*b
**2*d*n**4*x*x**(2*n)/(120*n**5 + 274*n**4 + 225*n**3 + 85*n**2 + 15*n + 1)
 + 107*b**2*d*n**3*x*x**(2*n)/(120*n**5 + 274*n**4 + 225*n**3 + 85*n**2 + 1
5*n + 1) + 59*b**2*d*n**2*x*x**(2*n)/(120*n**5 + 274*n**4 + 225*n**3 + 85*n
**2 + 15*n + 1) + 13*b**2*d*n*x*x**(2*n)/(120*n**5 + 274*n**4 + 225*n**3 +
85*n**2 + 15*n + 1) + b**2*d*x*x**(2*n)/(120*n**5 + 274*n**4 + 225*n**3 + 8
5*n**2 + 15*n + 1) + 40*b**2*e*n**4*x*x**(3*n)/(120*n**5 + 274*n**4 + 225*n
**3 + 85*n**2 + 15*n + 1) + 78*b**2*e*n**3*x*x**(3*n)/(120*n**5 + 274*n**4
+ 225*n**3 + 85*n**2 + 15*n + 1) + 49*b**2*e*n**2*x*x**(3*n)/(120*n**5 + 27
4*n**4 + 225*n**3 + 85*n**2 + 15*n + 1) + 12*b**2*e*n*x*x**(3*n)/(120*n**5
+ 274*n**4 + 225*n**3 + 85*n**2 + 15*n + 1) + b**2*e*x*x**(3*n)/(120*n**5 +
 274*n**4 + 225*n**3 + 85*n**2 + 15*n + 1) + 80*b*c*d*n**4*x*x**(3*n)/(120*
n**5 + 274*n**4 + 225*n**3 + 85*n**2 + 15*n + 1) + 156*b*c*d*n**3*x*x**(3*n
)/(120*n**5 + 274*n**4 + 225*n**3 + 85*n**2 + 15*n + 1) + 98*b*c*d*n**2*x*x
**(3*n)/(120*n**5 + 274*n**4 + 225*n**3 + 85*n**2 + 15*n + 1) + 24*b*c*d*n*
x*x**(3*n)/(120*n**5 + 274*n**4 + 225*n**3 + 85*n**2 + 15*n + 1) + 2*b*c*d*
x*x**(3*n)/(120*n**5 + 274*n**4 + 225*n**3 + 85*n**2 + 15*n + 1) + 60*b*c*e
*n**4*x*x**(4*n)/(120*n**5 + 274*n**4 + 225*n**3 + 85*n**2 + 15*n + 1) + 12
2*b*c*e*n**3*x*x**(4*n)/(120*n**5 + 274*n**4 + 225*n**3 + 85*n**2 + 15*n +
1) + 82*b*c*e*n**2*x*x**(4*n)/(120*n**5 + 274*n**4 + 225*n**3 + 85*n**2 + 1
5*n + 1) + 22*b*c*e*n*x*x**(4*n)/(120*n**5 + 27...
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 828 vs. 2(138) = 276.
time = 3.28, size = 828, normalized size = 6.27

$$\frac{120 a^2 d n^5 x + 30 c^2 d n^4 x x^{4n} + 80 b c d n^4 x x^{3n} + 60 b^2 d n^4 x x^{2n} + 120 a c d n^4 x x^{2n} + 240 a b d n^4 x x^n + 24 c^2 n^4 x x^{5n} e + 60 b c n^4 x x^{4n} e + 40 b^2 n^4 x x^{3n} e + 80 a c n^4 x x^{3n} e + 120 a b n^4 x x^{2n} e + 120 a^2 n^4 x x^n e + 274 a^2 d n^4 x + 61 c^2 d n^3 x x^{4n} + 156 b c d n^3 x x^{3n} + 107 b^2 d n^3 x x^{2n} + 214 a c d n^3 x x^{2n} + 308 a b d n^3 x x^n + 50 c^2 n^3 x x^{5n} e + 122 b c n^3 x x^{4n} e + 78 b^2 n^3 x x^{3n} e + 156 a c n^3 x x^{3n} e + 214 a b n^3 x x^{2n} e + 154 a^2 n^3 x x^n e + 225 a^2 d n^3 x + 41 c^2 d n^2 x x^{4n} + 98 b c d n^2 x x^{3n} + 59 b^2 d n^2 x x^{2n} + 118 a c d n^2 x x^{2n} + 142 a b d n^2 x x^n + 35 c^2 n^2 x x^{5n} e + 82 b c n^2 x x^{4n} e + 49 b^2 n^2 x x^{3n} e + 98 a c n^2 x x^{3n} e + 118 a b n^2 x x^{2n} e + 71 a^2 n^2 x x^n e + 85 a^2 d n^2 x + 11 c^2 d n x x^{4n} + 24 b c d n x x^{3n} + 13 b^2 d n x x^{2n} + 26 a c d n x x^{2n} + 28 a b d n x x^n + 10 c^2 n x x^{5n} e + 22 b c n x x^{4n} e + 12 b^2 n x x^{3n} e + 24 a c n x x^{3n} e + 26 a b n x x^{2n} e + 14 a^2 n x x^n e + 15 a^2 d n x + c^2 d x x^{4n} + 2 b c d x x^{3n} + b^2 d x x^{2n} + 2 a c d x x^{2n} + 2 a b d x x^n + c^2 x x^{5n} e + 2 b c x x^{4n} e + b^2 x x^{3n} e + 2 a c x x^{3n} e + 2 a b x x^{2n} e + a^2 x x^n e + a^2 d x}{120 n^5 + 274 n^4 + 225 n^3 + 85 n^2 + 15 n + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+e*x^n)*(a+b*x^n+c*x^(2*n))^2,x, algorithm="giac")
```

```
[Out] (120*a^2*d*n^5*x + 30*c^2*d*n^4*x*x^(4*n) + 80*b*c*d*n^4*x*x^(3*n) + 60*b^2
*d*n^4*x*x^(2*n) + 120*a*c*d*n^4*x*x^(2*n) + 240*a*b*d*n^4*x*x^n + 24*c^2*n
^4*x*x^(5*n)*e + 60*b*c*n^4*x*x^(4*n)*e + 40*b^2*n^4*x*x^(3*n)*e + 80*a*c*n
^4*x*x^(3*n)*e + 120*a*b*n^4*x*x^(2*n)*e + 120*a^2*n^4*x*x^n*e + 274*a^2*d*
n^4*x + 61*c^2*d*n^3*x*x^(4*n) + 156*b*c*d*n^3*x*x^(3*n) + 107*b^2*d*n^3*x*
x^(2*n) + 214*a*c*d*n^3*x*x^(2*n) + 308*a*b*d*n^3*x*x^n + 50*c^2*n^3*x*x^(5
*n)*e + 122*b*c*n^3*x*x^(4*n)*e + 78*b^2*n^3*x*x^(3*n)*e + 156*a*c*n^3*x*x^
(3*n)*e + 214*a*b*n^3*x*x^(2*n)*e + 154*a^2*n^3*x*x^n*e + 225*a^2*d*n^3*x +
 41*c^2*d*n^2*x*x^(4*n) + 98*b*c*d*n^2*x*x^(3*n) + 59*b^2*d*n^2*x*x^(2*n) +
 118*a*c*d*n^2*x*x^(2*n) + 142*a*b*d*n^2*x*x^n + 35*c^2*n^2*x*x^(5*n)*e + 8
2*b*c*n^2*x*x^(4*n)*e + 49*b^2*n^2*x*x^(3*n)*e + 98*a*c*n^2*x*x^(3*n)*e + 1
18*a*b*n^2*x*x^(2*n)*e + 71*a^2*n^2*x*x^n*e + 85*a^2*d*n^2*x + 11*c^2*d*n*x
*x^(4*n) + 24*b*c*d*n*x*x^(3*n) + 13*b^2*d*n*x*x^(2*n) + 26*a*c*d*n*x*x^(2*
n) + 28*a*b*d*n*x*x^n + 10*c^2*n*x*x^(5*n)*e + 22*b*c*n*x*x^(4*n)*e + 12*b^
2*n*x*x^(3*n)*e + 24*a*c*n*x*x^(3*n)*e + 26*a*b*n*x*x^(2*n)*e + 14*a^2*n*x*
x^n*e + 15*a^2*d*n*x + c^2*d*x*x^(4*n) + 2*b*c*d*x*x^(3*n) + b^2*d*x*x^(2*n
) + 2*a*c*d*x*x^(2*n) + 2*a*b*d*x*x^n + c^2*x*x^(5*n)*e + 2*b*c*x*x^(4*n)*e
 + b^2*x*x^(3*n)*e + 2*a*c*x*x^(3*n)*e + 2*a*b*x*x^(2*n)*e + a^2*x*x^n*e +
a^2*d*x)/(120*n^5 + 274*n^4 + 225*n^3 + 85*n^2 + 15*n + 1)
```

Mupad [B]
time = 1.71, size = 131, normalized size = 0.99

$$a^2\,d\,x+\frac{x\,x^{4\,n}\,\left(d\,c^2+2\,b\,e\,c\right)}{4\,n+1}+\frac{x\,x^n\,\left(e\,a^2+2\,b\,d\,a\right)}{n+1}+\frac{x\,x^{2\,n}\,\left(d\,b^2+2\,a\,e\,b+2\,a\,c\,d\right)}{2\,n+1}+\frac{x\,x^{3\,n}\,\left(e\,b^2+2\,c\,d\,b+2\,a\,c\,e\right)}{3\,n+1}+\frac{c^2\,e\,x\,x^{5\,n}}{5\,n+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x^n)*(a + b*x^n + c*x^(2*n))^2,x)
```

```
[Out] a^2*d*x + (x*x^(4*n)*(c^2*d + 2*b*c*e))/(4*n + 1) + (x*x^n*(a^2*e + 2*a*b*d
))/(n + 1) + (x*x^(2*n)*(b^2*d + 2*a*b*e + 2*a*c*d))/(2*n + 1) + (x*x^(3*n)
*(b^2*e + 2*a*c*e + 2*b*c*d))/(3*n + 1) + (c^2*e*x*x^(5*n))/(5*n + 1)
```

3.68 $\int \left(d+ex^n\right)\left(a+bx^n+cx^{2n}\right)^3\,dx$

Optimal. Leaf size=218

$$a^3dx+\frac{a^2(3bd+ae)x^{1+n}}{1+n}+\frac{3a\left(b^2d+acd+abe\right)x^{1+2n}}{1+2n}+\frac{\left(b^3d+6abcd+3ab^2e+3a^2ce\right)x^{1+3n}}{1+3n}+\frac{(3b^2cd+3ac^2d}{1}$$

```
[Out] a^3*d*x+a^2*(a*e+3*b*d)*x^(1+n)/(1+n)+3*a*(a*b*e+a*c*d+b^2*d)*x^(1+2*n)/(1+
2*n)+(3*a^2*c*e+3*a*b^2*e+6*a*b*c*d+b^3*d)*x^(1+3*n)/(1+3*n)+(6*a*b*c*e+3*a
*c^2*d+b^3*e+3*b^2*c*d)*x^(1+4*n)/(1+4*n)+3*c*(a*c*e+b^2*e+b*c*d)*x^(1+5*n)
/(1+5*n)+c^2*(3*b*e+c*d)*x^(1+6*n)/(1+6*n)+c^3*e*x^(1+7*n)/(1+7*n)
```

Rubi [A]
time = 0.14, antiderivative size = 218, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}=0.042$, Rules used = {1446}

$$a^3dx+\frac{x^{3n+1}\left(3a^2ce+3ab^2e+6abcd+b^3d\right)}{3n+1}+\frac{a^2x^{n+1}(ae+3bd)}{n+1}+\frac{3ax^{2n+1}\left(abe+acd+b^2d\right)}{2n+1}+\frac{3cx^{5n+1}\left(ace+b^2e+bcd\right)}{5n+1}+\frac{x^{4n+1}\left(6abce+3ac^2d+b^3e+3b^2cd\right)}{4n+1}+\frac{c^2x^{6n+1}(3be+cd)}{6n+1}+\frac{c^3ex^{7n+1}}{7n+1}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x^n)*(a + b*x^n + c*x^(2*n))^3,x]
```

```
[Out] a^3*d*x + (a^2*(3*b*d + a*e)*x^(1 + n))/(1 + n) + (3*a*(b^2*d + a*c*d + a*b
*e)*x^(1 + 2*n))/(1 + 2*n) + ((b^3*d + 6*a*b*c*d + 3*a*b^2*e + 3*a^2*c*e)*x
^(1 + 3*n))/(1 + 3*n) + ((3*b^2*c*d + 3*a*c^2*d + b^3*e + 6*a*b*c*e)*x^(1 +
 4*n))/(1 + 4*n) + (3*c*(b*c*d + b^2*e + a*c*e)*x^(1 + 5*n))/(1 + 5*n) + (c
^2*(c*d + 3*b*e)*x^(1 + 6*n))/(1 + 6*n) + (c^3*e*x^(1 + 7*n))/(1 + 7*n)
```

Rule 1446

```
Int[((d_) + (e_.)*(x_)^(n_))*((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_))^(p
_), x_Symbol] :> Int[ExpandIntegrand[(d + e*x^n)*(a + b*x^n + c*x^(2*n))^p,
 x], x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c,
 0]
```

Rubi steps

$$\int \left(d+ex^n\right)\left(a+bx^n+cx^{2n}\right)^3\,dx=\int \left(a^3d+a^2(3bd+ae)x^n+3a\left(b^2d+acd+abe\right)x^{2n}+\left(b^3d+6abcd+\right.\right.$$

$$=a^3dx+\frac{a^2(3bd+ae)x^{1+n}}{1+n}+\frac{3a\left(b^2d+acd+abe\right)x^{1+2n}}{1+2n}+\frac{\left(b^3d+6abcd\right.}{}$$

Mathematica [A]
time = 3.86, size = 205, normalized size = 0.94

$$x\left(a^3d+\frac{a^2(3bd+ae)x^n}{1+n}+\frac{3a\left(b^2d+acd+abe\right)x^{2n}}{1+2n}+\frac{\left(b^3d+6abcd+3ab^2e+3a^2ce\right)x^{3n}}{1+3n}+\frac{\left(3b^2cd+3ac^2d+b^3e+6abce\right)x^{4n}}{1+4n}+\frac{3c\left(bcd+b^2e+ace\right)x^{5n}}{1+5n}+\frac{c^2(cd+3be)x^{6n}}{1+6n}+\frac{c^3ex^{7n}}{1+7n}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x^n)*(a + b*x^n + c*x^(2*n))^3,x]
```

```
[Out] x*(a^3*d + (a^2*(3*b*d + a*e)*x^n)/(1 + n) + (3*a*(b^2*d + a*c*d + a*b*e)*x
^(2*n))/(1 + 2*n) + ((b^3*d + 6*a*b*c*d + 3*a*b^2*e + 3*a^2*c*e)*x^(3*n))/(
1 + 3*n) + ((3*b^2*c*d + 3*a*c^2*d + b^3*e + 6*a*b*c*e)*x^(4*n))/(1 + 4*n)
+ (3*c*(b*c*d + b^2*e + a*c*e)*x^(5*n))/(1 + 5*n) + (c^2*(c*d + 3*b*e)*x^(6
*n))/(1 + 6*n) + (c^3*e*x^(7*n))/(1 + 7*n))
```

Maple [A]
time = 0.03, size = 212, normalized size = 0.97

method	result
risch	$a^3dx + \frac{(6abce+3a\,c^2d+b^3e+3b^2cd)x\,x^{4n}}{1+4n} + \frac{(3a^2ce+3a\,b^2e+6abcd+b^3d)x\,x^{3n}}{1+3n} + \frac{a^2(ae+3bd)x\,x^n}{1+n} + \frac{c^2(3eb+cd)x\,x^{6n}}{1+6n} + c^3$
norman	$a^3dx + \frac{(6abce+3a\,c^2d+b^3e+3b^2cd)x\,\mathrm{e}^{4n\ln(x)}}{1+4n} + \frac{(3a^2ce+3a\,b^2e+6abcd+b^3d)x\,\mathrm{e}^{3n\ln(x)}}{1+3n} + \frac{a^2(ae+3bd)x\,\mathrm{e}^{n\ln(x)}}{1+n} + \frac{c^2(3eb+c}{1-}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d+e*x^n)*(a+b*x^n+c*x^(2*n))^3,x,method=_RETURNVERBOSE)
```

```
[Out] a^3*d*x+(6*a*b*c*e+3*a*c^2*d+b^3*e+3*b^2*c*d)/(1+4*n)*x*(x^n)^4+(3*a^2*c*e+
3*a*b^2*e+6*a*b*c*d+b^3*d)/(1+3*n)*x*(x^n)^3+a^2*(a*e+3*b*d)/(1+n)*x*x^n+c^
2*(3*b*e+c*d)/(1+6*n)*x*(x^n)^6+c^3*e/(1+7*n)*x*(x^n)^7+3*a*(a*b*e+a*c*d+b^
2*d)/(1+2*n)*x*(x^n)^2+3*c*(a*c*e+b^2*e+b*c*d)/(1+5*n)*x*(x^n)^5
```

Maxima [A]
time = 0.29, size = 386, normalized size = 1.77

$$a^3dx + \frac{c^3ex^{7n+1}}{7n+1} + \frac{c^3dx^{6n+1}}{6n+1} + \frac{3bc^2ex^{6n+1}}{6n+1} + \frac{3bc^2dx^{5n+1}}{5n+1} + \frac{3b^2cex^{5n+1}}{5n+1} + \frac{3ac^2ex^{5n+1}}{5n+1} + \frac{3b^2cdx^{4n+1}}{4n+1} + \frac{3ac^2dx^{4n+1}}{4n+1} + \frac{b^3ex^{4n+1}}{4n+1} + \frac{6abcex^{4n+1}}{4n+1} + \frac{b^3dx^{3n+1}}{3n+1} + \frac{6abcdx^{3n+1}}{3n+1} + \frac{3ab^2ex^{3n+1}}{3n+1} + \frac{3a^2cex^{3n+1}}{3n+1} + \frac{3ab^2dx^{2n+1}}{2n+1} + \frac{3a^2cdx^{2n+1}}{2n+1} + \frac{3a^2bex^{2n+1}}{2n+1} + \frac{3a^2bdx^{n+1}}{n+1} + \frac{a^3ex^{n+1}}{n+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+e*x^n)*(a+b*x^n+c*x^(2*n))^3,x, algorithm="maxima")
```

```
[Out] a^3*d*x + c^3*e*x^(7*n + 1)/(7*n + 1) + c^3*d*x^(6*n + 1)/(6*n + 1) + 3*b*c
^2*e*x^(6*n + 1)/(6*n + 1) + 3*b*c^2*d*x^(5*n + 1)/(5*n + 1) + 3*b^2*c*e*x^
(5*n + 1)/(5*n + 1) + 3*a*c^2*e*x^(5*n + 1)/(5*n + 1) + 3*b^2*c*d*x^(4*n +
1)/(4*n + 1) + 3*a*c^2*d*x^(4*n + 1)/(4*n + 1) + b^3*e*x^(4*n + 1)/(4*n + 1
) + 6*a*b*c*e*x^(4*n + 1)/(4*n + 1) + b^3*d*x^(3*n + 1)/(3*n + 1) + 6*a*b*c
*d*x^(3*n + 1)/(3*n + 1) + 3*a*b^2*e*x^(3*n + 1)/(3*n + 1) + 3*a^2*c*e*x^(3
*n + 1)/(3*n + 1) + 3*a*b^2*d*x^(2*n + 1)/(2*n + 1) + 3*a^2*c*d*x^(2*n + 1)
/(2*n + 1) + 3*a^2*b*e*x^(2*n + 1)/(2*n + 1) + 3*a^2*b*d*x^(n + 1)/(n + 1)
+ a^3*e*x^(n + 1)/(n + 1)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 1232 vs. $2(228) = 456$.

time = 0.36, size = 1232, normalized size = 5.65

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+e*x^n)*(a+b*x^n+c*x^(2*n))^3,x, algorithm="fricas")
```

```
[Out] ((720*c^3*n^6 + 1764*c^3*n^5 + 1624*c^3*n^4 + 735*c^3*n^3 + 175*c^3*n^2 + 2
1*c^3*n + c^3)*x*x^(7*n)*e + (5040*a^3*d*n^7 + 13068*a^3*d*n^6 + 13132*a^3*
d*n^5 + 6769*a^3*d*n^4 + 1960*a^3*d*n^3 + 322*a^3*d*n^2 + 28*a^3*d*n + a^3*
d)*x + (3*(840*b*c^2*n^6 + 2038*b*c^2*n^5 + 1849*b*c^2*n^4 + 820*b*c^2*n^3
+ 190*b*c^2*n^2 + 22*b*c^2*n + b*c^2)*x*e + (840*c^3*d*n^6 + 2038*c^3*d*n^5
 + 1849*c^3*d*n^4 + 820*c^3*d*n^3 + 190*c^3*d*n^2 + 22*c^3*d*n + c^3*d)*x)*
x^(6*n) + 3*((1008*(b^2*c + a*c^2)*n^6 + 2412*(b^2*c + a*c^2)*n^5 + 2144*(b
^2*c + a*c^2)*n^4 + 925*(b^2*c + a*c^2)*n^3 + b^2*c + a*c^2 + 207*(b^2*c +
a*c^2)*n^2 + 23*(b^2*c + a*c^2)*n)*x*e + (1008*b*c^2*d*n^6 + 2412*b*c^2*d*n
^5 + 2144*b*c^2*d*n^4 + 925*b*c^2*d*n^3 + 207*b*c^2*d*n^2 + 23*b*c^2*d*n +
b*c^2*d)*x)*x^(5*n) + ((1260*(b^3 + 6*a*b*c)*n^6 + 2952*(b^3 + 6*a*b*c)*n^5
 + 2545*(b^3 + 6*a*b*c)*n^4 + 1056*(b^3 + 6*a*b*c)*n^3 + b^3 + 6*a*b*c + 22
6*(b^3 + 6*a*b*c)*n^2 + 24*(b^3 + 6*a*b*c)*n)*x*e + 3*(1260*(b^2*c + a*c^2)
*d*n^6 + 2952*(b^2*c + a*c^2)*d*n^5 + 2545*(b^2*c + a*c^2)*d*n^4 + 1056*(b^
2*c + a*c^2)*d*n^3 + 226*(b^2*c + a*c^2)*d*n^2 + 24*(b^2*c + a*c^2)*d*n + (
b^2*c + a*c^2)*d)*x)*x^(4*n) + (3*(1680*(a*b^2 + a^2*c)*n^6 + 3796*(a*b^2 +
 a^2*c)*n^5 + 3112*(a*b^2 + a^2*c)*n^4 + 1219*(a*b^2 + a^2*c)*n^3 + a*b^2 +
 a^2*c + 247*(a*b^2 + a^2*c)*n^2 + 25*(a*b^2 + a^2*c)*n)*x*e + (1680*(b^3 +
 6*a*b*c)*d*n^6 + 3796*(b^3 + 6*a*b*c)*d*n^5 + 3112*(b^3 + 6*a*b*c)*d*n^4 +
 1219*(b^3 + 6*a*b*c)*d*n^3 + 247*(b^3 + 6*a*b*c)*d*n^2 + 25*(b^3 + 6*a*b*c
)*d*n + (b^3 + 6*a*b*c)*d)*x)*x^(3*n) + 3*((2520*a^2*b*n^6 + 5274*a^2*b*n^5
 + 3929*a^2*b*n^4 + 1420*a^2*b*n^3 + 270*a^2*b*n^2 + 26*a^2*b*n + a^2*b)*x*
e + (2520*(a*b^2 + a^2*c)*d*n^6 + 5274*(a*b^2 + a^2*c)*d*n^5 + 3929*(a*b^2
+ a^2*c)*d*n^4 + 1420*(a*b^2 + a^2*c)*d*n^3 + 270*(a*b^2 + a^2*c)*d*n^2 + 2
6*(a*b^2 + a^2*c)*d*n + (a*b^2 + a^2*c)*d)*x)*x^(2*n) + ((5040*a^3*n^6 + 80
28*a^3*n^5 + 5104*a^3*n^4 + 1665*a^3*n^3 + 295*a^3*n^2 + 27*a^3*n + a^3)*x*
e + 3*(5040*a^2*b*d*n^6 + 8028*a^2*b*d*n^5 + 5104*a^2*b*d*n^4 + 1665*a^2*b*
d*n^3 + 295*a^2*b*d*n^2 + 27*a^2*b*d*n + a^2*b*d)*x)*x^n)/(5040*n^7 + 13068
*n^6 + 13132*n^5 + 6769*n^4 + 1960*n^3 + 322*n^2 + 28*n + 1)
```

Sympy [**B**] Leaf count of result is larger than twice the leaf count of optimal. 9190 vs. $2(212) = 424$.
time = 8.41, size = 9190, normalized size = 42.16

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+e*x**n)*(a+b*x**n+c*x**(2*n))**3,x)
```

```
[Out] Piecewise((a**3*d*x + a**3*e*log(x) + 3*a**2*b*d*log(x) - 3*a**2*b*e/x - 3*
a**2*c*d/x - 3*a**2*c*e/(2*x**2) - 3*a*b**2*d/x - 3*a*b**2*e/(2*x**2) - 3*a
*b*c*d/x**2 - 2*a*b*c*e/x**3 - a*c**2*d/x**3 - 3*a*c**2*e/(4*x**4) - b**3*d
/(2*x**2) - b**3*e/(3*x**3) - b**2*c*d/x**3 - 3*b**2*c*e/(4*x**4) - 3*b*c**
2*d/(4*x**4) - 3*b*c**2*e/(5*x**5) - c**3*d/(5*x**5) - c**3*e/(6*x**6), Eq(
n, -1)), (a**3*d*x + 2*a**3*e*sqrt(x) + 6*a**2*b*d*sqrt(x) + 3*a**2*b*e*log
(x) + 3*a**2*c*d*log(x) - 6*a**2*c*e/sqrt(x) + 3*a*b**2*d*log(x) - 6*a*b**2
*e/sqrt(x) - 12*a*b*c*d/sqrt(x) - 6*a*b*c*e/x - 3*a*c**2*d/x - 2*a*c**2*e/x
**(3/2) - 2*b**3*d/sqrt(x) - b**3*e/x - 3*b**2*c*d/x - 2*b**2*c*e/x**(3/2)
- 2*b*c**2*d/x**(3/2) - 3*b*c**2*e/(2*x**2) - c**3*d/(2*x**2) - 2*c**3*e/(5
*x**(5/2)), Eq(n, -1/2)), (a**3*d*x + 3*a**3*e*x**(2/3)/2 + 9*a**2*b*d*x**(
2/3)/2 + 9*a**2*b*e*x**(1/3) + 9*a**2*c*d*x**(1/3) + 3*a**2*c*e*log(x) + 9*
a*b**2*d*x**(1/3) + 3*a*b**2*e*log(x) + 6*a*b*c*d*log(x) - 18*a*b*c*e/x**(1
/3) - 9*a*c**2*d/x**(1/3) - 9*a*c**2*e/(2*x**(2/3)) + b**3*d*log(x) - 3*b**
3*e/x**(1/3) - 9*b**2*c*d/x**(1/3) - 9*b**2*c*e/(2*x**(2/3)) - 9*b*c**2*d/(
2*x**(2/3)) - 3*b*c**2*e/x - c**3*d/x - 3*c**3*e/(4*x**(4/3)), Eq(n, -1/3))
, (a**3*d*x + 4*a**3*e*x**(3/4)/3 + 4*a**2*b*d*x**(3/4) + 6*a**2*b*e*sqrt(x
) + 6*a**2*c*d*sqrt(x) + 12*a**2*c*e*x**(1/4) + 6*a*b**2*d*sqrt(x) + 12*a*b
**2*e*x**(1/4) + 24*a*b*c*d*x**(1/4) + 6*a*b*c*e*log(x) + 3*a*c**2*d*log(x)
 - 12*a*c**2*e/x**(1/4) + 4*b**3*d*x**(1/4) + b**3*e*log(x) + 3*b**2*c*d*lo
g(x) - 12*b**2*c*e/x**(1/4) - 12*b*c**2*d/x**(1/4) - 6*b*c**2*e/sqrt(x) - 2
*c**3*d/sqrt(x) - 4*c**3*e/(3*x**(3/4)), Eq(n, -1/4)), (a**3*d*x + 5*a**3*e
*x**(4/5)/4 + 15*a**2*b*d*x**(4/5)/4 + 5*a**2*b*e*x**(3/5) + 5*a**2*c*d*x**
(3/5) + 15*a**2*c*e*x**(2/5)/2 + 5*a*b**2*d*x**(3/5) + 15*a*b**2*e*x**(2/5)
/2 + 15*a*b*c*d*x**(2/5) + 30*a*b*c*e*x**(1/5) + 15*a*c**2*d*x**(1/5) + 3*a
*c**2*e*log(x) + 5*b**3*d*x**(2/5)/2 + 5*b**3*e*x**(1/5) + 15*b**2*c*d*x**(
1/5) + 3*b**2*c*e*log(x) + 3*b*c**2*d*log(x) - 15*b*c**2*e/x**(1/5) - 5*c**
3*d/x**(1/5) - 5*c**3*e/(2*x**(2/5)), Eq(n, -1/5)), (a**3*d*x + 6*a**3*e*x*
*(5/6)/5 + 18*a**2*b*d*x**(5/6)/5 + 9*a**2*b*e*x**(2/3)/2 + 9*a**2*c*d*x**(
2/3)/2 + 6*a**2*c*e*sqrt(x) + 9*a*b**2*d*x**(2/3)/2 + 6*a*b**2*e*sqrt(x) +
12*a*b*c*d*sqrt(x) + 18*a*b*c*e*x**(1/3) + 9*a*c**2*d*x**(1/3) + 18*a*c**2*
e*x**(1/6) + 2*b**3*d*sqrt(x) + 3*b**3*e*x**(1/3) + 9*b**2*c*d*x**(1/3) + 1
8*b**2*c*e*x**(1/6) + 18*b*c**2*d*x**(1/6) + 3*b*c**2*e*log(x) + c**3*d*log
(x) - 6*c**3*e/x**(1/6), Eq(n, -1/6)), (a**3*d*x + 7*a**3*e*x**(6/7)/6 + 7*
a**2*b*d*x**(6/7)/2 + 21*a**2*b*e*x**(5/7)/5 + 21*a**2*c*d*x**(5/7)/5 + 21*
a**2*c*e*x**(4/7)/4 + 21*a*b**2*d*x**(5/7)/5 + 21*a*b**2*e*x**(4/7)/4 + 21*
a*b*c*d*x**(4/7)/2 + 14*a*b*c*e*x**(3/7) + 7*a*c**2*d*x**(3/7) + 21*a*c**2*
e*x**(2/7)/2 + 7*b**3*d*x**(4/7)/4 + 7*b**3*e*x**(3/7)/3 + 7*b**2*c*d*x**(3
/7) + 21*b**2*c*e*x**(2/7)/2 + 21*b*c**2*d*x**(2/7)/2 + 21*b*c**2*e*x**(1/7
) + 7*c**3*d*x**(1/7) + c**3*e*log(x), Eq(n, -1/7)), (5040*a**3*d*n**7*x/(5
040*n**7 + 13068*n**6 + 13132*n**5 + 6769*n**4 + 1960*n**3 + 322*n**2 + 28*
n + 1) + 13068*a**3*d*n**6*x/(5040*n**7 + 13068*n**6 + 13132*n**5 + 6769*n*
*4 + 1960*n**3 + 322*n**2 + 28*n + 1) + 13132*a**3*d*n**5*x/(5040*n**7 + 13
068*n**6 + 13132*n**5 + 6769*n**4 + 1960*n**3 + 322*n**2 + 28*n + 1) + 6769
*a**3*d*n**4*x/(5040*n**7 + 13068*n**6 + 13132*n**5 + 6769*n**4 + 1960*n**3
```

```
 + 322*n**2 + 28*n + 1) + 1960*a**3*d*n**3*x/(5040*n**7 + 13068*n**6 + 1313
2*n**5 + 6769*n**4 + 1960*n**3 + 322*n**2 + 28*n + 1) + 322*a**3*d*n**2*x/(
5040*n**7 + 13068*n**6 + 13132*n**5 + 6769*n**4 + 1960*n**3 + 322*n**2 + 28
*n + 1) + 28*a**3*d*n*x/(5040*n**7 + 13068*n**6 + 13132*n**5 + 6769*n**4 +
1960*n**3 + 322*n**2 + 28*n + 1) + a**3*d*x/(5040*n**7 + 13068*n**6 + 13132
*n**5 + 6769*n**4 + 1960*n**3 + 322*n**2 + 28*n + 1) + 5040*a**3*e*n**6*x*x
**n/(5040*n**7 + 13068*n**6 + 13132*n**5 + 6769*n**4 + 1960*n**3 + 322*n**2
 + 28*n + 1) + 8028*a**3*e*n**5*x*x**n/(5040*n**7 + 13068*n**6 + 13132*n**5
 + 6769*n**4 + 1960*n**3 + 322*n**2 + 28*n + 1) + 5104*a**3*e*n**4*x*x**n/(
5040*n**7 + 13068*n**6 + 13132*n**5 + 6769*n**4 + 1960*n**3 + 322*n**2 + 28
*n + 1) + 1665*a**3*e*n**3*x*x**n/(5040*n**7 + 13068*n**6 + 13132*n**5 + 67
69*n**4 + 1960*n**3 + 322*n**2 + 28*n + 1) + 295*a**3*e*n**2*x*x**n/(5040*n
**7 + 13068*n**6 + 13132*n**5 + 6769*n**4 + 1960*n**3 + 322*n**2 + 28*n + 1
) + 27*a**3*e*n*x*x**n/(5040*n**7 + 13068*n**6 + 13132*n**5 + 6769*n**4 + 1
960*n**3 + 322*n**2 + 28*n + 1) + a**3*e*x*x**n/(5040*n**7 + 13068*n**6 + 1
3132*n**5 + 6769*n**4 + 1960*n**3 + 322*n**2 + 28*n + 1) + 15120*a**2*b*d*n
**6*x*x**n/(5040*n**7 + 13068*n**6 + 13132*n**5 + 6769*n**4 + 1960*n**3 + 3
22*n**2 + 28*n + 1) + 24084*a**2*b*d*n**5*x*x**n/(5040*n**7 + 13068*n**6 +
13132*n**5 + 6769*n**4 + 1960*n**3 + 322*n**2 + 28*n + 1) + 15312*a**2*b*d*
n**4*x*x**n/(5040*n**7 + 13068*n**6 + 13132*n**5 + 6769*n**4 + 1960*n**3 +
322*n**2 + 28*n + 1) + 4995*a**2*b*d*n**3*x*x**n/(5040*n**7 + 13068*n**6 +
13132*n**5 + 6769*n**4 + 1960*n**3 + 322*n**2 +...
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 2134 vs. $2(228) = 456$.
time = 3.36, size = 2134, normalized size = 9.79

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+e*x^n)*(a+b*x^n+c*x^(2*n))^3,x, algorithm="giac")
```

```
[Out] (5040*a^3*d*n^7*x + 840*c^3*d*n^6*x*x^(6*n) + 3024*b*c^2*d*n^6*x*x^(5*n) +
3780*b^2*c*d*n^6*x*x^(4*n) + 3780*a*c^2*d*n^6*x*x^(4*n) + 1680*b^3*d*n^6*x*
x^(3*n) + 10080*a*b*c*d*n^6*x*x^(3*n) + 7560*a*b^2*d*n^6*x*x^(2*n) + 7560*a
^2*c*d*n^6*x*x^(2*n) + 15120*a^2*b*d*n^6*x*x^n + 720*c^3*n^6*x*x^(7*n)*e +
2520*b*c^2*n^6*x*x^(6*n)*e + 3024*b^2*c*n^6*x*x^(5*n)*e + 3024*a*c^2*n^6*x*
x^(5*n)*e + 1260*b^3*n^6*x*x^(4*n)*e + 7560*a*b*c*n^6*x*x^(4*n)*e + 5040*a*
b^2*n^6*x*x^(3*n)*e + 5040*a^2*c*n^6*x*x^(3*n)*e + 7560*a^2*b*n^6*x*x^(2*n)
*e + 5040*a^3*n^6*x*x^n*e + 13068*a^3*d*n^6*x + 2038*c^3*d*n^5*x*x^(6*n) +
7236*b*c^2*d*n^5*x*x^(5*n) + 8856*b^2*c*d*n^5*x*x^(4*n) + 8856*a*c^2*d*n^5*
x*x^(4*n) + 3796*b^3*d*n^5*x*x^(3*n) + 22776*a*b*c*d*n^5*x*x^(3*n) + 15822*
a*b^2*d*n^5*x*x^(2*n) + 15822*a^2*c*d*n^5*x*x^(2*n) + 24084*a^2*b*d*n^5*x*x
^n + 1764*c^3*n^5*x*x^(7*n)*e + 6114*b*c^2*n^5*x*x^(6*n)*e + 7236*b^2*c*n^5
*x*x^(5*n)*e + 7236*a*c^2*n^5*x*x^(5*n)*e + 2952*b^3*n^5*x*x^(4*n)*e + 1771
```

```
2*a*b*c*n^5*x*x^(4*n)*e + 11388*a*b^2*n^5*x*x^(3*n)*e + 11388*a^2*c*n^5*x*x
^(3*n)*e + 15822*a^2*b*n^5*x*x^(2*n)*e + 8028*a^3*n^5*x*x^n*e + 13132*a^3*d
*n^5*x + 1849*c^3*d*n^4*x*x^(6*n) + 6432*b*c^2*d*n^4*x*x^(5*n) + 7635*b^2*c
*d*n^4*x*x^(4*n) + 7635*a*c^2*d*n^4*x*x^(4*n) + 3112*b^3*d*n^4*x*x^(3*n) +
18672*a*b*c*d*n^4*x*x^(3*n) + 11787*a*b^2*d*n^4*x*x^(2*n) + 11787*a^2*c*d*n
^4*x*x^(2*n) + 15312*a^2*b*d*n^4*x*x^n + 1624*c^3*n^4*x*x^(7*n)*e + 5547*b*
c^2*n^4*x*x^(6*n)*e + 6432*b^2*c*n^4*x*x^(5*n)*e + 6432*a*c^2*n^4*x*x^(5*n)
*e + 2545*b^3*n^4*x*x^(4*n)*e + 15270*a*b*c*n^4*x*x^(4*n)*e + 9336*a*b^2*n^
4*x*x^(3*n)*e + 9336*a^2*c*n^4*x*x^(3*n)*e + 11787*a^2*b*n^4*x*x^(2*n)*e +
5104*a^3*n^4*x*x^n*e + 6769*a^3*d*n^4*x + 820*c^3*d*n^3*x*x^(6*n) + 2775*b*
c^2*d*n^3*x*x^(5*n) + 3168*b^2*c*d*n^3*x*x^(4*n) + 3168*a*c^2*d*n^3*x*x^(4*
n) + 1219*b^3*d*n^3*x*x^(3*n) + 7314*a*b*c*d*n^3*x*x^(3*n) + 4260*a*b^2*d*n
^3*x*x^(2*n) + 4260*a^2*c*d*n^3*x*x^(2*n) + 4995*a^2*b*d*n^3*x*x^n + 735*c^
3*n^3*x*x^(7*n)*e + 2460*b*c^2*n^3*x*x^(6*n)*e + 2775*b^2*c*n^3*x*x^(5*n)*e
 + 2775*a*c^2*n^3*x*x^(5*n)*e + 1056*b^3*n^3*x*x^(4*n)*e + 6336*a*b*c*n^3*x
*x^(4*n)*e + 3657*a*b^2*n^3*x*x^(3*n)*e + 3657*a^2*c*n^3*x*x^(3*n)*e + 4260
*a^2*b*n^3*x*x^(2*n)*e + 1665*a^3*n^3*x*x^n*e + 1960*a^3*d*n^3*x + 190*c^3*
d*n^2*x*x^(6*n) + 621*b*c^2*d*n^2*x*x^(5*n) + 678*b^2*c*d*n^2*x*x^(4*n) + 6
78*a*c^2*d*n^2*x*x^(4*n) + 247*b^3*d*n^2*x*x^(3*n) + 1482*a*b*c*d*n^2*x*x^(
3*n) + 810*a*b^2*d*n^2*x*x^(2*n) + 810*a^2*c*d*n^2*x*x^(2*n) + 885*a^2*b*d*
n^2*x*x^n + 175*c^3*n^2*x*x^(7*n)*e + 570*b*c^2*n^2*x*x^(6*n)*e + 621*b^2*c
*n^2*x*x^(5*n)*e + 621*a*c^2*n^2*x*x^(5*n)*e + 226*b^3*n^2*x*x^(4*n)*e + 13
56*a*b*c*n^2*x*x^(4*n)*e + 741*a*b^2*n^2*x*x^(3*n)*e + 741*a^2*c*n^2*x*x^(3
*n)*e + 810*a^2*b*n^2*x*x^(2*n)*e + 295*a^3*n^2*x*x^n*e + 322*a^3*d*n^2*x +
 22*c^3*d*n*x*x^(6*n) + 69*b*c^2*d*n*x*x^(5*n) + 72*b^2*c*d*n*x*x^(4*n) + 7
2*a*c^2*d*n*x*x^(4*n) + 25*b^3*d*n*x*x^(3*n) + 150*a*b*c*d*n*x*x^(3*n) + 78
*a*b^2*d*n*x*x^(2*n) + 78*a^2*c*d*n*x*x^(2*n) + 81*a^2*b*d*n*x*x^n + 21*c^3
*n*x*x^(7*n)*e + 66*b*c^2*n*x*x^(6*n)*e + 69*b^2*c*n*x*x^(5*n)*e + 69*a*c^2
*n*x*x^(5*n)*e + 24*b^3*n*x*x^(4*n)*e + 144*a*b*c*n*x*x^(4*n)*e + 75*a*b^2*
n*x*x^(3*n)*e + 75*a^2*c*n*x*x^(3*n)*e + 78*a^2*b*n*x*x^(2*n)*e + 27*a^3*n*
x*x^n*e + 28*a^3*d*n*x + c^3*d*x*x^(6*n) + 3*b*c^2*d*x*x^(5*n) + 3*b^2*c*d*
x*x^(4*n) + 3*a*c^2*d*x*x^(4*n) + b^3*d*x*x^(3*n) + 6*a*b*c*d*x*x^(3*n) + 3
*a*b^2*d*x*x^(2*n) + 3*a^2*c*d*x*x^(2*n) + 3*a^2*b*d*x*x^n + c^3*x*x^(7*n)*
e + 3*b*c^2*x*x^(6*n)*e + 3*b^2*c*x*x^(5*n)*e + 3*a*c^2*x*x^(5*n)*e + b^3*x
*x^(4*n)*e + 6*a*b*c*x*x^(4*n)*e + 3*a*b^2*x*x^(3*n)*e + 3*a^2*c*x*x^(3*n)*
e + 3*a^2*b*x*x^(2*n)*e + a^3*x*x^n*e + a^3*d*x)/(5040*n^7 + 13068*n^6 + 13
132*n^5 + 6769*n^4 + 1960*n^3 + 322*n^2 + 28*n + 1)
```

Mupad [B]
time = 1.85, size = 227, normalized size = 1.04

$$a^3\,d\,x+\frac{x\,x^n\,\left(e\,a^3+3\,b\,d\,a^2\right)}{n+1}+\frac{x\,x^{2\,n}\,\left(3\,e\,a^2\,b+3\,c\,d\,a^2+3\,d\,a\,b^2\right)}{2\,n+1}+\frac{x\,x^{5\,n}\,\left(3\,e\,b^2\,c+3\,d\,b\,c^2+3\,a\,e\,c^2\right)}{5\,n+1}+\frac{x\,x^{3\,n}\,\left(3\,c\,e\,a^2+3\,e\,a\,b^2+6\,c\,d\,a\,b+d\,b^3\right)}{3\,n+1}+\frac{x\,x^{4\,n}\,\left(e\,b^3+3\,d\,b^2\,c+6\,a\,e\,b\,c+3\,a\,d\,c^2\right)}{4\,n+1}+\frac{x\,x^{6\,n}\,\left(d\,c^3+3\,b\,e\,c^2\right)}{6\,n+1}+\frac{c^3\,e\,x\,x^{7\,n}}{7\,n+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x^n)*(a + b*x^n + c*x^(2*n))^3,x)
```

```
[Out] a^3*d*x + (x*x^n*(a^3*e + 3*a^2*b*d))/(n + 1) + (x*x^(2*n)*(3*a*b^2*d + 3*a
^2*b*e + 3*a^2*c*d))/(2*n + 1) + (x*x^(5*n)*(3*a*c^2*e + 3*b*c^2*d + 3*b^2*
c*e))/(5*n + 1) + (x*x^(3*n)*(b^3*d + 3*a*b^2*e + 3*a^2*c*e + 6*a*b*c*d))/(
3*n + 1) + (x*x^(4*n)*(b^3*e + 3*a*c^2*d + 3*b^2*c*d + 6*a*b*c*e))/(4*n + 1
) + (x*x^(6*n)*(c^3*d + 3*b*c^2*e))/(6*n + 1) + (c^3*e*x*x^(7*n))/(7*n + 1)
```

3.69 $\int \frac{(d+ex^n)^3}{a+bx^n+cx^{2n}}\,dx$

Optimal. Leaf size=308

$$\frac{e^2(3cd-be)x}{c^2}+\frac{e^3x^{1+n}}{c(1+n)}+\frac{\left(3c^2d^2e-3bcde^2+b^2e^3-ace^3+\frac{(2cd-be)(c^2d^2+b^2e^2-ce(bd+3ae))}{\sqrt{b^2-4ac}}\right)x\,{}_2F_1\left(1,\frac{1}{n};1+\frac{1}{n}\right.}{c^2\left(b-\sqrt{b^2-4ac}\right)}$$

```
[Out] e^2*(-b*e+3*c*d)*x/c^2+e^3*x^(1+n)/c/(1+n)+x*hypergeom([1, 1/n],[1+1/n],-2*
c*x^n/(b-(-4*a*c+b^2)^(1/2)))*(3*c^2*d^2*e-3*b*c*d*e^2+b^2*e^3-a*c*e^3+(-b*
e+2*c*d)*(c^2*d^2+b^2*e^2-c*e*(3*a*e+b*d))/(-4*a*c+b^2)^(1/2))/c^2/(b-(-4*a
*c+b^2)^(1/2))+x*hypergeom([1, 1/n],[1+1/n],-2*c*x^n/(b+(-4*a*c+b^2)^(1/2))
)*(3*c^2*d^2*e-3*b*c*d*e^2+b^2*e^3-a*c*e^3-(-b*e+2*c*d)*(c^2*d^2+b^2*e^2-c*
e*(3*a*e+b*d))/(-4*a*c+b^2)^(1/2))/c^2/(b+(-4*a*c+b^2)^(1/2))
```

Rubi [A]

time = 0.46, antiderivative size = 308, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.115$, Rules used = {1438, 1436, 251}

$$\frac{x\left(\frac{(2cd-be)(-ce(3ae+bd)+b^2e^2+c^2d^2)}{\sqrt{b^2-4ac}}-ace^3+b^2e^3-3bcde^2+3c^2d^2e\right)\,{}_2F_1\left(1,\frac{1}{n};1+\frac{1}{n};-\frac{2cx^n}{b-\sqrt{b^2-4ac}}\right)}{c^2\left(b-\sqrt{b^2-4ac}\right)}+\frac{x\left(-\frac{(2cd-be)(-ce(3ae+bd)+b^2e^2+c^2d^2)}{\sqrt{b^2-4ac}}-ace^3+b^2e^3-3bcde^2+3c^2d^2e\right)\,{}_2F_1\left(1,\frac{1}{n};1+\frac{1}{n};-\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)}{c^2\left(\sqrt{b^2-4ac}+b\right)}+\frac{e^2x(3cd-be)}{c^2}+\frac{e^3x^{n+1}}{c(n+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x^n)^3/(a + b*x^n + c*x^(2*n)),x]
```

```
[Out] (e^2*(3*c*d - b*e)*x)/c^2 + (e^3*x^(1 + n))/(c*(1 + n)) + ((3*c^2*d^2*e - 3
*b*c*d*e^2 + b^2*e^3 - a*c*e^3 + ((2*c*d - b*e)*(c^2*d^2 + b^2*e^2 - c*e*(b
*d + 3*a*e)))/Sqrt[b^2 - 4*a*c])*x*Hypergeometric2F1[1, n^(-1), 1 + n^(-1),
 (-2*c*x^n)/(b - Sqrt[b^2 - 4*a*c])])/(c^2*(b - Sqrt[b^2 - 4*a*c])) + ((3*c
^2*d^2*e - 3*b*c*d*e^2 + b^2*e^3 - a*c*e^3 - ((2*c*d - b*e)*(c^2*d^2 + b^2*
e^2 - c*e*(b*d + 3*a*e)))/Sqrt[b^2 - 4*a*c])*x*Hypergeometric2F1[1, n^(-1),
 1 + n^(-1), (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c])])/(c^2*(b + Sqrt[b^2 - 4*a*
c]))
```

Rule 251

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, (-b)*(x^n/a)], x] /; FreeQ[{a, b, n, p}, x] && !IGtQ[p
, 0] && !IntegerQ[1/n] && !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

Rule 1436

```
Int[((d_) + (e_.)*(x_)^(n_))/((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_)), x
_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q),
```

```
Int[1/(b/2 - q/2 + c*x^n), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(
b/2 + q/2 + c*x^n), x], x]] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[n2, 2*n]
 && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && (PosQ[b^2 - 4*a
*c] ||  !IGtQ[n/2, 0])
```

Rule 1438

```
Int[((d_) + (e_.)*(x_)^(n_))^(q_)/((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_
)), x_Symbol] :> Int[ExpandIntegrand[(d + e*x^n)^q/(a + b*x^n + c*x^(2*n)),
 x], x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c,
 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[q]
```

Rubi steps

$$\int \frac{(d+ex^n)^3}{a+bx^n+cx^{2n}}\,dx = \int \left(\frac{e^2(3cd-be)}{c^2}+\frac{e^3x^n}{c}+\frac{c^2d^3-3acde^2+abe^3+\left(3c^2d^2e-3bcde^2+b^2e^3-ace^3\right)}{c^2\left(a+bx^n+cx^{2n}\right)}\right.$$

$$= \frac{e^2(3cd-be)x}{c^2}+\frac{e^3x^{1+n}}{c(1+n)}+\frac{\int \frac{c^2d^3-3acde^2+abe^3+\left(3c^2d^2e-3bcde^2+b^2e^3-ace^3\right)x^n}{a+bx^n+cx^{2n}}\,dx}{c^2}$$

$$= \frac{e^2(3cd-be)x}{c^2}+\frac{e^3x^{1+n}}{c(1+n)}+\frac{\left(3c^2d^2e-3bcde^2+b^2e^3-ace^3-\frac{(2cd-be)\left(c^2d^2+b^2e^2-ce(bd\right.}{\sqrt{b^2-4ac}}\right.}{2c^2}$$

$$= \frac{e^2(3cd-be)x}{c^2}+\frac{e^3x^{1+n}}{c(1+n)}+\frac{\left(3c^2d^2e-3bcde^2+b^2e^3-ace^3+\frac{(2cd-be)\left(c^2d^2+b^2e^2-ce(bd\right.}{\sqrt{b^2-4ac}}\right.}{c^2\left(b-\sqrt{b^2-4\right.}}$$

Mathematica [A]
time = 2.67, size = 455, normalized size = 1.48

$$-\frac{2^{-\frac{1+n}{n}}x\left(-\frac{2^{1+\frac{1}{n}}c\sqrt{b^2-4ac}\left(cd^3(1+n)+ae^3x^n\right)}{1+n}+\left(-ab^2e^3+b\left(c^2d^3+3acde^2+a\sqrt{b^2-4ac}\,e^3\right)+c\left(cd^2\left(\sqrt{b^2-4ac}\,d-6ae\right)+ae^2\left(-3\sqrt{b^2-4ac}\,d+2ae\right)\right)\right)\left(\frac{cx^n}{b-\sqrt{b^2-4ac}+2cx^n}\right)^{-1/n}{}_2F_1\left(-\frac{1}{n},-\frac{1}{n};\frac{-1+n}{n};\frac{b-\sqrt{b^2-4ac}}{b-\sqrt{b^2-4ac}+2cx^n}\right)+\left(ab^2e^3+b\left(-c^2d^3-3acde^2+a\sqrt{b^2-4ac}\,e^3\right)+c\left(-ae^2\left(3\sqrt{b^2-4ac}\,d+2ae\right)+cd^2\left(\sqrt{b^2-4ac}\,d+6ae\right)\right)\right)\left(\frac{cx^n}{b+\sqrt{b^2-4ac}+2cx^n}\right)^{-1/n}{}_2F_1\left(-\frac{1}{n},-\frac{1}{n};\frac{-1+n}{n};\frac{b+\sqrt{b^2-4ac}}{b+\sqrt{b^2-4ac}+2cx^n}\right)\right)}{ac^2\sqrt{b^2-4ac}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x^n)^3/(a + b*x^n + c*x^(2*n)),x]
```

```
[Out] -((x*(-((2^(1 + n^(-1))*c*Sqrt[b^2 - 4*a*c]*(c*d^3*(1 + n) + a*e^3*x^n))/(1
 + n)) + ((-(a*b^2*e^3) + b*(c^2*d^3 + 3*a*c*d*e^2 + a*Sqrt[b^2 - 4*a*c]*e^
3) + c*(c*d^2*(Sqrt[b^2 - 4*a*c]*d - 6*a*e) + a*e^2*(-3*Sqrt[b^2 - 4*a*c]*d
 + 2*a*e)))*Hypergeometric2F1[-n^(-1), -n^(-1), (-1 + n)/n, (b - Sqrt[b^2 -
 4*a*c])/(b - Sqrt[b^2 - 4*a*c] + 2*c*x^n)])/((c*x^n)/(b - Sqrt[b^2 - 4*a*c
] + 2*c*x^n))^n^(-1) + ((a*b^2*e^3 + b*(-(c^2*d^3) - 3*a*c*d*e^2 + a*Sqrt[b
^2 - 4*a*c]*e^3) + c*(-(a*e^2*(3*Sqrt[b^2 - 4*a*c]*d + 2*a*e)) + c*d^2*(Sqr
```

```
t[b^2 - 4*a*c]*d + 6*a*e)))*Hypergeometric2F1[-n^(-1), -n^(-1), (-1 + n)/n,
 (b + Sqrt[b^2 - 4*a*c])/(b + Sqrt[b^2 - 4*a*c] + 2*c*x^n)])/((c*x^n)/(b +
Sqrt[b^2 - 4*a*c] + 2*c*x^n))^n^(-1)))/(2^((1 + n)/n)*a*c^2*Sqrt[b^2 - 4*a*
c]))
```

Maple [**F**]
time = 0.01, size = 0, normalized size = 0.00

$$\int \frac{\left(d+e\,x^{n}\right)^{3}}{a+b\,x^{n}+c\,x^{2n}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d+e*x^n)^3/(a+b*x^n+c*x^(2*n)),x)
```

```
[Out] int((d+e*x^n)^3/(a+b*x^n+c*x^(2*n)),x)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+e*x^n)^3/(a+b*x^n+c*x^(2*n)),x, algorithm="maxima")
```

```
[Out] (c*x*e^(n*log(x) + 3) + (3*c*d*(n + 1)*e^2 - b*(n + 1)*e^3)*x)/(c^2*(n + 1)
) - integrate(-(c^2*d^3 - 3*a*c*d*e^2 + a*b*e^3 + (3*c^2*d^2*e - 3*b*c*d*e^
2 + (b^2 - a*c)*e^3)*x^n)/(c^3*x^(2*n) + b*c^2*x^n + a*c^2), x)
```

Fricas [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+e*x^n)^3/(a+b*x^n+c*x^(2*n)),x, algorithm="fricas")
```

```
[Out] integral((3*d^2*x^n*e + d^3 + 3*d*x^(2*n)*e^2 + x^(3*n)*e^3)/(c*x^(2*n) + b
*x^n + a), x)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+e*x**n)**3/(a+b*x**n+c*x**(2*n)),x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+e*x^n)^3/(a+b*x^n+c*x^(2*n)),x, algorithm="giac")
```

```
[Out] integrate((x^n*e + d)^3/(c*x^(2*n) + b*x^n + a), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(d+e\,x^n)^3}{a+b\,x^n+c\,x^{2\,n}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x^n)^3/(a + b*x^n + c*x^(2*n)),x)
```

```
[Out] int((d + e*x^n)^3/(a + b*x^n + c*x^(2*n)), x)
```

3.70 $\int \frac{(d+ex^n)^2}{a+bx^n+cx^{2n}}\,dx$

Optimal. Leaf size=224

$$\frac{e^2x}{c}+\frac{\left(2cde-be^2+\frac{2c^2d^2+b^2e^2-2ce(bd+ae)}{\sqrt{b^2-4ac}}\right)x\,{}_2F_1\left(1,\frac{1}{n};1+\frac{1}{n};-\frac{2cx^n}{b-\sqrt{b^2-4ac}}\right)}{c\left(b-\sqrt{b^2-4ac}\right)}+\frac{\left(2cde-be^2-\frac{2c^2d^2+b^2e^2-2c}{\sqrt{b^2-4}}\right.}{c}$$

```
[Out] e^2*x/c+x*hypergeom([1, 1/n],[1+1/n],-2*c*x^n/(b-(-4*a*c+b^2)^(1/2)))*(2*c*
d*e-b*e^2+(2*c^2*d^2+b^2*e^2-2*c*e*(a*e+b*d))/(-4*a*c+b^2)^(1/2))/c/(b-(-4*
a*c+b^2)^(1/2))+x*hypergeom([1, 1/n],[1+1/n],-2*c*x^n/(b+(-4*a*c+b^2)^(1/2)
))*(2*c*d*e-b*e^2+(-2*c^2*d^2-b^2*e^2+2*c*e*(a*e+b*d))/(-4*a*c+b^2)^(1/2))/
c/(b+(-4*a*c+b^2)^(1/2))
```

Rubi [A]
time = 0.31, antiderivative size = 224, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.115$, Rules used = {1438, 1436, 251}

$$\frac{x\left(\frac{-2ce(ae+bd)+b^2e^2+2c^2d^2}{\sqrt{b^2-4ac}}-be^2+2cde\right){}_2F_1\left(1,\frac{1}{n};1+\frac{1}{n};-\frac{2cx^n}{b-\sqrt{b^2-4ac}}\right)}{c\left(b-\sqrt{b^2-4ac}\right)}+\frac{x\left(-\frac{-2ce(ae+bd)+b^2e^2+2c^2d^2}{\sqrt{b^2-4ac}}-be^2+2cde\right){}_2F_1\left(1,\frac{1}{n};1+\frac{1}{n};-\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)}{c\left(\sqrt{b^2-4ac}+b\right)}+\frac{e^2x}{c}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x^n)^2/(a + b*x^n + c*x^(2*n)),x]
```

```
[Out] (e^2*x)/c + ((2*c*d*e - b*e^2 + (2*c^2*d^2 + b^2*e^2 - 2*c*e*(b*d + a*e))/S
qrt[b^2 - 4*a*c])*x*Hypergeometric2F1[1, n^(-1), 1 + n^(-1), (-2*c*x^n)/(b
- Sqrt[b^2 - 4*a*c])])/(c*(b - Sqrt[b^2 - 4*a*c])) + ((2*c*d*e - b*e^2 - (2
*c^2*d^2 + b^2*e^2 - 2*c*e*(b*d + a*e))/Sqrt[b^2 - 4*a*c])*x*Hypergeometric
2F1[1, n^(-1), 1 + n^(-1), (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c])])/(c*(b + Sqr
t[b^2 - 4*a*c]))
```

Rule 251

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, (-b)*(x^n/a)], x] /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p
, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

Rule 1436

```
Int[((d_) + (e_.)*(x_)^(n_))/((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_)), x
_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q),
Int[1/(b/2 - q/2 + c*x^n), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(
b/2 + q/2 + c*x^n), x], x]] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[n2, 2*n]
```

```
 && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && (PosQ[b^2 - 4*a
*c] ||  !IGtQ[n/2, 0])
```

Rule 1438

```
Int[((d_) + (e_.)*(x_)^(n_))^(q_)/((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_
)), x_Symbol] :> Int[ExpandIntegrand[(d + e*x^n)^q/(a + b*x^n + c*x^(2*n)),
 x], x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c,
 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[q]
```

Rubi steps

$$\begin{aligned}\int \frac{(d+ex^n)^2}{a+bx^n+cx^{2n}}\,dx &= \int \left(\frac{e^2}{c}+\frac{cd^2-ae^2+(2cde-be^2)x^n}{c(a+bx^n+cx^{2n})}\right)dx\\ &= \frac{e^2x}{c}+\frac{\int \frac{cd^2-ae^2+(2cde-be^2)x^n}{a+bx^n+cx^{2n}}\,dx}{c}\\ &= \frac{e^2x}{c}+\frac{\left(2cde-be^2-\frac{2c^2d^2+b^2e^2-2ce(bd+ae)}{\sqrt{b^2-4ac}}\right)\int \frac{1}{\frac{b}{2}+\frac{1}{2}\sqrt{b^2-4ac}+cx^n}\,dx}{2c}+\frac{\left(2cde-be^2\right.\cdots}{\cdots}\\ &= \frac{e^2x}{c}+\frac{\left(2cde-be^2+\frac{2c^2d^2+b^2e^2-2ce(bd+ae)}{\sqrt{b^2-4ac}}\right)x\,{}_2F_1\left(1,\frac{1}{n};1+\frac{1}{n};-\frac{2cx^n}{b-\sqrt{b^2-4ac}}\right)}{c\left(b-\sqrt{b^2-4ac}\right)}+\cdots\end{aligned}$$

Mathematica [A]
time = 0.74, size = 348, normalized size = 1.55

$$\frac{2^{-\frac{1+n}{n}}x\left(2^{1+\frac{1}{n}}c\sqrt{b^2-4ac}\,d^2-\left(-a\sqrt{b^2-4ac}\,e^2+cd\left(\sqrt{b^2-4ac}\,d-4ae\right)+b(cd^2+ae^2)\right)\left(\frac{cx^n}{b-\sqrt{b^2-4ac}+2cx^n}\right)^{-1/n}{}_2F_1\left(-\frac{1}{n},-\frac{1}{n};\frac{-1+n}{n};\frac{b-\sqrt{b^2-4ac}}{b-\sqrt{b^2-4ac}+2cx^n}\right)+\left(a\sqrt{b^2-4ac}\,e^2-cd\left(\sqrt{b^2-4ac}\,d+4ae\right)+b(cd^2+ae^2)\right)\left(\frac{cx^n}{b+\sqrt{b^2-4ac}+2cx^n}\right)^{-1/n}{}_2F_1\left(-\frac{1}{n},-\frac{1}{n};\frac{-1+n}{n};\frac{b+\sqrt{b^2-4ac}}{b+\sqrt{b^2-4ac}+2cx^n}\right)\right)}{ac\sqrt{b^2-4ac}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x^n)^2/(a + b*x^n + c*x^(2*n)),x]
```

```
[Out] (x*(2^(1 + n^(-1))*c*Sqrt[b^2 - 4*a*c]*d^2 - ((-(a*Sqrt[b^2 - 4*a*c]*e^2) +
 c*d*(Sqrt[b^2 - 4*a*c]*d - 4*a*e) + b*(c*d^2 + a*e^2))*Hypergeometric2F1[-
n^(-1), -n^(-1), (-1 + n)/n, (b - Sqrt[b^2 - 4*a*c])/(b - Sqrt[b^2 - 4*a*c]
 + 2*c*x^n)])/((c*x^n)/(b - Sqrt[b^2 - 4*a*c] + 2*c*x^n))^n^(-1) + ((a*Sqrt
[b^2 - 4*a*c]*e^2 - c*d*(Sqrt[b^2 - 4*a*c]*d + 4*a*e) + b*(c*d^2 + a*e^2))*
Hypergeometric2F1[-n^(-1), -n^(-1), (-1 + n)/n, (b + Sqrt[b^2 - 4*a*c])/(b
+ Sqrt[b^2 - 4*a*c] + 2*c*x^n)])/((c*x^n)/(b + Sqrt[b^2 - 4*a*c] + 2*c*x^n)
)^n^(-1)))/(2^((1 + n)/n)*a*c*Sqrt[b^2 - 4*a*c])
```

Maple [F]
time = 0.01, size = 0, normalized size = 0.00

$$\int \frac{\left(d+e\,x^{n}\right)^{2}}{a+b\,x^{n}+c\,x^{2 n}}d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d+e*x^n)^2/(a+b*x^n+c*x^(2*n)),x)
[Out] int((d+e*x^n)^2/(a+b*x^n+c*x^(2*n)),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+e*x^n)^2/(a+b*x^n+c*x^(2*n)),x, algorithm="maxima")
[Out] x*e^2/c - integrate(-(c*d^2 + (2*c*d*e - b*e^2)*x^n - a*e^2)/(c^2*x^(2*n) +
 b*c*x^n + a*c), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+e*x^n)^2/(a+b*x^n+c*x^(2*n)),x, algorithm="fricas")
[Out] integral((2*d*x^n*e + d^2 + x^(2*n)*e^2)/(c*x^(2*n) + b*x^n + a), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(d + e x^{n}\right)^{2}}{a + b x^{n} + c x^{2 n}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+e*x**n)**2/(a+b*x**n+c*x**(2*n)),x)
[Out] Integral((d + e*x**n)**2/(a + b*x**n + c*x**(2*n)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+e*x^n)^2/(a+b*x^n+c*x^(2*n)),x, algorithm="giac")
```

```
[Out] integrate((x^n*e + d)^2/(c*x^(2*n) + b*x^n + a), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(d+e\,x^n)^2}{a+b\,x^n+c\,x^{2\,n}} \,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x^n)^2/(a + b*x^n + c*x^(2*n)),x)
```

```
[Out] int((d + e*x^n)^2/(a + b*x^n + c*x^(2*n)), x)
```

3.71 $\int \frac{d+ex^n}{a+bx^n+cx^{2n}}\,dx$

Optimal. Leaf size=154

$$\frac{\left(e+\frac{2cd-be}{\sqrt{b^2-4ac}}\right)x\,{}_2F_1\left(1,\frac{1}{n};1+\frac{1}{n};-\frac{2cx^n}{b-\sqrt{b^2-4ac}}\right)}{b-\sqrt{b^2-4ac}}+\frac{\left(e-\frac{2cd-be}{\sqrt{b^2-4ac}}\right)x\,{}_2F_1\left(1,\frac{1}{n};1+\frac{1}{n};-\frac{2cx^n}{b+\sqrt{b^2-}}\right.}{b+\sqrt{b^2-4ac}}$$

```
[Out] x*hypergeom([1, 1/n],[1+1/n],-2*c*x^n/(b-(-4*a*c+b^2)^(1/2)))*(e+(-b*e+2*c*
d)/(-4*a*c+b^2)^(1/2))/(b-(-4*a*c+b^2)^(1/2))+x*hypergeom([1, 1/n],[1+1/n],
-2*c*x^n/(b+(-4*a*c+b^2)^(1/2)))*(e+(b*e-2*c*d)/(-4*a*c+b^2)^(1/2))/(b+(-4*
a*c+b^2)^(1/2))
```

Rubi [A]

time = 0.08, antiderivative size = 154, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {1436, 251}

$$\frac{x\left(\frac{2cd-be}{\sqrt{b^2-4ac}}+e\right){}_2F_1\left(1,\frac{1}{n};1+\frac{1}{n};-\frac{2cx^n}{b-\sqrt{b^2-4ac}}\right)}{b-\sqrt{b^2-4ac}}+\frac{x\left(e-\frac{2cd-be}{\sqrt{b^2-4ac}}\right){}_2F_1\left(1,\frac{1}{n};1+\frac{1}{n};-\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)}{\sqrt{b^2-4ac}+b}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x^n)/(a + b*x^n + c*x^(2*n)),x]
```

```
[Out] ((e + (2*c*d - b*e)/Sqrt[b^2 - 4*a*c])*x*Hypergeometric2F1[1, n^(-1), 1 + n
^(-1), (-2*c*x^n)/(b - Sqrt[b^2 - 4*a*c])])/(b - Sqrt[b^2 - 4*a*c]) + ((e -
 (2*c*d - b*e)/Sqrt[b^2 - 4*a*c])*x*Hypergeometric2F1[1, n^(-1), 1 + n^(-1)
, (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c])])/(b + Sqrt[b^2 - 4*a*c])
```

Rule 251

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, (-b)*(x^n/a)], x] /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p
, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

Rule 1436

```
Int[((d_) + (e_.)*(x_)^(n_))/((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_)), x
_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q),
Int[1/(b/2 - q/2 + c*x^n), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(
b/2 + q/2 + c*x^n), x], x]] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[n2, 2*n]
 && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && (PosQ[b^2 - 4*a
*c] ||  !IGtQ[n/2, 0])
```

Rubi steps

$$\int \frac{d+ex^n}{a+bx^n+cx^{2n}}\,dx = \frac{1}{2}\left(e-\frac{2cd-be}{\sqrt{b^2-4ac}}\right)\int \frac{1}{\frac{b}{2}+\frac{1}{2}\sqrt{b^2-4ac}+cx^n}\,dx + \frac{1}{2}\left(e+\frac{2cd-be}{\sqrt{b^2-4ac}}\right)\int \frac{}{\frac{b}{2}-\frac{1}{2}}$$

$$= \frac{\left(e+\frac{2cd-be}{\sqrt{b^2-4ac}}\right)x\,{}_2F_1\left(1,\frac{1}{n};1+\frac{1}{n};-\frac{2cx^n}{b-\sqrt{b^2-4ac}}\right)}{b-\sqrt{b^2-4ac}} + \frac{\left(e-\frac{2cd-be}{\sqrt{b^2-4ac}}\right)x\,{}_2F_1}{b+}$$

Mathematica [A]
time = 0.37, size = 279, normalized size = 1.81

$$\frac{2^{-\frac{1+n}{n}}x\left(2^{1+\frac{1}{n}}\sqrt{b^2-4ac}\,d-\left(bd+\sqrt{b^2-4ac}\,d-2ae\right)\left(\frac{cx^n}{b-\sqrt{b^2-4ac}+2cx^n}\right)^{-1/n}{}_2F_1\left(-\frac{1}{n},-\frac{1}{n};\frac{-1+n}{n};\frac{b-\sqrt{b^2-4ac}}{b-\sqrt{b^2-4ac}+2cx^n}\right)+\left(bd-\sqrt{b^2-4ac}\,d-2ae\right)\left(\frac{cx^n}{b+\sqrt{b^2-4ac}+2cx^n}\right)^{-1/n}{}_2F_1\left(-\frac{1}{n},-\frac{1}{n};\frac{-1+n}{n};\frac{b+\sqrt{b^2-4ac}}{b+\sqrt{b^2-4ac}+2cx^n}\right)\right)}{a\sqrt{b^2-4ac}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x^n)/(a + b*x^n + c*x^(2*n)),x]
```

```
[Out] (x*(2^(1 + n^(-1))*Sqrt[b^2 - 4*a*c]*d - ((b*d + Sqrt[b^2 - 4*a*c]*d - 2*a*
e)*Hypergeometric2F1[-n^(-1), -n^(-1), (-1 + n)/n, (b - Sqrt[b^2 - 4*a*c])/
(b - Sqrt[b^2 - 4*a*c] + 2*c*x^n)])/((c*x^n)/(b - Sqrt[b^2 - 4*a*c] + 2*c*x
^n))^n^(-1) + ((b*d - Sqrt[b^2 - 4*a*c]*d - 2*a*e)*Hypergeometric2F1[-n^(-1
), -n^(-1), (-1 + n)/n, (b + Sqrt[b^2 - 4*a*c])/(b + Sqrt[b^2 - 4*a*c] + 2*
c*x^n)])/((c*x^n)/(b + Sqrt[b^2 - 4*a*c] + 2*c*x^n))^n^(-1)))/(2^((1 + n)/n
)*a*Sqrt[b^2 - 4*a*c])
```

Maple [F]
time = 0.01, size = 0, normalized size = 0.00

$$\int \frac{d+e\,x^n}{a+b\,x^n+c\,x^{2n}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d+e*x^n)/(a+b*x^n+c*x^(2*n)),x)
```

```
[Out] int((d+e*x^n)/(a+b*x^n+c*x^(2*n)),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+e*x^n)/(a+b*x^n+c*x^(2*n)),x, algorithm="maxima")
```

```
[Out] integrate((x^n*e + d)/(c*x^(2*n) + b*x^n + a), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+e*x^n)/(a+b*x^n+c*x^(2*n)),x, algorithm="fricas")
[Out] integral((x^n*e + d)/(c*x^(2*n) + b*x^n + a), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{d + ex^n}{a + bx^n + cx^{2n}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+e*x**n)/(a+b*x**n+c*x**(2*n)),x)
[Out] Integral((d + e*x**n)/(a + b*x**n + c*x**(2*n)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+e*x^n)/(a+b*x^n+c*x^(2*n)),x, algorithm="giac")
[Out] integrate((x^n*e + d)/(c*x^(2*n) + b*x^n + a), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{d + e\,x^n}{a + b\,x^n + c\,x^{2\,n}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x^n)/(a + b*x^n + c*x^(2*n)),x)
[Out] int((d + e*x^n)/(a + b*x^n + c*x^(2*n)), x)
```

3.72 $\int \frac{1}{(d+ex^n)(a+bx^n+cx^{2n})}\,dx$

Optimal. Leaf size=243

$$-\frac{c\left(2cd-\left(b+\sqrt{b^2-4ac}\right)e\right)x\,{}_2F_1\left(1,\frac{1}{n};1+\frac{1}{n};-\frac{2cx^n}{b-\sqrt{b^2-4ac}}\right)}{\left(b^2-4ac-b\sqrt{b^2-4ac}\right)(cd^2-bde+ae^2)}-\frac{c\left(e+\frac{2cd-be}{\sqrt{b^2-4ac}}\right)x\,{}_2F_1\left(1,\frac{1}{n};1+\frac{1}{n}\right.}{\left(b+\sqrt{b^2-4ac}\right)(cd^2-bd}$$

```
[Out] e^2*x*hypergeom([1, 1/n],[1+1/n],-e*x^n/d)/d/(a*e^2-b*d*e+c*d^2)-c*x*hyperg
eom([1, 1/n],[1+1/n],-2*c*x^n/(b+(-4*a*c+b^2)^(1/2)))*(e+(-b*e+2*c*d)/(-4*a
*c+b^2)^(1/2))/(a*e^2-b*d*e+c*d^2)/(b+(-4*a*c+b^2)^(1/2))-c*x*hypergeom([1,
 1/n],[1+1/n],-2*c*x^n/(b-(-4*a*c+b^2)^(1/2)))*(2*c*d-e*(b+(-4*a*c+b^2)^(1/
2)))/(a*e^2-b*d*e+c*d^2)/(b^2-4*a*c-b*(-4*a*c+b^2)^(1/2))
```

Rubi [A]
time = 0.29, antiderivative size = 243, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.115$, Rules used = {1438, 251, 1436}

$$-\frac{cx\left(2cd-e\left(\sqrt{b^2-4ac}+b\right)\right){}_2F_1\left(1,\frac{1}{n};1+\frac{1}{n};-\frac{2cx^n}{b-\sqrt{b^2-4ac}}\right)}{\left(-b\sqrt{b^2-4ac}-4ac+b^2\right)(ae^2-bde+cd^2)}-\frac{cx\left(\frac{2cd-be}{\sqrt{b^2-4ac}}+e\right){}_2F_1\left(1,\frac{1}{n};1+\frac{1}{n};-\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)}{\left(\sqrt{b^2-4ac}+b\right)(ae^2-bde+cd^2)}+\frac{e^2x\,{}_2F_1\left(1,\frac{1}{n};1+\frac{1}{n};-\frac{ex^n}{d}\right)}{d\,(ae^2-bde+cd^2)}$$

Antiderivative was successfully verified.

```
[In] Int[1/((d + e*x^n)*(a + b*x^n + c*x^(2*n))),x]
```

```
[Out] -((c*(2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)*x*Hypergeometric2F1[1, n^(-1), 1 +
 n^(-1), (-2*c*x^n)/(b - Sqrt[b^2 - 4*a*c])])/((b^2 - 4*a*c - b*Sqrt[b^2 -
4*a*c])*(c*d^2 - b*d*e + a*e^2))) - (c*(e + (2*c*d - b*e)/Sqrt[b^2 - 4*a*c]
)*x*Hypergeometric2F1[1, n^(-1), 1 + n^(-1), (-2*c*x^n)/(b + Sqrt[b^2 - 4*a
*c])])/((b + Sqrt[b^2 - 4*a*c])*(c*d^2 - b*d*e + a*e^2)) + (e^2*x*Hypergeom
etric2F1[1, n^(-1), 1 + n^(-1), -((e*x^n)/d)])/(d*(c*d^2 - b*d*e + a*e^2))
```

Rule 251

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, (-b)*(x^n/a)], x] /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p
, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

Rule 1436

```
Int[((d_) + (e_.)*(x_)^(n_))/((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_)), x
_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q),
Int[1/(b/2 - q/2 + c*x^n), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(
b/2 + q/2 + c*x^n), x], x]] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[n2, 2*n]
```

```
 && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && (PosQ[b^2 - 4*a
*c] ||  !IGtQ[n/2, 0])
```

Rule 1438

```
Int[((d_) + (e_.)*(x_)^(n_))^(q_)/((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_
)), x_Symbol] :> Int[ExpandIntegrand[(d + e*x^n)^q/(a + b*x^n + c*x^(2*n)),
 x], x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c,
 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[q]
```

Rubi steps

$$
\begin{aligned}
\int \frac{1}{(d+ex^n)(a+bx^n+cx^{2n})}\,dx &= \int \left(\frac{e^2}{(cd^2-bde+ae^2)(d+ex^n)}+\frac{cd-be-cex^n}{(cd^2-bde+ae^2)(a+bx^n+cx^{2n})}\right. \\
&= \frac{\int \frac{cd-be-cex^n}{a+bx^n+cx^{2n}}\,dx}{cd^2-bde+ae^2}+\frac{e^2\int \frac{1}{d+ex^n}\,dx}{cd^2-bde+ae^2} \\
&= \frac{e^2x\,{}_2F_1\left(1,\frac{1}{n};1+\frac{1}{n};-\frac{ex^n}{d}\right)}{d(cd^2-bde+ae^2)}-\frac{\left(c\left(e-\frac{2cd-be}{\sqrt{b^2-4ac}}\right)\right)\int \frac{1}{\frac{b}{2}-\frac{1}{2}\sqrt{b^2-4ac}}}{2(cd^2-bde+ae^2)} \\
&= -\frac{c\left(e-\frac{2cd-be}{\sqrt{b^2-4ac}}\right)x\,{}_2F_1\left(1,\frac{1}{n};1+\frac{1}{n};-\frac{2cx^n}{b-\sqrt{b^2-4ac}}\right)}{\left(b-\sqrt{b^2-4ac}\right)(cd^2-bde+ae^2)}-\frac{c\left(e+\right.}{}
\end{aligned}
$$

Mathematica [A]
time = 0.98, size = 379, normalized size = 1.56

$$
\frac{x\left(2cd-2be+\frac{2ae^2\,{}_2F_1\left(1,\frac{1}{n};1+\frac{1}{n};-\frac{ex^n}{d}\right)}{d}+\frac{2^{-1/n}\left(-bcd-c\sqrt{b^2-4ac}\,d+b^2e-2ace+b\sqrt{b^2-4ac}\,e\right)\left(\frac{cx^n}{b-\sqrt{b^2-4ac}+2cx^n}\right)^{-1/n}{}_2F_1\left(-\frac{1}{n},-\frac{1}{n};\frac{-1+n}{n};\frac{b-\sqrt{b^2-4ac}}{b-\sqrt{b^2-4ac}+2cx^n}\right)}{\sqrt{b^2-4ac}}+\frac{2^{-1/n}\left(bcd-c\sqrt{b^2-4ac}\,d-b^2e+2ace+b\sqrt{b^2-4ac}\,e\right)\left(\frac{cx^n}{b+\sqrt{b^2-4ac}+2cx^n}\right)^{-1/n}{}_2F_1\left(-\frac{1}{n},-\frac{1}{n};\frac{-1+n}{n};\frac{b+\sqrt{b^2-4ac}}{b+\sqrt{b^2-4ac}+2cx^n}\right)}{\sqrt{b^2-4ac}}\right)}{2a\left(cd^2+e(-bd+ae)\right)}
$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((d + e*x^n)*(a + b*x^n + c*x^(2*n))),x]
```

```
[Out] (x*(2*c*d - 2*b*e + (2*a*e^2*Hypergeometric2F1[1, n^(-1), 1 + n^(-1), -((e*
x^n)/d)])/d + ((-(b*c*d) - c*Sqrt[b^2 - 4*a*c]*d + b^2*e - 2*a*c*e + b*Sqrt
[b^2 - 4*a*c]*e)*Hypergeometric2F1[-n^(-1), -n^(-1), (-1 + n)/n, (b - Sqrt[
b^2 - 4*a*c])/(b - Sqrt[b^2 - 4*a*c] + 2*c*x^n)])/(2^n^(-1)*Sqrt[b^2 - 4*a*
c]*((c*x^n)/(b - Sqrt[b^2 - 4*a*c] + 2*c*x^n))^n^(-1)) + ((b*c*d - c*Sqrt[b
^2 - 4*a*c]*d - b^2*e + 2*a*c*e + b*Sqrt[b^2 - 4*a*c]*e)*Hypergeometric2F1[
-n^(-1), -n^(-1), (-1 + n)/n, (b + Sqrt[b^2 - 4*a*c])/(b + Sqrt[b^2 - 4*a*c
] + 2*c*x^n)])/(2^n^(-1)*Sqrt[b^2 - 4*a*c]*((c*x^n)/(b + Sqrt[b^2 - 4*a*c]
+ 2*c*x^n))^n^(-1))))/(2*a*(c*d^2 + e*(-(b*d) + a*e)))
```

Maple [F]
time = 0.05, size = 0, normalized size = 0.00

$$\int \frac{1}{(d+e\,x^{n})\,(a+b\,x^{n}+c\,x^{2n})}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(d+e*x^n)/(a+b*x^n+c*x^(2*n)),x)
[Out] int(1/(d+e*x^n)/(a+b*x^n+c*x^(2*n)),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d+e*x^n)/(a+b*x^n+c*x^(2*n)),x, algorithm="maxima")
[Out] integrate(1/((c*x^(2*n) + b*x^n + a)*(x^n*e + d)), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d+e*x^n)/(a+b*x^n+c*x^(2*n)),x, algorithm="fricas")
[Out] integral(1/(b*x^(2*n)*e + a*d + (c*x^n*e + c*d)*x^(2*n) + (b*d + a*e)*x^n),
 x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(d+ex^{n})\,(a+bx^{n}+cx^{2n})}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d+e*x**n)/(a+b*x**n+c*x**(2*n)),x)
[Out] Integral(1/((d + e*x**n)*(a + b*x**n + c*x**(2*n))), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d+e*x^n)/(a+b*x^n+c*x^(2*n)),x, algorithm="giac")
```

```
[Out] integrate(1/((c*x^(2*n) + b*x^n + a)*(x^n*e + d)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{(d+e\,x^n)\,\left(a+b\,x^n+c\,x^{2\,n}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((d + e*x^n)*(a + b*x^n + c*x^(2*n))),x)
```

```
[Out] int(1/((d + e*x^n)*(a + b*x^n + c*x^(2*n))), x)
```

3.73 $\int \frac{1}{(d+ex^n)^2(a+bx^n+cx^{2n})}\,dx$

Optimal. Leaf size=368

$$-\frac{c\left(2c^2d^2+b\left(b+\sqrt{b^2-4ac}\right)e^2-2ce\left(bd+\sqrt{b^2-4ac}\,d+ae\right)\right)x\,{}_2F_1\left(1,\frac{1}{n};1+\frac{1}{n};-\frac{2cx^n}{b-\sqrt{b^2-4ac}}\right)}{\left(b^2-4ac-b\sqrt{b^2-4ac}\right)\left(cd^2-bde+ae^2\right)^2}-\frac{c\left(\right.}{}$$

```
[Out] e^2*(-b*e+2*c*d)*x*hypergeom([1, 1/n],[1+1/n],-e*x^n/d)/d/(a*e^2-b*d*e+c*d^
2)^2+e^2*x*hypergeom([2, 1/n],[1+1/n],-e*x^n/d)/d^2/(a*e^2-b*d*e+c*d^2)-c*x
*hypergeom([1, 1/n],[1+1/n],-2*c*x^n/(b+(-4*a*c+b^2)^(1/2)))*(2*c^2*d^2+b*e
^2*(b-(-4*a*c+b^2)^(1/2))-2*c*e*(b*d+a*e-d*(-4*a*c+b^2)^(1/2)))/(a*e^2-b*d*
e+c*d^2)^2/(b^2-4*a*c+b*(-4*a*c+b^2)^(1/2))-c*x*hypergeom([1, 1/n],[1+1/n],
-2*c*x^n/(b-(-4*a*c+b^2)^(1/2)))*(2*c^2*d^2+b*e^2*(b+(-4*a*c+b^2)^(1/2))-2*
c*e*(b*d+a*e+d*(-4*a*c+b^2)^(1/2)))/(a*e^2-b*d*e+c*d^2)^2/(b^2-4*a*c-b*(-4*
a*c+b^2)^(1/2))
```

Rubi [A]
time = 0.46, antiderivative size = 368, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.115$, Rules used = {1438, 251, 1436}

$$-\frac{cx\left(-2ce\left(d\sqrt{b^2-4ac}+ae+bd\right)+be^2\left(\sqrt{b^2-4ac}+b\right)+2c^2d^2\right)\,{}_2F_1\left(1,\frac{1}{n};1+\frac{1}{n};-\frac{2cx^n}{b-\sqrt{b^2-4ac}}\right)}{\left(-b\sqrt{b^2-4ac}-4ac+b^2\right)\left(ae^2-bde+cd^2\right)^2}-\frac{cx\left(-2ce\left(-d\sqrt{b^2-4ac}+ae+bd\right)+be^2\left(b-\sqrt{b^2-4ac}\right)+2c^2d^2\right)\,{}_2F_1\left(1,\frac{1}{n};1+\frac{1}{n};-\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)}{\left(b\sqrt{b^2-4ac}-4ac+b^2\right)\left(ae^2-bde+cd^2\right)^2}+\frac{e^2x(2cd-be)\,{}_2F_1\left(1,\frac{1}{n};1+\frac{1}{n};-\frac{ex^n}{d}\right)}{d\left(ae^2-bde+cd^2\right)^2}+\frac{e^2x\,{}_2F_1\left(2,\frac{1}{n};1+\frac{1}{n};-\frac{ex^n}{d}\right)}{d^2\left(ae^2-bde+cd^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[1/((d + e*x^n)^2*(a + b*x^n + c*x^(2*n))),x]
```

```
[Out] -((c*(2*c^2*d^2 + b*(b + Sqrt[b^2 - 4*a*c])*e^2 - 2*c*e*(b*d + Sqrt[b^2 - 4
*a*c]*d + a*e))*x*Hypergeometric2F1[1, n^(-1), 1 + n^(-1), (-2*c*x^n)/(b -
Sqrt[b^2 - 4*a*c])])/((b^2 - 4*a*c - b*Sqrt[b^2 - 4*a*c])*(c*d^2 - b*d*e +
a*e^2)^2)) - (c*(2*c^2*d^2 + b*(b - Sqrt[b^2 - 4*a*c])*e^2 - 2*c*e*(b*d - S
qrt[b^2 - 4*a*c]*d + a*e))*x*Hypergeometric2F1[1, n^(-1), 1 + n^(-1), (-2*c
*x^n)/(b + Sqrt[b^2 - 4*a*c])])/((b^2 - 4*a*c + b*Sqrt[b^2 - 4*a*c])*(c*d^2
 - b*d*e + a*e^2)^2) + (e^2*(2*c*d - b*e)*x*Hypergeometric2F1[1, n^(-1), 1
+ n^(-1), -((e*x^n)/d)])/(d*(c*d^2 - b*d*e + a*e^2)^2) + (e^2*x*Hypergeomet
ric2F1[2, n^(-1), 1 + n^(-1), -((e*x^n)/d)])/(d^2*(c*d^2 - b*d*e + a*e^2))
```

Rule 251

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, (-b)*(x^n/a)], x] /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p
, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

Rule 1436

```
Int[((d_) + (e_.)*(x_)^(n_))/((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_)), x
_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q),
Int[1/(b/2 - q/2 + c*x^n), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(
b/2 + q/2 + c*x^n), x], x]] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[n2, 2*n]
 && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && (PosQ[b^2 - 4*a
*c] ||  !IGtQ[n/2, 0])
```

Rule 1438

```
Int[((d_) + (e_.)*(x_)^(n_))^(q_)/((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_
)), x_Symbol] :> Int[ExpandIntegrand[(d + e*x^n)^q/(a + b*x^n + c*x^(2*n)),
 x], x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c,
 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[q]
```

Rubi steps

$$\int \frac{1}{(d+ex^n)^2(a+bx^n+cx^{2n})}\,dx = \int \left(\frac{e^2}{(cd^2-bde+ae^2)(d+ex^n)^2} - \frac{e^2(-2cd+be)}{(cd^2-bde+ae^2)^2(d+ex^n)} + \frac{c^2 \cdots}{\cdots}\right.$$

$$= \frac{\int \frac{c^2d^2-2bcde+b^2e^2-ace^2-(2c^2de-bce^2)x^n}{a+bx^n+cx^{2n}}\,dx}{(cd^2-bde+ae^2)^2} + \frac{(e^2(2cd-be))\int \frac{1}{d+ex^n}\,dx}{(cd^2-bde+ae^2)^2} + \frac{e^2 \cdots}{c \cdots}$$

$$= \frac{e^2(2cd-be)x\,{}_2F_1\left(1,\frac{1}{n};1+\frac{1}{n};-\frac{ex^n}{d}\right)}{d(cd^2-bde+ae^2)^2} + \frac{e^2x\,{}_2F_1\left(2,\frac{1}{n};1+\frac{1}{n};-\frac{ex^n}{d}\right)}{d^2(cd^2-bde+ae^2)} - \cdots$$

$$= \frac{c\left(2c^2d^2+b\left(b+\sqrt{b^2-4ac}\right)e^2-2ce\left(bd+\sqrt{b^2-4ac}\,d+ae\right)\right)x\,{}_2F \cdots}{\sqrt{b^2-4ac}\left(b-\sqrt{b^2-4ac}\right)(cd^2-bde+ \cdots}$$

Mathematica [A]
time = 5.32, size = 564, normalized size = 1.53

$$\frac{x\left(\frac{a^2e^4+d(cd-be)^2n(d+ex^n)-ade^2(be+cd(-1+n)+cenx^n)}{adn(d+ex^n)} + \frac{e^2(cd^2(-1+3n)+e(ae(-1+n)+b(d-2dn)))\,{}_2F_1\left(1,\frac{1}{n};1+\frac{1}{n};-\frac{ex^n}{d}\right)}{d^2n} - \frac{2^{-1/n}c\left(b^2\left(b+\sqrt{b^2-4ac}\right)e^2+2c^2d\left(\sqrt{b^2-4ac}\,d+4ae\right)-2ce\left(b^2d+b\sqrt{b^2-4ac}\,d+2abe+a\sqrt{b^2-4ac}\,e\right)\right)\left(\frac{cx^n}{b-\sqrt{b^2-4ac}+2cx^n}\right)^{-1/n}{}_2F_1\left(-\frac{1}{n},-\frac{1}{n};\frac{-1+n}{n};\frac{b-\sqrt{b^2-4ac}}{b-\sqrt{b^2-4ac}+2cx^n}\right)}{\sqrt{b^2-4ac}\left(-b^2+4ac+b\sqrt{b^2-4ac}\right)} + \frac{2^{-1/n}c\left(2c^2d^2+b\left(b-\sqrt{b^2-4ac}\right)e^2-2ce\left(bd-\sqrt{b^2-4ac}\,d+ae\right)\right)\left(\frac{cx^n}{b+\sqrt{b^2-4ac}+2cx^n}\right)^{-1/n}{}_2F_1\left(-\frac{1}{n},-\frac{1}{n};\frac{-1+n}{n};\frac{b+\sqrt{b^2-4ac}}{b+\sqrt{b^2-4ac}+2cx^n}\right)}{\sqrt{b^2-4ac}\left(b+\sqrt{b^2-4ac}\right)}\right)}{(cd^2+e(-bd+ae))^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((d + e*x^n)^2*(a + b*x^n + c*x^(2*n))),x]
```

```
[Out] (x*((a^2*e^4 + d*(c*d - b*e)^2*n*(d + e*x^n) - a*d*e^2*(b*e + c*d*(-1 + n)
+ c*e*n*x^n))/(a*d*n*(d + e*x^n)) + (e^2*(c*d^2*(-1 + 3*n) + e*(a*e*(-1 + n
) + b*(d - 2*d*n)))*Hypergeometric2F1[1, n^(-1), 1 + n^(-1), -((e*x^n)/d)])
/(d^2*n) - (c*(b^2*(b + Sqrt[b^2 - 4*a*c])*e^2 + 2*c^2*d*(Sqrt[b^2 - 4*a*c]
*d + 4*a*e) - 2*c*e*(b^2*d + b*Sqrt[b^2 - 4*a*c]*d + 2*a*b*e + a*Sqrt[b^2 -
```

```
 4*a*c]*e))*Hypergeometric2F1[-n^(-1), -n^(-1), (-1 + n)/n, (b - Sqrt[b^2 -
 4*a*c])/(b - Sqrt[b^2 - 4*a*c] + 2*c*x^n)])/(2^n^(-1)*Sqrt[b^2 - 4*a*c]*(-
b^2 + 4*a*c + b*Sqrt[b^2 - 4*a*c])*((c*x^n)/(b - Sqrt[b^2 - 4*a*c] + 2*c*x^
n))^n^(-1)) + (c*(2*c^2*d^2 + b*(b - Sqrt[b^2 - 4*a*c])*e^2 - 2*c*e*(b*d -
Sqrt[b^2 - 4*a*c]*d + a*e))*Hypergeometric2F1[-n^(-1), -n^(-1), (-1 + n)/n,
 (b + Sqrt[b^2 - 4*a*c])/(b + Sqrt[b^2 - 4*a*c] + 2*c*x^n)])/(2^n^(-1)*Sqrt
[b^2 - 4*a*c]*(b + Sqrt[b^2 - 4*a*c])*((c*x^n)/(b + Sqrt[b^2 - 4*a*c] + 2*c
*x^n))^n^(-1))))/(c*d^2 + e*(-(b*d) + a*e))^2
```

Maple [F]
time = 0.02, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(d+e\,x^{n}\right)^{2}\left(a+b\,x^{n}+c\,x^{2 n}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(d+e*x^n)^2/(a+b*x^n+c*x^(2*n)),x)
```

```
[Out] int(1/(d+e*x^n)^2/(a+b*x^n+c*x^(2*n)),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d+e*x^n)^2/(a+b*x^n+c*x^(2*n)),x, algorithm="maxima")
```

```
[Out] (c*d^2*(3*n - 1)*e^2 - b*d*(2*n - 1)*e^3 + a*(n - 1)*e^4)*integrate(1/(c^2*
d^6*n - 2*b*c*d^5*n*e - 2*a*b*d^3*n*e^3 + a^2*d^2*n*e^4 + (c^2*d^5*n*e - 2*
b*c*d^4*n*e^2 - 2*a*b*d^2*n*e^4 + a^2*d*n*e^5 + (b^2*d^3*n + 2*a*c*d^3*n)*e
^3)*x^n + (b^2*d^4*n + 2*a*c*d^4*n)*e^2), x) + x*e^2/(c*d^4*n - b*d^3*n*e +
 a*d^2*n*e^2 + (c*d^3*n*e - b*d^2*n*e^2 + a*d*n*e^3)*x^n) + integrate((c^2*
d^2 - 2*b*c*d*e - (2*c^2*d*e - b*c*e^2)*x^n + (b^2 - a*c)*e^2)/(a*c^2*d^4 -
 2*a*b*c*d^3*e - 2*a^2*b*d*e^3 + a^3*e^4 + (c^3*d^4 - 2*b*c^2*d^3*e - 2*a*b
*c*d*e^3 + a^2*c*e^4 + (b^2*c*d^2 + 2*a*c^2*d^2)*e^2)*x^(2*n) + (b*c^2*d^4
- 2*b^2*c*d^3*e - 2*a*b^2*d*e^3 + a^2*b*e^4 + (b^3*d^2 + 2*a*b*c*d^2)*e^2)*
x^n + (a*b^2*d^2 + 2*a^2*c*d^2)*e^2), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d+e*x^n)^2/(a+b*x^n+c*x^(2*n)),x, algorithm="fricas")
```

```
[Out] integral(1/(a*d^2 + b*x^(3*n)*e^2 + (2*c*d*x^n*e + c*d^2 + c*x^(2*n)*e^2)*x
^(2*n) + (2*b*d*e + a*e^2)*x^(2*n) + (b*d^2 + 2*a*d*e)*x^n), x)
```

Sympy [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: HeuristicGCDFailed

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d+e*x**n)**2/(a+b*x**n+c*x**(2*n)),x)
```

```
[Out] Exception raised: HeuristicGCDFailed >> no luck
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d+e*x^n)^2/(a+b*x^n+c*x^(2*n)),x, algorithm="giac")
```

```
[Out] integrate(1/((c*x^(2*n) + b*x^n + a)*(x^n*e + d)^2), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{\left(d+e\,x^n\right)^2\,\left(a+b\,x^n+c\,x^{2\,n}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((d + e*x^n)^2*(a + b*x^n + c*x^(2*n))),x)
```

```
[Out] int(1/((d + e*x^n)^2*(a + b*x^n + c*x^(2*n))), x)
```

3.74 $\int \frac{1}{(d+ex^n)^3(a+bx^n+cx^{2n})}\,dx$

Optimal. Leaf size=552

$$-\frac{c\left(2c^3d^3-b^2\left(b+\sqrt{b^2-4ac}\right)e^3-3c^2de\left(bd+\sqrt{b^2-4ac}\,d+2ae\right)+ce^2\left(3b^2d+a\sqrt{b^2-4ac}\,e+3b\left(\sqrt{b\right.\right.\right.}{\left(b^2-4ac-b\sqrt{b^2-4ac}\right)\left(cd^2-bde+ae^2\right)^3}$$

```
[Out] e^2*(3*c^2*d^2+b^2*e^2-c*e*(a*e+3*b*d))*x*hypergeom([1, 1/n],[1+1/n],-e*x^n
/d)/d/(a*e^2-b*d*e+c*d^2)^3+e^2*(-b*e+2*c*d)*x*hypergeom([2, 1/n],[1+1/n],-
e*x^n/d)/d^2/(a*e^2-b*d*e+c*d^2)^2+e^2*x*hypergeom([3, 1/n],[1+1/n],-e*x^n/
d)/d^3/(a*e^2-b*d*e+c*d^2)-c*x*hypergeom([1, 1/n],[1+1/n],-2*c*x^n/(b+(-4*a
*c+b^2)^(1/2)))*(2*c^3*d^3-b^2*e^3*(b-(-4*a*c+b^2)^(1/2))-3*c^2*d*e*(b*d+2*
a*e-d*(-4*a*c+b^2)^(1/2))+c*e^2*(3*b^2*d+3*a*b*e-3*b*d*(-4*a*c+b^2)^(1/2)-(
-4*a*c+b^2)^(1/2)*a*e))/(a*e^2-b*d*e+c*d^2)^3/(b^2-4*a*c+b*(-4*a*c+b^2)^(1/
2))-c*x*hypergeom([1, 1/n],[1+1/n],-2*c*x^n/(b-(-4*a*c+b^2)^(1/2)))*(2*c^3*
d^3-b^2*e^3*(b+(-4*a*c+b^2)^(1/2))-3*c^2*d*e*(b*d+2*a*e+d*(-4*a*c+b^2)^(1/2
))+c*e^2*(3*b^2*d+(-4*a*c+b^2)^(1/2)*a*e+3*b*(a*e+d*(-4*a*c+b^2)^(1/2))))/(
a*e^2-b*d*e+c*d^2)^3/(b^2-4*a*c-b*(-4*a*c+b^2)^(1/2))
```

Rubi [A]
time = 0.68, antiderivative size = 552, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.115$, Rules used = {1438, 251, 1436}

$$\frac{e^2x\left(-ce(ae+3bd)+b^2e^2+3c^2d^2\right){}_2F_1\left(1,\frac{1}{n};1+\frac{1}{n};-\frac{ex^n}{d}\right)}{d\left(ae^2-bde+cd^2\right)^3}-\frac{cx\left(-3c^2de\left(d\sqrt{b^2-4ac}+2ae+bd\right)+ce^2\left(3b\left(d\sqrt{b^2-4ac}+ae\right)+ae\sqrt{b^2-4ac}+3b^2d\right)-b^2e^3\left(\sqrt{b^2-4ac}+b\right)+2c^3d^3\right){}_2F_1\left(1,\frac{1}{n};1+\frac{1}{n};-\frac{2cx^n}{b-\sqrt{b^2-4ac}}\right)}{\left(-b\sqrt{b^2-4ac}-4ac+b^2\right)\left(ae^2-bde+cd^2\right)^3}-\frac{cx\left(-3c^2de\left(-d\sqrt{b^2-4ac}+2ae+bd\right)+ce^2\left(-3bd\sqrt{b^2-4ac}-ae\sqrt{b^2-4ac}+3abe+3b^2d\right)-b^2e^3\left(b-\sqrt{b^2-4ac}\right)+2c^3d^3\right){}_2F_1\left(1,\frac{1}{n};1+\frac{1}{n};-\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)}{\left(b\sqrt{b^2-4ac}-4ac+b^2\right)\left(ae^2-bde+cd^2\right)^3}+\frac{e^2x(2cd-be){}_2F_1\left(2,\frac{1}{n};1+\frac{1}{n};-\frac{ex^n}{d}\right)}{d^2\left(ae^2-bde+cd^2\right)^2}+\frac{e^2x\,{}_2F_1\left(3,\frac{1}{n};1+\frac{1}{n};-\frac{ex^n}{d}\right)}{d^3\left(ae^2-bde+cd^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[1/((d + e*x^n)^3*(a + b*x^n + c*x^(2*n))),x]
```

```
[Out] -((c*(2*c^3*d^3 - b^2*(b + Sqrt[b^2 - 4*a*c])*e^3 - 3*c^2*d*e*(b*d + Sqrt[b
^2 - 4*a*c]*d + 2*a*e) + c*e^2*(3*b^2*d + a*Sqrt[b^2 - 4*a*c]*e + 3*b*(Sqrt
[b^2 - 4*a*c]*d + a*e)))*x*Hypergeometric2F1[1, n^(-1), 1 + n^(-1), (-2*c*x
^n)/(b - Sqrt[b^2 - 4*a*c])])/((b^2 - 4*a*c - b*Sqrt[b^2 - 4*a*c])*(c*d^2 -
 b*d*e + a*e^2)^3)) - (c*(2*c^3*d^3 - b^2*(b - Sqrt[b^2 - 4*a*c])*e^3 - 3*c
^2*d*e*(b*d - Sqrt[b^2 - 4*a*c]*d + 2*a*e) + c*e^2*(3*b^2*d - 3*b*Sqrt[b^2
- 4*a*c]*d + 3*a*b*e - a*Sqrt[b^2 - 4*a*c]*e))*x*Hypergeometric2F1[1, n^(-1
), 1 + n^(-1), (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c])])/((b^2 - 4*a*c + b*Sqrt[
b^2 - 4*a*c])*(c*d^2 - b*d*e + a*e^2)^3) + (e^2*(3*c^2*d^2 + b^2*e^2 - c*e*
(3*b*d + a*e))*x*Hypergeometric2F1[1, n^(-1), 1 + n^(-1), -((e*x^n)/d)])/(d
*(c*d^2 - b*d*e + a*e^2)^3) + (e^2*(2*c*d - b*e)*x*Hypergeometric2F1[2, n^(
-1), 1 + n^(-1), -((e*x^n)/d)])/(d^2*(c*d^2 - b*d*e + a*e^2)^2) + (e^2*x*Hy
pergeometric2F1[3, n^(-1), 1 + n^(-1), -((e*x^n)/d)])/(d^3*(c*d^2 - b*d*e +
 a*e^2))
```

Rule 251

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, (-b)*(x^n/a)], x] /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p
, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

Rule 1436

```
Int[((d_) + (e_.)*(x_)^(n_))/((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_)), x
_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q),
Int[1/(b/2 - q/2 + c*x^n), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(
b/2 + q/2 + c*x^n), x], x]] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[n2, 2*n]
 && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && (PosQ[b^2 - 4*a
*c] ||  !IGtQ[n/2, 0])
```

Rule 1438

```
Int[((d_) + (e_.)*(x_)^(n_))^(q_)/((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_
)), x_Symbol] :> Int[ExpandIntegrand[(d + e*x^n)^q/(a + b*x^n + c*x^(2*n)),
 x], x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c,
 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[q]
```

Rubi steps

$$\int \frac{1}{(d+ex^n)^3\,(a+bx^n+cx^{2n})}\,dx = \int \left(\frac{e^2}{(cd^2-bde+ae^2)\,(d+ex^n)^3} - \frac{e^2(-2cd+be)}{(cd^2-bde+ae^2)^2\,(d+ex^n)^2} + \frac{e}{}\right.$$

$$= \frac{\int \frac{c^3d^3-3bc^2d^2e+3b^2cde^2-3ac^2de^2-b^3e^3+2abce^3-(3c^3d^2e-3bc^2de^2+b^2ce^3-ac^2e^3)x^n}{a+bx^n+cx^{2n}}\,dx}{(cd^2-bde+ae^2)^3} +$$

$$= \frac{e^2(3c^2d^2+b^2e^2-ce(3bd+ae))\,x\,{}_2F_1\left(1,\frac{1}{n};1+\frac{1}{n};-\frac{ex^n}{d}\right)}{d\,(cd^2-bde+ae^2)^3} + \frac{e^2(2cd-b}{d^2(}$$

$$= \frac{c\left(2c^3d^3-b^2\left(b+\sqrt{b^2-4ac}\right)e^3-3c^2de\left(bd+\sqrt{b^2-4ac}\,d+2ae\right)+\right.}{\sqrt{b^2-4ac}\,\left(\right.}$$

Mathematica [**B**] Leaf count is larger than twice the leaf count of optimal. 2482 vs. $2(552) = 1104$.
time = 5.54, size = 2482, normalized size = 4.50

Result too large to show

Antiderivative was successfully verified.

```
[In] Integrate[1/((d + e*x^n)^3*(a + b*x^n + c*x^(2*n))),x]
```

```
[Out] (x*((2*c^3*d^3)/a - (6*b*c^2*d^2*e)/a + (6*b^2*c*d*e^2)/a - 6*c^2*d*e^2 - (
2*b^3*e^3)/a + 4*b*c*e^3 + (c^2*d^3*e^2)/(n*(d + e*x^n)^2) - (2*b*c*d^2*e^3
)/(n*(d + e*x^n)^2) + (b^2*d*e^4)/(n*(d + e*x^n)^2) + (2*a*c*d*e^4)/(n*(d +
 e*x^n)^2) - (2*a*b*e^5)/(n*(d + e*x^n)^2) + (a^2*e^6)/(d*n*(d + e*x^n)^2)
- (c^2*d^2*e^2)/(n^2*(d + e*x^n)) + (2*b*c*d*e^3)/(n^2*(d + e*x^n)) - (b^2*
e^4)/(n^2*(d + e*x^n)) - (2*a*c*e^4)/(n^2*(d + e*x^n)) + (2*a*b*e^5)/(d*n^2
*(d + e*x^n)) - (a^2*e^6)/(d^2*n^2*(d + e*x^n)) + (2*a^2*e^6)/(d^2*n*(d + e
*x^n)) + (6*c^2*d^2*e^2)/(d*n + e*n*x^n) - (10*b*c*d*e^3)/(d*n + e*n*x^n) +
 (4*b^2*e^4)/(d*n + e*n*x^n) + (8*a*c*e^4)/(d*n + e*n*x^n) - (6*a*b*e^5)/(d
^2*n + d*e*n*x^n) + (e^2*(c^2*d^4*(1 - 7*n + 12*n^2) + e^2*(a^2*e^2*(1 - 3*
n + 2*n^2) - 2*a*b*d*e*(1 - 4*n + 3*n^2) + b^2*d^2*(1 - 5*n + 6*n^2)) - 2*c
*d^2*e*(a*e*(-1 + 5*n - 3*n^2) + b*d*(1 - 6*n + 8*n^2)))*Hypergeometric2F1[
1, n^(-1), 1 + n^(-1), -((e*x^n)/d)])/(d^3*n^2) + ((b^4*e^3 + b^3*e^2*(-3*c
*d + Sqrt[b^2 - 4*a*c]*e) + b*c*(-(c^2*d^3) - 2*a*Sqrt[b^2 - 4*a*c]*e^3 + 3
*c*d*e*(Sqrt[b^2 - 4*a*c]*d + 3*a*e)) + b^2*c*e*(3*c*d^2 - e*(3*Sqrt[b^2 -
4*a*c]*d + 4*a*e)) + c^2*(a*e^2*(3*Sqrt[b^2 - 4*a*c]*d + 2*a*e) - c*d^2*(Sq
rt[b^2 - 4*a*c]*d + 6*a*e)))*Hypergeometric2F1[-n^(-1), -n^(-1), (-1 + n)/n
, (b - Sqrt[b^2 - 4*a*c])/(b - Sqrt[b^2 - 4*a*c] + 2*c*x^n)])/(2^n^(-1)*a*S
qrt[b^2 - 4*a*c]*((c*x^n)/(b - Sqrt[b^2 - 4*a*c] + 2*c*x^n))^n^(-1)) - (c^3
*d^3*Hypergeometric2F1[-n^(-1), -n^(-1), (-1 + n)/n, (b + Sqrt[b^2 - 4*a*c]
)/(b + Sqrt[b^2 - 4*a*c] + 2*c*x^n)])/(2^n^(-1)*a*((c*x^n)/(b + Sqrt[b^2 -
4*a*c] + 2*c*x^n))^n^(-1)) + (b*c^3*d^3*Hypergeometric2F1[-n^(-1), -n^(-1),
 (-1 + n)/n, (b + Sqrt[b^2 - 4*a*c])/(b + Sqrt[b^2 - 4*a*c] + 2*c*x^n)])/(2
^n^(-1)*a*Sqrt[b^2 - 4*a*c]*((c*x^n)/(b + Sqrt[b^2 - 4*a*c] + 2*c*x^n))^n^(
-1)) + (3*b*c^2*d^2*e*Hypergeometric2F1[-n^(-1), -n^(-1), (-1 + n)/n, (b +
Sqrt[b^2 - 4*a*c])/(b + Sqrt[b^2 - 4*a*c] + 2*c*x^n)])/(2^n^(-1)*a*((c*x^n)
/(b + Sqrt[b^2 - 4*a*c] + 2*c*x^n))^n^(-1)) - (3*b^2*c^2*d^2*e*Hypergeometr
ic2F1[-n^(-1), -n^(-1), (-1 + n)/n, (b + Sqrt[b^2 - 4*a*c])/(b + Sqrt[b^2 -
 4*a*c] + 2*c*x^n)])/(2^n^(-1)*a*Sqrt[b^2 - 4*a*c]*((c*x^n)/(b + Sqrt[b^2 -
 4*a*c] + 2*c*x^n))^n^(-1)) + (3*2^((-1 + n)/n)*c^3*d^2*e*Hypergeometric2F1
[-n^(-1), -n^(-1), (-1 + n)/n, (b + Sqrt[b^2 - 4*a*c])/(b + Sqrt[b^2 - 4*a*
c] + 2*c*x^n)])/(Sqrt[b^2 - 4*a*c]*((c*x^n)/(b + Sqrt[b^2 - 4*a*c] + 2*c*x^
n))^n^(-1)) - (3*b^2*c*d*e^2*Hypergeometric2F1[-n^(-1), -n^(-1), (-1 + n)/n
, (b + Sqrt[b^2 - 4*a*c])/(b + Sqrt[b^2 - 4*a*c] + 2*c*x^n)])/(2^n^(-1)*a*(
(c*x^n)/(b + Sqrt[b^2 - 4*a*c] + 2*c*x^n))^n^(-1)) + (3*c^2*d*e^2*Hypergeom
etric2F1[-n^(-1), -n^(-1), (-1 + n)/n, (b + Sqrt[b^2 - 4*a*c])/(b + Sqrt[b^
2 - 4*a*c] + 2*c*x^n)])/(2^n^(-1)*((c*x^n)/(b + Sqrt[b^2 - 4*a*c] + 2*c*x^n
))^n^(-1)) + (3*b^3*c*d*e^2*Hypergeometric2F1[-n^(-1), -n^(-1), (-1 + n)/n,
 (b + Sqrt[b^2 - 4*a*c])/(b + Sqrt[b^2 - 4*a*c] + 2*c*x^n)])/(2^n^(-1)*a*Sq
rt[b^2 - 4*a*c]*((c*x^n)/(b + Sqrt[b^2 - 4*a*c] + 2*c*x^n))^n^(-1)) - (9*b*
c^2*d*e^2*Hypergeometric2F1[-n^(-1), -n^(-1), (-1 + n)/n, (b + Sqrt[b^2 - 4
```

```
*a*c])/(b + Sqrt[b^2 - 4*a*c] + 2*c*x^n)])/(2^n^(-1)*Sqrt[b^2 - 4*a*c]*((c*
x^n)/(b + Sqrt[b^2 - 4*a*c] + 2*c*x^n))^n^(-1)) + (b^3*e^3*Hypergeometric2F
1[-n^(-1), -n^(-1), (-1 + n)/n, (b + Sqrt[b^2 - 4*a*c])/(b + Sqrt[b^2 - 4*a
*c] + 2*c*x^n)])/(2^n^(-1)*a*((c*x^n)/(b + Sqrt[b^2 - 4*a*c] + 2*c*x^n))^n^
(-1)) - (2^((-1 + n)/n)*b*c*e^3*Hypergeometric2F1[-n^(-1), -n^(-1), (-1 + n
)/n, (b + Sqrt[b^2 - 4*a*c])/(b + Sqrt[b^2 - 4*a*c] + 2*c*x^n)])/((c*x^n)/(
b + Sqrt[b^2 - 4*a*c] + 2*c*x^n))^n^(-1) - (b^4*e^3*Hypergeometric2F1[-n^(-
1), -n^(-1), (-1 + n)/n, (b + Sqrt[b^2 - 4*a*c])/(b + Sqrt[b^2 - 4*a*c] + 2
*c*x^n)])/(2^n^(-1)*a*Sqrt[b^2 - 4*a*c]*((c*x^n)/(b + Sqrt[b^2 - 4*a*c] + 2
*c*x^n))^n^(-1)) + (2^(2 - n^(-1))*b^2*c*e^3*Hypergeometric2F1[-n^(-1), -n^
(-1), (-1 + n)/n, (b + Sqrt[b^2 - 4*a*c])/(b + Sqrt[b^2 - 4*a*c] + 2*c*x^n)
])/(Sqrt[b^2 - 4*a*c]*((c*x^n)/(b + Sqrt[b^2 - 4*a*c] + 2*c*x^n))^n^(-1)) -
 (2^((-1 + n)/n)*a*c^2*e^3*Hypergeometric2F1[-n^(-1), -n^(-1), (-1 + n)/n,
(b + Sqrt[b^2 - 4*a*c])/(b + Sqrt[b^2 - 4*a*c] + 2*c*x^n)])/(Sqrt[b^2 - 4*a
*c]*((c*x^n)/(b + Sqrt[b^2 - 4*a*c] + 2*c*x^n))^n^(-1))))/(2*(c*d^2 + e*(-(
b*d) + a*e))^3)
```

Maple [F]
time = 0.03, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(d+e\,x^{n}\right)^{3}\left(a+b\,x^{n}+c\,x^{2 n}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(d+e*x^n)^3/(a+b*x^n+c*x^(2*n)),x)
```

```
[Out] int(1/(d+e*x^n)^3/(a+b*x^n+c*x^(2*n)),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d+e*x^n)^3/(a+b*x^n+c*x^(2*n)),x, algorithm="maxima")
```

```
[Out] ((12*n^2 - 7*n + 1)*c^2*d^4*e^2 - 2*(8*n^2 - 6*n + 1)*b*c*d^3*e^3 - 2*(3*n^
2 - 4*n + 1)*a*b*d*e^5 + (2*n^2 - 3*n + 1)*a^2*e^6 + ((6*n^2 - 5*n + 1)*b^2
*d^2 + 2*(3*n^2 - 5*n + 1)*a*c*d^2)*e^4)*integrate(1/2/(c^3*d^9*n^2 - 3*b*c
^2*d^8*n^2*e - 3*a^2*b*d^4*n^2*e^5 + a^3*d^3*n^2*e^6 + (c^3*d^8*n^2*e - 3*b
*c^2*d^7*n^2*e^2 - 3*a^2*b*d^3*n^2*e^6 + a^3*d^2*n^2*e^7 + 3*(a*b^2*d^4*n^2
 + a^2*c*d^4*n^2)*e^5 - (b^3*d^5*n^2 + 6*a*b*c*d^5*n^2)*e^4 + 3*(b^2*c*d^6*
n^2 + a*c^2*d^6*n^2)*e^3)*x^n + 3*(a*b^2*d^5*n^2 + a^2*c*d^5*n^2)*e^4 - (b^
3*d^6*n^2 + 6*a*b*c*d^6*n^2)*e^3 + 3*(b^2*c*d^7*n^2 + a*c^2*d^7*n^2)*e^2),
x) + 1/2*((c*d^2*(6*n - 1)*e^3 - b*d*(4*n - 1)*e^4 + a*(2*n - 1)*e^5)*x*x^n
 + (c*d^3*(7*n - 1)*e^2 - b*d^2*(5*n - 1)*e^3 + a*d*(3*n - 1)*e^4)*x)/(c^2*
```

```
d^8*n^2 - 2*b*c*d^7*n^2*e - 2*a*b*d^5*n^2*e^3 + a^2*d^4*n^2*e^4 + (c^2*d^6*
n^2*e^2 - 2*b*c*d^5*n^2*e^3 - 2*a*b*d^3*n^2*e^5 + a^2*d^2*n^2*e^6 + (b^2*d^
4*n^2 + 2*a*c*d^4*n^2)*e^4)*x^(2*n) + 2*(c^2*d^7*n^2*e - 2*b*c*d^6*n^2*e^2
- 2*a*b*d^4*n^2*e^4 + a^2*d^3*n^2*e^5 + (b^2*d^5*n^2 + 2*a*c*d^5*n^2)*e^3)*
x^n + (b^2*d^6*n^2 + 2*a*c*d^6*n^2)*e^2) + integrate((c^3*d^3 - 3*b*c^2*d^2
*e - (3*c^3*d^2*e - 3*b*c^2*d*e^2 + (b^2*c - a*c^2)*e^3)*x^n - (b^3 - 2*a*b
*c)*e^3 + 3*(b^2*c*d - a*c^2*d)*e^2)/(a*c^3*d^6 - 3*a*b*c^2*d^5*e - 3*a^3*b
*d*e^5 + a^4*e^6 + (c^4*d^6 - 3*b*c^3*d^5*e - 3*a^2*b*c*d*e^5 + a^3*c*e^6 +
 3*(a*b^2*c*d^2 + a^2*c^2*d^2)*e^4 - (b^3*c*d^3 + 6*a*b*c^2*d^3)*e^3 + 3*(b
^2*c^2*d^4 + a*c^3*d^4)*e^2)*x^(2*n) + (b*c^3*d^6 - 3*b^2*c^2*d^5*e - 3*a^2
*b^2*d*e^5 + a^3*b*e^6 + 3*(a*b^3*d^2 + a^2*b*c*d^2)*e^4 - (b^4*d^3 + 6*a*b
^2*c*d^3)*e^3 + 3*(b^3*c*d^4 + a*b*c^2*d^4)*e^2)*x^n + 3*(a^2*b^2*d^2 + a^3
*c*d^2)*e^4 - (a*b^3*d^3 + 6*a^2*b*c*d^3)*e^3 + 3*(a*b^2*c*d^4 + a^2*c^2*d^
4)*e^2), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d+e*x^n)^3/(a+b*x^n+c*x^(2*n)),x, algorithm="fricas")
```

```
[Out] integral(1/(a*d^3 + b*x^(4*n)*e^3 + (3*b*d*e^2 + a*e^3)*x^(3*n) + (3*c*d^2*
x^n*e + c*d^3 + 3*c*d*x^(2*n)*e^2 + c*x^(3*n)*e^3)*x^(2*n) + 3*(b*d^2*e + a
*d*e^2)*x^(2*n) + (b*d^3 + 3*a*d^2*e)*x^n), x)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: HeuristicGCDFailed

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d+e*x**n)**3/(a+b*x**n+c*x**(2*n)),x)
```

```
[Out] Exception raised: HeuristicGCDFailed >> no luck
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d+e*x^n)^3/(a+b*x^n+c*x^(2*n)),x, algorithm="giac")
```

```
[Out] integrate(1/((c*x^(2*n) + b*x^n + a)*(x^n*e + d)^3), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{(d+e\,x^n)^3\,\left(a+b\,x^n+c\,x^{2\,n}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((d + e*x^n)^3*(a + b*x^n + c*x^(2*n))),x)
[Out] int(1/((d + e*x^n)^3*(a + b*x^n + c*x^(2*n))), x)
```

3.75 $\int \frac{(d+ex^n)^3}{(a+bx^n+cx^{2n})^2}\,dx$

Optimal. Leaf size=750

$$\frac{x\left(b^2cd^3-2acd(cd^2-3ae^2)-abe(3cd^2+ae^2)-\left(ab^2e^3+2ace(3cd^2-ae^2)-bcd(cd^2+3ae^2)\right)x^n\right)}{ac\,(b^2-4ac)\,n\,(a+bx^n+cx^{2n})}+\frac{e^2\Big(e+}{}$$

```
[Out] x*(b^2*c*d^3-2*a*c*d*(-3*a*e^2+c*d^2)-a*b*e*(a*e^2+3*c*d^2)-(a*b^2*e^3+2*a*
c*e*(-a*e^2+3*c*d^2)-b*c*d*(3*a*e^2+c*d^2))*x^n)/a/c/(-4*a*c+b^2)/n/(a+b*x^
n+c*x^(2*n))+e^2*x*hypergeom([1, 1/n],[1+1/n],-2*c*x^n/(b-(-4*a*c+b^2)^(1/2
)))*(e+(-3*b*e+6*c*d)/(-4*a*c+b^2)^(1/2))/c/(b-(-4*a*c+b^2)^(1/2))+x*hyperg
eom([1, 1/n],[1+1/n],-2*c*x^n/(b-(-4*a*c+b^2)^(1/2)))*((a*b^2*e^3+2*a*c*e*(
-a*e^2+3*c*d^2)-b*c*d*(3*a*e^2+c*d^2))*(1-n)+(b^2*c*d*(3*a*e^2*(1-3*n)-c*d^
2*(1-n))-a*b^3*e^3*(1-3*n)+4*a*c^2*d*(-3*a*e^2+c*d^2)*(1-2*n)+2*a*b*c*e*(a*
e^2*(2-5*n)+3*c*d^2*n))/(-4*a*c+b^2)^(1/2))/a/c/(-4*a*c+b^2)/n/(b-(-4*a*c+b
^2)^(1/2))+e^2*x*hypergeom([1, 1/n],[1+1/n],-2*c*x^n/(b+(-4*a*c+b^2)^(1/2))
)*(e-3*(-b*e+2*c*d)/(-4*a*c+b^2)^(1/2))/c/(b+(-4*a*c+b^2)^(1/2))+x*hypergeo
m([1, 1/n],[1+1/n],-2*c*x^n/(b+(-4*a*c+b^2)^(1/2)))*((a*b^2*e^3+2*a*c*e*(-a
*e^2+3*c*d^2)-b*c*d*(3*a*e^2+c*d^2))*(1-n)+(-b^2*c*d*(3*a*e^2*(1-3*n)-c*d^2
*(1-n))+a*b^3*e^3*(1-3*n)-4*a*c^2*d*(-3*a*e^2+c*d^2)*(1-2*n)-2*a*b*c*e*(a*e
^2*(2-5*n)+3*c*d^2*n))/(-4*a*c+b^2)^(1/2))/a/c/(-4*a*c+b^2)/n/(b+(-4*a*c+b^
2)^(1/2))
```

Rubi [A]

time = 1.97, antiderivative size = 750, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {1450, 1444, 1436, 251}

Antiderivative was successfully verified.

```
[In] Int[(d + e*x^n)^3/(a + b*x^n + c*x^(2*n))^2,x]
```

```
[Out] (x*(b^2*c*d^3 - 2*a*c*d*(c*d^2 - 3*a*e^2) - a*b*e*(3*c*d^2 + a*e^2) - (a*b^
2*e^3 + 2*a*c*e*(3*c*d^2 - a*e^2) - b*c*d*(c*d^2 + 3*a*e^2))*x^n))/(a*c*(b^
2 - 4*a*c)*n*(a + b*x^n + c*x^(2*n))) + (e^2*(e + (6*c*d - 3*b*e)/Sqrt[b^2
- 4*a*c])*x*Hypergeometric2F1[1, n^(-1), 1 + n^(-1), (-2*c*x^n)/(b - Sqrt[b
^2 - 4*a*c])])/(c*(b - Sqrt[b^2 - 4*a*c])) + (((a*b^2*e^3 + 2*a*c*e*(3*c*d^
2 - a*e^2) - b*c*d*(c*d^2 + 3*a*e^2))*(1 - n) + (b^2*c*d*(3*a*e^2*(1 - 3*n)
 - c*d^2*(1 - n)) - a*b^3*e^3*(1 - 3*n) + 4*a*c^2*d*(c*d^2 - 3*a*e^2)*(1 -
2*n) + 2*a*b*c*e*(a*e^2*(2 - 5*n) + 3*c*d^2*n))/Sqrt[b^2 - 4*a*c])*x*Hyperg
eometric2F1[1, n^(-1), 1 + n^(-1), (-2*c*x^n)/(b - Sqrt[b^2 - 4*a*c])])/(a*
```

```
c*(b^2 - 4*a*c)*(b - Sqrt[b^2 - 4*a*c])*n) + (e^2*(e - (3*(2*c*d - b*e))/Sq
rt[b^2 - 4*a*c])*x*Hypergeometric2F1[1, n^(-1), 1 + n^(-1), (-2*c*x^n)/(b +
 Sqrt[b^2 - 4*a*c])])/(c*(b + Sqrt[b^2 - 4*a*c])) + (((a*b^2*e^3 + 2*a*c*e*
(3*c*d^2 - a*e^2) - b*c*d*(c*d^2 + 3*a*e^2))*(1 - n) - (b^2*c*d*(3*a*e^2*(1
 - 3*n) - c*d^2*(1 - n)) - a*b^3*e^3*(1 - 3*n) + 4*a*c^2*d*(c*d^2 - 3*a*e^2
)*(1 - 2*n) + 2*a*b*c*e*(a*e^2*(2 - 5*n) + 3*c*d^2*n))/Sqrt[b^2 - 4*a*c])*x
*Hypergeometric2F1[1, n^(-1), 1 + n^(-1), (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c]
)])/(a*c*(b^2 - 4*a*c)*(b + Sqrt[b^2 - 4*a*c])*n)
```

Rule 251

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, (-b)*(x^n/a)], x] /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p
, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

Rule 1436

```
Int[((d_) + (e_.)*(x_)^(n_))/((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_)), x
_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q),
Int[1/(b/2 - q/2 + c*x^n), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(
b/2 + q/2 + c*x^n), x], x]] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[n2, 2*n]
 && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && (PosQ[b^2 - 4*a
*c] ||  !IGtQ[n/2, 0])
```

Rule 1444

```
Int[((d_) + (e_.)*(x_)^(n_))*((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_))^(p
_), x_Symbol] :> Simp[(-x)*(d*b^2 - a*b*e - 2*a*c*d + (b*d - 2*a*e)*c*x^n)*
((a + b*x^n + c*x^(2*n))^(p + 1)/(a*n*(p + 1)*(b^2 - 4*a*c))), x] + Dist[1/
(a*n*(p + 1)*(b^2 - 4*a*c)), Int[Simp[(n*p + n + 1)*d*b^2 - a*b*e - 2*a*c*d
*(2*n*p + 2*n + 1) + (2*n*p + 3*n + 1)*(d*b - 2*a*e)*c*x^n, x]*(a + b*x^n +
 c*x^(2*n))^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[n2, 2*n]
 && NeQ[b^2 - 4*a*c, 0] && ILtQ[p, -1]
```

Rule 1450

```
Int[((d_) + (e_.)*(x_)^(n_))^(q_)*((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_
))^(p_), x_Symbol] :> Int[ExpandIntegrand[(d + e*x^n)^q*(a + b*x^n + c*x^(2
*n))^p, x], x] /; FreeQ[{a, b, c, d, e, n, p, q}, x] && EqQ[n2, 2*n] && NeQ
[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && ((IntegersQ[p, q] &&
!IntegerQ[n]) || IGtQ[p, 0] || (IGtQ[q, 0] &&  !IntegerQ[n]))
```

Rubi steps

$$\int \frac{(d+ex^n)^3}{(a+bx^n+cx^{2n})^2}\,dx = \int \left(\frac{c^2d^3-3acde^2+abe^3+(3c^2d^2e-3bcde^2+b^2e^3-ace^3)\,x^n}{c^2\,(a+bx^n+cx^{2n})^2}+\frac{e^2(3cd-be+c}{c^2\,(a+bx^n+c}\right.$$

$$=\frac{\int \frac{c^2d^3-3acde^2+abe^3+(3c^2d^2e-3bcde^2+b^2e^3-ace^3)x^n}{(a+bx^n+cx^{2n})^2}\,dx}{c^2}+\frac{e^2\int \frac{3cd-be+cex^n}{a+bx^n+cx^{2n}}\,dx}{c^2}$$

$$=\frac{x(b^2cd^3-2acd(cd^2-3ae^2)-abe(3cd^2+ae^2)-(ab^2e^3+2ace(3cd^2-ae^2)-bcd}{ac\,(b^2-4ac)\,n\,(a+bx^n+cx^{2n})}$$

$$=\frac{x(b^2cd^3-2acd(cd^2-3ae^2)-abe(3cd^2+ae^2)-(ab^2e^3+2ace(3cd^2-ae^2)-bcd}{ac\,(b^2-4ac)\,n\,(a+bx^n+cx^{2n})}$$

$$=\frac{x(b^2cd^3-2acd(cd^2-3ae^2)-abe(3cd^2+ae^2)-(ab^2e^3+2ace(3cd^2-ae^2)-bcd}{ac\,(b^2-4ac)\,n\,(a+bx^n+cx^{2n})}$$

Mathematica [**B**] Leaf count is larger than twice the leaf count of optimal. 5537 vs. $2(750) = 1500$.
time = 7.26, size = 5537, normalized size = 7.38

Result too large to show

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x^n)^3/(a + b*x^n + c*x^(2*n))^2,x]
```

```
[Out] Result too large to show
```

Maple [**F**]
time = 0.02, size = 0, normalized size = 0.00

$$\int \frac{(d+e\,x^n)^3}{(a+b\,x^n+c\,x^{2n})^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d+e*x^n)^3/(a+b*x^n+c*x^(2*n))^2,x)
```

```
[Out] int((d+e*x^n)^3/(a+b*x^n+c*x^(2*n))^2,x)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+e*x^n)^3/(a+b*x^n+c*x^(2*n))^2,x, algorithm="maxima")
```

```
[Out] ((b*c^2*d^3 - 6*a*c^2*d^2*e + 3*a*b*c*d*e^2 - (a*b^2 - 2*a^2*c)*e^3)*x*x^n
+ (b^2*c*d^3 - 2*a*c^2*d^3 - 3*a*b*c*d^2*e + 6*a^2*c*d*e^2 - a^2*b*e^3)*x)/
(a^2*b^2*c*n - 4*a^3*c^2*n + (a*b^2*c^2*n - 4*a^2*c^3*n)*x^(2*n) + (a*b^3*c
*n - 4*a^2*b*c^2*n)*x^n) + integrate(-(2*a*c^2*d^3*(2*n - 1) - b^2*c*d^3*(n
 - 1) - 3*a*b*c*d^2*e + 6*a^2*c*d*e^2 - a^2*b*e^3 - (b*c^2*d^3*(n - 1) - 6*
a*c^2*d^2*(n - 1)*e + 3*a*b*c*d*(n - 1)*e^2 - (2*a^2*c*(n + 1) - a*b^2)*e^3
)*x^n)/(a^2*b^2*c*n - 4*a^3*c^2*n + (a*b^2*c^2*n - 4*a^2*c^3*n)*x^(2*n) + (
a*b^3*c*n - 4*a^2*b*c^2*n)*x^n), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+e*x^n)^3/(a+b*x^n+c*x^(2*n))^2,x, algorithm="fricas")
```

```
[Out] integral((3*d^2*x^n*e + d^3 + 3*d*x^(2*n)*e^2 + x^(3*n)*e^3)/(c^2*x^(4*n) +
 b^2*x^(2*n) + 2*a*b*x^n + a^2 + 2*(b*c*x^n + a*c)*x^(2*n)), x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+e*x**n)**3/(a+b*x**n+c*x**(2*n))**2,x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+e*x^n)^3/(a+b*x^n+c*x^(2*n))^2,x, algorithm="giac")
```

```
[Out] integrate((x^n*e + d)^3/(c*x^(2*n) + b*x^n + a)^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(d+e\,x^n)^3}{(a+b\,x^n+c\,x^{2\,n})^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x^n)^3/(a + b*x^n + c*x^(2*n))^2,x)
[Out] int((d + e*x^n)^3/(a + b*x^n + c*x^(2*n))^2, x)
```

3.76 $\int \frac{(d+ex^n)^2}{(a+bx^n+cx^{2n})^2}\,dx$

Optimal. Leaf size=543

$$\frac{x\left(b^2d^2-2abde-2a(cd^2-ae^2)+(bcd^2-4acde+abe^2)\,x^n\right)}{a\left(b^2-4ac\right)n\left(a+bx^n+cx^{2n}\right)}-\frac{2e^2x\,{}_2F_1\left(1,\frac{1}{n};1+\frac{1}{n};-\frac{2cx^n}{b-\sqrt{b^2-4ac}}\right)}{b^2-4ac-b\sqrt{b^2-4ac}}-\frac{\left(\left(b\right.\right.}{}$$

```
[Out] x*(b^2*d^2-2*a*b*d*e-2*a*(-a*e^2+c*d^2)+(a*b*e^2-4*a*c*d*e+b*c*d^2)*x^n)/a/
(-4*a*c+b^2)/n/(a+b*x^n+c*x^(2*n))-x*hypergeom([1, 1/n],[1+1/n],-2*c*x^n/(b
-(-4*a*c+b^2)^(1/2)))*((a*b*e^2-4*a*c*d*e+b*c*d^2)*(1-n)+(-b^2*(a*e^2*(1-3*
n)-c*d^2*(1-n))-4*a*c*(-a*e^2+c*d^2)*(1-2*n)-4*a*b*c*d*e*n)/(-4*a*c+b^2)^(1
/2))/a/(-4*a*c+b^2)/n/(b-(-4*a*c+b^2)^(1/2))-x*hypergeom([1, 1/n],[1+1/n],-
2*c*x^n/(b+(-4*a*c+b^2)^(1/2)))*((a*b*e^2-4*a*c*d*e+b*c*d^2)*(1-n)+(b^2*(a*
e^2*(1-3*n)-c*d^2*(1-n))+4*a*c*(-a*e^2+c*d^2)*(1-2*n)+4*a*b*c*d*e*n)/(-4*a*
c+b^2)^(1/2))/a/(-4*a*c+b^2)/n/(b+(-4*a*c+b^2)^(1/2))-2*e^2*x*hypergeom([1,
 1/n],[1+1/n],-2*c*x^n/(b-(-4*a*c+b^2)^(1/2)))/(b^2-4*a*c-b*(-4*a*c+b^2)^(1
/2))-2*e^2*x*hypergeom([1, 1/n],[1+1/n],-2*c*x^n/(b+(-4*a*c+b^2)^(1/2)))/(b
^2-4*a*c+b*(-4*a*c+b^2)^(1/2))
```

Rubi [A]
time = 1.25, antiderivative size = 543, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 5, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.192, Rules used = {1450, 1444, 1436, 251, 1361}

$$-\frac{x\left((1-n)\left(abe^2-4acde+bcd^2\right)-\frac{b^2\left(ae^2(1-3n)-cd^2(1-n)\right)+4abcden+4ac(1-2n)\left(cd^2-ae^2\right)}{\sqrt{b^2-4ac}}\right){}_2F_1\left(1,\frac{1}{n};1+\frac{1}{n};-\frac{2cx^n}{b-\sqrt{b^2-4ac}}\right)}{an\left(b^2-4ac\right)\left(b-\sqrt{b^2-4ac}\right)}-\frac{x\left(\frac{b^2\left(ae^2(1-3n)-cd^2(1-n)\right)+4abcden+4ac(1-2n)\left(cd^2-ae^2\right)}{\sqrt{b^2-4ac}}+(1-n)\left(abe^2-4acde+bcd^2\right)\right){}_2F_1\left(1,\frac{1}{n};1+\frac{1}{n};-\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)}{an\left(b^2-4ac\right)\left(\sqrt{b^2-4ac}+b\right)}+\frac{x\left(x^n\left(abe^2-4acde+bcd^2\right)-2abde-2a\left(cd^2-ae^2\right)+b^2d^2\right)}{an\left(b^2-4ac\right)\left(a+bx^n+cx^{2n}\right)}-\frac{2e^2x\,{}_2F_1\left(1,\frac{1}{n};1+\frac{1}{n};-\frac{2cx^n}{b-\sqrt{b^2-4ac}}\right)}{-b\sqrt{b^2-4ac}-4ac+b^2}-\frac{2e^2x\,{}_2F_1\left(1,\frac{1}{n};1+\frac{1}{n};-\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)}{b\sqrt{b^2-4ac}-4ac+b^2}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x^n)^2/(a + b*x^n + c*x^(2*n))^2,x]
```

```
[Out] (x*(b^2*d^2 - 2*a*b*d*e - 2*a*(c*d^2 - a*e^2) + (b*c*d^2 - 4*a*c*d*e + a*b*
e^2)*x^n))/(a*(b^2 - 4*a*c)*n*(a + b*x^n + c*x^(2*n))) - (2*e^2*x*Hypergeom
etric2F1[1, n^(-1), 1 + n^(-1), (-2*c*x^n)/(b - Sqrt[b^2 - 4*a*c])])/(b^2 -
 4*a*c - b*Sqrt[b^2 - 4*a*c]) - (((b*c*d^2 - 4*a*c*d*e + a*b*e^2)*(1 - n) -
 (b^2*(a*e^2*(1 - 3*n) - c*d^2*(1 - n)) + 4*a*c*(c*d^2 - a*e^2)*(1 - 2*n) +
 4*a*b*c*d*e*n)/Sqrt[b^2 - 4*a*c])*x*Hypergeometric2F1[1, n^(-1), 1 + n^(-1
), (-2*c*x^n)/(b - Sqrt[b^2 - 4*a*c])])/(a*(b^2 - 4*a*c)*(b - Sqrt[b^2 - 4*
a*c])*n) - (2*e^2*x*Hypergeometric2F1[1, n^(-1), 1 + n^(-1), (-2*c*x^n)/(b
+ Sqrt[b^2 - 4*a*c])])/(b^2 - 4*a*c + b*Sqrt[b^2 - 4*a*c]) - (((b*c*d^2 - 4
*a*c*d*e + a*b*e^2)*(1 - n) + (b^2*(a*e^2*(1 - 3*n) - c*d^2*(1 - n)) + 4*a*
c*(c*d^2 - a*e^2)*(1 - 2*n) + 4*a*b*c*d*e*n)/Sqrt[b^2 - 4*a*c])*x*Hypergeom
etric2F1[1, n^(-1), 1 + n^(-1), (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c])])/(a*(b^
2 - 4*a*c)*(b + Sqrt[b^2 - 4*a*c])*n)
```

Rule 251

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, (-b)*(x^n/a)], x] /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p
, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

Rule 1361

```
Int[((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_))^(-1), x_Symbol] :> With[{q
= Rt[b^2 - 4*a*c, 2]}, Dist[c/q, Int[1/(b/2 - q/2 + c*x^n), x], x] - Dist[c
/q, Int[1/(b/2 + q/2 + c*x^n), x], x]] /; FreeQ[{a, b, c}, x] && EqQ[n2, 2*
n] && NeQ[b^2 - 4*a*c, 0]
```

Rule 1436

```
Int[((d_) + (e_.)*(x_)^(n_))/((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_)), x
_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q),
Int[1/(b/2 - q/2 + c*x^n), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(
b/2 + q/2 + c*x^n), x], x]] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[n2, 2*n]
 && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && (PosQ[b^2 - 4*a
*c] ||  !IGtQ[n/2, 0])
```

Rule 1444

```
Int[((d_) + (e_.)*(x_)^(n_))*((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_))^(p
_), x_Symbol] :> Simp[(-x)*(d*b^2 - a*b*e - 2*a*c*d + (b*d - 2*a*e)*c*x^n)*
((a + b*x^n + c*x^(2*n))^(p + 1)/(a*n*(p + 1)*(b^2 - 4*a*c))), x] + Dist[1/
(a*n*(p + 1)*(b^2 - 4*a*c)), Int[Simp[(n*p + n + 1)*d*b^2 - a*b*e - 2*a*c*d
*(2*n*p + 2*n + 1) + (2*n*p + 3*n + 1)*(d*b - 2*a*e)*c*x^n, x]*(a + b*x^n +
 c*x^(2*n))^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[n2, 2*n]
 && NeQ[b^2 - 4*a*c, 0] && ILtQ[p, -1]
```

Rule 1450

```
Int[((d_) + (e_.)*(x_)^(n_))^(q_)*((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_
))^(p_), x_Symbol] :> Int[ExpandIntegrand[(d + e*x^n)^q*(a + b*x^n + c*x^(2
*n))^p, x], x] /; FreeQ[{a, b, c, d, e, n, p, q}, x] && EqQ[n2, 2*n] && NeQ
[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && ((IntegersQ[p, q] &&
!IntegerQ[n]) || IGtQ[p, 0] || (IGtQ[q, 0] &&  !IntegerQ[n]))
```

Rubi steps

$$\int \frac{(d+ex^n)^2}{(a+bx^n+cx^{2n})^2}\,dx = \int \left(\frac{cd^2-ae^2+(2cde-be^2)\,x^n}{c\,(a+bx^n+cx^{2n})^2} + \frac{e^2}{c\,(a+bx^n+cx^{2n})}\right) dx$$

$$= \frac{\int \frac{cd^2-ae^2+(2cde-be^2)x^n}{(a+bx^n+cx^{2n})^2}\,dx}{c} + \frac{e^2\int \frac{1}{a+bx^n+cx^{2n}}\,dx}{c}$$

$$= \frac{x(b^2d^2-2abde-2a(cd^2-ae^2)+(bcd^2-4acde+abe^2)\,x^n)}{a\,(b^2-4ac)\,n\,(a+bx^n+cx^{2n})} + \frac{e^2\int \frac{1}{\frac{b}{2}-\frac{1}{2}\sqrt{b^2-4}}}{\sqrt{b^2-4}}$$

$$= \frac{x(b^2d^2-2abde-2a(cd^2-ae^2)+(bcd^2-4acde+abe^2)\,x^n)}{a\,(b^2-4ac)\,n\,(a+bx^n+cx^{2n})} - \frac{2e^2x\,{}_2F_1\left(1,\frac{1}{n};1\right.}{b^2-4ac}$$

$$= \frac{x(b^2d^2-2abde-2a(cd^2-ae^2)+(bcd^2-4acde+abe^2)\,x^n)}{a\,(b^2-4ac)\,n\,(a+bx^n+cx^{2n})} - \frac{2e^2x\,{}_2F_1\left(1,\frac{1}{n};1\right.}{b^2-4ac}$$

Mathematica [**B**] Leaf count is larger than twice the leaf count of optimal. 2980 vs. $2(543) = 1086$.
time = 3.48, size = 2980, normalized size = 5.49

Result too large to show

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x^n)^2/(a + b*x^n + c*x^(2*n))^2,x]
```

```
[Out] -((x*(-(a*Sqrt[b^2 - 4*a*c]*(b^2*d^2 + 2*a^2*e^2 + b*c*d^2*x^n + a*b*e*(-2*
d + e*x^n) - 2*a*c*d*(d + 2*e*x^n))) + (a*b*c*d^2*(a + x^n*(b + c*x^n))*(Hy
pergeometric2F1[-n^(-1), -n^(-1), (-1 + n)/n, (b - Sqrt[b^2 - 4*a*c])/(b -
Sqrt[b^2 - 4*a*c] + 2*c*x^n)]/((c*x^n)/(b - Sqrt[b^2 - 4*a*c] + 2*c*x^n))^n
^(-1) - Hypergeometric2F1[-n^(-1), -n^(-1), (-1 + n)/n, (b + Sqrt[b^2 - 4*a
*c])/(b + Sqrt[b^2 - 4*a*c] + 2*c*x^n)]/((c*x^n)/(b + Sqrt[b^2 - 4*a*c] + 2
*c*x^n))^n^(-1)))/2^n^(-1) - 2^(2 - n^(-1))*a^2*c*d*e*(a + x^n*(b + c*x^n))
*(Hypergeometric2F1[-n^(-1), -n^(-1), (-1 + n)/n, (b - Sqrt[b^2 - 4*a*c])/(
b - Sqrt[b^2 - 4*a*c] + 2*c*x^n)]/((c*x^n)/(b - Sqrt[b^2 - 4*a*c] + 2*c*x^n
))^n^(-1) - Hypergeometric2F1[-n^(-1), -n^(-1), (-1 + n)/n, (b + Sqrt[b^2 -
 4*a*c])/(b + Sqrt[b^2 - 4*a*c] + 2*c*x^n)]/((c*x^n)/(b + Sqrt[b^2 - 4*a*c]
 + 2*c*x^n))^n^(-1)) + (a^2*b*e^2*(a + x^n*(b + c*x^n))*(Hypergeometric2F1[
-n^(-1), -n^(-1), (-1 + n)/n, (b - Sqrt[b^2 - 4*a*c])/(b - Sqrt[b^2 - 4*a*c
] + 2*c*x^n)]/((c*x^n)/(b - Sqrt[b^2 - 4*a*c] + 2*c*x^n))^n^(-1) - Hypergeo
metric2F1[-n^(-1), -n^(-1), (-1 + n)/n, (b + Sqrt[b^2 - 4*a*c])/(b + Sqrt[b
^2 - 4*a*c] + 2*c*x^n)]/((c*x^n)/(b + Sqrt[b^2 - 4*a*c] + 2*c*x^n))^n^(-1))
)/2^n^(-1) - (a*b*c*d^2*n*(a + x^n*(b + c*x^n))*(Hypergeometric2F1[-n^(-1),
```

```
 -n^(-1), (-1 + n)/n, (b - Sqrt[b^2 - 4*a*c])/(b - Sqrt[b^2 - 4*a*c] + 2*c*
x^n)]/((c*x^n)/(b - Sqrt[b^2 - 4*a*c] + 2*c*x^n))^n^(-1) - Hypergeometric2F
1[-n^(-1), -n^(-1), (-1 + n)/n, (b + Sqrt[b^2 - 4*a*c])/(b + Sqrt[b^2 - 4*a
*c] + 2*c*x^n)]/((c*x^n)/(b + Sqrt[b^2 - 4*a*c] + 2*c*x^n))^n^(-1)))/2^n^(-
1) + 2^(2 - n^(-1))*a^2*c*d*e*n*(a + x^n*(b + c*x^n))*(Hypergeometric2F1[-n
^(-1), -n^(-1), (-1 + n)/n, (b - Sqrt[b^2 - 4*a*c])/(b - Sqrt[b^2 - 4*a*c]
+ 2*c*x^n)]/((c*x^n)/(b - Sqrt[b^2 - 4*a*c] + 2*c*x^n))^n^(-1) - Hypergeome
tric2F1[-n^(-1), -n^(-1), (-1 + n)/n, (b + Sqrt[b^2 - 4*a*c])/(b + Sqrt[b^2
 - 4*a*c] + 2*c*x^n)]/((c*x^n)/(b + Sqrt[b^2 - 4*a*c] + 2*c*x^n))^n^(-1)) -
 (a^2*b*e^2*n*(a + x^n*(b + c*x^n))*(Hypergeometric2F1[-n^(-1), -n^(-1), (-
1 + n)/n, (b - Sqrt[b^2 - 4*a*c])/(b - Sqrt[b^2 - 4*a*c] + 2*c*x^n)]/((c*x^
n)/(b - Sqrt[b^2 - 4*a*c] + 2*c*x^n))^n^(-1) - Hypergeometric2F1[-n^(-1), -
n^(-1), (-1 + n)/n, (b + Sqrt[b^2 - 4*a*c])/(b + Sqrt[b^2 - 4*a*c] + 2*c*x^
n)]/((c*x^n)/(b + Sqrt[b^2 - 4*a*c] + 2*c*x^n))^n^(-1)))/2^n^(-1) + (b^2*d^
2*(a + x^n*(b + c*x^n))*(2^(1 + n^(-1))*Sqrt[b^2 - 4*a*c] - ((b + Sqrt[b^2
- 4*a*c])*Hypergeometric2F1[-n^(-1), -n^(-1), (-1 + n)/n, (b - Sqrt[b^2 - 4
*a*c])/(b - Sqrt[b^2 - 4*a*c] + 2*c*x^n)])/((c*x^n)/(b - Sqrt[b^2 - 4*a*c]
+ 2*c*x^n))^n^(-1) + ((b - Sqrt[b^2 - 4*a*c])*Hypergeometric2F1[-n^(-1), -n
^(-1), (-1 + n)/n, (b + Sqrt[b^2 - 4*a*c])/(b + Sqrt[b^2 - 4*a*c] + 2*c*x^n
)])/((c*x^n)/(b + Sqrt[b^2 - 4*a*c] + 2*c*x^n))^n^(-1)))/2^((1 + n)/n) - (a
*c*d^2*(a + x^n*(b + c*x^n))*(2^(1 + n^(-1))*Sqrt[b^2 - 4*a*c] - ((b + Sqrt
[b^2 - 4*a*c])*Hypergeometric2F1[-n^(-1), -n^(-1), (-1 + n)/n, (b - Sqrt[b^
2 - 4*a*c])/(b - Sqrt[b^2 - 4*a*c] + 2*c*x^n)])/((c*x^n)/(b - Sqrt[b^2 - 4*
a*c] + 2*c*x^n))^n^(-1) + ((b - Sqrt[b^2 - 4*a*c])*Hypergeometric2F1[-n^(-1
), -n^(-1), (-1 + n)/n, (b + Sqrt[b^2 - 4*a*c])/(b + Sqrt[b^2 - 4*a*c] + 2*
c*x^n)])/((c*x^n)/(b + Sqrt[b^2 - 4*a*c] + 2*c*x^n))^n^(-1)))/2^n^(-1) - (a
*b*d*e*(a + x^n*(b + c*x^n))*(2^(1 + n^(-1))*Sqrt[b^2 - 4*a*c] - ((b + Sqrt
[b^2 - 4*a*c])*Hypergeometric2F1[-n^(-1), -n^(-1), (-1 + n)/n, (b - Sqrt[b^
2 - 4*a*c])/(b - Sqrt[b^2 - 4*a*c] + 2*c*x^n)])/((c*x^n)/(b - Sqrt[b^2 - 4*
a*c] + 2*c*x^n))^n^(-1) + ((b - Sqrt[b^2 - 4*a*c])*Hypergeometric2F1[-n^(-1
), -n^(-1), (-1 + n)/n, (b + Sqrt[b^2 - 4*a*c])/(b + Sqrt[b^2 - 4*a*c] + 2*
c*x^n)])/((c*x^n)/(b + Sqrt[b^2 - 4*a*c] + 2*c*x^n))^n^(-1)))/2^n^(-1) + (a
^2*e^2*(a + x^n*(b + c*x^n))*(2^(1 + n^(-1))*Sqrt[b^2 - 4*a*c] - ((b + Sqrt
[b^2 - 4*a*c])*Hypergeometric2F1[-n^(-1), -n^(-1), (-1 + n)/n, (b - Sqrt[b^
2 - 4*a*c])/(b - Sqrt[b^2 - 4*a*c] + 2*c*x^n)])/((c*x^n)/(b - Sqrt[b^2 - 4*
a*c] + 2*c*x^n))^n^(-1) + ((b - Sqrt[b^2 - 4*a*c])*Hypergeometric2F1[-n^(-1
), -n^(-1), (-1 + n)/n, (b + Sqrt[b^2 - 4*a*c])/(b + Sqrt[b^2 - 4*a*c] + 2*
c*x^n)])/((c*x^n)/(b + Sqrt[b^2 - 4*a*c] + 2*c*x^n))^n^(-1)))/2^n^(-1) - (b
^2*d^2*n*(a + x^n*(b + c*x^n))*(2^(1 + n^(-1))*Sqrt[b^2 - 4*a*c] - ((b + Sq
rt[b^2 - 4*a*c])*Hypergeometric2F1[-n^(-1), -n^(-1), (-1 + n)/n, (b - Sqrt[
b^2 - 4*a*c])/(b - Sqrt[b^2 - 4*a*c] + 2*c*x^n)])/((c*x^n)/(b - Sqrt[b^2 -
4*a*c] + 2*c*x^n))^n^(-1) + ((b - Sqrt[b^2 - 4*a*c])*Hypergeometric2F1[-n^(
-1), -n^(-1), (-1 + n)/n, (b + Sqrt[b^2 - 4*a*c])/(b + Sqrt[b^2 - 4*a*c] +
2*c*x^n)])/((c*x^n)/(b + Sqrt[b^2 - 4*a*c] + 2*c*x^n))^n^(-1)))/2^((1 + n)/
n) + 2^((-1 + n)/n)*a*c*d^2*n*(a + x^n*(b + c*x^n))*(2^(1 + n^(-1))*Sqrt[b^
```

```
2 - 4*a*c] - ((b + Sqrt[b^2 - 4*a*c])*Hypergeometric2F1[-n^(-1), -n^(-1), (
-1 + n)/n, (b - Sqrt[b^2 - 4*a*c])/(b - Sqrt[b^2 - 4*a*c] + 2*c*x^n)])/((c*
x^n)/(b - Sqrt[b^2 - 4*a*c] + 2*c*x^n))^n^(-1) + ((b - Sqrt[b^2 - 4*a*c])*H
ypergeometric2F1[-n^(-1), -n^(-1), (-1 + n)/n, (b + Sqrt[b^2 - 4*a*c])/(b +
 Sqrt[b^2 - 4*a*c] + 2*c*x^n)])/((c*x^n)/(b + S...
```

Maple [F]
time = 0.02, size = 0, normalized size = 0.00

$$\int \frac{(d+e\,x^{n})^{2}}{(a+b\,x^{n}+c\,x^{2n})^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d+e*x^n)^2/(a+b*x^n+c*x^(2*n))^2,x)
```

```
[Out] int((d+e*x^n)^2/(a+b*x^n+c*x^(2*n))^2,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+e*x^n)^2/(a+b*x^n+c*x^(2*n))^2,x, algorithm="maxima")
```

```
[Out] ((b*c*d^2 - 4*a*c*d*e + a*b*e^2)*x*x^n + (b^2*d^2 - 2*a*c*d^2 - 2*a*b*d*e +
 2*a^2*e^2)*x)/(a^2*b^2*n - 4*a^3*c*n + (a*b^2*c*n - 4*a^2*c^2*n)*x^(2*n) +
 (a*b^3*n - 4*a^2*b*c*n)*x^n) - integrate((2*a*c*d^2*(2*n - 1) - b^2*d^2*(n
 - 1) - 2*a*b*d*e + 2*a^2*e^2 - (b*c*d^2*(n - 1) - 4*a*c*d*(n - 1)*e + a*b*
(n - 1)*e^2)*x^n)/(a^2*b^2*n - 4*a^3*c*n + (a*b^2*c*n - 4*a^2*c^2*n)*x^(2*n
) + (a*b^3*n - 4*a^2*b*c*n)*x^n), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+e*x^n)^2/(a+b*x^n+c*x^(2*n))^2,x, algorithm="fricas")
```

```
[Out] integral((2*d*x^n*e + d^2 + x^(2*n)*e^2)/(c^2*x^(4*n) + b^2*x^(2*n) + 2*a*b
*x^n + a^2 + 2*(b*c*x^n + a*c)*x^(2*n)), x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+e*x**n)**2/(a+b*x**n+c*x**(2*n))**2,x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+e*x^n)^2/(a+b*x^n+c*x^(2*n))^2,x, algorithm="giac")
```

```
[Out] integrate((x^n*e + d)^2/(c*x^(2*n) + b*x^n + a)^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(d+e\,x^n)^2}{(a+b\,x^n+c\,x^{2\,n})^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x^n)^2/(a + b*x^n + c*x^(2*n))^2,x)
```

```
[Out] int((d + e*x^n)^2/(a + b*x^n + c*x^(2*n))^2, x)
```

3.77 $\int \frac{d+ex^n}{(a+bx^n+cx^{2n})^2} dx$

Optimal. Leaf size=362

$$\frac{x(b^2d-2acd-abe+c(bd-2ae)x^n)}{a\,(b^2-4ac)\,n\,(a+bx^n+cx^{2n})}-\frac{c\Big(2a\Big(2cd(1-2n)+\sqrt{b^2-4ac}\,e(1-n)\Big)-b^2(d-dn)-b\Big(\sqrt{b^2-\cdots}}{a\,(b^2-4ac)\,\Big(b^2-4ac-\cdots}$$

```
[Out] x*(b^2*d-2*a*c*d-a*b*e+c*(-2*a*e+b*d)*x^n)/a/(-4*a*c+b^2)/n/(a+b*x^n+c*x^(2
*n))-c*x*hypergeom([1, 1/n],[1+1/n],-2*c*x^n/(b+(-4*a*c+b^2)^(1/2)))*(-b^2*
d*(1-n)+b*(2*a*e*n+d*(1-n)*(-4*a*c+b^2)^(1/2))+2*a*(c*d*(2-4*n)-e*(1-n)*(-4
*a*c+b^2)^(1/2)))/a/(-4*a*c+b^2)/n/(b^2-4*a*c+b*(-4*a*c+b^2)^(1/2))-c*x*hyp
ergeom([1, 1/n],[1+1/n],-2*c*x^n/(b-(-4*a*c+b^2)^(1/2)))*(-b^2*(-d*n+d)-b*(
-2*a*e*n+d*(1-n)*(-4*a*c+b^2)^(1/2))+2*a*(2*c*d*(1-2*n)+e*(1-n)*(-4*a*c+b^2
)^(1/2)))/a/(-4*a*c+b^2)/n/(b^2-4*a*c-b*(-4*a*c+b^2)^(1/2))
```

Rubi [A]
time = 0.42, antiderivative size = 328, normalized size of antiderivative = 0.91, number of steps used = 4, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {1444, 1436, 251}

$$-\frac{cx\left(-(1-n)\sqrt{b^2-4ac}\,(bd-2ae)+2aben+2acd(2-4n)+b^2(-d)(1-n)\right)\,{}_2F_1\left(1,\frac{1}{n};1+\frac{1}{n};-\frac{2cx^n}{b-\sqrt{b^2-4ac}}\right)}{an\,(b^2-4ac)\left(-b\sqrt{b^2-4ac}-4ac+b^2\right)}-\frac{cx\left((1-n)\sqrt{b^2-4ac}\,(bd-2ae)+2aben+4acd(1-2n)+b^2(-d)(1-n)\right)\,{}_2F_1\left(1,\frac{1}{n};1+\frac{1}{n};-\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)}{an\,(b^2-4ac)\left(b\sqrt{b^2-4ac}-4ac+b^2\right)}+\frac{x(cx^n(bd-2ae)-abe-2acd+b^2d)}{an\,(b^2-4ac)\,(a+bx^n+cx^{2n})}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x^n)/(a + b*x^n + c*x^(2*n))^2,x]
```

```
[Out] (x*(b^2*d - 2*a*c*d - a*b*e + c*(b*d - 2*a*e)*x^n))/(a*(b^2 - 4*a*c)*n*(a +
 b*x^n + c*x^(2*n))) - (c*(2*a*c*d*(2 - 4*n) - b^2*d*(1 - n) - Sqrt[b^2 - 4
*a*c]*(b*d - 2*a*e)*(1 - n) + 2*a*b*e*n)*x*Hypergeometric2F1[1, n^(-1), 1 +
 n^(-1), (-2*c*x^n)/(b - Sqrt[b^2 - 4*a*c])])/(a*(b^2 - 4*a*c)*(b^2 - 4*a*c
 - b*Sqrt[b^2 - 4*a*c])*n) - (c*(4*a*c*d*(1 - 2*n) - b^2*d*(1 - n) + Sqrt[b
^2 - 4*a*c]*(b*d - 2*a*e)*(1 - n) + 2*a*b*e*n)*x*Hypergeometric2F1[1, n^(-1
), 1 + n^(-1), (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c])])/(a*(b^2 - 4*a*c)*(b^2 -
 4*a*c + b*Sqrt[b^2 - 4*a*c])*n)
```

Rule 251

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, (-b)*(x^n/a)], x] /; FreeQ[{a, b, n, p}, x] && !IGtQ[p
, 0] && !IntegerQ[1/n] && !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

Rule 1436

```
Int[((d_) + (e_.)*(x_)^(n_))/((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_)), x
_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q),
```

```
Int[1/(b/2 - q/2 + c*x^n), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(
b/2 + q/2 + c*x^n), x], x]] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[n2, 2*n]
 && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && (PosQ[b^2 - 4*a
*c] ||  !IGtQ[n/2, 0])
```

Rule 1444

```
Int[((d_) + (e_.)*(x_)^(n_))*((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_))^(p
_), x_Symbol] :> Simp[(-x)*(d*b^2 - a*b*e - 2*a*c*d + (b*d - 2*a*e)*c*x^n)*
((a + b*x^n + c*x^(2*n))^(p + 1)/(a*n*(p + 1)*(b^2 - 4*a*c))), x] + Dist[1/
(a*n*(p + 1)*(b^2 - 4*a*c)), Int[Simp[(n*p + n + 1)*d*b^2 - a*b*e - 2*a*c*d
*(2*n*p + 2*n + 1) + (2*n*p + 3*n + 1)*(d*b - 2*a*e)*c*x^n, x]*(a + b*x^n +
 c*x^(2*n))^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[n2, 2*n]
 && NeQ[b^2 - 4*a*c, 0] && ILtQ[p, -1]
```

Rubi steps

$$\int \frac{d+ex^n}{(a+bx^n+cx^{2n})^2}\,dx = \frac{x(b^2d-2acd-abe+c(bd-2ae)x^n)}{a(b^2-4ac)n(a+bx^n+cx^{2n})} - \frac{\int \frac{-abe-2acd(1-2n)+b^2(d-dn)+c(bd-2ae)(1-n)x^n}{a+bx^n+cx^{2n}}\,d}{a(b^2-4ac)n}$$

$$= \frac{x(b^2d-2acd-abe+c(bd-2ae)x^n)}{a(b^2-4ac)n(a+bx^n+cx^{2n})} - \frac{\Big(c\Big(2acd(2-4n)-b^2d(1-n)+\sqrt{b^2-}}{}$$

$$= \frac{x(b^2d-2acd-abe+c(bd-2ae)x^n)}{a(b^2-4ac)n(a+bx^n+cx^{2n})} - \frac{c\Big((bd-2ae)(1-n)-\frac{4acd(1-2n)+2aben-b^2}{\sqrt{b^2-4ac}}}{a(b^2-4ac)\Big(b}$$

Mathematica [A]
time = 3.86, size = 603, normalized size = 1.67

$$\frac{cx\left(\frac{4(b^2-4ac)(b^2d(-1+n)x^n(b+cx^n)-2a^2c(2dn+ex^n)+a(-2c^2d(-1+2n)x^{2n}+bcx^n(3d-4dn+ex^n)+b^2(dn+ex^n)))}{\left(b^2-4ac-b\sqrt{b^2-4ac}\right)\left(b^2-4ac+b\sqrt{b^2-4ac}\right)(a+x^n(b+cx^n))}+\frac{2^{-1/n}\left(4ac\left(\sqrt{b^2-4ac}\,d(1-2n)+2ae(-1+n)\right)+b^3d(-1+n)+b^2\left(\sqrt{b^2-4ac}\,d-2ae\right)(-1+n)+2ab\left(-2cd(-1+n)+\sqrt{b^2-4ac}\,en\right)\right)\left(\frac{cx^n}{b-\sqrt{b^2-4ac}+2cx^n}\right)^{-1/n}{}_2F_1\left(-\frac{1}{n},-\frac{1}{n};\frac{-1+n}{n};\frac{b-\sqrt{b^2-4ac}}{b-\sqrt{b^2-4ac}+2cx^n}\right)}{\sqrt{b^2-4ac}\left(-b^2+4ac+b\sqrt{b^2-4ac}\right)}+\frac{2^{-1/n}\left(b\sqrt{b^2-4ac}\,d(-1+n)-2a\sqrt{b^2-4ac}\,e(-1+n)-2aben+4acd(-1+2n)+b^2(d-dn)\right)\left(\frac{cx^n}{b+\sqrt{b^2-4ac}+2cx^n}\right)^{-1/n}{}_2F_1\left(-\frac{1}{n},-\frac{1}{n};\frac{-1+n}{n};\frac{b+\sqrt{b^2-4ac}}{b+\sqrt{b^2-4ac}+2cx^n}\right)}{\sqrt{b^2-4ac}\left(b+\sqrt{b^2-4ac}\right)}\right)}{a(-b^2+4ac)n}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x^n)/(a + b*x^n + c*x^(2*n))^2,x]
```

```
[Out] (c*x*((4*(b^2 - 4*a*c)*(b^2*d*(-1 + n)*x^n*(b + c*x^n) - 2*a^2*c*(2*d*n + e
*x^n) + a*(-2*c^2*d*(-1 + 2*n)*x^(2*n) + b*c*x^n*(3*d - 4*d*n + e*x^n) + b^
2*(d*n + e*x^n))))/((b^2 - 4*a*c - b*Sqrt[b^2 - 4*a*c])*(b^2 - 4*a*c + b*Sq
rt[b^2 - 4*a*c])*(a + x^n*(b + c*x^n))) + ((4*a*c*(Sqrt[b^2 - 4*a*c]*d*(1 -
 2*n) + 2*a*e*(-1 + n)) + b^3*d*(-1 + n) + b^2*(Sqrt[b^2 - 4*a*c]*d - 2*a*e
)*(-1 + n) + 2*a*b*(-2*c*d*(-1 + n) + Sqrt[b^2 - 4*a*c]*e*n))*Hypergeometri
```

```
c2F1[-n^(-1), -n^(-1), (-1 + n)/n, (b - Sqrt[b^2 - 4*a*c])/(b - Sqrt[b^2 -
4*a*c] + 2*c*x^n)])/(2^n^(-1)*Sqrt[b^2 - 4*a*c]*(-b^2 + 4*a*c + b*Sqrt[b^2
- 4*a*c])*((c*x^n)/(b - Sqrt[b^2 - 4*a*c] + 2*c*x^n))^n^(-1)) + ((b*Sqrt[b^
2 - 4*a*c]*d*(-1 + n) - 2*a*Sqrt[b^2 - 4*a*c]*e*(-1 + n) - 2*a*b*e*n + 4*a*
c*d*(-1 + 2*n) + b^2*(d - d*n))*Hypergeometric2F1[-n^(-1), -n^(-1), (-1 + n
)/n, (b + Sqrt[b^2 - 4*a*c])/(b + Sqrt[b^2 - 4*a*c] + 2*c*x^n)])/(2^n^(-1)*
Sqrt[b^2 - 4*a*c]*(b + Sqrt[b^2 - 4*a*c])*((c*x^n)/(b + Sqrt[b^2 - 4*a*c] +
 2*c*x^n))^n^(-1))))/(a*(-b^2 + 4*a*c)*n)
```

Maple [**F**]
time = 0.02, size = 0, normalized size = 0.00

$$\int \frac{d + e\,x^{n}}{\left(a + b\,x^{n} + c\,x^{2n}\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d+e*x^n)/(a+b*x^n+c*x^(2*n))^2,x)
```

```
[Out] int((d+e*x^n)/(a+b*x^n+c*x^(2*n))^2,x)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+e*x^n)/(a+b*x^n+c*x^(2*n))^2,x, algorithm="maxima")
```

```
[Out] ((b*c*d - 2*a*c*e)*x*x^n + (b^2*d - 2*a*c*d - a*b*e)*x)/(a^2*b^2*n - 4*a^3*
c*n + (a*b^2*c*n - 4*a^2*c^2*n)*x^(2*n) + (a*b^3*n - 4*a^2*b*c*n)*x^n) + in
tegrate(-(2*a*c*d*(2*n - 1) - b^2*d*(n - 1) - a*b*e - (b*c*d*(n - 1) - 2*a*
c*(n - 1)*e)*x^n)/(a^2*b^2*n - 4*a^3*c*n + (a*b^2*c*n - 4*a^2*c^2*n)*x^(2*n
) + (a*b^3*n - 4*a^2*b*c*n)*x^n), x)
```

Fricas [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+e*x^n)/(a+b*x^n+c*x^(2*n))^2,x, algorithm="fricas")
```

```
[Out] integral((x^n*e + d)/(c^2*x^(4*n) + b^2*x^(2*n) + 2*a*b*x^n + a^2 + 2*(b*c*
x^n + a*c)*x^(2*n)), x)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+e*x**n)/(a+b*x**n+c*x**(2*n))**2,x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+e*x^n)/(a+b*x^n+c*x^(2*n))^2,x, algorithm="giac")
```

```
[Out] integrate((x^n*e + d)/(c*x^(2*n) + b*x^n + a)^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{d+e\,x^n}{{\left(a+b\,x^n+c\,x^{2\,n}\right)}^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x^n)/(a + b*x^n + c*x^(2*n))^2,x)
```

```
[Out] int((d + e*x^n)/(a + b*x^n + c*x^(2*n))^2, x)
```

3.78 $\int \frac{1}{(d+ex^n)(a+bx^n+cx^{2n})^2}\,dx$

Optimal. Leaf size=726

$$\frac{x(b^2cd-2ac^2d-b^3e+3abce+c(bcd-b^2e+2ace)\,x^n)}{a\,(b^2-4ac)\,(cd^2-bde+ae^2)\,n\,(a+bx^n+cx^{2n})}-\frac{ce^2\left(2cd-\left(b+\sqrt{b^2-4ac}\right)e\right)x\,{}_2F_1\left(1,\frac{1}{n};1+\frac{1}{n}\right.}{\left(b^2-4ac-b\sqrt{b^2-4ac}\right)(cd^2-bde}$$

```
[Out] x*(b^2*c*d-2*a*c^2*d-b^3*e+3*a*b*c*e+c*(2*a*c*e-b^2*e+b*c*d)*x^n)/a/(-4*a*c
+b^2)/(a*e^2-b*d*e+c*d^2)/n/(a+b*x^n+c*x^(2*n))+e^4*x*hypergeom([1, 1/n],[1
+1/n],-e*x^n/d)/d/(a*e^2-b*d*e+c*d^2)^2-c*x*hypergeom([1, 1/n],[1+1/n],-2*c
*x^n/(b-(-4*a*c+b^2)^(1/2)))*((2*a*c*e-b^2*e+b*c*d)*(1-n)+(2*a*b*c*e*(2-3*n
)-4*a*c^2*d*(1-2*n)+b^2*c*d*(1-n)-b^3*e*(1-n))/(-4*a*c+b^2)^(1/2))/a/(-4*a*
c+b^2)/(a*e^2-b*d*e+c*d^2)/n/(b-(-4*a*c+b^2)^(1/2))-c*e^2*x*hypergeom([1, 1
/n],[1+1/n],-2*c*x^n/(b+(-4*a*c+b^2)^(1/2)))*(2*c*d-e*(b-(-4*a*c+b^2)^(1/2)
))/(a*e^2-b*d*e+c*d^2)^2/(b^2-4*a*c+b*(-4*a*c+b^2)^(1/2))-c*e^2*x*hypergeom
([1, 1/n],[1+1/n],-2*c*x^n/(b-(-4*a*c+b^2)^(1/2)))*(2*c*d-e*(b+(-4*a*c+b^2)
^(1/2)))/(a*e^2-b*d*e+c*d^2)^2/(b^2-4*a*c-b*(-4*a*c+b^2)^(1/2))+c*x*hyperge
om([1, 1/n],[1+1/n],-2*c*x^n/(b+(-4*a*c+b^2)^(1/2)))*(-b^3*e*(1-n)+b^2*(1-n
)*(c*d+e*(-4*a*c+b^2)^(1/2))+b*c*(2*a*e*(2-3*n)-d*(1-n)*(-4*a*c+b^2)^(1/2))
-2*a*c*(2*c*d*(1-2*n)+e*(1-n)*(-4*a*c+b^2)^(1/2)))/a/(-4*a*c+b^2)/(a*e^2-b*
d*e+c*d^2)/n/(b^2-4*a*c+b*(-4*a*c+b^2)^(1/2))
```

Rubi [A]

time = 1.27, antiderivative size = 726, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {1450, 251, 1444, 1436}

$$-\frac{ce^2x\left(2cd-e\left(\sqrt{b^2-4ac}+b\right)\right){}_2F_1\left(1,\frac{1}{n};1+\frac{1}{n};-\frac{2cx^n}{b-\sqrt{b^2-4ac}}\right)}{\left(-b\sqrt{b^2-4ac}-4ac+b^2\right)(ae^2-bde+cd^2)^2}-\frac{ce^2x\left(2cd-e\left(b-\sqrt{b^2-4ac}\right)\right){}_2F_1\left(1,\frac{1}{n};1+\frac{1}{n};-\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)}{\left(b\sqrt{b^2-4ac}-4ac+b^2\right)(ae^2-bde+cd^2)^2}-\frac{cx\left((1-n)(2ace+b^2(-e)+bcd)+\frac{2abce(2-3n)-4ac^2d(1-2n)+b^3(-e)(1-n)+b^2cd(1-n)}{\sqrt{b^2-4ac}}\right){}_2F_1\left(1,\frac{1}{n};1+\frac{1}{n};-\frac{2cx^n}{b-\sqrt{b^2-4ac}}\right)}{an(b^2-4ac)\left(b-\sqrt{b^2-4ac}\right)(ae^2-bde+cd^2)}+\frac{x(cx^n(2ace+b^2(-e)+bcd)+3abce-2ac^2d-b^3e+b^2cd)}{an(b^2-4ac)(ae^2-bde+cd^2)(a+bx^n+cx^{2n})}+\frac{cx\left(b^2(1-n)\left(e\sqrt{b^2-4ac}+cd\right)+bc\left(2ae(2-3n)-d(1-n)\sqrt{b^2-4ac}\right)-2ac\left(e(1-n)\sqrt{b^2-4ac}+2cd(1-2n)\right)+b^3(-e)(1-n)\right){}_2F_1\left(1,\frac{1}{n};1+\frac{1}{n};-\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)}{an(b^2-4ac)\left(b\sqrt{b^2-4ac}-4ac+b^2\right)(ae^2-bde+cd^2)}+\frac{e^4x\,{}_2F_1\left(1,\frac{1}{n};1+\frac{1}{n};-\frac{ex^n}{d}\right)}{d\,(ae^2-bde+cd^2)^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/((d + e*x^n)*(a + b*x^n + c*x^(2*n))^2),x]
```

```
[Out] (x*(b^2*c*d - 2*a*c^2*d - b^3*e + 3*a*b*c*e + c*(b*c*d - b^2*e + 2*a*c*e)*x
^n))/(a*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)*n*(a + b*x^n + c*x^(2*n))) -
(c*e^2*(2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)*x*Hypergeometric2F1[1, n^(-1), 1
 + n^(-1), (-2*c*x^n)/(b - Sqrt[b^2 - 4*a*c])])/((b^2 - 4*a*c - b*Sqrt[b^2
- 4*a*c])*(c*d^2 - b*d*e + a*e^2)^2) - (c*((2*a*b*c*e*(2 - 3*n) - 4*a*c^2*d
*(1 - 2*n) + b^2*c*d*(1 - n) - b^3*e*(1 - n))/Sqrt[b^2 - 4*a*c] + (b*c*d -
b^2*e + 2*a*c*e)*(1 - n))*x*Hypergeometric2F1[1, n^(-1), 1 + n^(-1), (-2*c*
x^n)/(b - Sqrt[b^2 - 4*a*c])])/(a*(b^2 - 4*a*c)*(b - Sqrt[b^2 - 4*a*c])*(c*
d^2 - b*d*e + a*e^2)*n) - (c*e^2*(2*c*d - (b - Sqrt[b^2 - 4*a*c])*e)*x*Hype
rgeometric2F1[1, n^(-1), 1 + n^(-1), (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c])])/(
(b^2 - 4*a*c + b*Sqrt[b^2 - 4*a*c])*(c*d^2 - b*d*e + a*e^2)^2) + (c*(b*c*(2
```

```
*a*e*(2 - 3*n) - Sqrt[b^2 - 4*a*c]*d*(1 - n)) - 2*a*c*(2*c*d*(1 - 2*n) + Sq
rt[b^2 - 4*a*c]*e*(1 - n)) - b^3*e*(1 - n) + b^2*(c*d + Sqrt[b^2 - 4*a*c]*e
)*(1 - n))*x*Hypergeometric2F1[1, n^(-1), 1 + n^(-1), (-2*c*x^n)/(b + Sqrt[
b^2 - 4*a*c])])/(a*(b^2 - 4*a*c)*(b^2 - 4*a*c + b*Sqrt[b^2 - 4*a*c])*(c*d^2
 - b*d*e + a*e^2)*n) + (e^4*x*Hypergeometric2F1[1, n^(-1), 1 + n^(-1), -((e
*x^n)/d)])/(d*(c*d^2 - b*d*e + a*e^2)^2)
```

Rule 251

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, (-b)*(x^n/a)], x] /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p
, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

Rule 1436

```
Int[((d_) + (e_.)*(x_)^(n_))/((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_)), x
_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q),
Int[1/(b/2 - q/2 + c*x^n), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(
b/2 + q/2 + c*x^n), x], x]] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[n2, 2*n]
 && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && (PosQ[b^2 - 4*a
*c] ||  !IGtQ[n/2, 0])
```

Rule 1444

```
Int[((d_) + (e_.)*(x_)^(n_))*((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_))^(p
_), x_Symbol] :> Simp[(-x)*(d*b^2 - a*b*e - 2*a*c*d + (b*d - 2*a*e)*c*x^n)*
((a + b*x^n + c*x^(2*n))^(p + 1)/(a*n*(p + 1)*(b^2 - 4*a*c))), x] + Dist[1/
(a*n*(p + 1)*(b^2 - 4*a*c)), Int[Simp[(n*p + n + 1)*d*b^2 - a*b*e - 2*a*c*d
*(2*n*p + 2*n + 1) + (2*n*p + 3*n + 1)*(d*b - 2*a*e)*c*x^n, x]*(a + b*x^n +
 c*x^(2*n))^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[n2, 2*n]
 && NeQ[b^2 - 4*a*c, 0] && ILtQ[p, -1]
```

Rule 1450

```
Int[((d_) + (e_.)*(x_)^(n_))^(q_)*((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_
))^(p_), x_Symbol] :> Int[ExpandIntegrand[(d + e*x^n)^q*(a + b*x^n + c*x^(2
*n))^p, x], x] /; FreeQ[{a, b, c, d, e, n, p, q}, x] && EqQ[n2, 2*n] && NeQ
[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && ((IntegersQ[p, q] &&
!IntegerQ[n]) || IGtQ[p, 0] || (IGtQ[q, 0] &&  !IntegerQ[n]))
```

Rubi steps

$$\int \frac{1}{(d+ex^n)(a+bx^n+cx^{2n})^2}\,dx = \int \left(\frac{e^4}{(cd^2-bde+ae^2)^2(d+ex^n)} + \frac{cd-be-cex^n}{(cd^2-bde+ae^2)(a+bx^n+cx^{2n}\right.$$

$$= -\frac{e^2\int \frac{-cd+be+cex^n}{a+bx^n+cx^{2n}}\,dx}{(cd^2-bde+ae^2)^2} + \frac{e^4\int \frac{1}{d+ex^n}\,dx}{(cd^2-bde+ae^2)^2} + \frac{\int \frac{cd-be-cex^n}{(a+bx^n+cx^{2n})^2}\,dx}{cd^2-bde+ae^2}$$

$$= \frac{x(b^2cd-2ac^2d-b^3e+3abce+c(bcd-b^2e+2ace)\,x^n)}{a(b^2-4ac)(cd^2-bde+ae^2)\,n\,(a+bx^n+cx^{2n})} + \frac{e^4x\,{}_2F_1(1,}{d(cd^2-}$$

$$= \frac{x(b^2cd-2ac^2d-b^3e+3abce+c(bcd-b^2e+2ace)\,x^n)}{a(b^2-4ac)(cd^2-bde+ae^2)\,n\,(a+bx^n+cx^{2n})} - \frac{ce^2\left(e-\frac{}{\sqrt{}}\right.}{}$$

$$= \frac{x(b^2cd-2ac^2d-b^3e+3abce+c(bcd-b^2e+2ace)\,x^n)}{a(b^2-4ac)(cd^2-bde+ae^2)\,n\,(a+bx^n+cx^{2n})} - \frac{ce^2\left(e-\frac{}{\sqrt{}}\right.}{}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 11767 vs. 2(726) = 1452.
time = 6.80, size = 11767, normalized size = 16.21

Result too large to show

Antiderivative was successfully verified.

```
[In] Integrate[1/((d + e*x^n)*(a + b*x^n + c*x^(2*n))^2),x]
```

```
[Out] Result too large to show
```

Maple [F]
time = 0.03, size = 0, normalized size = 0.00

$$\int \frac{1}{(d+e\,x^n)(a+b\,x^n+c\,x^{2n})^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(d+e*x^n)/(a+b*x^n+c*x^(2*n))^2,x)
```

```
[Out] int(1/(d+e*x^n)/(a+b*x^n+c*x^(2*n))^2,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d+e*x^n)/(a+b*x^n+c*x^(2*n))^2,x, algorithm="maxima")
```

```
[Out] e^4*integrate(1/(c^2*d^5 - 2*b*c*d^4*e - 2*a*b*d^2*e^3 + a^2*d*e^4 + (c^2*d
^4*e - 2*b*c*d^3*e^2 - 2*a*b*d*e^4 + a^2*e^5 + (b^2*d^2 + 2*a*c*d^2)*e^3)*x
^n + (b^2*d^3 + 2*a*c*d^3)*e^2), x) + ((b*c^2*d - (b^2*c - 2*a*c^2)*e)*x*x^
n + (b^2*c*d - 2*a*c^2*d - (b^3 - 3*a*b*c)*e)*x)/(a^2*b^2*c*d^2*n - 4*a^3*c
^2*d^2*n + (a*b^2*c^2*d^2*n - 4*a^2*c^3*d^2*n + (a^2*b^2*c*n - 4*a^3*c^2*n)
*e^2 - (a*b^3*c*d*n - 4*a^2*b*c^2*d*n)*e)*x^(2*n) + (a*b^3*c*d^2*n - 4*a^2*
b*c^2*d^2*n + (a^2*b^3*n - 4*a^3*b*c*n)*e^2 - (a*b^4*d*n - 4*a^2*b^2*c*d*n)
*e)*x^n + (a^3*b^2*n - 4*a^4*c*n)*e^2 - (a^2*b^3*d*n - 4*a^3*b*c*d*n)*e) -
integrate((2*a*c^3*d^3*(2*n - 1) - b^2*c^2*d^3*(n - 1) - (b*c^3*d^3*(n - 1)
 + (2*a^2*c^2*(3*n - 1) - a*b^2*c*(2*n - 1))*e^3 + (b^3*c*d*(n - 1) - a*b*c
^2*d*(n - 1))*e^2 - 2*(b^2*c^2*d^2*(n - 1) - a*c^3*d^2*(n - 1))*e)*x^n - (a
^2*b*c*(8*n - 3) - a*b^3*(2*n - 1))*e^3 + (2*a^2*c^2*d*(4*n - 1) - b^4*d*(n
 - 1) + 2*a*b^2*c*d*(n - 1))*e^2 - (a*b*c^2*d^2*(8*n - 5) - 2*b^3*c*d^2*(n
- 1))*e)/(a^2*b^2*c^2*d^4*n - 4*a^3*c^3*d^4*n + (a*b^2*c^3*d^4*n - 4*a^2*c^
4*d^4*n + (a^3*b^2*c*n - 4*a^4*c^2*n)*e^4 - 2*(a^2*b^3*c*d*n - 4*a^3*b*c^2*
d*n)*e^3 + (a*b^4*c*d^2*n - 2*a^2*b^2*c^2*d^2*n - 8*a^3*c^3*d^2*n)*e^2 - 2*
(a*b^3*c^2*d^3*n - 4*a^2*b*c^3*d^3*n)*e)*x^(2*n) + (a*b^3*c^2*d^4*n - 4*a^2
*b*c^3*d^4*n + (a^3*b^3*n - 4*a^4*b*c*n)*e^4 - 2*(a^2*b^4*d*n - 4*a^3*b^2*c
*d*n)*e^3 + (a*b^5*d^2*n - 2*a^2*b^3*c*d^2*n - 8*a^3*b*c^2*d^2*n)*e^2 - 2*(
a*b^4*c*d^3*n - 4*a^2*b^2*c^2*d^3*n)*e)*x^n + (a^4*b^2*n - 4*a^5*c*n)*e^4 -
 2*(a^3*b^3*d*n - 4*a^4*b*c*d*n)*e^3 + (a^2*b^4*d^2*n - 2*a^3*b^2*c*d^2*n -
 8*a^4*c^2*d^2*n)*e^2 - 2*(a^2*b^3*c*d^3*n - 4*a^3*b*c^2*d^3*n)*e), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d+e*x^n)/(a+b*x^n+c*x^(2*n))^2,x, algorithm="fricas")
```

```
[Out] integral(1/(b^2*x^(3*n)*e + a^2*d + (c^2*x^n*e + c^2*d)*x^(4*n) + 2*(b*c*x^
(2*n)*e + a*c*d + (b*c*d + a*c*e)*x^n)*x^(2*n) + (b^2*d + 2*a*b*e)*x^(2*n)
+ (2*a*b*d + a^2*e)*x^n), x)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: HeuristicGCDFailed

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d+e*x**n)/(a+b*x**n+c*x**(2*n))**2,x)
```

```
[Out] Exception raised: HeuristicGCDFailed >> no luck
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d+e*x^n)/(a+b*x^n+c*x^(2*n))^2,x, algorithm="giac")
[Out] integrate(1/((c*x^(2*n) + b*x^n + a)^2*(x^n*e + d)), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{(d+e\,x^n)\,\left(a+b\,x^n+c\,x^{2\,n}\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((d + e*x^n)*(a + b*x^n + c*x^(2*n))^2),x)
[Out] int(1/((d + e*x^n)*(a + b*x^n + c*x^(2*n))^2), x)
```

3.79 $\int \frac{1}{(d+ex^n)^2(a+bx^n+cx^{2n})^2}\,dx$

Optimal. Leaf size=1129

$$-\frac{x(2b^3cde-6abc^2de-b^4e^2-b^2c(cd^2-4ae^2)+2ac^2(cd^2-ae^2)+c(2b^2cde-4ac^2de-b^3e^2-bc(cd^2-3ae}{a\,(b^2-4ac)\,(cd^2-bde+ae^2)^2\,n\,(a+bx^n+cx^{2n})}$$

```
[Out] -x*(2*b^3*c*d*e-6*a*b*c^2*d*e-b^4*e^2-b^2*c*(-4*a*e^2+c*d^2)+2*a*c^2*(-a*e^
2+c*d^2)+c*(2*b^2*c*d*e-4*a*c^2*d*e-b^3*e^2-b*c*(-3*a*e^2+c*d^2))*x^n)/a/(-
4*a*c+b^2)/(a*e^2-b*d*e+c*d^2)^2/n/(a+b*x^n+c*x^(2*n))+2*e^4*(-b*e+2*c*d)*x
*hypergeom([1, 1/n],[1+1/n],-e*x^n/d)/d/(a*e^2-b*d*e+c*d^2)^3+e^4*x*hyperge
om([2, 1/n],[1+1/n],-e*x^n/d)/d^2/(a*e^2-b*d*e+c*d^2)^2-2*c*e^2*x*hypergeom
([1, 1/n],[1+1/n],-2*c*x^n/(b+(-4*a*c+b^2)^(1/2)))*(3*c^2*d^2+b*e^2*(b-(-4*
a*c+b^2)^(1/2))-c*e*(3*b*d+a*e-2*d*(-4*a*c+b^2)^(1/2)))/(a*e^2-b*d*e+c*d^2)
^3/(b^2-4*a*c+b*(-4*a*c+b^2)^(1/2))-2*c*e^2*x*hypergeom([1, 1/n],[1+1/n],-2
*c*x^n/(b-(-4*a*c+b^2)^(1/2)))*(3*c^2*d^2+b*e^2*(b+(-4*a*c+b^2)^(1/2))-c*e*
(3*b*d+a*e+2*d*(-4*a*c+b^2)^(1/2)))/(a*e^2-b*d*e+c*d^2)^3/(b^2-4*a*c-b*(-4*
a*c+b^2)^(1/2))+c*x*hypergeom([1, 1/n],[1+1/n],-2*c*x^n/(b+(-4*a*c+b^2)^(1/
2)))*(b^4*e^2*(1-n)-b^3*e*(1-n)*(2*c*d+e*(-4*a*c+b^2)^(1/2))-b^2*c*(-c*d^2*
(1-n)+e*(a*e*(5-7*n)-2*d*(1-n)*(-4*a*c+b^2)^(1/2)))+b*c*(3*a*e^2*(1-n)*(-4*
a*c+b^2)^(1/2)+c*d*(4*a*e*(2-3*n)-d*(1-n)*(-4*a*c+b^2)^(1/2)))+4*a*c^2*(-c*
d^2*(1-2*n)+e*(a*e*(1-2*n)-d*(1-n)*(-4*a*c+b^2)^(1/2))))/a/(-4*a*c+b^2)/(a*
e^2-b*d*e+c*d^2)^2/n/(b^2-4*a*c+b*(-4*a*c+b^2)^(1/2))+c*x*hypergeom([1, 1/n
],[1+1/n],-2*c*x^n/(b-(-4*a*c+b^2)^(1/2)))*(b^4*e^2*(1-n)-b^3*e*(1-n)*(2*c*
d-e*(-4*a*c+b^2)^(1/2))+b*c*(-3*a*e^2*(1-n)*(-4*a*c+b^2)^(1/2)+c*d*(4*a*e*(
2-3*n)+d*(1-n)*(-4*a*c+b^2)^(1/2)))+4*a*c^2*(-c*d^2*(1-2*n)+e*(a*e*(1-2*n)+
d*(1-n)*(-4*a*c+b^2)^(1/2)))-b^2*c*(-c*d^2*(1-n)+e*(a*e*(5-7*n)+2*d*(1-n)*(
-4*a*c+b^2)^(1/2))))/a/(-4*a*c+b^2)/(a*e^2-b*d*e+c*d^2)^2/n/(b^2-4*a*c-b*(-
4*a*c+b^2)^(1/2))
```

Rubi [A]
time = 2.18, antiderivative size = 1129, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {1450, 251, 1444, 1436}

Antiderivative was successfully verified.

```
[In] Int[1/((d + e*x^n)^2*(a + b*x^n + c*x^(2*n))^2),x]
```

```
[Out] -((x*(2*b^3*c*d*e - 6*a*b*c^2*d*e - b^4*e^2 - b^2*c*(c*d^2 - 4*a*e^2) + 2*a
*c^2*(c*d^2 - a*e^2) + c*(2*b^2*c*d*e - 4*a*c^2*d*e - b^3*e^2 - b*c*(c*d^2
- 3*a*e^2))*x^n))/(a*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)^2*n*(a + b*x^n +
```

```
 c*x^(2*n)))) - (2*c*e^2*(3*c^2*d^2 + b*(b + Sqrt[b^2 - 4*a*c])*e^2 - c*e*(
3*b*d + 2*Sqrt[b^2 - 4*a*c]*d + a*e))*x*Hypergeometric2F1[1, n^(-1), 1 + n^
(-1), (-2*c*x^n)/(b - Sqrt[b^2 - 4*a*c])])/((b^2 - 4*a*c - b*Sqrt[b^2 - 4*a
*c])*(c*d^2 - b*d*e + a*e^2)^3) + (c*(4*a*c^2*(e*(a*e*(1 - 2*n) + Sqrt[b^2
- 4*a*c]*d*(1 - n)) - c*d^2*(1 - 2*n)) - b^2*c*(e*(a*e*(5 - 7*n) + 2*Sqrt[b
^2 - 4*a*c]*d*(1 - n)) - c*d^2*(1 - n)) + b*c*(c*d*(4*a*e*(2 - 3*n) + Sqrt[
b^2 - 4*a*c]*d*(1 - n)) - 3*a*Sqrt[b^2 - 4*a*c]*e^2*(1 - n)) + b^4*e^2*(1 -
 n) - b^3*e*(2*c*d - Sqrt[b^2 - 4*a*c]*e)*(1 - n))*x*Hypergeometric2F1[1, n
^(-1), 1 + n^(-1), (-2*c*x^n)/(b - Sqrt[b^2 - 4*a*c])])/(a*(b^2 - 4*a*c)*(b
^2 - 4*a*c - b*Sqrt[b^2 - 4*a*c])*(c*d^2 - b*d*e + a*e^2)^2*n) - (2*c*e^2*(
3*c^2*d^2 + b*(b - Sqrt[b^2 - 4*a*c])*e^2 - c*e*(3*b*d - 2*Sqrt[b^2 - 4*a*c
]*d + a*e))*x*Hypergeometric2F1[1, n^(-1), 1 + n^(-1), (-2*c*x^n)/(b + Sqrt
[b^2 - 4*a*c])])/((b^2 - 4*a*c + b*Sqrt[b^2 - 4*a*c])*(c*d^2 - b*d*e + a*e^
2)^3) + (c*(4*a*c^2*(e*(a*e*(1 - 2*n) - Sqrt[b^2 - 4*a*c]*d*(1 - n)) - c*d^
2*(1 - 2*n)) - b^2*c*(e*(a*e*(5 - 7*n) - 2*Sqrt[b^2 - 4*a*c]*d*(1 - n)) - c
*d^2*(1 - n)) + b*c*(c*d*(4*a*e*(2 - 3*n) - Sqrt[b^2 - 4*a*c]*d*(1 - n)) +
3*a*Sqrt[b^2 - 4*a*c]*e^2*(1 - n)) + b^4*e^2*(1 - n) - b^3*e*(2*c*d + Sqrt[
b^2 - 4*a*c]*e)*(1 - n))*x*Hypergeometric2F1[1, n^(-1), 1 + n^(-1), (-2*c*x
^n)/(b + Sqrt[b^2 - 4*a*c])])/(a*(b^2 - 4*a*c)*(b^2 - 4*a*c + b*Sqrt[b^2 -
4*a*c])*(c*d^2 - b*d*e + a*e^2)^2*n) + (2*e^4*(2*c*d - b*e)*x*Hypergeometri
c2F1[1, n^(-1), 1 + n^(-1), -((e*x^n)/d)])/(d*(c*d^2 - b*d*e + a*e^2)^3) +
(e^4*x*Hypergeometric2F1[2, n^(-1), 1 + n^(-1), -((e*x^n)/d)])/(d^2*(c*d^2
- b*d*e + a*e^2)^2)
```

Rule 251

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, (-b)*(x^n/a)], x] /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p
, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

Rule 1436

```
Int[((d_) + (e_.)*(x_)^(n_))/((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_)), x
_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q),
Int[1/(b/2 - q/2 + c*x^n), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(
b/2 + q/2 + c*x^n), x], x]] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[n2, 2*n]
 && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && (PosQ[b^2 - 4*a
*c] ||  !IGtQ[n/2, 0])
```

Rule 1444

```
Int[((d_) + (e_.)*(x_)^(n_))*((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_))^(p
_), x_Symbol] :> Simp[(-x)*(d*b^2 - a*b*e - 2*a*c*d + (b*d - 2*a*e)*c*x^n)*
((a + b*x^n + c*x^(2*n))^(p + 1)/(a*n*(p + 1)*(b^2 - 4*a*c))), x] + Dist[1/
(a*n*(p + 1)*(b^2 - 4*a*c)), Int[Simp[(n*p + n + 1)*d*b^2 - a*b*e - 2*a*c*d
*(2*n*p + 2*n + 1) + (2*n*p + 3*n + 1)*(d*b - 2*a*e)*c*x^n, x]*(a + b*x^n +
```

```
c*x^(2*n))^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[n2, 2*n]
&& NeQ[b^2 - 4*a*c, 0] && ILtQ[p, -1]
```

Rule 1450

```
Int[((d_) + (e_.)*(x_)^(n_))^(q_)*((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_
))^(p_), x_Symbol] :> Int[ExpandIntegrand[(d + e*x^n)^q*(a + b*x^n + c*x^(2
*n))^p, x], x] /; FreeQ[{a, b, c, d, e, n, p, q}, x] && EqQ[n2, 2*n] && NeQ
[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && ((IntegersQ[p, q] &&
!IntegerQ[n]) || IGtQ[p, 0] || (IGtQ[q, 0] && !IntegerQ[n]))
```

Rubi steps

$$\int \frac{1}{(d+ex^n)^2 (a+bx^n+cx^{2n})^2} dx = \int \left(\frac{e^4}{(cd^2-bde+ae^2)^2 (d+ex^n)^2} - \frac{2e^4(-2cd+be)}{(cd^2-bde+ae^2)^3 (d+ex^n)} + \frac{c^2 \cdots}{\cdots}\right.$$

$$= \frac{e^2 \int \frac{3c^2d^2-5bcde+2b^2e^2-ace^2+(-4c^2de+2bce^2)x^n}{a+bx^n+cx^{2n}} dx}{(cd^2-bde+ae^2)^3} + \frac{(2e^4(2cd-be)) \int \frac{1}{d+ex^n} d \cdots}{(cd^2-bde+ae^2)^3}$$

$$= -\frac{x(2b^3cde-6abc^2de-b^4e^2-b^2c(cd^2-4ae^2)+2ac^2(cd^2-ae^2)+c(2 \cdots}{a(b^2-4ac)(cd^2-bde+ae^2)^2 n (a+ \cdots}$$

$$= -\frac{x(2b^3cde-6abc^2de-b^4e^2-b^2c(cd^2-4ae^2)+2ac^2(cd^2-ae^2)+c(2 \cdots}{a(b^2-4ac)(cd^2-bde+ae^2)^2 n (a+ \cdots}$$

$$= -\frac{x(2b^3cde-6abc^2de-b^4e^2-b^2c(cd^2-4ae^2)+2ac^2(cd^2-ae^2)+c(2 \cdots}{a(b^2-4ac)(cd^2-bde+ae^2)^2 n (a+ \cdots}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 16855 vs. 2(1129) = 2258.
time = 7.37, size = 16855, normalized size = 14.93

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/((d + e*x^n)^2*(a + b*x^n + c*x^(2*n))^2),x]
[Out] Result too large to show
```

Maple [F]
time = 0.04, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(d+e\,x^{n}\right)^{2}\left(a+b\,x^{n}+c\,x^{2n}\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(d+e*x^n)^2/(a+b*x^n+c*x^(2*n))^2,x)
```

```
[Out] int(1/(d+e*x^n)^2/(a+b*x^n+c*x^(2*n))^2,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d+e*x^n)^2/(a+b*x^n+c*x^(2*n))^2,x, algorithm="maxima")
```

```
[Out] (c*d^2*(5*n - 1)*e^4 - b*d*(3*n - 1)*e^5 + a*(n - 1)*e^6)*integrate(1/(c^3*
d^8*n - 3*b*c^2*d^7*n*e - 3*a^2*b*d^3*n*e^5 + a^3*d^2*n*e^6 + (c^3*d^7*n*e
- 3*b*c^2*d^6*n*e^2 - 3*a^2*b*d^2*n*e^6 + a^3*d*n*e^7 + 3*(a*b^2*d^3*n + a^
2*c*d^3*n)*e^5 - (b^3*d^4*n + 6*a*b*c*d^4*n)*e^4 + 3*(b^2*c*d^5*n + a*c^2*d
^5*n)*e^3)*x^n + 3*(a*b^2*d^4*n + a^2*c*d^4*n)*e^4 - (b^3*d^5*n + 6*a*b*c*d
^5*n)*e^3 + 3*(b^2*c*d^6*n + a*c^2*d^6*n)*e^2), x) + ((b*c^3*d^3*e + (a*b^2
*c - 4*a^2*c^2)*e^4 + (b^3*c*d - 3*a*b*c^2*d)*e^3 - 2*(b^2*c^2*d^2 - 2*a*c^
3*d^2)*e^2)*x*x^(2*n) + (b*c^3*d^4 + (a*b^3 - 4*a^2*b*c)*e^4 + (b^4*d - 4*a
*b^2*c*d + 2*a^2*c^2*d)*e^3 - (b^3*c*d^2 - 3*a*b*c^2*d^2)*e^2 - (b^2*c^2*d^
3 - 2*a*c^3*d^3)*e)*x*x^n + (b^2*c^2*d^4 - 2*a*c^3*d^4 + (a^2*b^2 - 4*a^3*c
)*e^4 + (b^4*d^2 - 4*a*b^2*c*d^2 + 2*a^2*c^2*d^2)*e^2 - 2*(b^3*c*d^3 - 3*a*
b*c^2*d^3)*e)*x)/(a^2*b^2*c^2*d^6*n - 4*a^3*c^3*d^6*n + ((a^3*b^2*c*d*n - 4
*a^4*c^2*d*n)*e^5 - 2*(a^2*b^3*c*d^2*n - 4*a^3*b*c^2*d^2*n)*e^4 + (a*b^4*c*
d^3*n - 2*a^2*b^2*c^2*d^3*n - 8*a^3*c^3*d^3*n)*e^3 - 2*(a*b^3*c^2*d^4*n - 4
*a^2*b*c^3*d^4*n)*e^2 + (a*b^2*c^3*d^5*n - 4*a^2*c^4*d^5*n)*e)*x^(3*n) + (a
*b^2*c^3*d^6*n - 4*a^2*c^4*d^6*n + (a^3*b^3*d*n - 4*a^4*b*c*d*n)*e^5 - (2*a
^2*b^4*d^2*n - 9*a^3*b^2*c*d^2*n + 4*a^4*c^2*d^2*n)*e^4 + (a*b^5*d^3*n - 4*
a^2*b^3*c*d^3*n)*e^3 - (a*b^4*c*d^4*n - 6*a^2*b^2*c^2*d^4*n + 8*a^3*c^3*d^4
*n)*e^2 - (a*b^3*c^2*d^5*n - 4*a^2*b*c^3*d^5*n)*e)*x^(2*n) + (a*b^3*c^2*d^6
*n - 4*a^2*b*c^3*d^6*n + (a^4*b^2*d*n - 4*a^5*c*d*n)*e^5 - (a^3*b^3*d^2*n -
 4*a^4*b*c*d^2*n)*e^4 - (a^2*b^4*d^3*n - 6*a^3*b^2*c*d^3*n + 8*a^4*c^2*d^3*
n)*e^3 + (a*b^5*d^4*n - 4*a^2*b^3*c*d^4*n)*e^2 - (2*a*b^4*c*d^5*n - 9*a^2*b
^2*c^2*d^5*n + 4*a^3*c^3*d^5*n)*e)*x^n + (a^4*b^2*d^2*n - 4*a^5*c*d^2*n)*e^
4 - 2*(a^3*b^3*d^3*n - 4*a^4*b*c*d^3*n)*e^3 + (a^2*b^4*d^4*n - 2*a^3*b^2*c*
d^4*n - 8*a^4*c^2*d^4*n)*e^2 - 2*(a^2*b^3*c*d^5*n - 4*a^3*b*c^2*d^5*n)*e) +
 integrate(-(2*a*c^4*d^4*(2*n - 1) - b^2*c^3*d^4*(n - 1) - (b*c^4*d^4*(n -
```

```
1) - (a^2*b*c^2*(11*n - 3) - a*b^3*c*(3*n - 1))*e^4 + (4*a^2*c^3*d*(5*n - 1
) - a*b^2*c^2*d*(3*n + 1) - b^4*c*d*(n - 1))*e^3 + 3*(b^3*c^2*d^2*(n - 1) -
 2*a*b*c^3*d^2*(n - 1))*e^2 - (3*b^2*c^3*d^3*(n - 1) - 4*a*c^4*d^3*(n - 1))
*e)*x^n + (2*a^2*b^2*c*(7*n - 2) - 2*a^3*c^2*(4*n - 1) - a*b^4*(3*n - 1))*e
^4 - (4*a^2*b*c^2*d*(6*n - 1) - 2*a*b^3*c*d*(n + 1) - b^5*d*(n - 1))*e^3 -
3*(b^4*c*d^2*(n - 1) - 3*a*b^2*c^2*d^2*(n - 1) - 4*a^2*c^3*d^2*n)*e^2 - (4*
a*b*c^3*d^3*(3*n - 2) - 3*b^3*c^2*d^3*(n - 1))*e)/(a^2*b^2*c^3*d^6*n - 4*a^
3*c^4*d^6*n + (a*b^2*c^4*d^6*n - 4*a^2*c^5*d^6*n + (a^4*b^2*c*n - 4*a^5*c^2
*n)*e^6 - 3*(a^3*b^3*c*d*n - 4*a^4*b*c^2*d*n)*e^5 + 3*(a^2*b^4*c*d^2*n - 3*
a^3*b^2*c^2*d^2*n - 4*a^4*c^3*d^2*n)*e^4 - (a*b^5*c*d^3*n + 2*a^2*b^3*c^2*d
^3*n - 24*a^3*b*c^3*d^3*n)*e^3 + 3*(a*b^4*c^2*d^4*n - 3*a^2*b^2*c^3*d^4*n -
 4*a^3*c^4*d^4*n)*e^2 - 3*(a*b^3*c^3*d^5*n - 4*a^2*b*c^4*d^5*n)*e)*x^(2*n)
+ (a*b^3*c^3*d^6*n - 4*a^2*b*c^4*d^6*n + (a^4*b^3*n - 4*a^5*b*c*n)*e^6 - 3*
(a^3*b^4*d*n - 4*a^4*b^2*c*d*n)*e^5 + 3*(a^2*b^5*d^2*n - 3*a^3*b^3*c*d^2*n
- 4*a^4*b*c^2*d^2*n)*e^4 - (a*b^6*d^3*n + 2*a^2*b^4*c*d^3*n - 24*a^3*b^2*c^
2*d^3*n)*e^3 + 3*(a*b^5*c*d^4*n - 3*a^2*b^3*c^2*d^4*n - 4*a^3*b*c^3*d^4*n)*
e^2 - 3*(a*b^4*c^2*d^5*n - 4*a^2*b^2*c^3*d^5*n)*e)*x^n + (a^5*b^2*n - 4*a^6
*c*n)*e^6 - 3*(a^4*b^3*d*n - 4*a^5*b*c*d*n)*e^5 + 3*(a^3*b^4*d^2*n - 3*a^4*
b^2*c*d^2*n - 4*a^5*c^2*d^2*n)*e^4 - (a^2*b^5*d^3*n + 2*a^3*b^3*c*d^3*n - 2
4*a^4*b*c^2*d^3*n)*e^3 + 3*(a^2*b^4*c*d^4*n - 3*a^3*b^2*c^2*d^4*n - 4*a^4*c
^3*d^4*n)*e^2 - 3*(a^2*b^3*c^2*d^5*n - 4*a^3*b*c^3*d^5*n)*e), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d+e*x^n)^2/(a+b*x^n+c*x^(2*n))^2,x, algorithm="fricas")
```

```
[Out] integral(1/(a^2*d^2 + b^2*x^(4*n)*e^2 + (2*c^2*d*x^n*e + c^2*d^2 + c^2*x^(2
*n)*e^2)*x^(4*n) + 2*(b^2*d*e + a*b*e^2)*x^(3*n) + (b^2*d^2 + 4*a*b*d*e + a
^2*e^2)*x^(2*n) + 2*(a*c*d^2 + b*c*x^(3*n)*e^2 + (2*b*c*d*e + a*c*e^2)*x^(2
*n) + (b*c*d^2 + 2*a*c*d*e)*x^n)*x^(2*n) + 2*(a*b*d^2 + a^2*d*e)*x^n), x)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: HeuristicGCDFailed

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d+e*x**n)**2/(a+b*x**n+c*x**(2*n))**2,x)
```

```
[Out] Exception raised: HeuristicGCDFailed >> no luck
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d+e*x^n)^2/(a+b*x^n+c*x^(2*n))^2,x, algorithm="giac")
[Out] integrate(1/((c*x^(2*n) + b*x^n + a)^2*(x^n*e + d)^2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{{\left(d+e\,x^n\right)}^2\,{\left(a+b\,x^n+c\,x^{2\,n}\right)}^2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((d + e*x^n)^2*(a + b*x^n + c*x^(2*n))^2),x)
[Out] int(1/((d + e*x^n)^2*(a + b*x^n + c*x^(2*n))^2), x)
```

3.80 $\int \frac{(d+ex^n)^3}{(a+bx^n+cx^{2n})^3}\, dx$

Optimal. Leaf size=1707

$$\frac{x(b^2cd^3 - 2acd(cd^2 - 3ae^2) - abe(3cd^2 + ae^2) - (ab^2e^3 + 2ace(3cd^2 - ae^2) - bcd(cd^2 + 3ae^2))\, x^n)}{2ac\,(b^2 - 4ac)\, n\, (a + bx^n + cx^{2n})^2} + \frac{e^2x(3b^2c}{}$$

```
[Out] 1/2*x*(b^2*c*d^3-2*a*c*d*(-3*a*e^2+c*d^2)-a*b*e*(a*e^2+3*c*d^2)-(a*b^2*e^3+
2*a*c*e*(-a*e^2+3*c*d^2)-b*c*d*(3*a*e^2+c*d^2))*x^n)/a/c/(-4*a*c+b^2)/n/(a+
b*x^n+c*x^(2*n))^2+e^2*x*(3*b^2*c*d-6*a*c^2*d-b^3*e+a*b*c*e+c*(-2*a*c*e-b^2
*e+3*b*c*d)*x^n)/a/c^2/(-4*a*c+b^2)/n/(a+b*x^n+c*x^(2*n))-1/2*x*(a*b^2*c^2*
d*(3*a*e^2*(1-9*n)-5*c*d^2*(1-3*n))+4*a^2*c^3*d*(-3*a*e^2+c*d^2)*(1-4*n)-2*
a*b^5*e^3*n+2*a^2*b*c^2*e*(3*c*d^2*(2-3*n)-5*a*e^2*n)-3*a*b^3*c*e*(-3*a*e^2
*n+c*d^2)+b^4*c*d*(c*d^2*(1-2*n)+6*a*e^2*n)+c*(4*a^2*c^2*e*(-a*e^2+3*c*d^2)
*(1-3*n)-2*a*b^4*e^3*n-2*a*b*c^2*d*(c*d^2*(2-7*n)+3*a*e^2*n)+b^3*c*d*(c*d^2
*(1-2*n)+6*a*e^2*n)-a*b^2*c*e*(3*c*d^2-a*e^2*(1+2*n)))*x^n)/a^2/c^2/(-4*a*c
+b^2)^2/n^2/(a+b*x^n+c*x^(2*n))+1/2*x*hypergeom([1, 1/n],[1+1/n],-2*c*x^n/(
b-(-4*a*c+b^2)^(1/2)))*((1-n)*(4*a^2*c^2*e*(-a*e^2+3*c*d^2)*(1-3*n)-2*a*b^4
*e^3*n-2*a*b*c^2*d*(c*d^2*(2-7*n)+3*a*e^2*n)+b^3*c*d*(c*d^2*(1-2*n)+6*a*e^2
*n)-a*b^2*c*e*(3*c*d^2-a*e^2*(1+2*n)))+(-2*a*b^5*e^3*(1-n)*n+b^4*c*d*(1-n)*
(c*d^2*(1-2*n)+6*a*e^2*n)+8*a^2*c^3*d*(-3*a*e^2+c*d^2)*(8*n^2-6*n+1)-6*a*b^
2*c^2*d*(c*d^2*(3*n^2-4*n+1)-a*e^2*(15*n^2-10*n+1))+4*a^2*b*c^2*e*(3*c*d^2*
(-3*n^2-n+1)+a*e^2*(19*n^2-11*n+1))-a*b^3*c*e*(3*c*d^2*(1-n)+a*e^2*(30*n^2-
19*n+1)))/(-4*a*c+b^2)^(1/2))/a^2/c/(-4*a*c+b^2)^2/n^2/(b-(-4*a*c+b^2)^(1/2
))+1/2*x*hypergeom([1, 1/n],[1+1/n],-2*c*x^n/(b+(-4*a*c+b^2)^(1/2)))*((1-n)
*(4*a^2*c^2*e*(-a*e^2+3*c*d^2)*(1-3*n)-2*a*b^4*e^3*n-2*a*b*c^2*d*(c*d^2*(2-
7*n)+3*a*e^2*n)+b^3*c*d*(c*d^2*(1-2*n)+6*a*e^2*n)-a*b^2*c*e*(3*c*d^2-a*e^2*
(1+2*n)))+(2*a*b^5*e^3*(1-n)*n-b^4*c*d*(1-n)*(c*d^2*(1-2*n)+6*a*e^2*n)-8*a^
2*c^3*d*(-3*a*e^2+c*d^2)*(8*n^2-6*n+1)+6*a*b^2*c^2*d*(c*d^2*(3*n^2-4*n+1)-a
*e^2*(15*n^2-10*n+1))-4*a^2*b*c^2*e*(3*c*d^2*(-3*n^2-n+1)+a*e^2*(19*n^2-11*
n+1))+a*b^3*c*e*(3*c*d^2*(1-n)+a*e^2*(30*n^2-19*n+1)))/(-4*a*c+b^2)^(1/2))/
a^2/c/(-4*a*c+b^2)^2/n^2/(b+(-4*a*c+b^2)^(1/2))+e^2*x*hypergeom([1, 1/n],[1
+1/n],-2*c*x^n/(b+(-4*a*c+b^2)^(1/2)))*(-b^3*e*(1-n)+b^2*(1-n)*(3*c*d+e*(-4
*a*c+b^2)^(1/2))+b*c*(2*a*e*(2-5*n)-3*d*(1-n)*(-4*a*c+b^2)^(1/2))-2*a*c*(6*
c*d*(1-2*n)-e*(1-n)*(-4*a*c+b^2)^(1/2)))/a/c/(-4*a*c+b^2)/n/(b^2-4*a*c+b*(-
4*a*c+b^2)^(1/2))+e^2*x*hypergeom([1, 1/n],[1+1/n],-2*c*x^n/(b-(-4*a*c+b^2)
^(1/2)))*(-b^3*e*(1-n)+b^2*(1-n)*(3*c*d-e*(-4*a*c+b^2)^(1/2))+b*c*(2*a*e*(2
-5*n)+3*d*(1-n)*(-4*a*c+b^2)^(1/2))-2*a*c*(6*c*d*(1-2*n)+e*(1-n)*(-4*a*c+b^
2)^(1/2)))/a/c/(-4*a*c+b^2)/n/(b^2-4*a*c-b*(-4*a*c+b^2)^(1/2))
```

Rubi [A]
time = 3.31, antiderivative size = 1707, normalized size of antiderivative = 1.00, number of

steps used = 11, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {1450, 1444, 1436, 251}

Antiderivative was successfully verified.

```
[In] Int[(d + e*x^n)^3/(a + b*x^n + c*x^(2*n))^3,x]
```

```
[Out] (x*(b^2*c*d^3 - 2*a*c*d*(c*d^2 - 3*a*e^2) - a*b*e*(3*c*d^2 + a*e^2) - (a*b^
2*e^3 + 2*a*c*e*(3*c*d^2 - a*e^2) - b*c*d*(c*d^2 + 3*a*e^2))*x^n))/(2*a*c*(
b^2 - 4*a*c)*n*(a + b*x^n + c*x^(2*n))^2) + (e^2*x*(3*b^2*c*d - 6*a*c^2*d -
 b^3*e + a*b*c*e + c*(3*b*c*d - b^2*e - 2*a*c*e)*x^n))/(a*c^2*(b^2 - 4*a*c)
*n*(a + b*x^n + c*x^(2*n))) - (x*(a*b^2*c^2*d*(3*a*e^2*(1 - 9*n) - 5*c*d^2*
(1 - 3*n)) + 4*a^2*c^3*d*(c*d^2 - 3*a*e^2)*(1 - 4*n) - 2*a*b^5*e^3*n + 2*a^
2*b*c^2*e*(3*c*d^2*(2 - 3*n) - 5*a*e^2*n) - 3*a*b^3*c*e*(c*d^2 - 3*a*e^2*n)
 + b^4*c*d*(c*d^2*(1 - 2*n) + 6*a*e^2*n) + c*(4*a^2*c^2*e*(3*c*d^2 - a*e^2)
*(1 - 3*n) - 2*a*b^4*e^3*n - 2*a*b*c^2*d*(c*d^2*(2 - 7*n) + 3*a*e^2*n) + b^
3*c*d*(c*d^2*(1 - 2*n) + 6*a*e^2*n) - a*b^2*c*e*(3*c*d^2 - a*e^2*(1 + 2*n))
)*x^n))/(2*a^2*c^2*(b^2 - 4*a*c)^2*n^2*(a + b*x^n + c*x^(2*n))) + (e^2*(b*c
*(2*a*e*(2 - 5*n) + 3*Sqrt[b^2 - 4*a*c]*d*(1 - n)) - 2*a*c*(6*c*d*(1 - 2*n)
 + Sqrt[b^2 - 4*a*c]*e*(1 - n)) - b^3*e*(1 - n) + b^2*(3*c*d - Sqrt[b^2 - 4
*a*c]*e)*(1 - n))*x*Hypergeometric2F1[1, n^(-1), 1 + n^(-1), (-2*c*x^n)/(b
- Sqrt[b^2 - 4*a*c])])/(a*c*(b^2 - 4*a*c)*(b^2 - 4*a*c - b*Sqrt[b^2 - 4*a*c
])*n) + (((1 - n)*(4*a^2*c^2*e*(3*c*d^2 - a*e^2)*(1 - 3*n) - 2*a*b^4*e^3*n
- 2*a*b*c^2*d*(c*d^2*(2 - 7*n) + 3*a*e^2*n) + b^3*c*d*(c*d^2*(1 - 2*n) + 6*
a*e^2*n) - a*b^2*c*e*(3*c*d^2 - a*e^2*(1 + 2*n))) - (2*a*b^5*e^3*(1 - n)*n
- b^4*c*d*(1 - n)*(c*d^2*(1 - 2*n) + 6*a*e^2*n) - 8*a^2*c^3*d*(c*d^2 - 3*a*
e^2)*(1 - 6*n + 8*n^2) + 6*a*b^2*c^2*d*(c*d^2*(1 - 4*n + 3*n^2) - a*e^2*(1
- 10*n + 15*n^2)) - 4*a^2*b*c^2*e*(3*c*d^2*(1 - n - 3*n^2) + a*e^2*(1 - 11*
n + 19*n^2)) + a*b^3*c*e*(3*c*d^2*(1 - n) + a*e^2*(1 - 19*n + 30*n^2)))/Sqr
t[b^2 - 4*a*c])*x*Hypergeometric2F1[1, n^(-1), 1 + n^(-1), (-2*c*x^n)/(b -
Sqrt[b^2 - 4*a*c])])/(2*a^2*c*(b^2 - 4*a*c)^2*(b - Sqrt[b^2 - 4*a*c])*n^2)
+ (e^2*(b*c*(2*a*e*(2 - 5*n) - 3*Sqrt[b^2 - 4*a*c]*d*(1 - n)) - 2*a*c*(6*c*
d*(1 - 2*n) - Sqrt[b^2 - 4*a*c]*e*(1 - n)) - b^3*e*(1 - n) + b^2*(3*c*d + S
qrt[b^2 - 4*a*c]*e)*(1 - n))*x*Hypergeometric2F1[1, n^(-1), 1 + n^(-1), (-2
*c*x^n)/(b + Sqrt[b^2 - 4*a*c])])/(a*c*(b^2 - 4*a*c)*(b^2 - 4*a*c + b*Sqrt[
b^2 - 4*a*c])*n) + (((1 - n)*(4*a^2*c^2*e*(3*c*d^2 - a*e^2)*(1 - 3*n) - 2*a
*b^4*e^3*n - 2*a*b*c^2*d*(c*d^2*(2 - 7*n) + 3*a*e^2*n) + b^3*c*d*(c*d^2*(1
- 2*n) + 6*a*e^2*n) - a*b^2*c*e*(3*c*d^2 - a*e^2*(1 + 2*n))) + (2*a*b^5*e^3
*(1 - n)*n - b^4*c*d*(1 - n)*(c*d^2*(1 - 2*n) + 6*a*e^2*n) - 8*a^2*c^3*d*(c
*d^2 - 3*a*e^2)*(1 - 6*n + 8*n^2) + 6*a*b^2*c^2*d*(c*d^2*(1 - 4*n + 3*n^2)
- a*e^2*(1 - 10*n + 15*n^2)) - 4*a^2*b*c^2*e*(3*c*d^2*(1 - n - 3*n^2) + a*e
^2*(1 - 11*n + 19*n^2)) + a*b^3*c*e*(3*c*d^2*(1 - n) + a*e^2*(1 - 19*n + 30
*n^2)))/Sqrt[b^2 - 4*a*c])*x*Hypergeometric2F1[1, n^(-1), 1 + n^(-1), (-2*c
*x^n)/(b + Sqrt[b^2 - 4*a*c])])/(2*a^2*c*(b^2 - 4*a*c)^2*(b + Sqrt[b^2 - 4*
a*c])*n^2)
```

Rule 251

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, (-b)*(x^n/a)], x] /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p
, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

Rule 1436

```
Int[((d_) + (e_.)*(x_)^(n_))/((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_)), x
_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q),
Int[1/(b/2 - q/2 + c*x^n), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(
b/2 + q/2 + c*x^n), x], x]] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[n2, 2*n]
 && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && (PosQ[b^2 - 4*a
*c] ||  !IGtQ[n/2, 0])
```

Rule 1444

```
Int[((d_) + (e_.)*(x_)^(n_))*((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_))^(p
_), x_Symbol] :> Simp[(-x)*(d*b^2 - a*b*e - 2*a*c*d + (b*d - 2*a*e)*c*x^n)*
((a + b*x^n + c*x^(2*n))^(p + 1)/(a*n*(p + 1)*(b^2 - 4*a*c))), x] + Dist[1/
(a*n*(p + 1)*(b^2 - 4*a*c)), Int[Simp[(n*p + n + 1)*d*b^2 - a*b*e - 2*a*c*d
*(2*n*p + 2*n + 1) + (2*n*p + 3*n + 1)*(d*b - 2*a*e)*c*x^n, x]*(a + b*x^n +
 c*x^(2*n))^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[n2, 2*n]
 && NeQ[b^2 - 4*a*c, 0] && ILtQ[p, -1]
```

Rule 1450

```
Int[((d_) + (e_.)*(x_)^(n_))^(q_)*((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_
))^(p_), x_Symbol] :> Int[ExpandIntegrand[(d + e*x^n)^q*(a + b*x^n + c*x^(2
*n))^p, x], x] /; FreeQ[{a, b, c, d, e, n, p, q}, x] && EqQ[n2, 2*n] && NeQ
[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && ((IntegersQ[p, q] &&
!IntegerQ[n]) || IGtQ[p, 0] || (IGtQ[q, 0] &&  !IntegerQ[n]))
```

Rubi steps

$$\int \frac{(d+ex^n)^3}{(a+bx^n+cx^{2n})^3}\,dx = \int \left(\frac{c^2d^3-3acde^2+abe^3+(3c^2d^2e-3bcde^2+b^2e^3-ace^3)\,x^n}{c^2\,(a+bx^n+cx^{2n})^3} + \frac{e^2(3cd-be+}{c^2\,(a+bx^n+}\right.$$

$$= \frac{\int \frac{c^2d^3-3acde^2+abe^3+(3c^2d^2e-3bcde^2+b^2e^3-ace^3)x^n}{(a+bx^n+cx^{2n})^3}\,dx}{c^2} + \frac{e^2\int \frac{3cd-be+cex^n}{(a+bx^n+cx^{2n})^2}\,dx}{c^2}$$

$$= \frac{x(b^2cd^3-2acd(cd^2-3ae^2)-abe(3cd^2+ae^2)-(ab^2e^3+2ace(3cd^2-ae^2)-bc}{2ac\,(b^2-4ac)\,n\,(a+bx^n+cx^{2n})^2}$$

$$= \frac{x(b^2cd^3-2acd(cd^2-3ae^2)-abe(3cd^2+ae^2)-(ab^2e^3+2ace(3cd^2-ae^2)-bc}{2ac\,(b^2-4ac)\,n\,(a+bx^n+cx^{2n})^2}$$

$$= \frac{x(b^2cd^3-2acd(cd^2-3ae^2)-abe(3cd^2+ae^2)-(ab^2e^3+2ace(3cd^2-ae^2)-bc}{2ac\,(b^2-4ac)\,n\,(a+bx^n+cx^{2n})^2}$$

$$= \frac{x(b^2cd^3-2acd(cd^2-3ae^2)-abe(3cd^2+ae^2)-(ab^2e^3+2ace(3cd^2-ae^2)-bc}{2ac\,(b^2-4ac)\,n\,(a+bx^n+cx^{2n})^2}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 13018 vs. $2(1707) = 3414$.
time = 7.93, size = 13018, normalized size = 7.63

Result too large to show

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x^n)^3/(a + b*x^n + c*x^(2*n))^3,x]
```

```
[Out] Result too large to show
```

Maple [F]
time = 0.04, size = 0, normalized size = 0.00

$$\int \frac{(d+e\,x^n)^3}{(a+b\,x^n+c\,x^{2n})^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d+e*x^n)^3/(a+b*x^n+c*x^(2*n))^3,x)
```

```
[Out] int((d+e*x^n)^3/(a+b*x^n+c*x^(2*n))^3,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+e*x^n)^3/(a+b*x^n+c*x^(2*n))^3,x, algorithm="maxima")
```

```
[Out] -1/2*((2*a*b*c^3*d^3*(7*n - 2) - b^3*c^2*d^3*(2*n - 1) + 18*a^2*b*c^2*d*n*e
^2 - (a^2*b^2*c*(2*n - 1) + 4*a^3*c^2*(n + 1))*e^3 - 3*(4*a^2*c^3*d^2*(3*n
- 1) + a*b^2*c^2*d^2)*e)*x*x^(3*n) + (a*b^2*c^2*d^3*(29*n - 9) - 4*a^2*c^3*
d^3*(4*n - 1) - 2*b^4*c*d^3*(2*n - 1) - (2*a^3*b*c*(3*n + 2) + a^2*b^3*(3*n
 - 1))*e^3 + 3*(a^2*b^2*c*d*(9*n + 1) - 4*a^3*c^2*d)*e^2 - 6*(a^2*b*c^2*d^2
*(9*n - 4) + a*b^3*c*d^2)*e)*x*x^(2*n) + (4*a*b^3*c*d^3*(3*n - 1) - b^5*d^3
*(2*n - 1) + 2*a^2*b*c^2*d^3*n - (a^3*b^2*(10*n - 1) - 4*a^4*c*(n - 1))*e^3
 + 3*(2*a^3*b*c*d*(5*n - 2) + a^2*b^3*d*(2*n + 1))*e^2 - 3*(4*a^3*c^2*d^2*(
5*n - 1) + a^2*b^2*c*d^2*(4*n - 3) + a*b^4*d^2)*e)*x*x^n + (a^2*b^2*c*d^3*(
21*n - 5) - 4*a^3*c^2*d^3*(6*n - 1) - a*b^4*d^3*(3*n - 1) - 6*a^4*b*n*e^3 +
 3*(4*a^4*c*d*(2*n - 1) + a^3*b^2*d*(n + 1))*e^2 - 3*(2*a^3*b*c*d^2*(5*n -
2) - a^2*b^3*d^2*(n - 1))*e)*x)/(a^4*b^4*n^2 - 8*a^5*b^2*c*n^2 + 16*a^6*c^2
*n^2 + (a^2*b^4*c^2*n^2 - 8*a^3*b^2*c^3*n^2 + 16*a^4*c^4*n^2)*x^(4*n) + 2*(
a^2*b^5*c*n^2 - 8*a^3*b^3*c^2*n^2 + 16*a^4*b*c^3*n^2)*x^(3*n) + (a^2*b^6*n^
2 - 6*a^3*b^4*c*n^2 + 32*a^5*c^3*n^2)*x^(2*n) + 2*(a^3*b^5*n^2 - 8*a^4*b^3*
c*n^2 + 16*a^5*b*c^2*n^2)*x^n) + integrate(1/2*((2*n^2 - 3*n + 1)*b^4*d^3 -
 (16*n^2 - 21*n + 5)*a*b^2*c*d^3 + 4*(8*n^2 - 6*n + 1)*a^2*c^2*d^3 - 6*a^3*
b*n*e^3 + ((2*n^2 - 3*n + 1)*b^3*c*d^3 - 2*(7*n^2 - 9*n + 2)*a*b*c^2*d^3 -
18*(n^2 - n)*a^2*b*c*d*e^2 + ((2*n^2 - 3*n + 1)*a^2*b^2 + 4*(n^2 - 1)*a^3*c
)*e^3 + 3*(4*(3*n^2 - 4*n + 1)*a^2*c^2*d^2 + a*b^2*c*d^2*(n - 1))*e)*x^n +
3*(4*a^3*c*d*(2*n - 1) + a^2*b^2*d*(n + 1))*e^2 - 3*(2*a^2*b*c*d^2*(5*n - 2
) - a*b^3*d^2*(n - 1))*e)/(a^3*b^4*n^2 - 8*a^4*b^2*c*n^2 + 16*a^5*c^2*n^2 +
 (a^2*b^4*c*n^2 - 8*a^3*b^2*c^2*n^2 + 16*a^4*c^3*n^2)*x^(2*n) + (a^2*b^5*n^
2 - 8*a^3*b^3*c*n^2 + 16*a^4*b*c^2*n^2)*x^n), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+e*x^n)^3/(a+b*x^n+c*x^(2*n))^3,x, algorithm="fricas")
```

```
[Out] integral((3*d^2*x^n*e + d^3 + 3*d*x^(2*n)*e^2 + x^(3*n)*e^3)/(c^3*x^(6*n) +
 b^3*x^(3*n) + 3*a*b^2*x^(2*n) + 3*a^2*b*x^n + a^3 + 3*(b*c^2*x^n + a*c^2)*
x^(4*n) + 3*(b^2*c*x^(2*n) + 2*a*b*c*x^n + a^2*c)*x^(2*n)), x)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+e*x**n)**3/(a+b*x**n+c*x**(2*n))**3,x)
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+e*x^n)^3/(a+b*x^n+c*x^(2*n))^3,x, algorithm="giac")
[Out] integrate((x^n*e + d)^3/(c*x^(2*n) + b*x^n + a)^3, x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(d+e\,x^n)^3}{(a+b\,x^n+c\,x^{2\,n})^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x^n)^3/(a + b*x^n + c*x^(2*n))^3,x)
[Out] int((d + e*x^n)^3/(a + b*x^n + c*x^(2*n))^3, x)
```

3.81 $\int \frac{(d+ex^n)^2}{(a+bx^n+cx^{2n})^3}\,dx$

Optimal. Leaf size=1191

$$\frac{x(b^2d^2-2abde-2a(cd^2-ae^2)+(bcd^2-4acde+abe^2)\,x^n)}{2a\,(b^2-4ac)\,n\,(a+bx^n+cx^{2n})^2}+\frac{e^2x(b^2-2ac+bcx^n)}{ac\,(b^2-4ac)\,n\,(a+bx^n+cx^{2n})}+\frac{x(2ab^3cde-}{}$$

```
[Out] 1/2*x*(b^2*d^2-2*a*b*d*e-2*a*(-a*e^2+c*d^2)+(a*b*e^2-4*a*c*d*e+b*c*d^2)*x^n
)/a/(-4*a*c+b^2)/n/(a+b*x^n+c*x^(2*n))^2+e^2*x*(b^2-2*a*c+b*c*x^n)/a/c/(-4*
a*c+b^2)/n/(a+b*x^n+c*x^(2*n))+1/2*x*(2*a*b^3*c*d*e-a*b^2*c*(a*e^2*(1-9*n)-
5*c*d^2*(1-3*n))-4*a^2*c^2*(-a*e^2+c*d^2)*(1-4*n)-4*a^2*b*c^2*d*e*(2-3*n)-b
^4*(c*d^2*(1-2*n)+2*a*e^2*n)+c*(2*a*b^2*c*d*e-8*a^2*c^2*d*e*(1-3*n)+2*a*b*c
*(c*d^2*(2-7*n)+a*e^2*n)-b^3*(c*d^2*(1-2*n)+2*a*e^2*n))*x^n)/a^2/c/(-4*a*c+
b^2)^2/n^2/(a+b*x^n+c*x^(2*n))-1/2*x*hypergeom([1, 1/n],[1+1/n],-2*c*x^n/(b
-(-4*a*c+b^2)^(1/2)))*((1-n)*(2*a*b^2*c*d*e-8*a^2*c^2*d*e*(1-3*n)+2*a*b*c*(
c*d^2*(2-7*n)+a*e^2*n)-b^3*(c*d^2*(1-2*n)+2*a*e^2*n))+(2*a*b^3*c*d*e*(1-n)-
b^4*(1-n)*(c*d^2*(1-2*n)+2*a*e^2*n)-8*a^2*b*c^2*d*e*(-3*n^2-n+1)-8*a^2*c^2*
(-a*e^2+c*d^2)*(8*n^2-6*n+1)+2*a*b^2*c*(3*c*d^2*(3*n^2-4*n+1)-a*e^2*(15*n^2
-10*n+1)))/(-4*a*c+b^2)^(1/2))/a^2/(-4*a*c+b^2)^2/n^2/(b-(-4*a*c+b^2)^(1/2)
)-1/2*x*hypergeom([1, 1/n],[1+1/n],-2*c*x^n/(b+(-4*a*c+b^2)^(1/2)))*((1-n)*
(2*a*b^2*c*d*e-8*a^2*c^2*d*e*(1-3*n)+2*a*b*c*(c*d^2*(2-7*n)+a*e^2*n)-b^3*(c
*d^2*(1-2*n)+2*a*e^2*n))+(-2*a*b^3*c*d*e*(1-n)+b^4*(1-n)*(c*d^2*(1-2*n)+2*a
*e^2*n)+8*a^2*b*c^2*d*e*(-3*n^2-n+1)+8*a^2*c^2*(-a*e^2+c*d^2)*(8*n^2-6*n+1)
-2*a*b^2*c*(3*c*d^2*(3*n^2-4*n+1)-a*e^2*(15*n^2-10*n+1)))/(-4*a*c+b^2)^(1/2
))/a^2/(-4*a*c+b^2)^2/n^2/(b+(-4*a*c+b^2)^(1/2))-e^2*x*hypergeom([1, 1/n],[
1+1/n],-2*c*x^n/(b-(-4*a*c+b^2)^(1/2)))*(4*a*c*(1-2*n)-b^2*(1-n)-b*(1-n)*(-
4*a*c+b^2)^(1/2))/a/(-4*a*c+b^2)/n/(b^2-4*a*c-b*(-4*a*c+b^2)^(1/2))-e^2*x*h
ypergeom([1, 1/n],[1+1/n],-2*c*x^n/(b+(-4*a*c+b^2)^(1/2)))*(4*a*c*(1-2*n)-b
^2*(1-n)+b*(1-n)*(-4*a*c+b^2)^(1/2))/a/(-4*a*c+b^2)/n/(b^2-4*a*c+b*(-4*a*c+
b^2)^(1/2))
```

Rubi [A]
time = 2.55, antiderivative size = 1191, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 5, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.192$, Rules used = {1450, 1444, 1436, 251, 1359}

Antiderivative was successfully verified.

```
[In] Int[(d + e*x^n)^2/(a + b*x^n + c*x^(2*n))^3,x]
```

```
[Out] (x*(b^2*d^2 - 2*a*b*d*e - 2*a*(c*d^2 - a*e^2) + (b*c*d^2 - 4*a*c*d*e + a*b*
e^2)*x^n))/(2*a*(b^2 - 4*a*c)*n*(a + b*x^n + c*x^(2*n))^2) + (e^2*x*(b^2 -
```

```
2*a*c + b*c*x^n))/(a*c*(b^2 - 4*a*c)*n*(a + b*x^n + c*x^(2*n))) + (x*(2*a*b
^3*c*d*e - a*b^2*c*(a*e^2*(1 - 9*n) - 5*c*d^2*(1 - 3*n)) - 4*a^2*c^2*(c*d^2
 - a*e^2)*(1 - 4*n) - 4*a^2*b*c^2*d*e*(2 - 3*n) - b^4*(c*d^2*(1 - 2*n) + 2*
a*e^2*n) + c*(2*a*b^2*c*d*e - 8*a^2*c^2*d*e*(1 - 3*n) + 2*a*b*c*(c*d^2*(2 -
 7*n) + a*e^2*n) - b^3*(c*d^2*(1 - 2*n) + 2*a*e^2*n))*x^n))/(2*a^2*c*(b^2 -
 4*a*c)^2*n^2*(a + b*x^n + c*x^(2*n))) - (e^2*(4*a*c*(1 - 2*n) - b^2*(1 - n
) - b*Sqrt[b^2 - 4*a*c]*(1 - n))*x*Hypergeometric2F1[1, n^(-1), 1 + n^(-1),
 (-2*c*x^n)/(b - Sqrt[b^2 - 4*a*c])])/(a*(b^2 - 4*a*c)*(b^2 - 4*a*c - b*Sqr
t[b^2 - 4*a*c])*n) - (((1 - n)*(2*a*b^2*c*d*e - 8*a^2*c^2*d*e*(1 - 3*n) + 2
*a*b*c*(c*d^2*(2 - 7*n) + a*e^2*n) - b^3*(c*d^2*(1 - 2*n) + 2*a*e^2*n)) + (
2*a*b^3*c*d*e*(1 - n) - b^4*(1 - n)*(c*d^2*(1 - 2*n) + 2*a*e^2*n) - 8*a^2*b
*c^2*d*e*(1 - n - 3*n^2) - 8*a^2*c^2*(c*d^2 - a*e^2)*(1 - 6*n + 8*n^2) + 2*
a*b^2*c*(3*c*d^2*(1 - 4*n + 3*n^2) - a*e^2*(1 - 10*n + 15*n^2)))/Sqrt[b^2 -
 4*a*c])*x*Hypergeometric2F1[1, n^(-1), 1 + n^(-1), (-2*c*x^n)/(b - Sqrt[b^
2 - 4*a*c])])/(2*a^2*(b^2 - 4*a*c)^2*(b - Sqrt[b^2 - 4*a*c])*n^2) - (e^2*(4
*a*c*(1 - 2*n) - b^2*(1 - n) + b*Sqrt[b^2 - 4*a*c]*(1 - n))*x*Hypergeometri
c2F1[1, n^(-1), 1 + n^(-1), (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c])])/(a*(b^2 -
4*a*c)*(b^2 - 4*a*c + b*Sqrt[b^2 - 4*a*c])*n) - (((1 - n)*(2*a*b^2*c*d*e -
8*a^2*c^2*d*e*(1 - 3*n) + 2*a*b*c*(c*d^2*(2 - 7*n) + a*e^2*n) - b^3*(c*d^2*
(1 - 2*n) + 2*a*e^2*n)) - (2*a*b^3*c*d*e*(1 - n) - b^4*(1 - n)*(c*d^2*(1 -
2*n) + 2*a*e^2*n) - 8*a^2*b*c^2*d*e*(1 - n - 3*n^2) - 8*a^2*c^2*(c*d^2 - a*
e^2)*(1 - 6*n + 8*n^2) + 2*a*b^2*c*(3*c*d^2*(1 - 4*n + 3*n^2) - a*e^2*(1 -
10*n + 15*n^2)))/Sqrt[b^2 - 4*a*c])*x*Hypergeometric2F1[1, n^(-1), 1 + n^(-
1), (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c])])/(2*a^2*(b^2 - 4*a*c)^2*(b + Sqrt[b
^2 - 4*a*c])*n^2)
```

Rule 251

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, (-b)*(x^n/a)], x] /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p
, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

Rule 1359

```
Int[((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-
x)*(b^2 - 2*a*c + b*c*x^n)*((a + b*x^n + c*x^(2*n))^(p + 1)/(a*n*(p + 1)*(b
^2 - 4*a*c))), x] + Dist[1/(a*n*(p + 1)*(b^2 - 4*a*c)), Int[(b^2 - 2*a*c +
n*(p + 1)*(b^2 - 4*a*c) + b*c*(n*(2*p + 3) + 1)*x^n)*(a + b*x^n + c*x^(2*n)
)^(p + 1), x], x] /; FreeQ[{a, b, c, n}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*
a*c, 0] && ILtQ[p, -1]
```

Rule 1436

```
Int[((d_) + (e_.)*(x_)^(n_))/((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_)), x
_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q),
Int[1/(b/2 - q/2 + c*x^n), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(
```

```
b/2 + q/2 + c*x^n), x], x]] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[n2, 2*n]
 && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && (PosQ[b^2 - 4*a
*c] ||  !IGtQ[n/2, 0])
```

Rule 1444

```
Int[((d_) + (e_.)*(x_)^(n_))*((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_))^(p
_), x_Symbol] :> Simp[(-x)*(d*b^2 - a*b*e - 2*a*c*d + (b*d - 2*a*e)*c*x^n)*
((a + b*x^n + c*x^(2*n))^(p + 1)/(a*n*(p + 1)*(b^2 - 4*a*c))), x] + Dist[1/
(a*n*(p + 1)*(b^2 - 4*a*c)), Int[Simp[(n*p + n + 1)*d*b^2 - a*b*e - 2*a*c*d
*(2*n*p + 2*n + 1) + (2*n*p + 3*n + 1)*(d*b - 2*a*e)*c*x^n, x]*(a + b*x^n +
 c*x^(2*n))^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[n2, 2*n]
 && NeQ[b^2 - 4*a*c, 0] && ILtQ[p, -1]
```

Rule 1450

```
Int[((d_) + (e_.)*(x_)^(n_))^(q_)*((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_
))^(p_), x_Symbol] :> Int[ExpandIntegrand[(d + e*x^n)^q*(a + b*x^n + c*x^(2
*n))^p, x], x] /; FreeQ[{a, b, c, d, e, n, p, q}, x] && EqQ[n2, 2*n] && NeQ
[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && ((IntegersQ[p, q] &&
!IntegerQ[n]) || IGtQ[p, 0] || (IGtQ[q, 0] &&  !IntegerQ[n]))
```

Rubi steps

$$\int \frac{(d+ex^n)^2}{(a+bx^n+cx^{2n})^3}\,dx = \int \left(\frac{cd^2-ae^2+(2cde-be^2)\,x^n}{c\,(a+bx^n+cx^{2n})^3}+\frac{e^2}{c\,(a+bx^n+cx^{2n})^2}\right)dx$$

$$= \frac{\int \frac{cd^2-ae^2+(2cde-be^2)x^n}{(a+bx^n+cx^{2n})^3}\,dx}{c}+\frac{e^2\int \frac{1}{(a+bx^n+cx^{2n})^2}\,dx}{c}$$

$$= \frac{x(b^2d^2-2abde-2a(cd^2-ae^2)+(bcd^2-4acde+abe^2)\,x^n)}{2a\,(b^2-4ac)\,n\,(a+bx^n+cx^{2n})^2}+\frac{e^2x(b^2-2ac}{ac\,(b^2-4ac)\,n\,(a}$$

$$= \frac{x(b^2d^2-2abde-2a(cd^2-ae^2)+(bcd^2-4acde+abe^2)\,x^n)}{2a\,(b^2-4ac)\,n\,(a+bx^n+cx^{2n})^2}+\frac{e^2x(b^2-2ac}{ac\,(b^2-4ac)\,n\,(a}$$

$$= \frac{x(b^2d^2-2abde-2a(cd^2-ae^2)+(bcd^2-4acde+abe^2)\,x^n)}{2a\,(b^2-4ac)\,n\,(a+bx^n+cx^{2n})^2}+\frac{e^2x(b^2-2ac}{ac\,(b^2-4ac)\,n\,(a}$$

$$= \frac{x(b^2d^2-2abde-2a(cd^2-ae^2)+(bcd^2-4acde+abe^2)\,x^n)}{2a\,(b^2-4ac)\,n\,(a+bx^n+cx^{2n})^2}+\frac{e^2x(b^2-2ac}{ac\,(b^2-4ac)\,n\,(a}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 10910 vs. $2(1191) = 2382$.
time = 7.41, size = 10910, normalized size = 9.16

Result too large to show

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x^n)^2/(a + b*x^n + c*x^(2*n))^3,x]
[Out] Result too large to show
```

Maple [F]
time = 0.04, size = 0, normalized size = 0.00

$$\int \frac{(d + e\, x^n)^2}{(a + b\, x^n + c\, x^{2n})^3}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d+e*x^n)^2/(a+b*x^n+c*x^(2*n))^3,x)
[Out] int((d+e*x^n)^2/(a+b*x^n+c*x^(2*n))^3,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+e*x^n)^2/(a+b*x^n+c*x^(2*n))^3,x, algorithm="maxima")
[Out] -1/2*((2*a*b*c^3*d^2*(7*n - 2) - b^3*c^2*d^2*(2*n - 1) + 6*a^2*b*c^2*n*e^2
- 2*(4*a^2*c^3*d*(3*n - 1) + a*b^2*c^2*d)*e)*x*x^(3*n) + (a*b^2*c^2*d^2*(29
*n - 9) - 4*a^2*c^3*d^2*(4*n - 1) - 2*b^4*c*d^2*(2*n - 1) + (a^2*b^2*c*(9*n
 + 1) - 4*a^3*c^2)*e^2 - 4*(a^2*b*c^2*d*(9*n - 4) + a*b^3*c*d)*e)*x*x^(2*n)
 + (4*a*b^3*c*d^2*(3*n - 1) - b^5*d^2*(2*n - 1) + 2*a^2*b*c^2*d^2*n + (2*a^
3*b*c*(5*n - 2) + a^2*b^3*(2*n + 1))*e^2 - 2*(4*a^3*c^2*d*(5*n - 1) + a^2*b
^2*c*d*(4*n - 3) + a*b^4*d)*e)*x*x^n + (a^2*b^2*c*d^2*(21*n - 5) - 4*a^3*c^
2*d^2*(6*n - 1) - a*b^4*d^2*(3*n - 1) + (4*a^4*c*(2*n - 1) + a^3*b^2*(n + 1
))*e^2 - 2*(2*a^3*b*c*d*(5*n - 2) - a^2*b^3*d*(n - 1))*e)*x)/(a^4*b^4*n^2 -
 8*a^5*b^2*c*n^2 + 16*a^6*c^2*n^2 + (a^2*b^4*c^2*n^2 - 8*a^3*b^2*c^3*n^2 +
16*a^4*c^4*n^2)*x^(4*n) + 2*(a^2*b^5*c*n^2 - 8*a^3*b^3*c^2*n^2 + 16*a^4*b*c
^3*n^2)*x^(3*n) + (a^2*b^6*n^2 - 6*a^3*b^4*c*n^2 + 32*a^5*c^3*n^2)*x^(2*n)
+ 2*(a^3*b^5*n^2 - 8*a^4*b^3*c*n^2 + 16*a^5*b*c^2*n^2)*x^n) - integrate(-1/
2*((2*n^2 - 3*n + 1)*b^4*d^2 - (16*n^2 - 21*n + 5)*a*b^2*c*d^2 + 4*(8*n^2 -
 6*n + 1)*a^2*c^2*d^2 + ((2*n^2 - 3*n + 1)*b^3*c*d^2 - 2*(7*n^2 - 9*n + 2)*
```

```
a*b*c^2*d^2 - 6*(n^2 - n)*a^2*b*c*e^2 + 2*(4*(3*n^2 - 4*n + 1)*a^2*c^2*d +
a*b^2*c*d*(n - 1))*e)*x^n + (4*a^3*c*(2*n - 1) + a^2*b^2*(n + 1))*e^2 - 2*(
2*a^2*b*c*d*(5*n - 2) - a*b^3*d*(n - 1))*e)/(a^3*b^4*n^2 - 8*a^4*b^2*c*n^2
+ 16*a^5*c^2*n^2 + (a^2*b^4*c*n^2 - 8*a^3*b^2*c^2*n^2 + 16*a^4*c^3*n^2)*x^(
2*n) + (a^2*b^5*n^2 - 8*a^3*b^3*c*n^2 + 16*a^4*b*c^2*n^2)*x^n), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+e*x^n)^2/(a+b*x^n+c*x^(2*n))^3,x, algorithm="fricas")
```

```
[Out] integral((2*d*x^n*e + d^2 + x^(2*n)*e^2)/(c^3*x^(6*n) + b^3*x^(3*n) + 3*a*b
^2*x^(2*n) + 3*a^2*b*x^n + a^3 + 3*(b*c^2*x^n + a*c^2)*x^(4*n) + 3*(b^2*c*x
^(2*n) + 2*a*b*c*x^n + a^2*c)*x^(2*n)), x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+e*x**n)**2/(a+b*x**n+c*x**(2*n))**3,x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+e*x^n)^2/(a+b*x^n+c*x^(2*n))^3,x, algorithm="giac")
```

```
[Out] integrate((x^n*e + d)^2/(c*x^(2*n) + b*x^n + a)^3, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{{\left(d+e\,x^n\right)}^2}{{\left(a+b\,x^n+c\,x^{2\,n}\right)}^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x^n)^2/(a + b*x^n + c*x^(2*n))^3,x)
```

```
[Out] int((d + e*x^n)^2/(a + b*x^n + c*x^(2*n))^3, x)
```

3.82 $\int \frac{d+ex^n}{(a+bx^n+cx^{2n})^3}\,dx$

Optimal. Leaf size=713

$$\frac{x(b^2d-2acd-abe+c(bd-2ae)x^n)}{2a\,(b^2-4ac)\,n\,(a+bx^n+cx^{2n})^2}+\frac{x(ab^3e-4a^2c^2d(1-4n)+5ab^2cd(1-3n)-2a^2bce(2-3n)-b^4d(}{2a^2\,(b^2-4ac)^2\,r}$$

```
[Out] 1/2*x*(b^2*d-2*a*c*d-a*b*e+c*(-2*a*e+b*d)*x^n)/a/(-4*a*c+b^2)/n/(a+b*x^n+c*
x^(2*n))^2+1/2*x*(a*b^3*e-4*a^2*c^2*d*(1-4*n)+5*a*b^2*c*d*(1-3*n)-2*a^2*b*c
*e*(2-3*n)-b^4*d*(1-2*n)+c*(a*b^2*e+2*a*b*c*d*(2-7*n)-4*a^2*c*e*(1-3*n)-b^3
*d*(1-2*n))*x^n)/a^2/(-4*a*c+b^2)^2/n^2/(a+b*x^n+c*x^(2*n))+1/2*c*x*hyperge
om([1, 1/n],[1+1/n],-2*c*x^n/(b-(-4*a*c+b^2)^(1/2)))*(-b^4*d*(2*n^2-3*n+1)+
a*b^2*(1-n)*(6*c*d*(1-3*n)+e*(-4*a*c+b^2)^(1/2))+b^3*(1-n)*(a*e-d*(1-2*n)*(
-4*a*c+b^2)^(1/2))-4*a^2*c*(2*c*d*(8*n^2-6*n+1)+e*(3*n^2-4*n+1)*(-4*a*c+b^2
)^(1/2))-2*a*b*c*(2*a*e*(-3*n^2-n+1)-d*(7*n^2-9*n+2)*(-4*a*c+b^2)^(1/2)))/a
^2/(-4*a*c+b^2)^2/n^2/(b^2-4*a*c-b*(-4*a*c+b^2)^(1/2))-1/2*c*x*hypergeom([1
, 1/n],[1+1/n],-2*c*x^n/(b+(-4*a*c+b^2)^(1/2)))*(b^4*d*(2*n^2-3*n+1)+a*b^2*
(1-n)*(-6*c*d*(1-3*n)+e*(-4*a*c+b^2)^(1/2))-b^3*(1-n)*(a*e+d*(1-2*n)*(-4*a*
c+b^2)^(1/2))-4*a^2*c*(-2*c*d*(8*n^2-6*n+1)+e*(3*n^2-4*n+1)*(-4*a*c+b^2)^(1
/2))+2*a*b*c*(2*a*e*(-3*n^2-n+1)+d*(7*n^2-9*n+2)*(-4*a*c+b^2)^(1/2)))/a^2/(
-4*a*c+b^2)^2/n^2/(b^2-4*a*c+b*(-4*a*c+b^2)^(1/2))
```

Rubi [A]
time = 1.14, antiderivative size = 713, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {1444, 1436, 251}

Antiderivative was successfully verified.

```
[In] Int[(d + e*x^n)/(a + b*x^n + c*x^(2*n))^3,x]
```

```
[Out] (x*(b^2*d - 2*a*c*d - a*b*e + c*(b*d - 2*a*e)*x^n))/(2*a*(b^2 - 4*a*c)*n*(a
 + b*x^n + c*x^(2*n))^2) + (x*(a*b^3*e - 4*a^2*c^2*d*(1 - 4*n) + 5*a*b^2*c*
d*(1 - 3*n) - 2*a^2*b*c*e*(2 - 3*n) - b^4*d*(1 - 2*n) + c*(a*b^2*e + 2*a*b*
c*d*(2 - 7*n) - 4*a^2*c*e*(1 - 3*n) - b^3*d*(1 - 2*n))*x^n))/(2*a^2*(b^2 -
4*a*c)^2*n^2*(a + b*x^n + c*x^(2*n))) + (c*(a*b^2*(Sqrt[b^2 - 4*a*c]*e + 6*
c*d*(1 - 3*n))*(1 - n) + b^3*(a*e - Sqrt[b^2 - 4*a*c]*d*(1 - 2*n))*(1 - n)
- b^4*d*(1 - 3*n + 2*n^2) - 2*a*b*c*(2*a*e*(1 - n - 3*n^2) - Sqrt[b^2 - 4*a
*c]*d*(2 - 9*n + 7*n^2)) - 4*a^2*c*(Sqrt[b^2 - 4*a*c]*e*(1 - 4*n + 3*n^2) +
 2*c*d*(1 - 6*n + 8*n^2)))*x*Hypergeometric2F1[1, n^(-1), 1 + n^(-1), (-2*c
*x^n)/(b - Sqrt[b^2 - 4*a*c])])/(2*a^2*(b^2 - 4*a*c)^2*(b^2 - 4*a*c - b*Sqr
t[b^2 - 4*a*c])*n^2) - (c*(a*b^2*(Sqrt[b^2 - 4*a*c]*e - 6*c*d*(1 - 3*n))*(1
```

```
- n) - b^3*(a*e + Sqrt[b^2 - 4*a*c]*d*(1 - 2*n))*(1 - n) + b^4*d*(1 - 3*n
+ 2*n^2) + 2*a*b*c*(2*a*e*(1 - n - 3*n^2) + Sqrt[b^2 - 4*a*c]*d*(2 - 9*n +
7*n^2)) - 4*a^2*c*(Sqrt[b^2 - 4*a*c]*e*(1 - 4*n + 3*n^2) - 2*c*d*(1 - 6*n +
 8*n^2)))*x*Hypergeometric2F1[1, n^(-1), 1 + n^(-1), (-2*c*x^n)/(b + Sqrt[b
^2 - 4*a*c])])/(2*a^2*(b^2 - 4*a*c)^2*(b^2 - 4*a*c + b*Sqrt[b^2 - 4*a*c])*n
^2)
```

Rule 251

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, (-b)*(x^n/a)], x] /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p
, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

Rule 1436

```
Int[((d_) + (e_.)*(x_)^(n_))/((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_)), x
_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q),
Int[1/(b/2 - q/2 + c*x^n), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(
b/2 + q/2 + c*x^n), x], x]] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[n2, 2*n]
 && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && (PosQ[b^2 - 4*a
*c] ||  !IGtQ[n/2, 0])
```

Rule 1444

```
Int[((d_) + (e_.)*(x_)^(n_))*((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_))^(p
_), x_Symbol] :> Simp[(-x)*(d*b^2 - a*b*e - 2*a*c*d + (b*d - 2*a*e)*c*x^n)*
((a + b*x^n + c*x^(2*n))^(p + 1)/(a*n*(p + 1)*(b^2 - 4*a*c))), x] + Dist[1/
(a*n*(p + 1)*(b^2 - 4*a*c)), Int[Simp[(n*p + n + 1)*d*b^2 - a*b*e - 2*a*c*d
*(2*n*p + 2*n + 1) + (2*n*p + 3*n + 1)*(d*b - 2*a*e)*c*x^n, x]*(a + b*x^n +
 c*x^(2*n))^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[n2, 2*n]
 && NeQ[b^2 - 4*a*c, 0] && ILtQ[p, -1]
```

Rubi steps

$$\int \frac{d+ex^n}{(a+bx^n+cx^{2n})^3}\,dx = \frac{x(b^2d-2acd-abe+c(bd-2ae)x^n)}{2a\,(b^2-4ac)\,n\,(a+bx^n+cx^{2n})^2} - \frac{\int \frac{-abe-2acd(1-4n)+b^2(d-2dn)+c(bd-2ae)(1-3n)}{(a+bx^n+cx^{2n})^2}}{2a\,(b^2-4ac)\,n}$$

$$= \frac{x(b^2d-2acd-abe+c(bd-2ae)x^n)}{2a\,(b^2-4ac)\,n\,(a+bx^n+cx^{2n})^2} + \frac{x(ab^3e-4a^2c^2d(1-4n)+5ab^2cd(1-}{}$$

$$= \frac{x(b^2d-2acd-abe+c(bd-2ae)x^n)}{2a\,(b^2-4ac)\,n\,(a+bx^n+cx^{2n})^2} + \frac{x(ab^3e-4a^2c^2d(1-4n)+5ab^2cd(1-}{}$$

$$= \frac{x(b^2d-2acd-abe+c(bd-2ae)x^n)}{2a\,(b^2-4ac)\,n\,(a+bx^n+cx^{2n})^2} + \frac{x(ab^3e-4a^2c^2d(1-4n)+5ab^2cd(1-}{}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 8593 vs. $2(713) = 1426$.
time = 6.77, size = 8593, normalized size = 12.05

Result too large to show

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x^n)/(a + b*x^n + c*x^(2*n))^3,x]
[Out] Result too large to show
```

Maple [F]
time = 0.04, size = 0, normalized size = 0.00

$$\int \frac{d+e\,x^n}{(a+b\,x^n+c\,x^{2n})^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d+e*x^n)/(a+b*x^n+c*x^(2*n))^3,x)
[Out] int((d+e*x^n)/(a+b*x^n+c*x^(2*n))^3,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+e*x^n)/(a+b*x^n+c*x^(2*n))^3,x, algorithm="maxima")
```

```
[Out] -1/2*((2*a*b*c^3*d*(7*n - 2) - b^3*c^2*d*(2*n - 1) - (4*a^2*c^3*(3*n - 1) +
 a*b^2*c^2)*e)*x*x^(3*n) + (a*b^2*c^2*d*(29*n - 9) - 4*a^2*c^3*d*(4*n - 1)
- 2*b^4*c*d*(2*n - 1) - 2*(a^2*b*c^2*(9*n - 4) + a*b^3*c)*e)*x*x^(2*n) + (4
*a*b^3*c*d*(3*n - 1) - b^5*d*(2*n - 1) + 2*a^2*b*c^2*d*n - (4*a^3*c^2*(5*n
- 1) + a^2*b^2*c*(4*n - 3) + a*b^4)*e)*x*x^n + (a^2*b^2*c*d*(21*n - 5) - 4*
a^3*c^2*d*(6*n - 1) - a*b^4*d*(3*n - 1) - (2*a^3*b*c*(5*n - 2) - a^2*b^3*(n
 - 1))*e)*x)/(a^4*b^4*n^2 - 8*a^5*b^2*c*n^2 + 16*a^6*c^2*n^2 + (a^2*b^4*c^2
*n^2 - 8*a^3*b^2*c^3*n^2 + 16*a^4*c^4*n^2)*x^(4*n) + 2*(a^2*b^5*c*n^2 - 8*a
^3*b^3*c^2*n^2 + 16*a^4*b*c^3*n^2)*x^(3*n) + (a^2*b^6*n^2 - 6*a^3*b^4*c*n^2
 + 32*a^5*c^3*n^2)*x^(2*n) + 2*(a^3*b^5*n^2 - 8*a^4*b^3*c*n^2 + 16*a^5*b*c^
2*n^2)*x^n) + integrate(1/2*((2*n^2 - 3*n + 1)*b^4*d - (16*n^2 - 21*n + 5)*
a*b^2*c*d + 4*(8*n^2 - 6*n + 1)*a^2*c^2*d + ((2*n^2 - 3*n + 1)*b^3*c*d - 2*
(7*n^2 - 9*n + 2)*a*b*c^2*d + (4*(3*n^2 - 4*n + 1)*a^2*c^2 + a*b^2*c*(n - 1
))*e)*x^n - (2*a^2*b*c*(5*n - 2) - a*b^3*(n - 1))*e)/(a^3*b^4*n^2 - 8*a^4*b
^2*c*n^2 + 16*a^5*c^2*n^2 + (a^2*b^4*c*n^2 - 8*a^3*b^2*c^2*n^2 + 16*a^4*c^3
*n^2)*x^(2*n) + (a^2*b^5*n^2 - 8*a^3*b^3*c*n^2 + 16*a^4*b*c^2*n^2)*x^n), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+e*x^n)/(a+b*x^n+c*x^(2*n))^3,x, algorithm="fricas")
```

```
[Out] integral((x^n*e + d)/(c^3*x^(6*n) + b^3*x^(3*n) + 3*a*b^2*x^(2*n) + 3*a^2*b
*x^n + a^3 + 3*(b*c^2*x^n + a*c^2)*x^(4*n) + 3*(b^2*c*x^(2*n) + 2*a*b*c*x^n
 + a^2*c)*x^(2*n)), x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+e*x**n)/(a+b*x**n+c*x**(2*n))**3,x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+e*x^n)/(a+b*x^n+c*x^(2*n))^3,x, algorithm="giac")
```

```
[Out] integrate((x^n*e + d)/(c*x^(2*n) + b*x^n + a)^3, x)
```

Mupad [F]

time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{d+e\,x^n}{{\left(a+b\,x^n+c\,x^{2\,n}\right)}^3} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x^n)/(a + b*x^n + c*x^(2*n))^3,x)
[Out] int((d + e*x^n)/(a + b*x^n + c*x^(2*n))^3, x)
```

3.83 $\int \frac{1}{(d+ex^n)(a+bx^n+cx^{2n})^3}\,dx$

Optimal. Leaf size=1708

$$\frac{x(b^2cd-2ac^2d-b^3e+3abce+c(bcd-b^2e+2ace)\,x^n)}{2a\,(b^2-4ac)\,(cd^2-bde+ae^2)\,n\,(a+bx^n+cx^{2n})^2}+\frac{e^2x(b^2cd-2ac^2d-b^3e+3abce+c(bcd-b^2e+2a}{a\,(b^2-4ac)\,(cd^2-bde+ae^2)^2\,n\,(a+bx^n+cx}$$

[Out] 1/2*x*(b^2*c*d-2*a*c^2*d-b^3*e+3*a*b*c*e+c*(2*a*c*e-b^2*e+b*c*d)*x^n)/a/(-4
*a*c+b^2)/(a*e^2-b*d*e+c*d^2)/n/(a+b*x^n+c*x^(2*n))^2+e^2*x*(b^2*c*d-2*a*c^
2*d-b^3*e+3*a*b*c*e+c*(2*a*c*e-b^2*e+b*c*d)*x^n)/a/(-4*a*c+b^2)/(a*e^2-b*d*
e+c*d^2)^2/n/(a+b*x^n+c*x^(2*n))+1/2*x*(2*a^2*b*c^2*e*(4-11*n)-3*a*b^3*c*e*
(2-5*n)-4*a^2*c^3*d*(1-4*n)+5*a*b^2*c^2*d*(1-3*n)-b^4*c*d*(1-2*n)+b^5*(-2*e
n+e)-c(a*b^2*c*e*(5-14*n)-2*a*b*c^2*d*(2-7*n)-4*a^2*c^2*e*(1-3*n)+b^3*c*d
*(1-2*n)-b^4*e*(1-2*n))*x^n)/a^2/(-4*a*c+b^2)^2/(a*e^2-b*d*e+c*d^2)/n^2/(a+
b*x^n+c*x^(2*n))+e^6*x*hypergeom([1, 1/n],[1+1/n],-e*x^n/d)/d/(a*e^2-b*d*e+
c*d^2)^3-c*e^4*x*hypergeom([1, 1/n],[1+1/n],-2*c*x^n/(b+(-4*a*c+b^2)^(1/2))
)*(2*c*d-e*(b-(-4*a*c+b^2)^(1/2)))/(a*e^2-b*d*e+c*d^2)^3/(b^2-4*a*c+b*(-4*a
*c+b^2)^(1/2))-c*e^4*x*hypergeom([1, 1/n],[1+1/n],-2*c*x^n/(b-(-4*a*c+b^2)^
(1/2)))*(2*c*d-e*(b+(-4*a*c+b^2)^(1/2)))/(a*e^2-b*d*e+c*d^2)^3/(b^2-4*a*c-b
*(-4*a*c+b^2)^(1/2))+c*e^2*x*hypergeom([1, 1/n],[1+1/n],-2*c*x^n/(b-(-4*a*c
+b^2)^(1/2)))*(-b^3*e*(1-n)+b^2*(1-n)*(c*d-e*(-4*a*c+b^2)^(1/2))+b*c*(2*a*e
*(2-3*n)+d*(1-n)*(-4*a*c+b^2)^(1/2))-2*a*c*(2*c*d*(1-2*n)-e*(1-n)*(-4*a*c+b
^2)^(1/2)))/a/(-4*a*c+b^2)/(a*e^2-b*d*e+c*d^2)^2/n/(b^2-4*a*c-b*(-4*a*c+b^2
)^(1/2))+c*e^2*x*hypergeom([1, 1/n],[1+1/n],-2*c*x^n/(b+(-4*a*c+b^2)^(1/2))
)*(-b^3*e*(1-n)+b^2*(1-n)*(c*d+e*(-4*a*c+b^2)^(1/2))+b*c*(2*a*e*(2-3*n)-d*(
1-n)*(-4*a*c+b^2)^(1/2))-2*a*c*(2*c*d*(1-2*n)+e*(1-n)*(-4*a*c+b^2)^(1/2)))/
a/(-4*a*c+b^2)/(a*e^2-b*d*e+c*d^2)^2/n/(b^2-4*a*c+b*(-4*a*c+b^2)^(1/2))+1/2
*c*x*hypergeom([1, 1/n],[1+1/n],-2*c*x^n/(b+(-4*a*c+b^2)^(1/2)))*(b^5*e*(2*
n^2-3*n+1)-b^4*(2*n^2-3*n+1)*(c*d+e*(-4*a*c+b^2)^(1/2))+a*b^2*c*(1-n)*(6*c*
d*(1-3*n)+e*(5-14*n)*(-4*a*c+b^2)^(1/2))-b^3*c*(1-n)*(a*e*(7-18*n)-d*(1-2*n
)*(-4*a*c+b^2)^(1/2))-4*a^2*c^2*(2*c*d*(8*n^2-6*n+1)+e*(3*n^2-4*n+1)*(-4*a*
c+b^2)^(1/2))-2*a*b*c^2*(-2*a*e*(13*n^2-13*n+3)+d*(7*n^2-9*n+2)*(-4*a*c+b^2
)^(1/2)))/a^2/(-4*a*c+b^2)^2/(a*e^2-b*d*e+c*d^2)/n^2/(b^2-4*a*c+b*(-4*a*c+b
^2)^(1/2))-1/2*c*x*hypergeom([1, 1/n],[1+1/n],-2*c*x^n/(b-(-4*a*c+b^2)^(1/2
)))*(-b^5*e*(2*n^2-3*n+1)+b^4*(2*n^2-3*n+1)*(c*d-e*(-4*a*c+b^2)^(1/2))+a*b^
2*c*(1-n)*(-6*c*d*(1-3*n)+e*(5-14*n)*(-4*a*c+b^2)^(1/2))+b^3*c*(1-n)*(a*e*(
7-18*n)+d*(1-2*n)*(-4*a*c+b^2)^(1/2))-4*a^2*c^2*(-2*c*d*(8*n^2-6*n+1)+e*(3*
n^2-4*n+1)*(-4*a*c+b^2)^(1/2))-2*a*b*c^2*(2*a*e*(13*n^2-13*n+3)+d*(7*n^2-9*
n+2)*(-4*a*c+b^2)^(1/2)))/a^2/(-4*a*c+b^2)^2/(a*e^2-b*d*e+c*d^2)/n^2/(b^2-4
*a*c-b*(-4*a*c+b^2)^(1/2))

Rubi [A]
time = 3.35, antiderivative size = 1708, normalized size of antiderivative = 1.00, number of

steps used = 15, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {1450, 251, 1444, 1436}

Antiderivative was successfully verified.

```
[In] Int[1/((d + e*x^n)*(a + b*x^n + c*x^(2*n))^3),x]
```

```
[Out] (x*(b^2*c*d - 2*a*c^2*d - b^3*e + 3*a*b*c*e + c*(b*c*d - b^2*e + 2*a*c*e)*x
^n))/(2*a*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)*n*(a + b*x^n + c*x^(2*n))^2
) + (e^2*x*(b^2*c*d - 2*a*c^2*d - b^3*e + 3*a*b*c*e + c*(b*c*d - b^2*e + 2*
a*c*e)*x^n))/(a*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)^2*n*(a + b*x^n + c*x^
(2*n))) + (x*(2*a^2*b*c^2*e*(4 - 11*n) - 3*a*b^3*c*e*(2 - 5*n) - 4*a^2*c^3*
d*(1 - 4*n) + 5*a*b^2*c^2*d*(1 - 3*n) - b^4*c*d*(1 - 2*n) + b^5*(e - 2*e*n)
 - c*(a*b^2*c*e*(5 - 14*n) - 2*a*b*c^2*d*(2 - 7*n) - 4*a^2*c^2*e*(1 - 3*n)
+ b^3*c*d*(1 - 2*n) - b^4*e*(1 - 2*n))*x^n))/(2*a^2*(b^2 - 4*a*c)^2*(c*d^2
- b*d*e + a*e^2)*n^2*(a + b*x^n + c*x^(2*n))) - (c*e^4*(2*c*d - (b + Sqrt[b
^2 - 4*a*c])*e)*x*Hypergeometric2F1[1, n^(-1), 1 + n^(-1), (-2*c*x^n)/(b -
Sqrt[b^2 - 4*a*c])])/((b^2 - 4*a*c - b*Sqrt[b^2 - 4*a*c])*(c*d^2 - b*d*e +
a*e^2)^3) + (c*e^2*(b*c*(2*a*e*(2 - 3*n) + Sqrt[b^2 - 4*a*c]*d*(1 - n)) - 2
*a*c*(2*c*d*(1 - 2*n) - Sqrt[b^2 - 4*a*c]*e*(1 - n)) - b^3*e*(1 - n) + b^2*
(c*d - Sqrt[b^2 - 4*a*c]*e)*(1 - n))*x*Hypergeometric2F1[1, n^(-1), 1 + n^(
-1), (-2*c*x^n)/(b - Sqrt[b^2 - 4*a*c])])/(a*(b^2 - 4*a*c)*(b^2 - 4*a*c - b
*Sqrt[b^2 - 4*a*c])*(c*d^2 - b*d*e + a*e^2)^2*n) - (c*(a*b^2*c*(Sqrt[b^2 -
4*a*c]*e*(5 - 14*n) - 6*c*d*(1 - 3*n))*(1 - n) + b^3*c*(a*e*(7 - 18*n) + Sq
rt[b^2 - 4*a*c]*d*(1 - 2*n))*(1 - n) - b^5*e*(1 - 3*n + 2*n^2) + b^4*(c*d -
 Sqrt[b^2 - 4*a*c]*e)*(1 - 3*n + 2*n^2) - 4*a^2*c^2*(Sqrt[b^2 - 4*a*c]*e*(1
 - 4*n + 3*n^2) - 2*c*d*(1 - 6*n + 8*n^2)) - 2*a*b*c^2*(Sqrt[b^2 - 4*a*c]*d
*(2 - 9*n + 7*n^2) + 2*a*e*(3 - 13*n + 13*n^2)))*x*Hypergeometric2F1[1, n^(
-1), 1 + n^(-1), (-2*c*x^n)/(b - Sqrt[b^2 - 4*a*c])])/(2*a^2*(b^2 - 4*a*c)^
2*(b^2 - 4*a*c - b*Sqrt[b^2 - 4*a*c])*(c*d^2 - b*d*e + a*e^2)*n^2) - (c*e^4
*(2*c*d - (b - Sqrt[b^2 - 4*a*c])*e)*x*Hypergeometric2F1[1, n^(-1), 1 + n^(
-1), (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c])])/((b^2 - 4*a*c + b*Sqrt[b^2 - 4*a*
c])*(c*d^2 - b*d*e + a*e^2)^3) + (c*e^2*(b*c*(2*a*e*(2 - 3*n) - Sqrt[b^2 -
4*a*c]*d*(1 - n)) - 2*a*c*(2*c*d*(1 - 2*n) + Sqrt[b^2 - 4*a*c]*e*(1 - n)) -
 b^3*e*(1 - n) + b^2*(c*d + Sqrt[b^2 - 4*a*c]*e)*(1 - n))*x*Hypergeometric2
F1[1, n^(-1), 1 + n^(-1), (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c])])/(a*(b^2 - 4*
a*c)*(b^2 - 4*a*c + b*Sqrt[b^2 - 4*a*c])*(c*d^2 - b*d*e + a*e^2)^2*n) + (c*
(a*b^2*c*(Sqrt[b^2 - 4*a*c]*e*(5 - 14*n) + 6*c*d*(1 - 3*n))*(1 - n) - b^3*c
*(a*e*(7 - 18*n) - Sqrt[b^2 - 4*a*c]*d*(1 - 2*n))*(1 - n) + b^5*e*(1 - 3*n
+ 2*n^2) - b^4*(c*d + Sqrt[b^2 - 4*a*c]*e)*(1 - 3*n + 2*n^2) - 4*a^2*c^2*(S
qrt[b^2 - 4*a*c]*e*(1 - 4*n + 3*n^2) + 2*c*d*(1 - 6*n + 8*n^2)) - 2*a*b*c^2
*(Sqrt[b^2 - 4*a*c]*d*(2 - 9*n + 7*n^2) - 2*a*e*(3 - 13*n + 13*n^2)))*x*Hyp
ergeometric2F1[1, n^(-1), 1 + n^(-1), (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c])])/
(2*a^2*(b^2 - 4*a*c)^2*(b^2 - 4*a*c + b*Sqrt[b^2 - 4*a*c])*(c*d^2 - b*d*e +
```

```
 a*e^2)*n^2) + (e^6*x*Hypergeometric2F1[1, n^(-1), 1 + n^(-1), -((e*x^n)/d)
])/(d*(c*d^2 - b*d*e + a*e^2)^3)
```

Rule 251

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, (-b)*(x^n/a)], x] /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p
, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

Rule 1436

```
Int[((d_) + (e_.)*(x_)^(n_))/((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_)), x
_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q),
Int[1/(b/2 - q/2 + c*x^n), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(
b/2 + q/2 + c*x^n), x], x]] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[n2, 2*n]
 && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && (PosQ[b^2 - 4*a
*c] ||  !IGtQ[n/2, 0])
```

Rule 1444

```
Int[((d_) + (e_.)*(x_)^(n_))*((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_))^(p
_), x_Symbol] :> Simp[(-x)*(d*b^2 - a*b*e - 2*a*c*d + (b*d - 2*a*e)*c*x^n)*
((a + b*x^n + c*x^(2*n))^(p + 1)/(a*n*(p + 1)*(b^2 - 4*a*c))), x] + Dist[1/
(a*n*(p + 1)*(b^2 - 4*a*c)), Int[Simp[(n*p + n + 1)*d*b^2 - a*b*e - 2*a*c*d
*(2*n*p + 2*n + 1) + (2*n*p + 3*n + 1)*(d*b - 2*a*e)*c*x^n, x]*(a + b*x^n +
 c*x^(2*n))^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[n2, 2*n]
 && NeQ[b^2 - 4*a*c, 0] && ILtQ[p, -1]
```

Rule 1450

```
Int[((d_) + (e_.)*(x_)^(n_))^(q_)*((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_
))^(p_), x_Symbol] :> Int[ExpandIntegrand[(d + e*x^n)^q*(a + b*x^n + c*x^(2
*n))^p, x], x] /; FreeQ[{a, b, c, d, e, n, p, q}, x] && EqQ[n2, 2*n] && NeQ
[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && ((IntegersQ[p, q] &&
!IntegerQ[n]) || IGtQ[p, 0] || (IGtQ[q, 0] &&  !IntegerQ[n]))
```

Rubi steps

$$\int \frac{1}{(d+ex^n)(a+bx^n+cx^{2n})^3}\,dx = \int \Bigg(\frac{e^6}{(cd^2-bde+ae^2)^3(d+ex^n)} + \frac{cd-be-cex^n}{(cd^2-bde+ae^2)(a+bx^n+cx^{2n}}$$

$$= -\frac{e^4\int \frac{-cd+be+cex^n}{a+bx^n+cx^{2n}}\,dx}{(cd^2-bde+ae^2)^3} + \frac{e^6\int \frac{1}{d+ex^n}\,dx}{(cd^2-bde+ae^2)^3} - \frac{e^2\int \frac{-cd+be+cex^n}{(a+bx^n+cx^{2n})^2}\,dx}{(cd^2-bde+ae^2)^2} + \frac{\int}{c}$$

$$= \frac{x(b^2cd-2ac^2d-b^3e+3abce+c(bcd-b^2e+2ace)\,x^n)}{2a(b^2-4ac)(cd^2-bde+ae^2)\,n\,(a+bx^n+cx^{2n})^2} + \frac{e^2x(b^2cd-}{a(b^2-}$$

$$= \frac{x(b^2cd-2ac^2d-b^3e+3abce+c(bcd-b^2e+2ace)\,x^n)}{2a(b^2-4ac)(cd^2-bde+ae^2)\,n\,(a+bx^n+cx^{2n})^2} + \frac{e^2x(b^2cd-}{a(b^2-}$$

$$= \frac{x(b^2cd-2ac^2d-b^3e+3abce+c(bcd-b^2e+2ace)\,x^n)}{2a(b^2-4ac)(cd^2-bde+ae^2)\,n\,(a+bx^n+cx^{2n})^2} + \frac{e^2x(b^2cd-}{a(b^2-}$$

$$= \frac{x(b^2cd-2ac^2d-b^3e+3abce+c(bcd-b^2e+2ace)\,x^n)}{2a(b^2-4ac)(cd^2-bde+ae^2)\,n\,(a+bx^n+cx^{2n})^2} + \frac{e^2x(b^2cd-}{a(b^2-}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 43535 vs. $2(1708) = 3416$.
time = 7.82, size = 43535, normalized size = 25.49

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/((d + e*x^n)*(a + b*x^n + c*x^(2*n))^3),x]
```

```
[Out] Result too large to show
```

Maple [F]
time = 0.06, size = 0, normalized size = 0.00

$$\int \frac{1}{(d+e\,x^n)(a+b\,x^n+c\,x^{2n})^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(d+e*x^n)/(a+b*x^n+c*x^(2*n))^3,x)
```

```
[Out] int(1/(d+e*x^n)/(a+b*x^n+c*x^(2*n))^3,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d+e*x^n)/(a+b*x^n+c*x^(2*n))^3,x, algorithm="maxima")
```

```
[Out] e^6*integrate(1/(c^3*d^7 - 3*b*c^2*d^6*e - 3*a^2*b*d^2*e^5 + a^3*d*e^6 + (c
^3*d^6*e - 3*b*c^2*d^5*e^2 - 3*a^2*b*d*e^6 + a^3*e^7 + 3*(a*b^2*d^2 + a^2*c
*d^2)*e^5 - (b^3*d^3 + 6*a*b*c*d^3)*e^4 + 3*(b^2*c*d^4 + a*c^2*d^4)*e^3)*x^
n + 3*(a*b^2*d^3 + a^2*c*d^3)*e^4 - (b^3*d^4 + 6*a*b*c*d^4)*e^3 + 3*(b^2*c*
d^5 + a*c^2*d^5)*e^2), x) - 1/2*((2*a*b*c^5*d^3*(7*n - 2) - b^3*c^4*d^3*(2*
n - 1) - (a^2*b^2*c^3*(26*n - 5) - 4*a^3*c^4*(7*n - 1) - a*b^4*c^2*(4*n - 1
))*e^3 + (2*a*b^3*c^3*d*(5*n - 2) - b^5*c^2*d*(2*n - 1) + 10*a^2*b*c^4*d*n)
*e^2 - (a*b^2*c^4*d^2*(28*n - 9) - 4*a^2*c^5*d^2*(3*n - 1) - 2*b^4*c^3*d^2*
(2*n - 1))*e)*x*x^(3*n) + (a*b^2*c^4*d^3*(29*n - 9) - 4*a^2*c^5*d^3*(4*n -
1) - 2*b^4*c^3*d^3*(2*n - 1) + (2*a^3*b*c^3*(37*n - 6) - 11*a^2*b^3*c^2*(5*
n - 1) + 2*a*b^5*c*(4*n - 1))*e^3 - (4*a^3*c^4*d*(8*n - 1) - 3*a*b^4*c^2*d*
(7*n - 3) - 3*a^2*b^2*c^3*d*(5*n + 1) + 2*b^6*c*d*(2*n - 1))*e^2 - 2*(a*b^3
*c^3*d^2*(29*n - 10) - a^2*b*c^4*d^2*(25*n - 8) - 2*b^5*c^2*d^2*(2*n - 1))*
e)*x*x^(2*n) + (4*a*b^3*c^3*d^3*(3*n - 1) - b^5*c^2*d^3*(2*n - 1) + 2*a^2*b
*c^4*d^3n - (a^2*b^4*c*(24*n - 5) - a^3*b^2*c^2*(14*n - 3) - 4*a^4*c^3*(9*
n - 1) - a*b^6*(4*n - 1))*e^3 + (a^2*b^3*c^2*d*(20*n - 1) - 2*a^3*b*c^3*d*(
13*n - 2) - b^7*d*(2*n - 1) + 4*a*b^5*c*d*(2*n - 1))*e^2 - (3*a*b^4*c^2*d^2
*(8*n - 3) - 4*a^3*c^4*d^2*(5*n - 1) - 2*b^6*c*d^2*(2*n - 1) + 3*a^2*b^2*c^
3*d^2)*e)*x*x^n + (a^2*b^2*c^3*d^3*(21*n - 5) - 4*a^3*c^4*d^3*(6*n - 1) - a
*b^4*c^2*d^3*(3*n - 1) + (2*a^4*b*c^2*(29*n - 4) - 6*a^3*b^3*c*(6*n - 1) +
a^2*b^5*(5*n - 1))*e^3 + (a^2*b^4*c*d*(17*n - 5) - 4*a^4*c^3*d*(10*n - 1) -
 a*b^6*d*(3*n - 1) - a^3*b^2*c^2*d*(n - 3))*e^2 - (a^2*b^3*c^2*d^2*(43*n -
11) - 2*a^3*b*c^3*d^2*(29*n - 6) - 2*a*b^5*c*d^2*(3*n - 1))*e)*x)/(a^4*b^4*
c^2*d^4*n^2 - 8*a^5*b^2*c^3*d^4*n^2 + 16*a^6*c^4*d^4*n^2 + (a^2*b^4*c^4*d^4
*n^2 - 8*a^3*b^2*c^5*d^4*n^2 + 16*a^4*c^6*d^4*n^2 + (a^4*b^4*c^2*n^2 - 8*a^
5*b^2*c^3*n^2 + 16*a^6*c^4*n^2)*e^4 - 2*(a^3*b^5*c^2*d*n^2 - 8*a^4*b^3*c^3*
d*n^2 + 16*a^5*b*c^4*d*n^2)*e^3 + (a^2*b^6*c^2*d^2*n^2 - 6*a^3*b^4*c^3*d^2*
n^2 + 32*a^5*c^5*d^2*n^2)*e^2 - 2*(a^2*b^5*c^3*d^3*n^2 - 8*a^3*b^3*c^4*d^3*
n^2 + 16*a^4*b*c^5*d^3*n^2)*e)*x^(4*n) + 2*(a^2*b^5*c^3*d^4*n^2 - 8*a^3*b^3
*c^4*d^4*n^2 + 16*a^4*b*c^5*d^4*n^2 + (a^4*b^5*c*n^2 - 8*a^5*b^3*c^2*n^2 +
16*a^6*b*c^3*n^2)*e^4 - 2*(a^3*b^6*c*d*n^2 - 8*a^4*b^4*c^2*d*n^2 + 16*a^5*b
^2*c^3*d*n^2)*e^3 + (a^2*b^7*c*d^2*n^2 - 6*a^3*b^5*c^2*d^2*n^2 + 32*a^5*b*c
^4*d^2*n^2)*e^2 - 2*(a^2*b^6*c^2*d^3*n^2 - 8*a^3*b^4*c^3*d^3*n^2 + 16*a^4*b
^2*c^4*d^3*n^2)*e)*x^(3*n) + (a^2*b^6*c^2*d^4*n^2 - 6*a^3*b^4*c^3*d^4*n^2 +
 32*a^5*c^5*d^4*n^2 + (a^4*b^6*n^2 - 6*a^5*b^4*c*n^2 + 32*a^7*c^3*n^2)*e^4
- 2*(a^3*b^7*d*n^2 - 6*a^4*b^5*c*d*n^2 + 32*a^6*b*c^3*d*n^2)*e^3 + (a^2*b^8
*d^2*n^2 - 4*a^3*b^6*c*d^2*n^2 - 12*a^4*b^4*c^2*d^2*n^2 + 32*a^5*b^2*c^3*d^
```

```
2*n^2 + 64*a^6*c^4*d^2*n^2)*e^2 - 2*(a^2*b^7*c*d^3*n^2 - 6*a^3*b^5*c^2*d^3*
n^2 + 32*a^5*b*c^4*d^3*n^2)*e)*x^(2*n) + 2*(a^3*b^5*c^2*d^4*n^2 - 8*a^4*b^3
*c^3*d^4*n^2 + 16*a^5*b*c^4*d^4*n^2 + (a^5*b^5*n^2 - 8*a^6*b^3*c*n^2 + 16*a
^7*b*c^2*n^2)*e^4 - 2*(a^4*b^6*d*n^2 - 8*a^5*b^4*c*d*n^2 + 16*a^6*b^2*c^2*d
*n^2)*e^3 + (a^3*b^7*d^2*n^2 - 6*a^4*b^5*c*d^2*n^2 + 32*a^6*b*c^3*d^2*n^2)*
e^2 - 2*(a^3*b^6*c*d^3*n^2 - 8*a^4*b^4*c^2*d^3*n^2 + 16*a^5*b^2*c^3*d^3*n^2
)*e)*x^n + (a^6*b^4*n^2 - 8*a^7*b^2*c*n^2 + 16*a^8*c^2*n^2)*e^4 - 2*(a^5*b^
5*d*n^2 - 8*a^6*b^3*c*d*n^2 + 16*a^7*b*c^2*d*n^2)*e^3 + (a^4*b^6*d^2*n^2 -
6*a^5*b^4*c*d^2*n^2 + 32*a^7*c^3*d^2*n^2)*e^2 - 2*(a^4*b^5*c*d^3*n^2 - 8*a^
5*b^3*c^2*d^3*n^2 + 16*a^6*b*c^3*d^3*n^2)*e) - integrate(-1/2*((2*n^2 - 3*n
 + 1)*b^4*c^3*d^5 - (16*n^2 - 21*n + 5)*a*b^2*c^4*d^5 + 4*(8*n^2 - 6*n + 1)
*a^2*c^5*d^5 + ((2*n^2 - 3*n + 1)*b^3*c^4*d^5 - 2*(7*n^2 - 9*n + 2)*a*b*c^5
*d^5 - ((6*n^2 - 5*n + 1)*a^2*b^4*c - (42*n^2 - 31*n + 5)*a^3*b^2*c^2 + 4*(
15*n^2 - 8*n + 1)*a^4*c^3)*e^5 + (2*(3*n^2 - 4*n + 1)*a*b^5*c*d - 9*(4*n^2
- 5*n + 1)*a^2*b^3*c^2*d + 2*(9*n^2 - 11*n + 2)*a^3*b*c^3*d)*e^4 - ((2*n^2
- 3*n + 1)*b^6*c*d^2 - (2*n^2 - 3*n + 1)*a*b^4*c^2*d^2 - 2*(32*n^2 - 39*n +
 7)*a^2*b^2*c^3*d^2 + 8*(5*n^2 - 6*n + 1)*a^3*c^4*d^2)*e^3 + 3*((2*n^2 - 3*
n + 1)*b^5*c^2*d^3 - 4*(3*n^2 - 4*n + 1)*a*b^3*c^3*d^3 - 4*(n^2 - n)*a^2*b*
c^4*d^3)*e^2 - (3*(2*n^2 - 3*n + 1)*b^4*c^3*d^4 - (42*n^2 - 55*n + 13)*a*b^
2*c^4*d^4 + 4*(3*n^2 - 4*n + 1)*a^2*c^5*d^4)*e)*x^n - ((6*n^2 - 5*n + 1)*a^
2*b^5 - 6*(8*n^2 - 6*n + 1)*a^3*b^3*c + 2*(48*n^2 - 29*n + 4)*a^4*b*c^2)*e^
5 + (2*(3*n^2 - 4*n + 1)*a*b^6*d - (42*n^2 - 53*n + 11)*a^2*b^4*c*d + (48*n
^2 - 59*n + 11)*a^3*b^2*c^2*d + 4*(24*n^2 - 10*n + 1)*a^4*c^3*d)*e^4 - ((2*
n^2 - 3*n + 1)*b^7*d^2 - 2*(2*n^2 - 3*n + 1)*a*b^5*c*d^2 - 2*(32*n^2 - 39*n
 + 7)*a^2*b^3*c^2*d^2 + 12*(16*n^2 - 13*n + 2)*a^3*b*c^3*d^2)*e^3 + (3*(2*n
^2 - 3*n + 1)*b^6*c*d^3 - 3*(14*n^2 - 19*n + 5)*a*b^4*c^2*d^3 + 2*(24*n^2 -
 19*n + 5)*a^2*b^2*c^3*d^3 + 8*(12*n^2 - 8*n + 1)*a^3*c^4*d^3)*e^2 - (3*(2*
n^2 - 3*n + 1)*b^5*c^2*d^4 - 16*(3*n^2 - 4*n + 1)*a*b^3*c^3*d^4 + 2*(48*n^2
 - 41*n + 8)*a^2*b*c^4*d^4)*e)/(a^3*b^4*c^3*d^6*n^2 - 8*a^4*b^2*c^4*d^6*n^2
 + 16*a^5*c^5*d^6*n^2 + (a^2*b^4*c^4*d^6*n^2 - ...
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d+e*x^n)/(a+b*x^n+c*x^(2*n))^3,x, algorithm="fricas")
```

```
[Out] integral(1/(b^3*x^(4*n)*e + a^3*d + (c^3*x^n*e + c^3*d)*x^(6*n) + 3*(b*c^2*
x^(2*n)*e + a*c^2*d + (b*c^2*d + a*c^2*e)*x^n)*x^(4*n) + (b^3*d + 3*a*b^2*e
)*x^(3*n) + 3*(b^2*c*x^(3*n)*e + a^2*c*d + (b^2*c*d + 2*a*b*c*e)*x^(2*n) +
(2*a*b*c*d + a^2*c*e)*x^n)*x^(2*n) + 3*(a*b^2*d + a^2*b*e)*x^(2*n) + (3*a^2
*b*d + a^3*e)*x^n), x)
```

Sympy [F(-2)]

time = 0.00, size = 0, normalized size = 0.00

Exception raised: HeuristicGCDFailed

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d+e*x**n)/(a+b*x**n+c*x**(2*n))**3,x)
[Out] Exception raised: HeuristicGCDFailed >> no luck
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d+e*x^n)/(a+b*x^n+c*x^(2*n))^3,x, algorithm="giac")
[Out] integrate(1/((c*x^(2*n) + b*x^n + a)^3*(x^n*e + d)), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{(d+e\,x^n)\,\left(a+b\,x^n+c\,x^{2\,n}\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((d + e*x^n)*(a + b*x^n + c*x^(2*n))^3),x)
[Out] int(1/((d + e*x^n)*(a + b*x^n + c*x^(2*n))^3), x)
```

3.84 $\int \frac{1}{(d+ex^n)^2 (a+bx^n+cx^{2n})^3} dx$

Optimal. Leaf size=2446

result too large to display

```
[Out] -1/2*x*(2*b^3*c*d*e-6*a*b*c^2*d*e-b^4*e^2-b^2*c*(-4*a*e^2+c*d^2)+2*a*c^2*(-
a*e^2+c*d^2)+c*(2*b^2*c*d*e-4*a*c^2*d*e-b^3*e^2-b*c*(-3*a*e^2+c*d^2))*x^n)/
a/(-4*a*c+b^2)/(a*e^2-b*d*e+c*d^2)^2/n/(a+b*x^n+c*x^(2*n))^2-e^2*x*(5*b^3*c
*d*e-14*a*b*c^2*d*e-2*b^4*e^2-b^2*c*(-7*a*e^2+3*c*d^2)+2*a*c^2*(-a*e^2+3*c*
d^2)+c*(5*b^2*c*d*e-8*a*c^2*d*e-2*b^3*e^2-b*c*(-5*a*e^2+3*c*d^2))*x^n)/a/(-
4*a*c+b^2)/(a*e^2-b*d*e+c*d^2)^3/n/(a+b*x^n+c*x^(2*n))-1/2*x*(a*b^2*c^2*(a*
e^2*(13-37*n)-5*c*d^2*(1-3*n))-b^4*c*(a*e^2*(7-17*n)-c*d^2*(1-2*n))-4*a^2*b
*c^3*d*e*(4-11*n)+6*a*b^3*c^2*d*e*(2-5*n)+4*a^2*c^3*(-a*e^2+c*d^2)*(1-4*n)-
2*b^5*c*d*e*(1-2*n)+b^6*e^2*(1-2*n)+c*(2*a*b*c^2*(a*e^2*(4-13*n)-c*d^2*(2-7
*n))-b^3*c*(2*a*e^2*(3-8*n)-c*d^2*(1-2*n))+2*a*b^2*c^2*d*e*(5-14*n)-8*a^2*c
^3*d*e*(1-3*n)-2*b^4*c*d*e*(1-2*n)+b^5*e^2*(1-2*n))*x^n)/a^2/(-4*a*c+b^2)^2
/(a*e^2-b*d*e+c*d^2)^2/n^2/(a+b*x^n+c*x^(2*n))+3*e^6*(-b*e+2*c*d)*x*hyperge
om([1, 1/n],[1+1/n],-e*x^n/d)/d/(a*e^2-b*d*e+c*d^2)^4+e^6*x*hypergeom([2, 1
/n],[1+1/n],-e*x^n/d)/d^2/(a*e^2-b*d*e+c*d^2)^3+1/2*c*x*hypergeom([1, 1/n],
[1+1/n],-2*c*x^n/(b-(-4*a*c+b^2)^(1/2)))*((2*a*b*c^2*(a*e^2*(4-13*n)-c*d^2*
(2-7*n))-b^3*c*(2*a*e^2*(3-8*n)-c*d^2*(1-2*n))+2*a*b^2*c^2*d*e*(5-14*n)-8*a
^2*c^3*d*e*(1-3*n)-2*b^4*c*d*e*(1-2*n)+b^5*e^2*(1-2*n))*(1-n)+(-b^4*c*(4*a*
e^2*(2-5*n)-c*d^2*(1-2*n))*(1-n)-2*b^5*c*d*e*(2*n^2-3*n+1)+b^6*e^2*(2*n^2-3
*n+1)+8*a^2*c^3*(-a*e^2+c*d^2)*(8*n^2-6*n+1)-8*a^2*b*c^3*d*e*(13*n^2-13*n+3
)+2*a*b^3*c^2*d*e*(18*n^2-25*n+7)-2*a*b^2*c^2*(3*c*d^2*(3*n^2-4*n+1)-a*e^2*
(35*n^2-38*n+9)))/(-4*a*c+b^2)^(1/2))/a^2/(-4*a*c+b^2)^2/(a*e^2-b*d*e+c*d^2
)^2/n^2/(b-(-4*a*c+b^2)^(1/2))+1/2*c*x*hypergeom([1, 1/n],[1+1/n],-2*c*x^n/
(b+(-4*a*c+b^2)^(1/2)))*((2*a*b*c^2*(a*e^2*(4-13*n)-c*d^2*(2-7*n))-b^3*c*(2
*a*e^2*(3-8*n)-c*d^2*(1-2*n))+2*a*b^2*c^2*d*e*(5-14*n)-8*a^2*c^3*d*e*(1-3*n
)-2*b^4*c*d*e*(1-2*n)+b^5*e^2*(1-2*n))*(1-n)+(b^4*c*(4*a*e^2*(2-5*n)-c*d^2*
(1-2*n))*(1-n)+2*b^5*c*d*e*(2*n^2-3*n+1)-b^6*e^2*(2*n^2-3*n+1)-8*a^2*c^3*(-
a*e^2+c*d^2)*(8*n^2-6*n+1)+8*a^2*b*c^3*d*e*(13*n^2-13*n+3)-2*a*b^3*c^2*d*e*
(18*n^2-25*n+7)+2*a*b^2*c^2*(3*c*d^2*(3*n^2-4*n+1)-a*e^2*(35*n^2-38*n+9)))/
(-4*a*c+b^2)^(1/2))/a^2/(-4*a*c+b^2)^2/(a*e^2-b*d*e+c*d^2)^2/n^2/(b+(-4*a*c
+b^2)^(1/2))-c*e^4*x*hypergeom([1, 1/n],[1+1/n],-2*c*x^n/(b+(-4*a*c+b^2)^(1
/2)))*(10*c^2*d^2+3*b*e^2*(b-(-4*a*c+b^2)^(1/2))-2*c*e*(5*b*d+a*e-3*d*(-4*a
*c+b^2)^(1/2)))/(a*e^2-b*d*e+c*d^2)^4/(b^2-4*a*c+b*(-4*a*c+b^2)^(1/2))-c*e^
4*x*hypergeom([1, 1/n],[1+1/n],-2*c*x^n/(b-(-4*a*c+b^2)^(1/2)))*(10*c^2*d^2
+3*b*e^2*(b+(-4*a*c+b^2)^(1/2))-2*c*e*(5*b*d+a*e+3*d*(-4*a*c+b^2)^(1/2)))/(
a*e^2-b*d*e+c*d^2)^4/(b^2-4*a*c-b*(-4*a*c+b^2)^(1/2))+c*e^2*x*hypergeom([1,
 1/n],[1+1/n],-2*c*x^n/(b+(-4*a*c+b^2)^(1/2)))*(2*b^4*e^2*(1-n)-b^3*e*(1-n)
*(5*c*d+2*e*(-4*a*c+b^2)^(1/2))-b^2*c*(-3*c*d^2*(1-n)+e*(a*e*(9-13*n)-5*d*(
1-n)*(-4*a*c+b^2)^(1/2)))+b*c*(5*a*e^2*(1-n)*(-4*a*c+b^2)^(1/2)+c*d*(4*a*e*
(5-8*n)-3*d*(1-n)*(-4*a*c+b^2)^(1/2)))+4*a*c^2*(-3*c*d^2*(1-2*n)+e*(a*e*(1-
```

```
2*n)-2*d*(1-n)*(-4*a*c+b^2)^(1/2))))/a/(-4*a*c+b^2)/(a*e^2-b*d*e+c*d^2)^3/n
/(b^2-4*a*c+b*(-4*a*c+b^2)^(1/2))+c*e^2*x*hypergeom([1, 1/n],[1+1/n],-2*c*x
^n/(b-(-4*a*c+b^2)^(1/2)))*(2*b^4*e^2*(1-n)-b^3*e*(1-n)*(5*c*d-2*e*(-4*a*c+
b^2)^(1/2))+4*a*c^2*(-3*c*d^2*(1-2*n)+e*(a*e*(1-2*n)+2*d*(1-n)*(-4*a*c+b^2)
^(1/2)))+b*c*(-5*a*e^2*(1-n)*(-4*a*c+b^2)^(1/2)+c*d*(4*a*e*(5-8*n)+3*d*(1-n
)*(-4*a*c+b^2)^(1/2)))-b^2*c*(-3*c*d^2*(1-n)+e*(a*e*(9-13*n)+5*d*(1-n)*(-4*
a*c+b^2)^(1/2))))/a/(-4*a*c+b^2)/(a*e^2-b*d*e+c*d^2)^3/n/(b^2-4*a*c-b*(-4*a
*c+b^2)^(1/2))
```

Rubi [A]

time = 5.75, antiderivative size = 2446, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {1450, 251, 1444, 1436}

Antiderivative was successfully verified.

```
[In] Int[1/((d + e*x^n)^2*(a + b*x^n + c*x^(2*n))^3),x]
```

```
[Out] -1/2*(x*(2*b^3*c*d*e - 6*a*b*c^2*d*e - b^4*e^2 - b^2*c*(c*d^2 - 4*a*e^2) +
2*a*c^2*(c*d^2 - a*e^2) + c*(2*b^2*c*d*e - 4*a*c^2*d*e - b^3*e^2 - b*c*(c*d
^2 - 3*a*e^2))*x^n))/(a*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)^2*n*(a + b*x^
n + c*x^(2*n))^2) - (e^2*x*(5*b^3*c*d*e - 14*a*b*c^2*d*e - 2*b^4*e^2 - b^2*
c*(3*c*d^2 - 7*a*e^2) + 2*a*c^2*(3*c*d^2 - a*e^2) + c*(5*b^2*c*d*e - 8*a*c^
2*d*e - 2*b^3*e^2 - b*c*(3*c*d^2 - 5*a*e^2))*x^n))/(a*(b^2 - 4*a*c)*(c*d^2
- b*d*e + a*e^2)^3*n*(a + b*x^n + c*x^(2*n))) - (x*(a*b^2*c^2*(a*e^2*(13 -
37*n) - 5*c*d^2*(1 - 3*n)) - b^4*c*(a*e^2*(7 - 17*n) - c*d^2*(1 - 2*n)) - 4
*a^2*b*c^3*d*e*(4 - 11*n) + 6*a*b^3*c^2*d*e*(2 - 5*n) + 4*a^2*c^3*(c*d^2 -
a*e^2)*(1 - 4*n) - 2*b^5*c*d*e*(1 - 2*n) + b^6*e^2*(1 - 2*n) + c*(2*a*b*c^2
*(a*e^2*(4 - 13*n) - c*d^2*(2 - 7*n)) - b^3*c*(2*a*e^2*(3 - 8*n) - c*d^2*(1
 - 2*n)) + 2*a*b^2*c^2*d*e*(5 - 14*n) - 8*a^2*c^3*d*e*(1 - 3*n) - 2*b^4*c*d
*e*(1 - 2*n) + b^5*e^2*(1 - 2*n))*x^n))/(2*a^2*(b^2 - 4*a*c)^2*(c*d^2 - b*d
*e + a*e^2)^2*n^2*(a + b*x^n + c*x^(2*n))) - (c*e^4*(10*c^2*d^2 + 3*b*(b +
Sqrt[b^2 - 4*a*c])*e^2 - 2*c*e*(5*b*d + 3*Sqrt[b^2 - 4*a*c]*d + a*e))*x*Hyp
ergeometric2F1[1, n^(-1), 1 + n^(-1), (-2*c*x^n)/(b - Sqrt[b^2 - 4*a*c])])/
((b^2 - 4*a*c - b*Sqrt[b^2 - 4*a*c])*(c*d^2 - b*d*e + a*e^2)^4) + (c*e^2*(4
*a*c^2*(e*(a*e*(1 - 2*n) + 2*Sqrt[b^2 - 4*a*c]*d*(1 - n)) - 3*c*d^2*(1 - 2*
n)) - b^2*c*(e*(a*e*(9 - 13*n) + 5*Sqrt[b^2 - 4*a*c]*d*(1 - n)) - 3*c*d^2*(
1 - n)) + b*c*(c*d*(4*a*e*(5 - 8*n) + 3*Sqrt[b^2 - 4*a*c]*d*(1 - n)) - 5*a*
Sqrt[b^2 - 4*a*c]*e^2*(1 - n)) + 2*b^4*e^2*(1 - n) - b^3*e*(5*c*d - 2*Sqrt[
b^2 - 4*a*c]*e)*(1 - n))*x*Hypergeometric2F1[1, n^(-1), 1 + n^(-1), (-2*c*x
^n)/(b - Sqrt[b^2 - 4*a*c])])/(a*(b^2 - 4*a*c)*(b^2 - 4*a*c - b*Sqrt[b^2 -
4*a*c])*(c*d^2 - b*d*e + a*e^2)^3*n) + (c*((2*a*b*c^2*(a*e^2*(4 - 13*n) - c
*d^2*(2 - 7*n)) - b^3*c*(2*a*e^2*(3 - 8*n) - c*d^2*(1 - 2*n)) + 2*a*b^2*c^2
*d*e*(5 - 14*n) - 8*a^2*c^3*d*e*(1 - 3*n) - 2*b^4*c*d*e*(1 - 2*n) + b^5*e^2
```

```
*(1 - 2*n))*(1 - n) - (b^4*c*(4*a*e^2*(2 - 5*n) - c*d^2*(1 - 2*n))*(1 - n)
+ 2*b^5*c*d*e*(1 - 3*n + 2*n^2) - b^6*e^2*(1 - 3*n + 2*n^2) - 8*a^2*c^3*(c*
d^2 - a*e^2)*(1 - 6*n + 8*n^2) + 8*a^2*b*c^3*d*e*(3 - 13*n + 13*n^2) - 2*a*
b^3*c^2*d*e*(7 - 25*n + 18*n^2) + 2*a*b^2*c^2*(3*c*d^2*(1 - 4*n + 3*n^2) -
a*e^2*(9 - 38*n + 35*n^2)))/Sqrt[b^2 - 4*a*c])*x*Hypergeometric2F1[1, n^(-1
), 1 + n^(-1), (-2*c*x^n)/(b - Sqrt[b^2 - 4*a*c])])/(2*a^2*(b^2 - 4*a*c)^2*
(b - Sqrt[b^2 - 4*a*c])*(c*d^2 - b*d*e + a*e^2)^2*n^2) - (c*e^4*(10*c^2*d^2
 + 3*b*(b - Sqrt[b^2 - 4*a*c])*e^2 - 2*c*e*(5*b*d - 3*Sqrt[b^2 - 4*a*c]*d +
 a*e))*x*Hypergeometric2F1[1, n^(-1), 1 + n^(-1), (-2*c*x^n)/(b + Sqrt[b^2
- 4*a*c])])/((b^2 - 4*a*c + b*Sqrt[b^2 - 4*a*c])*(c*d^2 - b*d*e + a*e^2)^4)
 + (c*e^2*(4*a*c^2*(e*(a*e*(1 - 2*n) - 2*Sqrt[b^2 - 4*a*c]*d*(1 - n)) - 3*c
*d^2*(1 - 2*n)) - b^2*c*(e*(a*e*(9 - 13*n) - 5*Sqrt[b^2 - 4*a*c]*d*(1 - n))
 - 3*c*d^2*(1 - n)) + b*c*(c*d*(4*a*e*(5 - 8*n) - 3*Sqrt[b^2 - 4*a*c]*d*(1
- n)) + 5*a*Sqrt[b^2 - 4*a*c]*e^2*(1 - n)) + 2*b^4*e^2*(1 - n) - b^3*e*(5*c
*d + 2*Sqrt[b^2 - 4*a*c]*e)*(1 - n))*x*Hypergeometric2F1[1, n^(-1), 1 + n^(
-1), (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c])])/(a*(b^2 - 4*a*c)*(b^2 - 4*a*c + b
*Sqrt[b^2 - 4*a*c])*(c*d^2 - b*d*e + a*e^2)^3*n) + (c*((2*a*b*c^2*(a*e^2*(4
 - 13*n) - c*d^2*(2 - 7*n)) - b^3*c*(2*a*e^2*(3 - 8*n) - c*d^2*(1 - 2*n)) +
 2*a*b^2*c^2*d*e*(5 - 14*n) - 8*a^2*c^3*d*e*(1 - 3*n) - 2*b^4*c*d*e*(1 - 2*
n) + b^5*e^2*(1 - 2*n))*(1 - n) + (b^4*c*(4*a*e^2*(2 - 5*n) - c*d^2*(1 - 2*
n))*(1 - n) + 2*b^5*c*d*e*(1 - 3*n + 2*n^2) - b^6*e^2*(1 - 3*n + 2*n^2) - 8
*a^2*c^3*(c*d^2 - a*e^2)*(1 - 6*n + 8*n^2) + 8*a^2*b*c^3*d*e*(3 - 13*n + 13
*n^2) - 2*a*b^3*c^2*d*e*(7 - 25*n + 18*n^2) + 2*a*b^2*c^2*(3*c*d^2*(1 - 4*n
 + 3*n^2) - a*e^2*(9 - 38*n + 35*n^2)))/Sqrt[b^2 - 4*a*c])*x*Hypergeometric
2F1[1, n^(-1), 1 + n^(-1), (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c])])/(2*a^2*(b^2
 - 4*a*c)^2*(b + Sqrt[b^2 - 4*a*c])*(c*d^2 - b*d*e + a*e^2)^2*n^2) + (3*e^6
*(2*c*d - b*e)*x*Hypergeometric2F1[1, n^(-1), 1 + n^(-1), -((e*x^n)/d)])/(d
*(c*d^2 - b*d*e + a*e^2)^4) + (e^6*x*Hypergeometric2F1[2, n^(-1), 1 + n^(-1
), -((e*x^n)/d)])/(d^2*(c*d^2 - b*d*e + a*e^2)^3)
```

Rule 251

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, (-b)*(x^n/a)], x] /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p
, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

Rule 1436

```
Int[((d_) + (e_.)*(x_)^(n_))/((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_)), x
_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q),
Int[1/(b/2 - q/2 + c*x^n), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(
b/2 + q/2 + c*x^n), x], x]] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[n2, 2*n]
 && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && (PosQ[b^2 - 4*a
*c] ||  !IGtQ[n/2, 0])
```

Rule 1444

```
Int[((d_) + (e_.)*(x_)^(n_))*((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_))^(p
_), x_Symbol] :> Simp[(-x)*(d*b^2 - a*b*e - 2*a*c*d + (b*d - 2*a*e)*c*x^n)*
((a + b*x^n + c*x^(2*n))^(p + 1)/(a*n*(p + 1)*(b^2 - 4*a*c))), x] + Dist[1/
(a*n*(p + 1)*(b^2 - 4*a*c)), Int[Simp[(n*p + n + 1)*d*b^2 - a*b*e - 2*a*c*d
*(2*n*p + 2*n + 1) + (2*n*p + 3*n + 1)*(d*b - 2*a*e)*c*x^n, x]*(a + b*x^n +
 c*x^(2*n))^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[n2, 2*n]
 && NeQ[b^2 - 4*a*c, 0] && ILtQ[p, -1]
```

Rule 1450

```
Int[((d_) + (e_.)*(x_)^(n_))^(q_)*((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_
))^(p_), x_Symbol] :> Int[ExpandIntegrand[(d + e*x^n)^q*(a + b*x^n + c*x^(2
*n))^p, x], x] /; FreeQ[{a, b, c, d, e, n, p, q}, x] && EqQ[n2, 2*n] && NeQ
[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && ((IntegersQ[p, q] &&
!IntegerQ[n]) || IGtQ[p, 0] || (IGtQ[q, 0] &&  !IntegerQ[n]))
```

Rubi steps

$$\int \frac{1}{(d+ex^n)^2\,(a+bx^n+cx^{2n})^3}\,dx = \int \left(\frac{e^6}{(cd^2-bde+ae^2)^3\,(d+ex^n)^2} - \frac{3e^6(-2cd+be)}{(cd^2-bde+ae^2)^4\,(d+ex^n)} + \frac{c^2}{}\right.$$

$$= \frac{e^4\int \frac{5c^2d^2-8bcde+3b^2e^2-ace^2+(-6c^2de+3bce^2)x^n}{a+bx^n+cx^{2n}}\,dx}{(cd^2-bde+ae^2)^4} + \frac{(3e^6(2cd-be))\int \frac{1}{d+ex^n}\,d}{(cd^2-bde+ae^2)^4}$$

$$= -\frac{x(2b^3cde-6abc^2de-b^4e^2-b^2c(cd^2-4ae^2)+2ac^2(cd^2-ae^2)+c(2}{2a\,(b^2-4ac)\,(cd^2-bde+ae^2)^2\,n\,(a+}$$

$$= -\frac{x(2b^3cde-6abc^2de-b^4e^2-b^2c(cd^2-4ae^2)+2ac^2(cd^2-ae^2)+c(2}{2a\,(b^2-4ac)\,(cd^2-bde+ae^2)^2\,n\,(a+}$$

$$= -\frac{x(2b^3cde-6abc^2de-b^4e^2-b^2c(cd^2-4ae^2)+2ac^2(cd^2-ae^2)+c(2}{2a\,(b^2-4ac)\,(cd^2-bde+ae^2)^2\,n\,(a+}$$

$$= -\frac{x(2b^3cde-6abc^2de-b^4e^2-b^2c(cd^2-4ae^2)+2ac^2(cd^2-ae^2)+c(2}{2a\,(b^2-4ac)\,(cd^2-bde+ae^2)^2\,n\,(a+}$$

Mathematica [**B**] Leaf count is larger than twice the leaf count of optimal. 56566 vs. $2(2446) = 4892$.

time = 8.89, size = 56566, normalized size = 23.13

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/((d + e*x^n)^2*(a + b*x^n + c*x^(2*n))^3),x]
```

```
[Out] Result too large to show
```

Maple [F]
time = 0.07, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(d+e\,x^{n}\right)^{2} \left(a+b\,x^{n}+c\,x^{2 n}\right)^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(d+e*x^n)^2/(a+b*x^n+c*x^(2*n))^3,x)
```

```
[Out] int(1/(d+e*x^n)^2/(a+b*x^n+c*x^(2*n))^3,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d+e*x^n)^2/(a+b*x^n+c*x^(2*n))^3,x, algorithm="maxima")
```

```
[Out] (c*d^2*(7*n - 1)*e^6 - b*d*(4*n - 1)*e^7 + a*(n - 1)*e^8)*integrate(1/(c^4*
d^10*n - 4*b*c^3*d^9*n*e - 4*a^3*b*d^3*n*e^7 + a^4*d^2*n*e^8 + (c^4*d^9*n*e
 - 4*b*c^3*d^8*n*e^2 - 4*a^3*b*d^2*n*e^8 + a^4*d*n*e^9 + 2*(3*a^2*b^2*d^3*n
 + 2*a^3*c*d^3*n)*e^7 - 4*(a*b^3*d^4*n + 3*a^2*b*c*d^4*n)*e^6 + (b^4*d^5*n
+ 12*a*b^2*c*d^5*n + 6*a^2*c^2*d^5*n)*e^5 - 4*(b^3*c*d^6*n + 3*a*b*c^2*d^6*
n)*e^4 + 2*(3*b^2*c^2*d^7*n + 2*a*c^3*d^7*n)*e^3)*x^n + 2*(3*a^2*b^2*d^4*n
+ 2*a^3*c*d^4*n)*e^6 - 4*(a*b^3*d^5*n + 3*a^2*b*c*d^5*n)*e^5 + (b^4*d^6*n +
 12*a*b^2*c*d^6*n + 6*a^2*c^2*d^6*n)*e^4 - 4*(b^3*c*d^7*n + 3*a*b*c^2*d^7*n
)*e^3 + 2*(3*b^2*c^2*d^8*n + 2*a*c^3*d^8*n)*e^2), x) + 1/2*((2*(a^2*b^4*c^2
*n - 8*a^3*b^2*c^3*n + 16*a^4*c^4*n)*e^6 + (2*a^3*b*c^4*d*(33*n - 4) - 6*a^
2*b^3*c^3*d*(7*n - 1) + a*b^5*c^2*d*(6*n - 1))*e^5 + (2*a^2*b^2*c^4*d^2*(29
*n - 1) - 8*a^3*c^5*d^2*(11*n - 1) - b^6*c^2*d^2*(2*n - 1) + 2*a*b^4*c^3*d^
2*(n - 2))*e^4 - 3*(a*b^3*c^4*d^3*(12*n - 5) - b^5*c^3*d^3*(2*n - 1) - 4*a^
2*b*c^5*d^3*(n - 1))*e^3 + (14*a*b^2*c^5*d^4*(3*n - 1) - 8*a^2*c^6*d^4*(3*n
 - 1) - 3*b^4*c^4*d^4*(2*n - 1))*e^2 - (2*a*b*c^6*d^5*(7*n - 2) - b^3*c^5*d
^5*(2*n - 1))*e)*x*x^(4*n) - (2*a*b*c^6*d^6*(7*n - 2) - b^3*c^5*d^6*(2*n -
1) - 4*(a^2*b^5*c*n - 8*a^3*b^3*c^2*n + 16*a^4*b*c^3*n)*e^6 - (a^3*b^2*c^3*
```

d*(163*n - 21) - a^2*b^4*c^2*d*(89*n - 13) - 4*a^4*c^4*d*(8*n - 1) + 2*a*b^
6*c*d*(6*n - 1))*e^5 - (a^2*b^3*c^3*d^2*(77*n + 5) - 6*a^3*b*c^4*d^2*(27*n
- 2) + a*b^5*c^2*d^2*(11*n - 10) - 2*b^7*c*d^2*(2*n - 1))*e^4 + (a*b^4*c^3*
d^3*(73*n - 29) - 2*a^2*b^2*c^4*d^3*(50*n - 19) + 8*a^3*c^5*d^3*(5*n - 1) -
5*b^6*c^2*d^3*(2*n - 1))*e^3 - (a*b^3*c^4*d^4*(51*n - 16) - 8*a^2*b*c^5*d^
4*(9*n - 2) - 3*b^5*c^3*d^4*(2*n - 1))*e^2 - (a*b^2*c^5*d^5*(13*n - 5) - b^
4*c^4*d^5*(2*n - 1) - 4*a^2*c^6*d^5*(2*n - 1))*e)*x*x^(3*n) - (a*b^2*c^5*d^
6*(29*n - 9) - 4*a^2*c^6*d^6*(4*n - 1) - 2*b^4*c^4*d^6*(2*n - 1) - 2*(a^2*b
^6*n - 6*a^3*b^4*c*n + 32*a^5*c^3*n)*e^6 - (a^3*b^3*c^2*d*(48*n - 7) + 2*a^
4*b*c^3*d*(23*n - 2) - 2*a^2*b^5*c*d*(20*n - 3) + a*b^7*d*(6*n - 1))*e^5 -
(a^3*b^2*c^3*d^2*(81*n - 11) - 4*a^4*c^4*d^2*(34*n - 3) - a^2*b^4*c^2*d^2*(
21*n - 10) - b^8*d^2*(2*n - 1) + 6*a*b^6*c*d^2*(2*n - 1))*e^4 + (a*b^5*c^2*
d^3*(25*n - 6) - 5*a^2*b^3*c^3*d^3*(19*n - 2) + 8*a^3*b*c^4*d^3*(18*n - 1)
- b^7*c*d^3*(2*n - 1))*e^3 + (a*b^4*c^3*d^4*(39*n - 19) - 10*a^2*b^2*c^4*d^
4*(4*n - 3) - 3*b^6*c^2*d^4*(2*n - 1) - 8*a^3*c^5*d^4*(n + 1))*e^2 + (2*a^2
*b*c^5*d^5*(43*n - 14) - 3*a*b^3*c^4*d^5*(25*n - 9) + 5*b^5*c^3*d^5*(2*n -
1))*e)*x*x^(2*n) - (4*a*b^3*c^4*d^6*(3*n - 1) - b^5*c^3*d^6*(2*n - 1) + 2*a
^2*b*c^5*d^6*n - 4*(a^3*b^5*n - 8*a^4*b^3*c*n + 16*a^5*b*c^2*n)*e^6 - (a^4*
b^2*c^2*d*(115*n - 13) - a^3*b^4*c*d*(55*n - 7) - 4*a^5*c^3*d*(10*n - 1) +
a^2*b^6*d*(7*n - 1))*e^5 + (2*a^4*b*c^3*d^2*(55*n - 4) - 3*a^3*b^3*c^2*d^2*
(35*n - 2) + a^2*b^5*c*d^2*(31*n - 1) - 3*a*b^7*d^2*n)*e^4 - (a*b^6*c*d^3*(
9*n - 7) + a^2*b^4*c^2*d^3*(8*n + 15) - 2*a^3*b^2*c^3*d^3*(8*n + 7) - 8*a^4
*c^4*d^3*(7*n - 1) - b^8*d^3*(2*n - 1))*e^3 - (a^2*b^3*c^3*d^4*(41*n - 26)
- 3*a*b^5*c^2*d^4*(13*n - 6) + 3*b^7*c*d^4*(2*n - 1) - 8*a^3*b*c^4*d^4*(n -
1))*e^2 + (a^2*b^2*c^4*d^5*(23*n - 11) - 3*a*b^4*c^3*d^5*(13*n - 5) + 4*a^
3*c^5*d^5*(4*n - 1) + 3*b^6*c^2*d^5*(2*n - 1))*e)*x*x^n - (a^2*b^2*c^4*d^6*
(21*n - 5) - 4*a^3*c^5*d^6*(6*n - 1) - a*b^4*c^3*d^6*(3*n - 1) - 2*(a^4*b^4
*n - 8*a^5*b^2*c*n + 16*a^6*c^2*n)*e^6 - (a^4*b^2*c^2*d^2*(115*n - 13) - a^
3*b^4*c*d^2*(55*n - 7) - 4*a^5*c^3*d^2*(10*n - 1) + a^2*b^6*d^2*(7*n - 1))*
e^4 - (a^3*b^3*c^2*d^3*(57*n + 1) - 12*a^4*b*c^3*d^3*(13*n - 1) + a^2*b^5*c
d^3(9*n - 5) - a*b^7*d^3*(3*n - 1))*e^3 - 3*(2*a^3*b^2*c^3*d^4*(11*n - 4)
- 2*a^2*b^4*c^2*d^4*(10*n - 3) + a*b^6*c*d^4*(3*n - 1) + 16*a^4*c^4*d^4*n)
*e^2 - (a^2*b^3*c^3*d^5*(65*n - 17) - 4*a^3*b*c^4*d^5*(23*n - 5) - 3*a*b^5*
c^2*d^5*(3*n - 1))*e)*x)/(a^4*b^4*c^3*d^8*n^2 - 8*a^5*b^2*c^4*d^8*n^2 + 16*
a^6*c^5*d^8*n^2 + ((a^5*b^4*c^2*d*n^2 - 8*a^6*b^2*c^3*d*n^2 + 16*a^7*c^4*d*
n^2)*e^7 - 3*(a^4*b^5*c^2*d^2*n^2 - 8*a^5*b^3*c^3*d^2*n^2 + 16*a^6*b*c^4*d^
2*n^2)*e^6 + 3*(a^3*b^6*c^2*d^3*n^2 - 7*a^4*b^4*c^3*d^3*n^2 + 8*a^5*b^2*c^4
*d^3*n^2 + 16*a^6*c^5*d^3*n^2)*e^5 - (a^2*b^7*c^2*d^4*n^2 - 2*a^3*b^5*c^3*d
^4*n^2 - 32*a^4*b^3*c^4*d^4*n^2 + 96*a^5*b*c^5*d^4*n^2)*e^4 + 3*(a^2*b^6*c^
3*d^5*n^2 - 7*a^3*b^4*c^4*d^5*n^2 + 8*a^4*b^2*c^5*d^5*n^2 + 16*a^5*c^6*d^5*
n^2)*e^3 - 3*(a^2*b^5*c^4*d^6*n^2 - 8*a^3*b^3*c^5*d^6*n^2 + 16*a^4*b*c^6*d^
6*n^2)*e^2 + (a^2*b^4*c^5*d^7*n^2 - 8*a^3*b^2*c^6*d^7*n^2 + 16*a^4*c^7*d^7*
n^2)*e)*x^(5*n) + (a^2*b^4*c^5*d^8*n^2 - 8*a^3*b^2*c^6*d^8*n^2 + 16*a^4*c^7
*d^8*n^2 + 2*(a^5*b^5*c*d*n^2 - 8*a^6*b^3*c^2*d*n^2 + 16*a^7*b*c^3*d*n^2)*e
^7 - (6*a^4*b^6*c*d^2*n^2 - 49*a^5*b^4*c^2*d^2*n^2 + 104*a^6*b^2*c^3*d^2*n^

```
2 - 16*a^7*c^4*d^2*n^2)*e^6 + 3*(2*a^3*b^7*c*d^3*n^2 - 15*a^4*b^5*c^2*d^3*n
^2 + 24*a^5*b^3*c^3*d^3*n^2 + 16*a^6*b*c^4*d^3*n^2)*e^5 - (2*a^2*b^8*c*d^4*
n^2 - 7*a^3*b^6*c^2*d^4*n^2 - 43*a^4*b^4*c^3*d^4*n^2 + 168*a^5*b^2*c^4*d^4*
n^2 - 48*a^6*c^5*d^4*n^2)*e^4 + 5*(a^2*b^7*c^2*d^5*n^2 - 8*a^3*b^5*c^3*d^5*
n^2 + 16*a^4*b^3*c^4*d^5*n^2)*e^3 - 3*(a^2*b^6*...
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d+e*x^n)^2/(a+b*x^n+c*x^(2*n))^3,x, algorithm="fricas")
```

```
[Out] integral(1/(a^3*d^2 + b^3*x^(5*n)*e^2 + (2*c^3*d*x^n*e + c^3*d^2 + c^3*x^(2
*n)*e^2)*x^(6*n) + 3*(a*c^2*d^2 + b*c^2*x^(3*n)*e^2 + (2*b*c^2*d*e + a*c^2*
e^2)*x^(2*n) + (b*c^2*d^2 + 2*a*c^2*d*e)*x^n)*x^(4*n) + (2*b^3*d*e + 3*a*b^
2*e^2)*x^(4*n) + (b^3*d^2 + 6*a*b^2*d*e + 3*a^2*b*e^2)*x^(3*n) + (3*a*b^2*d
^2 + 6*a^2*b*d*e + a^3*e^2)*x^(2*n) + 3*(a^2*c*d^2 + b^2*c*x^(4*n)*e^2 + 2*
(b^2*c*d*e + a*b*c*e^2)*x^(3*n) + (b^2*c*d^2 + 4*a*b*c*d*e + a^2*c*e^2)*x^(
2*n) + 2*(a*b*c*d^2 + a^2*c*d*e)*x^n)*x^(2*n) + (3*a^2*b*d^2 + 2*a^3*d*e)*x
^n), x)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: HeuristicGCDFailed

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d+e*x**n)**2/(a+b*x**n+c*x**(2*n))**3,x)
```

```
[Out] Exception raised: HeuristicGCDFailed >> no luck
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d+e*x^n)^2/(a+b*x^n+c*x^(2*n))^3,x, algorithm="giac")
```

```
[Out] integrate(1/((c*x^(2*n) + b*x^n + a)^3*(x^n*e + d)^2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{{\left(d+e\,x^n\right)}^2\,{\left(a+b\,x^n+c\,x^{2\,n}\right)}^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((d + e*x^n)^2*(a + b*x^n + c*x^(2*n))^3),x)
[Out] int(1/((d + e*x^n)^2*(a + b*x^n + c*x^(2*n))^3), x)
```

3.85 $\int (d+ex^n)\sqrt{a+bx^n+cx^{2n}}\,dx$

Optimal. Leaf size=292

$$\frac{ex^{1+n}\sqrt{a+bx^n+cx^{2n}}\,F_1\left(1+\frac{1}{n};-\frac{1}{2},-\frac{1}{2};2+\frac{1}{n};-\frac{2cx^n}{b-\sqrt{b^2-4ac}},-\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)}{(1+n)\sqrt{1+\frac{2cx^n}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^n}{b+\sqrt{b^2-4ac}}}}+\frac{dx\sqrt{a+bx^n+cx^{2n}}\,F}{\sqrt{1+}}$$

```
[Out] e*x^(1+n)*AppellF1(1+1/n,-1/2,-1/2,2+1/n,-2*c*x^n/(b-(-4*a*c+b^2)^(1/2)),-2
*c*x^n/(b+(-4*a*c+b^2)^(1/2)))*(a+b*x^n+c*x^(2*n))^(1/2)/(1+n)/(1+2*c*x^n/(
b-(-4*a*c+b^2)^(1/2)))^(1/2)/(1+2*c*x^n/(b+(-4*a*c+b^2)^(1/2)))^(1/2)+d*x*A
ppellF1(1/n,-1/2,-1/2,1+1/n,-2*c*x^n/(b-(-4*a*c+b^2)^(1/2)),-2*c*x^n/(b+(-4
*a*c+b^2)^(1/2)))*(a+b*x^n+c*x^(2*n))^(1/2)/(1+2*c*x^n/(b-(-4*a*c+b^2)^(1/2
)))^(1/2)/(1+2*c*x^n/(b+(-4*a*c+b^2)^(1/2)))^(1/2)
```

Rubi [A]
time = 0.24, antiderivative size = 292, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.192$, Rules used = {1446, 1362, 440, 1399, 524}

$$\frac{dx\sqrt{a+bx^n+cx^{2n}}\,F_1\left(\frac{1}{n};-\frac{1}{2},-\frac{1}{2};1+\frac{1}{n};-\frac{2cx^n}{b-\sqrt{b^2-4ac}},-\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)}{\sqrt{\frac{2cx^n}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^n}{\sqrt{b^2-4ac}+b}+1}}+\frac{ex^{n+1}\sqrt{a+bx^n+cx^{2n}}\,F_1\left(1+\frac{1}{n};-\frac{1}{2},-\frac{1}{2};2+\frac{1}{n};-\frac{2cx^n}{b-\sqrt{b^2-4ac}},-\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)}{(n+1)\sqrt{\frac{2cx^n}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^n}{\sqrt{b^2-4ac}+b}+1}}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x^n)*Sqrt[a + b*x^n + c*x^(2*n)],x]
```

```
[Out] (e*x^(1 + n)*Sqrt[a + b*x^n + c*x^(2*n)]*AppellF1[1 + n^(-1), -1/2, -1/2, 2
 + n^(-1), (-2*c*x^n)/(b - Sqrt[b^2 - 4*a*c]), (-2*c*x^n)/(b + Sqrt[b^2 - 4
*a*c])])/((1 + n)*Sqrt[1 + (2*c*x^n)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[1 + (2*c
*x^n)/(b + Sqrt[b^2 - 4*a*c])]) + (d*x*Sqrt[a + b*x^n + c*x^(2*n)]*AppellF1
[n^(-1), -1/2, -1/2, 1 + n^(-1), (-2*c*x^n)/(b - Sqrt[b^2 - 4*a*c]), (-2*c*
x^n)/(b + Sqrt[b^2 - 4*a*c])])/(Sqrt[1 + (2*c*x^n)/(b - Sqrt[b^2 - 4*a*c])]
*Sqrt[1 + (2*c*x^n)/(b + Sqrt[b^2 - 4*a*c])])
```

Rule 440

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, (-b)*(x^n/a), (-d)*(x^n/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 524

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
```

```
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 1362

```
Int[((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^
IntPart[p]*((a + b*x^n + c*x^(2*n))^FracPart[p]/((1 + 2*c*(x^n/(b + Rt[b^2
- 4*a*c, 2])))^FracPart[p]*(1 + 2*c*(x^n/(b - Rt[b^2 - 4*a*c, 2])))^FracPar
t[p])), Int[(1 + 2*c*(x^n/(b + Sqrt[b^2 - 4*a*c])))^p*(1 + 2*c*(x^n/(b - Sq
rt[b^2 - 4*a*c])))^p, x], x] /; FreeQ[{a, b, c, n, p}, x] && EqQ[n2, 2*n] &
& NeQ[b^2 - 4*a*c, 0] &&  !IntegerQ[p]
```

Rule 1399

```
Int[((d_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x
_Symbol] :> Dist[a^IntPart[p]*((a + b*x^n + c*x^(2*n))^FracPart[p]/((1 + 2*
c*(x^n/(b + Rt[b^2 - 4*a*c, 2])))^FracPart[p]*(1 + 2*c*(x^n/(b - Rt[b^2 - 4
*a*c, 2])))^FracPart[p])), Int[(d*x)^m*(1 + 2*c*(x^n/(b + Sqrt[b^2 - 4*a*c]
)))^p*(1 + 2*c*(x^n/(b - Sqrt[b^2 - 4*a*c])))^p, x], x] /; FreeQ[{a, b, c,
d, m, n, p}, x] && EqQ[n2, 2*n]
```

Rule 1446

```
Int[((d_) + (e_.)*(x_)^(n_))*((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_))^(p
_), x_Symbol] :> Int[ExpandIntegrand[(d + e*x^n)*(a + b*x^n + c*x^(2*n))^p,
 x], x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c,
 0]
```

Rubi steps

$$\int (d+ex^n)\sqrt{a+bx^n+cx^{2n}}\,dx = \int \left(d\sqrt{a+bx^n+cx^{2n}} + ex^n\sqrt{a+bx^n+cx^{2n}}\right)\,dx$$

$$= d\int \sqrt{a+bx^n+cx^{2n}}\,dx + e\int x^n\sqrt{a+bx^n+cx^{2n}}\,dx$$

$$= \frac{\left(d\sqrt{a+bx^n+cx^{2n}}\right)\int \sqrt{1+\frac{2cx^n}{b-\sqrt{b^2-4ac}}}\ \sqrt{1+\frac{2cx^n}{b+\sqrt{b^2-4ac}}}}{\sqrt{1+\frac{2cx^n}{b-\sqrt{b^2-4ac}}}\ \sqrt{1+\frac{2cx^n}{b+\sqrt{b^2-4ac}}}}$$

$$= \frac{ex^{1+n}\sqrt{a+bx^n+cx^{2n}}\,F_1\left(1+\frac{1}{n};-\frac{1}{2},-\frac{1}{2};2+\frac{1}{n};-\frac{2cx^n}{b-\sqrt{b^2-4ac}},-\frac{}{b+}\right.}{(1+n)\sqrt{1+\frac{2cx^n}{b-\sqrt{b^2-4ac}}}\ \sqrt{1+\frac{2cx^n}{b+\sqrt{b^2-4ac}}}}$$

Mathematica [**A**]
time = 1.07, size = 424, normalized size = 1.45

$$\frac{x\left(-n(-4ace(1+n)+b^2e(2+n)-2bcd(1+2n))x^n\sqrt{\frac{b-\sqrt{b^2-4ac}+2cx^n}{b-\sqrt{b^2-4ac}}}\sqrt{\frac{b+\sqrt{b^2-4ac}+2cx^n}{b+\sqrt{b^2-4ac}}}F_1\left(1+\frac{1}{n};\frac{1}{2},\frac{1}{2};2+\frac{1}{n};-\frac{2cx^n}{b+\sqrt{b^2-4ac}},\frac{2cx^n}{-b+\sqrt{b^2-4ac}}\right)+2(1+n)\left((a+x^n(b+cx^n))(ben+2c(d+2dn+e(1+n)x^n))+an(-be+2c(d+2dn))\sqrt{\frac{b-\sqrt{b^2-4ac}+2cx^n}{b-\sqrt{b^2-4ac}}}\sqrt{\frac{b+\sqrt{b^2-4ac}+2cx^n}{b+\sqrt{b^2-4ac}}}F_1\left(\frac{1}{n};\frac{1}{2},\frac{1}{2};1+\frac{1}{n};-\frac{2cx^n}{b+\sqrt{b^2-4ac}},\frac{2cx^n}{-b+\sqrt{b^2-4ac}}\right)\right)\right)}{4(1+n)^2(c+2cn)\sqrt{a+x^n(b+cx^n)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x^n)*Sqrt[a + b*x^n + c*x^(2*n)],x]
```

```
[Out] (x*(-(n*(-4*a*c*e*(1 + n) + b^2*e*(2 + n) - 2*b*c*d*(1 + 2*n))*x^n*Sqrt[(b
- Sqrt[b^2 - 4*a*c] + 2*c*x^n)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[(b + Sqrt[b^2
- 4*a*c] + 2*c*x^n)/(b + Sqrt[b^2 - 4*a*c])]*AppellF1[1 + n^(-1), 1/2, 1/2,
 2 + n^(-1), (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^n)/(-b + Sqrt[b^2 -
 4*a*c])]) + 2*(1 + n)*((a + x^n*(b + c*x^n))*(b*e*n + 2*c*(d + 2*d*n + e*(
1 + n)*x^n)) + a*n*(-(b*e) + 2*c*(d + 2*d*n))*Sqrt[(b - Sqrt[b^2 - 4*a*c] +
 2*c*x^n)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^n)/(
b + Sqrt[b^2 - 4*a*c])]*AppellF1[n^(-1), 1/2, 1/2, 1 + n^(-1), (-2*c*x^n)/(
b + Sqrt[b^2 - 4*a*c]), (2*c*x^n)/(-b + Sqrt[b^2 - 4*a*c])])))/(4*(1 + n)^2
*(c + 2*c*n)*Sqrt[a + x^n*(b + c*x^n)])
```

Maple [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \left(d+e\,x^{n}\right)\sqrt{a+b\,x^{n}+c\,x^{2n}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d+e*x^n)*(a+b*x^n+c*x^(2*n))^(1/2),x)
```

```
[Out] int((d+e*x^n)*(a+b*x^n+c*x^(2*n))^(1/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+e*x^n)*(a+b*x^n+c*x^(2*n))^(1/2),x, algorithm="maxima")
[Out] integrate(sqrt(c*x^(2*n) + b*x^n + a)*(x^n*e + d), x)
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+e*x^n)*(a+b*x^n+c*x^(2*n))^(1/2),x, algorithm="fricas")
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (has polynomial part)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (d + ex^n)\sqrt{a + bx^n + cx^{2n}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+e*x**n)*(a+b*x**n+c*x**(2*n))**(1/2),x)
[Out] Integral((d + e*x**n)*sqrt(a + b*x**n + c*x**(2*n)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+e*x^n)*(a+b*x^n+c*x^(2*n))^(1/2),x, algorithm="giac")
[Out] integrate(sqrt(c*x^(2*n) + b*x^n + a)*(x^n*e + d), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int (d + e\,x^n)\,\sqrt{a + b\,x^n + c\,x^{2\,n}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x^n)*(a + b*x^n + c*x^(2*n))^(1/2),x)
[Out] int((d + e*x^n)*(a + b*x^n + c*x^(2*n))^(1/2), x)
```

3.86 $\int (d+ex^n)\left(a+bx^n+cx^{2n}\right)^{3/2}\,dx$

Optimal. Leaf size=294

$$\frac{aex^{1+n}\sqrt{a+bx^n+cx^{2n}}\,F_1\left(1+\frac{1}{n};-\frac{3}{2},-\frac{3}{2};2+\frac{1}{n};-\frac{2cx^n}{b-\sqrt{b^2-4ac}},-\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)}{(1+n)\sqrt{1+\frac{2cx^n}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^n}{b+\sqrt{b^2-4ac}}}}+\frac{adx\sqrt{a+bx^n+cx^2}}{\sqrt{1}}$$

```
[Out] a*e*x^(1+n)*AppellF1(1+1/n,-3/2,-3/2,2+1/n,-2*c*x^n/(b-(-4*a*c+b^2)^(1/2)),
-2*c*x^n/(b+(-4*a*c+b^2)^(1/2)))*(a+b*x^n+c*x^(2*n))^(1/2)/(1+n)/(1+2*c*x^n
/(b-(-4*a*c+b^2)^(1/2)))^(1/2)/(1+2*c*x^n/(b+(-4*a*c+b^2)^(1/2)))^(1/2)+a*d
*x*AppellF1(1/n,-3/2,-3/2,1+1/n,-2*c*x^n/(b-(-4*a*c+b^2)^(1/2)),-2*c*x^n/(b
+(-4*a*c+b^2)^(1/2)))*(a+b*x^n+c*x^(2*n))^(1/2)/(1+2*c*x^n/(b-(-4*a*c+b^2)^
(1/2)))^(1/2)/(1+2*c*x^n/(b+(-4*a*c+b^2)^(1/2)))^(1/2)
```

Rubi [A]
time = 0.23, antiderivative size = 294, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.192$, Rules used = {1446, 1362, 440, 1399, 524}

$$\frac{adx\sqrt{a+bx^n+cx^{2n}}\,F_1\left(\frac{1}{n};-\frac{3}{2},-\frac{3}{2};1+\frac{1}{n};-\frac{2cx^n}{b-\sqrt{b^2-4ac}},-\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)}{\sqrt{\frac{2cx^n}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^n}{\sqrt{b^2-4ac}+b}+1}}+\frac{aex^{n+1}\sqrt{a+bx^n+cx^{2n}}\,F_1\left(1+\frac{1}{n};-\frac{3}{2},-\frac{3}{2};2+\frac{1}{n};-\frac{2cx^n}{b-\sqrt{b^2-4ac}},-\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)}{(n+1)\sqrt{\frac{2cx^n}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^n}{\sqrt{b^2-4ac}+b}+1}}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x^n)*(a + b*x^n + c*x^(2*n))^(3/2),x]
```

```
[Out] (a*e*x^(1 + n)*Sqrt[a + b*x^n + c*x^(2*n)]*AppellF1[1 + n^(-1), -3/2, -3/2,
 2 + n^(-1), (-2*c*x^n)/(b - Sqrt[b^2 - 4*a*c]), (-2*c*x^n)/(b + Sqrt[b^2 -
 4*a*c])])/((1 + n)*Sqrt[1 + (2*c*x^n)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[1 + (2
*c*x^n)/(b + Sqrt[b^2 - 4*a*c])]) + (a*d*x*Sqrt[a + b*x^n + c*x^(2*n)]*Appe
llF1[n^(-1), -3/2, -3/2, 1 + n^(-1), (-2*c*x^n)/(b - Sqrt[b^2 - 4*a*c]), (-
2*c*x^n)/(b + Sqrt[b^2 - 4*a*c])])/(Sqrt[1 + (2*c*x^n)/(b - Sqrt[b^2 - 4*a*
c])]*Sqrt[1 + (2*c*x^n)/(b + Sqrt[b^2 - 4*a*c])])
```

Rule 440

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, (-b)*(x^n/a), (-d)*(x^n/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 524

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
```

```
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 1362

```
Int[((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^
IntPart[p]*((a + b*x^n + c*x^(2*n))^FracPart[p]/((1 + 2*c*(x^n/(b + Rt[b^2
- 4*a*c, 2])))^FracPart[p]*(1 + 2*c*(x^n/(b - Rt[b^2 - 4*a*c, 2])))^FracPar
t[p])), Int[(1 + 2*c*(x^n/(b + Sqrt[b^2 - 4*a*c])))^p*(1 + 2*c*(x^n/(b - Sq
rt[b^2 - 4*a*c])))^p, x], x] /; FreeQ[{a, b, c, n, p}, x] && EqQ[n2, 2*n] &
& NeQ[b^2 - 4*a*c, 0] &&  !IntegerQ[p]
```

Rule 1399

```
Int[((d_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x
_Symbol] :> Dist[a^IntPart[p]*((a + b*x^n + c*x^(2*n))^FracPart[p]/((1 + 2*
c*(x^n/(b + Rt[b^2 - 4*a*c, 2])))^FracPart[p]*(1 + 2*c*(x^n/(b - Rt[b^2 - 4
*a*c, 2])))^FracPart[p])), Int[(d*x)^m*(1 + 2*c*(x^n/(b + Sqrt[b^2 - 4*a*c]
)))^p*(1 + 2*c*(x^n/(b - Sqrt[b^2 - 4*a*c])))^p, x], x] /; FreeQ[{a, b, c,
d, m, n, p}, x] && EqQ[n2, 2*n]
```

Rule 1446

```
Int[((d_) + (e_.)*(x_)^(n_))*((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_))^(p
_), x_Symbol] :> Int[ExpandIntegrand[(d + e*x^n)*(a + b*x^n + c*x^(2*n))^p,
 x], x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c,
 0]
```

Rubi steps

$$\begin{aligned}\int (d+ex^n)\left(a+bx^n+cx^{2n}\right)^{3/2}\,dx &= \int \left(d\left(a+bx^n+cx^{2n}\right)^{3/2}+ex^n\left(a+bx^n+cx^{2n}\right)^{3/2}\right)dx \\ &= d\int \left(a+bx^n+cx^{2n}\right)^{3/2}dx + e\int x^n\left(a+bx^n+cx^{2n}\right)^{3/2}dx \\ &= \frac{\left(ad\sqrt{a+bx^n+cx^{2n}}\right)\int\left(1+\frac{2cx^n}{b-\sqrt{b^2-4ac}}\right)^{3/2}\left(1+\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right.}{\sqrt{1+\frac{2cx^n}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^n}{b+\sqrt{b^2-4ac}}}} \\ &= \frac{aex^{1+n}\sqrt{a+bx^n+cx^{2n}}\,F_1\left(1+\frac{1}{n};-\frac{3}{2},-\frac{3}{2};2+\frac{1}{n};-\frac{2cx^n}{b-\sqrt{b^2-4ac}},\right.}{(1+n)\sqrt{1+\frac{2cx^n}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^n}{b+\sqrt{b^2-4ac}}}}\end{aligned}$$

Mathematica [**B**] Leaf count is larger than twice the leaf count of optimal. 690 vs. 2(294) = 588.
time = 3.27, size = 690, normalized size = 2.35

Warning: Unable to verify antiderivative.

```
[In] Integrate[(d + e*x^n)*(a + b*x^n + c*x^(2*n))^(3/2),x]
```

```
[Out] (x*(3*n^2*(16*a^2*c^2*e*(1 + 4*n + 3*n^2) + b^4*e*(4 + 8*n + 3*n^2) - 2*b^3
*c*d*(2 + 9*n + 4*n^2) - 4*a*b^2*c*e*(5 + 14*n + 6*n^2) + 8*a*b*c^2*d*(2 +
11*n + 12*n^2))*x^n*Sqrt[(b - Sqrt[b^2 - 4*a*c] + 2*c*x^n)/(b - Sqrt[b^2 -
4*a*c])]*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^n)/(b + Sqrt[b^2 - 4*a*c])]*Ap
pellF1[1 + n^(-1), 1/2, 1/2, 2 + n^(-1), (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c])
, (2*c*x^n)/(-b + Sqrt[b^2 - 4*a*c])] + 2*(1 + n)*((a + x^n*(b + c*x^n))*(-
3*b^3*e*n^2*(2 + 3*n) + 6*b^2*c*n^2*(d + 4*d*n + e*(1 + n)*x^n) + 8*c^3*(1
+ 3*n + 2*n^2)*x^(2*n)*(d + 4*d*n + e*(1 + 3*n)*x^n) + 4*b*c^2*(1 + n)*x^n*
(d*(2 + 15*n + 28*n^2) + e*(2 + 13*n + 18*n^2)*x^n) + 4*a*c*(3*b*e*n^2*(2 +
 5*n) + 2*c*(d*(1 + 2*n)*(1 + 4*n)^2 + e*(1 + 9*n + 23*n^2 + 15*n^3)*x^n)))
 + 3*a*n^2*(b^3*e*(2 + 3*n) - 2*b^2*c*d*(1 + 4*n) - 4*a*b*c*e*(2 + 5*n) + 8
*a*c^2*d*(1 + 6*n + 8*n^2))*Sqrt[(b - Sqrt[b^2 - 4*a*c] + 2*c*x^n)/(b - Sqr
t[b^2 - 4*a*c])]*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^n)/(b + Sqrt[b^2 - 4*a
*c])]*AppellF1[n^(-1), 1/2, 1/2, 1 + n^(-1), (-2*c*x^n)/(b + Sqrt[b^2 - 4*a
*c]), (2*c*x^n)/(-b + Sqrt[b^2 - 4*a*c])])))/(16*c^2*(1 + n)^2*(1 + 2*n)*(1
 + 3*n)*(1 + 4*n)*Sqrt[a + x^n*(b + c*x^n)])
```

Maple [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \left(d+e\,x^{n}\right) \left(a+b\,x^{n}+c\,x^{2n}\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d+e*x^n)*(a+b*x^n+c*x^(2*n))^(3/2),x)
[Out] int((d+e*x^n)*(a+b*x^n+c*x^(2*n))^(3/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+e*x^n)*(a+b*x^n+c*x^(2*n))^(3/2),x, algorithm="maxima")
[Out] integrate((c*x^(2*n) + b*x^n + a)^(3/2)*(x^n*e + d), x)
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+e*x^n)*(a+b*x^n+c*x^(2*n))^(3/2),x, algorithm="fricas")
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (has polynomial part)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \left(d + ex^{n}\right) \left(a + bx^{n} + cx^{2n}\right)^{\frac{3}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+e*x**n)*(a+b*x**n+c*x**(2*n))**(3/2),x)
[Out] Integral((d + e*x**n)*(a + b*x**n + c*x**(2*n))**(3/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+e*x^n)*(a+b*x^n+c*x^(2*n))^(3/2),x, algorithm="giac")
```

```
[Out] integrate((c*x^(2*n) + b*x^n + a)^(3/2)*(x^n*e + d), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int (d+e\,x^n)\,\left(a+b\,x^n+c\,x^{2\,n}\right)^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x^n)*(a + b*x^n + c*x^(2*n))^(3/2),x)
```

```
[Out] int((d + e*x^n)*(a + b*x^n + c*x^(2*n))^(3/2), x)
```

3.87 $\int \frac{d+ex^n}{\sqrt{a+bx^n+cx^{2n}}}\,dx$

Optimal. Leaf size=292

$$\frac{ex^{1+n}\sqrt{1+\frac{2cx^n}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^n}{b+\sqrt{b^2-4ac}}}\,F_1\left(1+\frac{1}{n};\frac{1}{2},\frac{1}{2};2+\frac{1}{n};-\frac{2cx^n}{b-\sqrt{b^2-4ac}},-\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)}{(1+n)\sqrt{a+bx^n+cx^{2n}}}$$

```
[Out] e*x^(1+n)*AppellF1(1+1/n,1/2,1/2,2+1/n,-2*c*x^n/(b-(-4*a*c+b^2)^(1/2)),-2*c
*x^n/(b+(-4*a*c+b^2)^(1/2)))*(1+2*c*x^n/(b-(-4*a*c+b^2)^(1/2)))^(1/2)*(1+2*
c*x^n/(b+(-4*a*c+b^2)^(1/2)))^(1/2)/(1+n)/(a+b*x^n+c*x^(2*n))^(1/2)+d*x*App
ellF1(1/n,1/2,1/2,1+1/n,-2*c*x^n/(b-(-4*a*c+b^2)^(1/2)),-2*c*x^n/(b+(-4*a*c
+b^2)^(1/2)))*(1+2*c*x^n/(b-(-4*a*c+b^2)^(1/2)))^(1/2)*(1+2*c*x^n/(b+(-4*a*
c+b^2)^(1/2)))^(1/2)/(a+b*x^n+c*x^(2*n))^(1/2)
```

Rubi [A]
time = 0.22, antiderivative size = 292, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.192$, Rules used = {1446, 1362, 440, 1399, 524}

$$\frac{dx\sqrt{\frac{2cx^n}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^n}{\sqrt{b^2-4ac}+b}+1}\,F_1\left(\frac{1}{n};\frac{1}{2},\frac{1}{2};1+\frac{1}{n};-\frac{2cx^n}{b-\sqrt{b^2-4ac}},-\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)}{\sqrt{a+bx^n+cx^{2n}}}+\frac{ex^{n+1}\sqrt{\frac{2cx^n}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^n}{\sqrt{b^2-4ac}+b}+1}\,F_1\left(1+\frac{1}{n};\frac{1}{2},\frac{1}{2};2+\frac{1}{n};-\frac{2cx^n}{b-\sqrt{b^2-4ac}},-\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)}{(n+1)\sqrt{a+bx^n+cx^{2n}}}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x^n)/Sqrt[a + b*x^n + c*x^(2*n)],x]
```

```
[Out] (e*x^(1 + n)*Sqrt[1 + (2*c*x^n)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[1 + (2*c*x^n)
/(b + Sqrt[b^2 - 4*a*c])]*AppellF1[1 + n^(-1), 1/2, 1/2, 2 + n^(-1), (-2*c*
x^n)/(b - Sqrt[b^2 - 4*a*c]), (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c])])/((1 + n)
*Sqrt[a + b*x^n + c*x^(2*n)]) + (d*x*Sqrt[1 + (2*c*x^n)/(b - Sqrt[b^2 - 4*a
*c])]*Sqrt[1 + (2*c*x^n)/(b + Sqrt[b^2 - 4*a*c])]*AppellF1[n^(-1), 1/2, 1/2
, 1 + n^(-1), (-2*c*x^n)/(b - Sqrt[b^2 - 4*a*c]), (-2*c*x^n)/(b + Sqrt[b^2
- 4*a*c])])/Sqrt[a + b*x^n + c*x^(2*n)]
```

Rule 440

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, (-b)*(x^n/a), (-d)*(x^n/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 524

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
```

```
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 1362

```
Int[((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^
IntPart[p]*((a + b*x^n + c*x^(2*n))^FracPart[p]/((1 + 2*c*(x^n/(b + Rt[b^2
- 4*a*c, 2])))^FracPart[p]*(1 + 2*c*(x^n/(b - Rt[b^2 - 4*a*c, 2])))^FracPar
t[p])), Int[(1 + 2*c*(x^n/(b + Sqrt[b^2 - 4*a*c])))^p*(1 + 2*c*(x^n/(b - Sq
rt[b^2 - 4*a*c])))^p, x], x] /; FreeQ[{a, b, c, n, p}, x] && EqQ[n2, 2*n] &
& NeQ[b^2 - 4*a*c, 0] &&  !IntegerQ[p]
```

Rule 1399

```
Int[((d_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x
_Symbol] :> Dist[a^IntPart[p]*((a + b*x^n + c*x^(2*n))^FracPart[p]/((1 + 2*
c*(x^n/(b + Rt[b^2 - 4*a*c, 2])))^FracPart[p]*(1 + 2*c*(x^n/(b - Rt[b^2 - 4
*a*c, 2])))^FracPart[p])), Int[(d*x)^m*(1 + 2*c*(x^n/(b + Sqrt[b^2 - 4*a*c]
)))^p*(1 + 2*c*(x^n/(b - Sqrt[b^2 - 4*a*c])))^p, x], x] /; FreeQ[{a, b, c,
d, m, n, p}, x] && EqQ[n2, 2*n]
```

Rule 1446

```
Int[((d_) + (e_.)*(x_)^(n_))*((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_))^(p
_), x_Symbol] :> Int[ExpandIntegrand[(d + e*x^n)*(a + b*x^n + c*x^(2*n))^p,
 x], x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c,
 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{d+ex^n}{\sqrt{a+bx^n+cx^{2n}}}\,dx &= \int \left(\frac{d}{\sqrt{a+bx^n+cx^{2n}}}+\frac{ex^n}{\sqrt{a+bx^n+cx^{2n}}}\right) dx\\
&= d\int \frac{1}{\sqrt{a+bx^n+cx^{2n}}}\,dx+e\int \frac{x^n}{\sqrt{a+bx^n+cx^{2n}}}\,dx\\
&= \frac{\left(d\sqrt{1+\frac{2cx^n}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^n}{b+\sqrt{b^2-4ac}}}\right)\int \frac{1}{\sqrt{1+\frac{2cx^n}{b-\sqrt{b^2-4ac}}}\sqrt{\cdots}}}{\sqrt{a+bx^n+cx^{2n}}}\\
&= \frac{ex^{1+n}\sqrt{1+\frac{2cx^n}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^n}{b+\sqrt{b^2-4ac}}}\,F_1\left(1+\frac{1}{n};\frac{1}{2},\frac{1}{2};2+\frac{1}{n};-\frac{\cdots}{b-\sqrt{\cdots}}\right.}{(1+n)\sqrt{a+bx^n+cx^{2n}}}
\end{aligned}$$

Mathematica [**A**]
time = 0.32, size = 245, normalized size = 0.84

$$\frac{x\sqrt{\frac{b-\sqrt{b^2-4ac}+2cx^n}{b-\sqrt{b^2-4ac}}}\sqrt{\frac{b+\sqrt{b^2-4ac}+2cx^n}{b+\sqrt{b^2-4ac}}}\left(ex^nF_1\left(1+\frac{1}{n};\frac{1}{2},\frac{1}{2};2+\frac{1}{n};-\frac{2cx^n}{b+\sqrt{b^2-4ac}},\frac{2cx^n}{-b+\sqrt{b^2-4ac}}\right)+d(1+n)F_1\left(\frac{1}{n};\frac{1}{2},\frac{1}{2};1+\frac{1}{n};-\frac{2cx^n}{b+\sqrt{b^2-4ac}},\frac{2cx^n}{-b+\sqrt{b^2-4ac}}\right)\right)}{(1+n)\sqrt{a+x^n(b+cx^n)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x^n)/Sqrt[a + b*x^n + c*x^(2*n)],x]
```

```
[Out] (x*Sqrt[(b - Sqrt[b^2 - 4*a*c] + 2*c*x^n)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[(b
+ Sqrt[b^2 - 4*a*c] + 2*c*x^n)/(b + Sqrt[b^2 - 4*a*c])]*(e*x^n*AppellF1[1 +
 n^(-1), 1/2, 1/2, 2 + n^(-1), (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^n
)/(-b + Sqrt[b^2 - 4*a*c])] + d*(1 + n)*AppellF1[n^(-1), 1/2, 1/2, 1 + n^(-
1), (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^n)/(-b + Sqrt[b^2 - 4*a*c])]
))/((1 + n)*Sqrt[a + x^n*(b + c*x^n)])
```

Maple [**F**]
time = 0.01, size = 0, normalized size = 0.00

$$\int \frac{d+e\,x^{n}}{\sqrt{a+b\,x^{n}+c\,x^{2n}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d+e*x^n)/(a+b*x^n+c*x^(2*n))^(1/2),x)
```

```
[Out] int((d+e*x^n)/(a+b*x^n+c*x^(2*n))^(1/2),x)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+e*x^n)/(a+b*x^n+c*x^(2*n))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((x^n*e + d)/sqrt(c*x^(2*n) + b*x^n + a), x)
```

Fricas [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+e*x^n)/(a+b*x^n+c*x^(2*n))^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{d+ex^n}{\sqrt{a+bx^n+cx^{2n}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+e*x**n)/(a+b*x**n+c*x**(2*n))**(1/2),x)
```

```
[Out] Integral((d + e*x**n)/sqrt(a + b*x**n + c*x**(2*n)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+e*x^n)/(a+b*x^n+c*x^(2*n))^(1/2),x, algorithm="giac")
```

```
[Out] integrate((x^n*e + d)/sqrt(c*x^(2*n) + b*x^n + a), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{d+e\,x^n}{\sqrt{a+b\,x^n+c\,x^{2\,n}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x^n)/(a + b*x^n + c*x^(2*n))^(1/2),x)
```

```
[Out] int((d + e*x^n)/(a + b*x^n + c*x^(2*n))^(1/2), x)
```

3.88 $\int \frac{d+ex^n}{(a+bx^n+cx^{2n})^{3/2}}\,dx$

Optimal. Leaf size=298

$$\frac{ex^{1+n}\sqrt{1+\frac{2cx^n}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^n}{b+\sqrt{b^2-4ac}}}\,F_1\left(1+\frac{1}{n};\frac{3}{2},\frac{3}{2};2+\frac{1}{n};-\frac{2cx^n}{b-\sqrt{b^2-4ac}},-\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)}{a(1+n)\sqrt{a+bx^n+cx^{2n}}}$$

```
[Out] e*x^(1+n)*AppellF1(1+1/n,3/2,3/2,2+1/n,-2*c*x^n/(b-(-4*a*c+b^2)^(1/2)),-2*c
*x^n/(b+(-4*a*c+b^2)^(1/2)))*(1+2*c*x^n/(b-(-4*a*c+b^2)^(1/2)))^(1/2)*(1+2*
c*x^n/(b+(-4*a*c+b^2)^(1/2)))^(1/2)/a/(1+n)/(a+b*x^n+c*x^(2*n))^(1/2)+d*x*A
ppellF1(1/n,3/2,3/2,1+1/n,-2*c*x^n/(b-(-4*a*c+b^2)^(1/2)),-2*c*x^n/(b+(-4*a
*c+b^2)^(1/2)))*(1+2*c*x^n/(b-(-4*a*c+b^2)^(1/2)))^(1/2)*(1+2*c*x^n/(b+(-4*
a*c+b^2)^(1/2)))^(1/2)/a/(a+b*x^n+c*x^(2*n))^(1/2)
```

Rubi [A]

time = 0.24, antiderivative size = 298, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.192$, Rules used = {1446, 1362, 440, 1399, 524}

$$\frac{dx\sqrt{\frac{2cx^n}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^n}{\sqrt{b^2-4ac}+b}+1}\,F_1\left(\frac{1}{n};\frac{3}{2},\frac{3}{2};1+\frac{1}{n};-\frac{2cx^n}{b-\sqrt{b^2-4ac}},-\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)}{a\sqrt{a+bx^n+cx^{2n}}}+\frac{ex^{n+1}\sqrt{\frac{2cx^n}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^n}{\sqrt{b^2-4ac}+b}+1}\,F_1\left(1+\frac{1}{n};\frac{3}{2},\frac{3}{2};2+\frac{1}{n};-\frac{2cx^n}{b-\sqrt{b^2-4ac}},-\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)}{a(n+1)\sqrt{a+bx^n+cx^{2n}}}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x^n)/(a + b*x^n + c*x^(2*n))^(3/2),x]
```

```
[Out] (e*x^(1 + n)*Sqrt[1 + (2*c*x^n)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[1 + (2*c*x^n)
/(b + Sqrt[b^2 - 4*a*c])]*AppellF1[1 + n^(-1), 3/2, 3/2, 2 + n^(-1), (-2*c*
x^n)/(b - Sqrt[b^2 - 4*a*c]), (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c])])/(a*(1 +
n)*Sqrt[a + b*x^n + c*x^(2*n)]) + (d*x*Sqrt[1 + (2*c*x^n)/(b - Sqrt[b^2 - 4
*a*c])]*Sqrt[1 + (2*c*x^n)/(b + Sqrt[b^2 - 4*a*c])]*AppellF1[n^(-1), 3/2, 3
/2, 1 + n^(-1), (-2*c*x^n)/(b - Sqrt[b^2 - 4*a*c]), (-2*c*x^n)/(b + Sqrt[b^
2 - 4*a*c])])/(a*Sqrt[a + b*x^n + c*x^(2*n)])
```

Rule 440

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, (-b)*(x^n/a), (-d)*(x^n/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 524

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
```

```
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 1362

```
Int[((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^
IntPart[p]*((a + b*x^n + c*x^(2*n))^FracPart[p]/((1 + 2*c*(x^n/(b + Rt[b^2
- 4*a*c, 2])))^FracPart[p]*(1 + 2*c*(x^n/(b - Rt[b^2 - 4*a*c, 2])))^FracPar
t[p])), Int[(1 + 2*c*(x^n/(b + Sqrt[b^2 - 4*a*c])))^p*(1 + 2*c*(x^n/(b - Sq
rt[b^2 - 4*a*c])))^p, x], x] /; FreeQ[{a, b, c, n, p}, x] && EqQ[n2, 2*n] &
& NeQ[b^2 - 4*a*c, 0] &&  !IntegerQ[p]
```

Rule 1399

```
Int[((d_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x
_Symbol] :> Dist[a^IntPart[p]*((a + b*x^n + c*x^(2*n))^FracPart[p]/((1 + 2*
c*(x^n/(b + Rt[b^2 - 4*a*c, 2])))^FracPart[p]*(1 + 2*c*(x^n/(b - Rt[b^2 - 4
*a*c, 2])))^FracPart[p])), Int[(d*x)^m*(1 + 2*c*(x^n/(b + Sqrt[b^2 - 4*a*c]
)))^p*(1 + 2*c*(x^n/(b - Sqrt[b^2 - 4*a*c])))^p, x], x] /; FreeQ[{a, b, c,
d, m, n, p}, x] && EqQ[n2, 2*n]
```

Rule 1446

```
Int[((d_) + (e_.)*(x_)^(n_))*((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_))^(p
_), x_Symbol] :> Int[ExpandIntegrand[(d + e*x^n)*(a + b*x^n + c*x^(2*n))^p,
 x], x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c,
 0]
```

Rubi steps

$$\int \frac{d+ex^n}{(a+bx^n+cx^{2n})^{3/2}}\,dx = \int \left(\frac{d}{(a+bx^n+cx^{2n})^{3/2}} + \frac{ex^n}{(a+bx^n+cx^{2n})^{3/2}}\right) dx$$

$$= d\int \frac{1}{(a+bx^n+cx^{2n})^{3/2}}\,dx + e\int \frac{x^n}{(a+bx^n+cx^{2n})^{3/2}}\,dx$$

$$= \frac{\left(d\sqrt{1+\frac{2cx^n}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^n}{b+\sqrt{b^2-4ac}}}\right)\int \frac{1}{\left(1+\frac{2cx^n}{b-\sqrt{b^2-4ac}}\right)^{3/2}\left(1+\frac{}{b-}\right.}}{a\sqrt{a+bx^n+cx^{2n}}}$$

$$= \frac{ex^{1+n}\sqrt{1+\frac{2cx^n}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^n}{b+\sqrt{b^2-4ac}}}\,F_1\left(1+\frac{1}{n};\frac{3}{2},\frac{3}{2};2+\frac{1}{n};-\frac{}{b-\sqrt{}}\right.}{a(1+n)\sqrt{a+bx^n+cx^{2n}}}$$

Mathematica [A]
time = 1.85, size = 414, normalized size = 1.39

$$\frac{x\left(2c(bd-2ae)x^n\sqrt{\frac{b-\sqrt{b^2-4ac}+2cx^n}{b-\sqrt{b^2-4ac}}}\sqrt{\frac{b+\sqrt{b^2-4ac}+2cx^n}{b+\sqrt{b^2-4ac}}}F_1\left(1+\frac{1}{n};\frac{1}{2},\frac{1}{2};2+\frac{1}{n};-\frac{2cx^n}{b+\sqrt{b^2-4ac}},\frac{2cx^n}{-b+\sqrt{b^2-4ac}}\right)-(1+n)\left(2(b^2d+b(-ae+cdx^n)-2ac(d+ex^n))+(2abe+b^2d(-2+n)-4acd(-1+n))\sqrt{\frac{b-\sqrt{b^2-4ac}+2cx^n}{b-\sqrt{b^2-4ac}}}\sqrt{\frac{b+\sqrt{b^2-4ac}+2cx^n}{b+\sqrt{b^2-4ac}}}F_1\left(\frac{1}{n};\frac{1}{2},\frac{1}{2};1+\frac{1}{n};-\frac{2cx^n}{b+\sqrt{b^2-4ac}},\frac{2cx^n}{-b+\sqrt{b^2-4ac}}\right)\right)\right)}{a(-b^2+4ac)n(1+n)\sqrt{a+x^n(b+cx^n)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x^n)/(a + b*x^n + c*x^(2*n))^(3/2),x]
```

```
[Out] (x*(2*c*(b*d - 2*a*e)*x^n*Sqrt[(b - Sqrt[b^2 - 4*a*c] + 2*c*x^n)/(b - Sqrt[
b^2 - 4*a*c])]*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^n)/(b + Sqrt[b^2 - 4*a*c
])]*AppellF1[1 + n^(-1), 1/2, 1/2, 2 + n^(-1), (-2*c*x^n)/(b + Sqrt[b^2 - 4
*a*c]), (2*c*x^n)/(-b + Sqrt[b^2 - 4*a*c])] - (1 + n)*(2*(b^2*d + b*(-(a*e)
 + c*d*x^n) - 2*a*c*(d + e*x^n)) + (2*a*b*e + b^2*d*(-2 + n) - 4*a*c*d*(-1
+ n))*Sqrt[(b - Sqrt[b^2 - 4*a*c] + 2*c*x^n)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[
(b + Sqrt[b^2 - 4*a*c] + 2*c*x^n)/(b + Sqrt[b^2 - 4*a*c])]*AppellF1[n^(-1),
 1/2, 1/2, 1 + n^(-1), (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^n)/(-b +
Sqrt[b^2 - 4*a*c])])))/(a*(-b^2 + 4*a*c)*n*(1 + n)*Sqrt[a + x^n*(b + c*x^n)
])
```

Maple [F]
time = 0.01, size = 0, normalized size = 0.00

$$\int \frac{d+e\,x^n}{\left(a+b\,x^n+c\,x^{2n}\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d+e*x^n)/(a+b*x^n+c*x^(2*n))^(3/2),x)
```

```
[Out] int((d+e*x^n)/(a+b*x^n+c*x^(2*n))^(3/2),x)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+e*x^n)/(a+b*x^n+c*x^(2*n))^(3/2),x, algorithm="maxima")
[Out] integrate((x^n*e + d)/(c*x^(2*n) + b*x^n + a)^(3/2), x)
```

Fricas [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+e*x^n)/(a+b*x^n+c*x^(2*n))^(3/2),x, algorithm="fricas")
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{d + ex^n}{\left(a + bx^n + cx^{2n}\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+e*x**n)/(a+b*x**n+c*x**(2*n))**(3/2),x)
[Out] Integral((d + e*x**n)/(a + b*x**n + c*x**(2*n))**(3/2), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+e*x^n)/(a+b*x^n+c*x^(2*n))^(3/2),x, algorithm="giac")
[Out] integrate((x^n*e + d)/(c*x^(2*n) + b*x^n + a)^(3/2), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{d+e\,x^n}{{\left(a+b\,x^n+c\,x^{2\,n}\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x^n)/(a + b*x^n + c*x^(2*n))^(3/2),x)
[Out] int((d + e*x^n)/(a + b*x^n + c*x^(2*n))^(3/2), x)
```

3.89 $\int \frac{d+ex^n}{(a+bx^n+cx^{2n})^{5/2}}\,dx$

Optimal. Leaf size=298

$$\frac{ex^{1+n}\sqrt{1+\frac{2cx^n}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^n}{b+\sqrt{b^2-4ac}}}\,F_1\left(1+\frac{1}{n};\frac{5}{2},\frac{5}{2};2+\frac{1}{n};-\frac{2cx^n}{b-\sqrt{b^2-4ac}},-\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)}{a^2(1+n)\sqrt{a+bx^n+cx^{2n}}}$$

```
[Out] e*x^(1+n)*AppellF1(1+1/n,5/2,5/2,2+1/n,-2*c*x^n/(b-(-4*a*c+b^2)^(1/2)),-2*c
*x^n/(b+(-4*a*c+b^2)^(1/2)))*(1+2*c*x^n/(b-(-4*a*c+b^2)^(1/2)))^(1/2)*(1+2*
c*x^n/(b+(-4*a*c+b^2)^(1/2)))^(1/2)/a^2/(1+n)/(a+b*x^n+c*x^(2*n))^(1/2)+d*x
*AppellF1(1/n,5/2,5/2,1+1/n,-2*c*x^n/(b-(-4*a*c+b^2)^(1/2)),-2*c*x^n/(b+(-4
*a*c+b^2)^(1/2)))*(1+2*c*x^n/(b-(-4*a*c+b^2)^(1/2)))^(1/2)*(1+2*c*x^n/(b+(-
4*a*c+b^2)^(1/2)))^(1/2)/a^2/(a+b*x^n+c*x^(2*n))^(1/2)
```

Rubi [A]

time = 0.23, antiderivative size = 298, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.192$, Rules used = {1446, 1362, 440, 1399, 524}

$$\frac{dx\sqrt{\frac{2cx^n}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^n}{\sqrt{b^2-4ac}+b}+1}\,F_1\left(\frac{1}{n};\frac{5}{2},\frac{5}{2};1+\frac{1}{n};-\frac{2cx^n}{b-\sqrt{b^2-4ac}},-\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)}{a^2\sqrt{a+bx^n+cx^{2n}}}+\frac{ex^{n+1}\sqrt{\frac{2cx^n}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^n}{\sqrt{b^2-4ac}+b}+1}\,F_1\left(1+\frac{1}{n};\frac{5}{2},\frac{5}{2};2+\frac{1}{n};-\frac{2cx^n}{b-\sqrt{b^2-4ac}},-\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)}{a^2(n+1)\sqrt{a+bx^n+cx^{2n}}}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x^n)/(a + b*x^n + c*x^(2*n))^(5/2),x]
```

```
[Out] (e*x^(1 + n)*Sqrt[1 + (2*c*x^n)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[1 + (2*c*x^n)
/(b + Sqrt[b^2 - 4*a*c])]*AppellF1[1 + n^(-1), 5/2, 5/2, 2 + n^(-1), (-2*c*
x^n)/(b - Sqrt[b^2 - 4*a*c]), (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c])])/(a^2*(1
+ n)*Sqrt[a + b*x^n + c*x^(2*n)]) + (d*x*Sqrt[1 + (2*c*x^n)/(b - Sqrt[b^2 -
 4*a*c])]*Sqrt[1 + (2*c*x^n)/(b + Sqrt[b^2 - 4*a*c])]*AppellF1[n^(-1), 5/2,
 5/2, 1 + n^(-1), (-2*c*x^n)/(b - Sqrt[b^2 - 4*a*c]), (-2*c*x^n)/(b + Sqrt[
b^2 - 4*a*c])])/(a^2*Sqrt[a + b*x^n + c*x^(2*n)])
```

Rule 440

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, (-b)*(x^n/a), (-d)*(x^n/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 524

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
```

```
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 1362

```
Int[((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^
IntPart[p]*((a + b*x^n + c*x^(2*n))^FracPart[p]/((1 + 2*c*(x^n/(b + Rt[b^2
- 4*a*c, 2])))^FracPart[p]*(1 + 2*c*(x^n/(b - Rt[b^2 - 4*a*c, 2])))^FracPar
t[p])), Int[(1 + 2*c*(x^n/(b + Sqrt[b^2 - 4*a*c])))^p*(1 + 2*c*(x^n/(b - Sq
rt[b^2 - 4*a*c])))^p, x], x] /; FreeQ[{a, b, c, n, p}, x] && EqQ[n2, 2*n] &
& NeQ[b^2 - 4*a*c, 0] &&  !IntegerQ[p]
```

Rule 1399

```
Int[((d_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x
_Symbol] :> Dist[a^IntPart[p]*((a + b*x^n + c*x^(2*n))^FracPart[p]/((1 + 2*
c*(x^n/(b + Rt[b^2 - 4*a*c, 2])))^FracPart[p]*(1 + 2*c*(x^n/(b - Rt[b^2 - 4
*a*c, 2])))^FracPart[p])), Int[(d*x)^m*(1 + 2*c*(x^n/(b + Sqrt[b^2 - 4*a*c]
)))^p*(1 + 2*c*(x^n/(b - Sqrt[b^2 - 4*a*c])))^p, x], x] /; FreeQ[{a, b, c,
d, m, n, p}, x] && EqQ[n2, 2*n]
```

Rule 1446

```
Int[((d_) + (e_.)*(x_)^(n_))*((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_))^(p
_), x_Symbol] :> Int[ExpandIntegrand[(d + e*x^n)*(a + b*x^n + c*x^(2*n))^p,
 x], x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c,
 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{d+ex^n}{(a+bx^n+cx^{2n})^{5/2}}\,dx &= \int \left(\frac{d}{(a+bx^n+cx^{2n})^{5/2}}+\frac{ex^n}{(a+bx^n+cx^{2n})^{5/2}}\right)dx \\
&= d\int \frac{1}{(a+bx^n+cx^{2n})^{5/2}}\,dx + e\int \frac{x^n}{(a+bx^n+cx^{2n})^{5/2}}\,dx \\
&= \frac{\left(d\sqrt{1+\frac{2cx^n}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^n}{b+\sqrt{b^2-4ac}}}\right)\int \frac{1}{\left(1+\frac{2cx^n}{b-\sqrt{b^2-4ac}}\right)^{5/2}\left(1+\right.}}{a^2\sqrt{a+bx^n+cx^{2n}}} \\
&= \frac{ex^{1+n}\sqrt{1+\frac{2cx^n}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^n}{b+\sqrt{b^2-4ac}}}\,F_1\left(1+\frac{1}{n};\frac{5}{2},\frac{5}{2};2+\frac{1}{n};-\frac{}{b-}\right.}{a^2(1+n)\sqrt{a+bx^n+cx^{2n}}}
\end{aligned}
$$

Mathematica [**B**] Leaf count is larger than twice the leaf count of optimal. 701 vs. 2(298) = 596.
time = 5.64, size = 701, normalized size = 2.35

Warning: Unable to verify antiderivative.

```
[In] Integrate[(d + e*x^n)/(a + b*x^n + c*x^(2*n))^(5/2),x]
```

```
[Out] (x*(-2*c*(2*a*b^2*e + 4*a*b*c*d*(2 - 5*n) + 8*a^2*c*e*(-1 + 2*n) + b^3*d*(-
2 + 3*n))*x^n*Sqrt[(b - Sqrt[b^2 - 4*a*c] + 2*c*x^n)/(b - Sqrt[b^2 - 4*a*c]
)]*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^n)/(b + Sqrt[b^2 - 4*a*c])]*(a + x^n
*(b + c*x^n))*AppellF1[1 + n^(-1), 1/2, 1/2, 2 + n^(-1), (-2*c*x^n)/(b + Sq
rt[b^2 - 4*a*c]), (2*c*x^n)/(-b + Sqrt[b^2 - 4*a*c])] + (1 + n)*(2*(b^3*d*(
-2 + 3*n)*x^n*(b + c*x^n)^2 + 4*a^3*c*(b*e*(-2 + 3*n) + c*d*(-2 + 8*n) + 2*
c*e*(-1 + 3*n)*x^n) + 2*a*b*(b + c*x^n)*(-2*c^2*d*(-2 + 5*n)*x^(2*n) + b*c*
x^n*(d*(5 - 11*n) + e*x^n) + b^2*(d*(-1 + 2*n) + e*x^n)) + a^2*(-(b^3*e*(-2
 + n)) + 8*b*c^2*e*(-2 + 3*n)*x^(2*n) - 2*b^2*c*(d*(-5 + 14*n) - 3*e*(-1 +
n)*x^n) + 8*c^3*x^(2*n)*(d*(-1 + 3*n) + e*(-1 + 2*n)*x^n))) + (2*a*b^3*e*(-
2 + n) - 8*a^2*b*c*e*(-2 + 3*n) + b^4*d*(4 - 8*n + 3*n^2) + 16*a^2*c^2*d*(1
 - 4*n + 3*n^2) - 4*a*b^2*c*d*(5 - 14*n + 6*n^2))*Sqrt[(b - Sqrt[b^2 - 4*a*
c] + 2*c*x^n)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^
n)/(b + Sqrt[b^2 - 4*a*c])]*(a + x^n*(b + c*x^n))*AppellF1[n^(-1), 1/2, 1/2
, 1 + n^(-1), (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^n)/(-b + Sqrt[b^2
- 4*a*c])])))/(3*a^2*(b^2 - 4*a*c)^2*n^2*(1 + n)*(a + x^n*(b + c*x^n))^(3/2
))
```

Maple [**F**]
time = 0.01, size = 0, normalized size = 0.00

$$\int \frac{d + e\,x^{n}}{\left(a + b\,x^{n} + c\,x^{2n}\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d+e*x^n)/(a+b*x^n+c*x^(2*n))^(5/2),x)
```

```
[Out] int((d+e*x^n)/(a+b*x^n+c*x^(2*n))^(5/2),x)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+e*x^n)/(a+b*x^n+c*x^(2*n))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((x^n*e + d)/(c*x^(2*n) + b*x^n + a)^(5/2), x)
```

Fricas [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+e*x^n)/(a+b*x^n+c*x^(2*n))^(5/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+e*x**n)/(a+b*x**n+c*x**(2*n))**(5/2),x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+e*x^n)/(a+b*x^n+c*x^(2*n))^(5/2),x, algorithm="giac")
```

```
[Out] integrate((x^n*e + d)/(c*x^(2*n) + b*x^n + a)^(5/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{d+e\,x^n}{{\left(a+b\,x^n+c\,x^{2\,n}\right)}^{5/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x^n)/(a + b*x^n + c*x^(2*n))^(5/2),x)
```

```
[Out] int((d + e*x^n)/(a + b*x^n + c*x^(2*n))^(5/2), x)
```

3.90 $\int (d+ex^n)^q \left(a+bx^n+cx^{2n}\right)^p \, dx$

Optimal. Leaf size=29

$$\text{Int}\left((d+ex^n)^q \left(a+bx^n+cx^{2n}\right)^p, x\right)$$

```
[Out] Unintegrable((d+e*x^n)^q*(a+b*x^n+c*x^(2*n))^p,x)
```

Rubi [A]
time = 0.01, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int (d+ex^n)^q \left(a+bx^n+cx^{2n}\right)^p \, dx$$

Verification is not applicable to the result.

```
[In] Int[(d + e*x^n)^q*(a + b*x^n + c*x^(2*n))^p,x]
[Out] Defer[Int][(d + e*x^n)^q*(a + b*x^n + c*x^(2*n))^p, x]
```

Rubi steps

$$\int (d+ex^n)^q \left(a+bx^n+cx^{2n}\right)^p \, dx = \int (d+ex^n)^q \left(a+bx^n+cx^{2n}\right)^p \, dx$$

Mathematica [A]
time = 0.26, size = 0, normalized size = 0.00

$$\int (d+ex^n)^q \left(a+bx^n+cx^{2n}\right)^p \, dx$$

Verification is not applicable to the result.

```
[In] Integrate[(d + e*x^n)^q*(a + b*x^n + c*x^(2*n))^p,x]
[Out] Integrate[(d + e*x^n)^q*(a + b*x^n + c*x^(2*n))^p, x]
```

Maple [A]
time = 0.05, size = 0, normalized size = 0.00

$$\int (d+e\,x^n)^q \left(a+b\,x^n+c\,x^{2n}\right)^p \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d+e*x^n)^q*(a+b*x^n+c*x^(2*n))^p,x)
[Out] int((d+e*x^n)^q*(a+b*x^n+c*x^(2*n))^p,x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+e*x^n)^q*(a+b*x^n+c*x^(2*n))^p,x, algorithm="maxima")
[Out] integrate((c*x^(2*n) + b*x^n + a)^p*(x^n*e + d)^q, x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+e*x^n)^q*(a+b*x^n+c*x^(2*n))^p,x, algorithm="fricas")
[Out] integral((c*x^(2*n) + b*x^n + a)^p*(x^n*e + d)^q, x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+e*x**n)**q*(a+b*x**n+c*x**(2*n))**p,x)
[Out] Timed out
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+e*x^n)^q*(a+b*x^n+c*x^(2*n))^p,x, algorithm="giac")
[Out] integrate((c*x^(2*n) + b*x^n + a)^p*(x^n*e + d)^q, x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.03

$$\int (d+e\,x^n)^q\,\left(a+b\,x^n+c\,x^{2\,n}\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x^n)^q*(a + b*x^n + c*x^(2*n))^p,x)
[Out] int((d + e*x^n)^q*(a + b*x^n + c*x^(2*n))^p, x)
```

3.91 $\int (d+ex^n)^3 (a+bx^n+cx^{2n})^p \, dx$

Optimal. Leaf size=606

$$\frac{3d^2ex^{1+n}\left(1+\frac{2cx^n}{b-\sqrt{b^2-4ac}}\right)^{-p}\left(1+\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)^{-p}(a+bx^n+cx^{2n})^p F_1\left(1+\frac{1}{n};-p,-p;2+\frac{1}{n};-\frac{2}{b-\sqrt{b}}\right.}{1+n}$$

```
[Out] 3*d^2*e*x^(1+n)*(a+b*x^n+c*x^(2*n))^p*AppellF1(1+1/n,-p,-p,2+1/n,-2*c*x^n/(
b-(-4*a*c+b^2)^(1/2)),-2*c*x^n/(b+(-4*a*c+b^2)^(1/2)))/(1+n)/((1+2*c*x^n/(b
-(-4*a*c+b^2)^(1/2)))^p)/((1+2*c*x^n/(b+(-4*a*c+b^2)^(1/2)))^p)+3*d*e^2*x^(
1+2*n)*(a+b*x^n+c*x^(2*n))^p*AppellF1(2+1/n,-p,-p,3+1/n,-2*c*x^n/(b-(-4*a*c
+b^2)^(1/2)),-2*c*x^n/(b+(-4*a*c+b^2)^(1/2)))/(1+2*n)/((1+2*c*x^n/(b-(-4*a*
c+b^2)^(1/2)))^p)/((1+2*c*x^n/(b+(-4*a*c+b^2)^(1/2)))^p)+e^3*x^(1+3*n)*(a+b
*x^n+c*x^(2*n))^p*AppellF1(3+1/n,-p,-p,4+1/n,-2*c*x^n/(b-(-4*a*c+b^2)^(1/2)
),-2*c*x^n/(b+(-4*a*c+b^2)^(1/2)))/(1+3*n)/((1+2*c*x^n/(b-(-4*a*c+b^2)^(1/2
)))^p)/((1+2*c*x^n/(b+(-4*a*c+b^2)^(1/2)))^p)+d^3*x*(a+b*x^n+c*x^(2*n))^p*A
ppellF1(1/n,-p,-p,1+1/n,-2*c*x^n/(b-(-4*a*c+b^2)^(1/2)),-2*c*x^n/(b+(-4*a*c
+b^2)^(1/2)))/((1+2*c*x^n/(b-(-4*a*c+b^2)^(1/2)))^p)/((1+2*c*x^n/(b+(-4*a*c
+b^2)^(1/2)))^p)
```

Rubi [A]

time = 0.41, antiderivative size = 606, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 5, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.192$, Rules used = {1450, 1362, 440, 1399, 524}

$$d^3x\left(\frac{2cx^n}{b-\sqrt{b^2-4ac}}+1\right)^{-p}\left(\frac{2cx^n}{\sqrt{b^2-4ac}+b}+1\right)^{-p}(a+bx^n+cx^{2n})^p F_1\left(\frac{1}{n};-p,-p;1+\frac{1}{n};-\frac{2cx^n}{b-\sqrt{b^2-4ac}},-\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)+\frac{3d^2ex^{n+1}\left(\frac{2cx^n}{b-\sqrt{b^2-4ac}}+1\right)^{-p}\left(\frac{2cx^n}{\sqrt{b^2-4ac}+b}+1\right)^{-p}(a+bx^n+cx^{2n})^p F_1\left(1+\frac{1}{n};-p,-p;2+\frac{1}{n};-\frac{2cx^n}{b-\sqrt{b^2-4ac}},-\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)}{n+1}+\frac{3de^2x^{2n+1}\left(\frac{2cx^n}{b-\sqrt{b^2-4ac}}+1\right)^{-p}\left(\frac{2cx^n}{\sqrt{b^2-4ac}+b}+1\right)^{-p}(a+bx^n+cx^{2n})^p F_1\left(2+\frac{1}{n};-p,-p;3+\frac{1}{n};-\frac{2cx^n}{b-\sqrt{b^2-4ac}},-\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)}{2n+1}+\frac{e^3x^{3n+1}\left(\frac{2cx^n}{b-\sqrt{b^2-4ac}}+1\right)^{-p}\left(\frac{2cx^n}{\sqrt{b^2-4ac}+b}+1\right)^{-p}(a+bx^n+cx^{2n})^p F_1\left(3+\frac{1}{n};-p,-p;4+\frac{1}{n};-\frac{2cx^n}{b-\sqrt{b^2-4ac}},-\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)}{3n+1}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x^n)^3*(a + b*x^n + c*x^(2*n))^p,x]
```

```
[Out] (3*d^2*e*x^(1 + n)*(a + b*x^n + c*x^(2*n))^p*AppellF1[1 + n^(-1), -p, -p, 2
 + n^(-1), (-2*c*x^n)/(b - Sqrt[b^2 - 4*a*c]), (-2*c*x^n)/(b + Sqrt[b^2 - 4
*a*c])])/((1 + n)*(1 + (2*c*x^n)/(b - Sqrt[b^2 - 4*a*c]))^p*(1 + (2*c*x^n)/
(b + Sqrt[b^2 - 4*a*c]))^p) + (3*d*e^2*x^(1 + 2*n)*(a + b*x^n + c*x^(2*n))^
p*AppellF1[2 + n^(-1), -p, -p, 3 + n^(-1), (-2*c*x^n)/(b - Sqrt[b^2 - 4*a*c
]), (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c])])/((1 + 2*n)*(1 + (2*c*x^n)/(b - Sqr
t[b^2 - 4*a*c]))^p*(1 + (2*c*x^n)/(b + Sqrt[b^2 - 4*a*c]))^p) + (e^3*x^(1 +
 3*n)*(a + b*x^n + c*x^(2*n))^p*AppellF1[3 + n^(-1), -p, -p, 4 + n^(-1), (-
2*c*x^n)/(b - Sqrt[b^2 - 4*a*c]), (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c])])/((1
+ 3*n)*(1 + (2*c*x^n)/(b - Sqrt[b^2 - 4*a*c]))^p*(1 + (2*c*x^n)/(b + Sqrt[b
^2 - 4*a*c]))^p) + (d^3*x*(a + b*x^n + c*x^(2*n))^p*AppellF1[n^(-1), -p, -p
, 1 + n^(-1), (-2*c*x^n)/(b - Sqrt[b^2 - 4*a*c]), (-2*c*x^n)/(b + Sqrt[b^2
- 4*a*c])])/((1 + (2*c*x^n)/(b - Sqrt[b^2 - 4*a*c]))^p*(1 + (2*c*x^n)/(b +
Sqrt[b^2 - 4*a*c]))^p)
```

Rule 440

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, (-b)*(x^n/a), (-d)*(x^n/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 524

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 1362

```
Int[((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^
IntPart[p]*((a + b*x^n + c*x^(2*n))^FracPart[p]/((1 + 2*c*(x^n/(b + Rt[b^2
- 4*a*c, 2])))^FracPart[p]*(1 + 2*c*(x^n/(b - Rt[b^2 - 4*a*c, 2])))^FracPar
t[p])), Int[(1 + 2*c*(x^n/(b + Sqrt[b^2 - 4*a*c])))^p*(1 + 2*c*(x^n/(b - Sq
rt[b^2 - 4*a*c])))^p, x], x] /; FreeQ[{a, b, c, n, p}, x] && EqQ[n2, 2*n] &
& NeQ[b^2 - 4*a*c, 0] &&  !IntegerQ[p]
```

Rule 1399

```
Int[((d_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x
_Symbol] :> Dist[a^IntPart[p]*((a + b*x^n + c*x^(2*n))^FracPart[p]/((1 + 2*
c*(x^n/(b + Rt[b^2 - 4*a*c, 2])))^FracPart[p]*(1 + 2*c*(x^n/(b - Rt[b^2 - 4
*a*c, 2])))^FracPart[p])), Int[(d*x)^m*(1 + 2*c*(x^n/(b + Sqrt[b^2 - 4*a*c]
)))^p*(1 + 2*c*(x^n/(b - Sqrt[b^2 - 4*a*c])))^p, x], x] /; FreeQ[{a, b, c,
d, m, n, p}, x] && EqQ[n2, 2*n]
```

Rule 1450

```
Int[((d_) + (e_.)*(x_)^(n_))^(q_)*((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_
))^(p_), x_Symbol] :> Int[ExpandIntegrand[(d + e*x^n)^q*(a + b*x^n + c*x^(2
*n))^p, x], x] /; FreeQ[{a, b, c, d, e, n, p, q}, x] && EqQ[n2, 2*n] && NeQ
[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && ((IntegersQ[p, q] &&
!IntegerQ[n]) || IGtQ[p, 0] || (IGtQ[q, 0] &&  !IntegerQ[n]))
```

Rubi steps

$$\int (d+ex^n)^3 \left(a+bx^n+cx^{2n}\right)^p \, dx = \int \left(d^3\left(a+bx^n+cx^{2n}\right)^p + 3d^2ex^n\left(a+bx^n+cx^{2n}\right)^p + 3de^2x^{2n}\left(a+bx\right.\right.$$
$$= d^3 \int \left(a+bx^n+cx^{2n}\right)^p \, dx + \left(3d^2e\right) \int x^n\left(a+bx^n+cx^{2n}\right)^p \, dx + (3de$$
$$= \left(d^3\left(1+\frac{2cx^n}{b-\sqrt{b^2-4ac}}\right)^{-p}\left(1+\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)^{-p}\left(a+bx^n+cx\right.\right.$$
$$= \frac{3d^2ex^{1+n}\left(1+\frac{2cx^n}{b-\sqrt{b^2-4ac}}\right)^{-p}\left(1+\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)^{-p}\left(a+bx^n+cx\right.}{1+n}$$

Mathematica [**A**]
time = 0.77, size = 438, normalized size = 0.72

$$\frac{x\left(\frac{b-\sqrt{b^2-4ac}+2cx^n}{b-\sqrt{b^2-4ac}}\right)^{-p}\left(\frac{b+\sqrt{b^2-4ac}+2cx^n}{b+\sqrt{b^2-4ac}}\right)^{-p}(a+x^n(b+cx^n))^p\left(3d^2e(1+5n+6n^2)\,x^nF_1\left(1+\frac{1}{n};-p,-p;2+\frac{1}{n};-\frac{2cx^n}{b+\sqrt{b^2-4ac}},\frac{2cx^n}{-b+\sqrt{b^2-4ac}}\right)+(1+n)\left(3de^2(1+3n)x^{2n}F_1\left(2+\frac{1}{n};-p,-p;3+\frac{1}{n};-\frac{2cx^n}{b+\sqrt{b^2-4ac}},\frac{2cx^n}{-b+\sqrt{b^2-4ac}}\right)+(1+2n)\left(e^3x^{3n}F_1\left(3+\frac{1}{n};-p,-p;4+\frac{1}{n};-\frac{2cx^n}{b+\sqrt{b^2-4ac}},\frac{2cx^n}{-b+\sqrt{b^2-4ac}}\right)+d^3(1+3n)F_1\left(\frac{1}{n};-p,-p;1+\frac{1}{n};-\frac{2cx^n}{b+\sqrt{b^2-4ac}},\frac{2cx^n}{-b+\sqrt{b^2-4ac}}\right)\right)\right)\right)}{(1+n)(1+2n)(1+3n)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x^n)^3*(a + b*x^n + c*x^(2*n))^p,x]
```

```
[Out] (x*(a + x^n*(b + c*x^n))^p*(3*d^2*e*(1 + 5*n + 6*n^2)*x^n*AppellF1[1 + n^(-
1), -p, -p, 2 + n^(-1), (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^n)/(-b +
 Sqrt[b^2 - 4*a*c])] + (1 + n)*(3*d*e^2*(1 + 3*n)*x^(2*n)*AppellF1[2 + n^(-
1), -p, -p, 3 + n^(-1), (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^n)/(-b +
 Sqrt[b^2 - 4*a*c])] + (1 + 2*n)*(e^3*x^(3*n)*AppellF1[3 + n^(-1), -p, -p,
4 + n^(-1), (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^n)/(-b + Sqrt[b^2 -
4*a*c])] + d^3*(1 + 3*n)*AppellF1[n^(-1), -p, -p, 1 + n^(-1), (-2*c*x^n)/(b
 + Sqrt[b^2 - 4*a*c]), (2*c*x^n)/(-b + Sqrt[b^2 - 4*a*c])]))))/((1 + n)*(1
+ 2*n)*(1 + 3*n)*((b - Sqrt[b^2 - 4*a*c] + 2*c*x^n)/(b - Sqrt[b^2 - 4*a*c])
)^p*((b + Sqrt[b^2 - 4*a*c] + 2*c*x^n)/(b + Sqrt[b^2 - 4*a*c]))^p)
```

Maple [**F**]
time = 0.02, size = 0, normalized size = 0.00

$$\int \left(d+e\,x^n\right)^3 \left(a+b\,x^n+c\,x^{2n}\right)^p \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d+e*x^n)^3*(a+b*x^n+c*x^(2*n))^p,x)
```

```
[Out] int((d+e*x^n)^3*(a+b*x^n+c*x^(2*n))^p,x)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+e*x^n)^3*(a+b*x^n+c*x^(2*n))^p,x, algorithm="maxima")
[Out] integrate((x^n*e + d)^3*(c*x^(2*n) + b*x^n + a)^p, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+e*x^n)^3*(a+b*x^n+c*x^(2*n))^p,x, algorithm="fricas")
[Out] integral((3*d^2*x^n*e + d^3 + 3*d*x^(2*n)*e^2 + x^(3*n)*e^3)*(c*x^(2*n) + b
*x^n + a)^p, x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+e*x**n)**3*(a+b*x**n+c*x**(2*n))**p,x)
[Out] Timed out
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+e*x^n)^3*(a+b*x^n+c*x^(2*n))^p,x, algorithm="giac")
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Unable to divide, perhaps due to roun
ding error%%%{-256,[1,0,7,4,7,5,2,8,1]%%%}+%%%{-1280,[1,0,7,4,7,4,2,8,1]%%%
}+%%%{
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int {\left(d+e\,x^n\right)}^3\,{\left(a+b\,x^n+c\,x^{2\,n}\right)}^p \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x^n)^3*(a + b*x^n + c*x^(2*n))^p,x)
[Out] int((d + e*x^n)^3*(a + b*x^n + c*x^(2*n))^p, x)
```

3.92 $\int (d+ex^n)^2 \left(a+bx^n+cx^{2n}\right)^p \, dx$

Optimal. Leaf size=447

$$\frac{2dex^{1+n}\left(1+\frac{2cx^n}{b-\sqrt{b^2-4ac}}\right)^{-p}\left(1+\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)^{-p}\left(a+bx^n+cx^{2n}\right)^p F_1\left(1+\frac{1}{n};-p,-p;2+\frac{1}{n};-\frac{2c}{b-\sqrt{b^2}}\right.}{1+n}$$

```
[Out] 2*d*e*x^(1+n)*(a+b*x^n+c*x^(2*n))^p*AppellF1(1+1/n,-p,-p,2+1/n,-2*c*x^n/(b-
(-4*a*c+b^2)^(1/2)),-2*c*x^n/(b+(-4*a*c+b^2)^(1/2)))/(1+n)/((1+2*c*x^n/(b-(
-4*a*c+b^2)^(1/2)))^p)/((1+2*c*x^n/(b+(-4*a*c+b^2)^(1/2)))^p)+e^2*x^(1+2*n)
*(a+b*x^n+c*x^(2*n))^p*AppellF1(2+1/n,-p,-p,3+1/n,-2*c*x^n/(b-(-4*a*c+b^2)^
(1/2)),-2*c*x^n/(b+(-4*a*c+b^2)^(1/2)))/(1+2*n)/((1+2*c*x^n/(b-(-4*a*c+b^2)
^(1/2)))^p)/((1+2*c*x^n/(b+(-4*a*c+b^2)^(1/2)))^p)+d^2*x*(a+b*x^n+c*x^(2*n)
)^p*AppellF1(1/n,-p,-p,1+1/n,-2*c*x^n/(b-(-4*a*c+b^2)^(1/2)),-2*c*x^n/(b+(-
4*a*c+b^2)^(1/2)))/((1+2*c*x^n/(b-(-4*a*c+b^2)^(1/2)))^p)/((1+2*c*x^n/(b+(-
4*a*c+b^2)^(1/2)))^p)
```

Rubi [A]
time = 0.30, antiderivative size = 447, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 5, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.192$, Rules used = {1450, 1362, 440, 1399, 524}

$$d^2x\left(\frac{2cx^n}{b-\sqrt{b^2-4ac}}+1\right)^{-p}\left(\frac{2cx^n}{\sqrt{b^2-4ac}+b}+1\right)^{-p}\left(a+bx^n+cx^{2n}\right)^p F_1\left(\frac{1}{n};-p,-p;1+\frac{1}{n};-\frac{2cx^n}{b-\sqrt{b^2-4ac}},-\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)+\frac{2dex^{n+1}\left(\frac{2cx^n}{b-\sqrt{b^2-4ac}}+1\right)^{-p}\left(\frac{2cx^n}{\sqrt{b^2-4ac}+b}+1\right)^{-p}\left(a+bx^n+cx^{2n}\right)^p F_1\left(1+\frac{1}{n};-p,-p;2+\frac{1}{n};-\frac{2cx^n}{b-\sqrt{b^2-4ac}},-\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)}{n+1}+\frac{e^2x^{2n+1}\left(\frac{2cx^n}{b-\sqrt{b^2-4ac}}+1\right)^{-p}\left(\frac{2cx^n}{\sqrt{b^2-4ac}+b}+1\right)^{-p}\left(a+bx^n+cx^{2n}\right)^p F_1\left(2+\frac{1}{n};-p,-p;3+\frac{1}{n};-\frac{2cx^n}{b-\sqrt{b^2-4ac}},-\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)}{2n+1}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x^n)^2*(a + b*x^n + c*x^(2*n))^p,x]
```

```
[Out] (2*d*e*x^(1 + n)*(a + b*x^n + c*x^(2*n))^p*AppellF1[1 + n^(-1), -p, -p, 2 +
 n^(-1), (-2*c*x^n)/(b - Sqrt[b^2 - 4*a*c]), (-2*c*x^n)/(b + Sqrt[b^2 - 4*a
*c])])/((1 + n)*(1 + (2*c*x^n)/(b - Sqrt[b^2 - 4*a*c]))^p*(1 + (2*c*x^n)/(b
 + Sqrt[b^2 - 4*a*c]))^p) + (e^2*x^(1 + 2*n)*(a + b*x^n + c*x^(2*n))^p*Appe
llF1[2 + n^(-1), -p, -p, 3 + n^(-1), (-2*c*x^n)/(b - Sqrt[b^2 - 4*a*c]), (-
2*c*x^n)/(b + Sqrt[b^2 - 4*a*c])])/((1 + 2*n)*(1 + (2*c*x^n)/(b - Sqrt[b^2
- 4*a*c]))^p*(1 + (2*c*x^n)/(b + Sqrt[b^2 - 4*a*c]))^p) + (d^2*x*(a + b*x^n
 + c*x^(2*n))^p*AppellF1[n^(-1), -p, -p, 1 + n^(-1), (-2*c*x^n)/(b - Sqrt[b
^2 - 4*a*c]), (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c])])/((1 + (2*c*x^n)/(b - Sqr
t[b^2 - 4*a*c]))^p*(1 + (2*c*x^n)/(b + Sqrt[b^2 - 4*a*c]))^p)
```

Rule 440

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, (-b)*(x^n/a), (-d)*(x^n/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 524

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 1362

```
Int[((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^
IntPart[p]*((a + b*x^n + c*x^(2*n))^FracPart[p]/((1 + 2*c*(x^n/(b + Rt[b^2
- 4*a*c, 2])))^FracPart[p]*(1 + 2*c*(x^n/(b - Rt[b^2 - 4*a*c, 2])))^FracPar
t[p])), Int[(1 + 2*c*(x^n/(b + Sqrt[b^2 - 4*a*c])))^p*(1 + 2*c*(x^n/(b - Sq
rt[b^2 - 4*a*c])))^p, x], x] /; FreeQ[{a, b, c, n, p}, x] && EqQ[n2, 2*n] &
& NeQ[b^2 - 4*a*c, 0] &&  !IntegerQ[p]
```

Rule 1399

```
Int[((d_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x
_Symbol] :> Dist[a^IntPart[p]*((a + b*x^n + c*x^(2*n))^FracPart[p]/((1 + 2*
c*(x^n/(b + Rt[b^2 - 4*a*c, 2])))^FracPart[p]*(1 + 2*c*(x^n/(b - Rt[b^2 - 4
*a*c, 2])))^FracPart[p])), Int[(d*x)^m*(1 + 2*c*(x^n/(b + Sqrt[b^2 - 4*a*c]
)))^p*(1 + 2*c*(x^n/(b - Sqrt[b^2 - 4*a*c])))^p, x], x] /; FreeQ[{a, b, c,
d, m, n, p}, x] && EqQ[n2, 2*n]
```

Rule 1450

```
Int[((d_) + (e_.)*(x_)^(n_))^(q_)*((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_
))^(p_), x_Symbol] :> Int[ExpandIntegrand[(d + e*x^n)^q*(a + b*x^n + c*x^(2
*n))^p, x], x] /; FreeQ[{a, b, c, d, e, n, p, q}, x] && EqQ[n2, 2*n] && NeQ
[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && ((IntegersQ[p, q] &&
!IntegerQ[n]) || IGtQ[p, 0] || (IGtQ[q, 0] &&  !IntegerQ[n]))
```

Rubi steps

$$\int (d+ex^n)^2 \left(a+bx^n+cx^{2n}\right)^p \, dx = \int \left(d^2\left(a+bx^n+cx^{2n}\right)^p + 2dex^n\left(a+bx^n+cx^{2n}\right)^p + e^2x^{2n}\left(a+bx^n+\right.\right.$$

$$= d^2 \int \left(a+bx^n+cx^{2n}\right)^p \, dx + (2de) \int x^n\left(a+bx^n+cx^{2n}\right)^p \, dx + e^2 \int$$

$$= \left(d^2\left(1+\frac{2cx^n}{b-\sqrt{b^2-4ac}}\right)^{-p}\left(1+\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)^{-p}\left(a+bx^n+cx\right.\right.$$

$$= \frac{2dex^{1+n}\left(1+\frac{2cx^n}{b-\sqrt{b^2-4ac}}\right)^{-p}\left(1+\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)^{-p}\left(a+bx^n+cx\right.}{1+n}$$

Mathematica [A]
time = 0.54, size = 338, normalized size = 0.76

$$\frac{x\left(\frac{b-\sqrt{b^2-4ac}+2cx^n}{b-\sqrt{b^2-4ac}}\right)^{-p}\left(\frac{b+\sqrt{b^2-4ac}+2cx^n}{b+\sqrt{b^2-4ac}}\right)^{-p}(a+x^n(b+cx^n))^p\left(2de(1+2n)x^nF_1\left(1+\frac{1}{n};-p,-p;2+\frac{1}{n};-\frac{2cx^n}{b+\sqrt{b^2-4ac}},\frac{2cx^n}{-b+\sqrt{b^2-4ac}}\right)+(1+n)\left(e^2x^{2n}F_1\left(2+\frac{1}{n};-p,-p;3+\frac{1}{n};-\frac{2cx^n}{b+\sqrt{b^2-4ac}},\frac{2cx^n}{-b+\sqrt{b^2-4ac}}\right)+d^2(1+2n)F_1\left(\frac{1}{n};-p,-p;1+\frac{1}{n};-\frac{2cx^n}{b+\sqrt{b^2-4ac}},\frac{2cx^n}{-b+\sqrt{b^2-4ac}}\right)\right)\right)}{(1+n)(1+2n)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x^n)^2*(a + b*x^n + c*x^(2*n))^p,x]
```

```
[Out] (x*(a + x^n*(b + c*x^n))^p*(2*d*e*(1 + 2*n)*x^n*AppellF1[1 + n^(-1), -p, -p
, 2 + n^(-1), (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^n)/(-b + Sqrt[b^2
- 4*a*c])] + (1 + n)*(e^2*x^(2*n)*AppellF1[2 + n^(-1), -p, -p, 3 + n^(-1),
(-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^n)/(-b + Sqrt[b^2 - 4*a*c])] + d
^2*(1 + 2*n)*AppellF1[n^(-1), -p, -p, 1 + n^(-1), (-2*c*x^n)/(b + Sqrt[b^2
- 4*a*c]), (2*c*x^n)/(-b + Sqrt[b^2 - 4*a*c])])))/((1 + n)*(1 + 2*n)*((b -
Sqrt[b^2 - 4*a*c] + 2*c*x^n)/(b - Sqrt[b^2 - 4*a*c]))^p*((b + Sqrt[b^2 - 4*
a*c] + 2*c*x^n)/(b + Sqrt[b^2 - 4*a*c]))^p)
```

Maple [F]
time = 0.02, size = 0, normalized size = 0.00

$$\int (d+e\,x^n)^2 \left(a+b\,x^n+c\,x^{2n}\right)^p \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d+e*x^n)^2*(a+b*x^n+c*x^(2*n))^p,x)
```

```
[Out] int((d+e*x^n)^2*(a+b*x^n+c*x^(2*n))^p,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+e*x^n)^2*(a+b*x^n+c*x^(2*n))^p,x, algorithm="maxima")
```

```
[Out] integrate((x^n*e + d)^2*(c*x^(2*n) + b*x^n + a)^p, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+e*x^n)^2*(a+b*x^n+c*x^(2*n))^p,x, algorithm="fricas")
```

```
[Out] integral((2*d*x^n*e + d^2 + x^(2*n)*e^2)*(c*x^(2*n) + b*x^n + a)^p, x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+e*x**n)**2*(a+b*x**n+c*x**(2*n))**p,x)
```

```
[Out] Timed out
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+e*x^n)^2*(a+b*x^n+c*x^(2*n))^p,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Unable to divide, perhaps due to roun
ding error%%%{128,[1,0,5,3,5,4,1,6,1]%%%}+%%%{512,[1,0,5,3,5,3,1,6,1]%%%}+%
%%{768
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int {\left(d+e\,x^n\right)}^2\,{\left(a+b\,x^n+c\,x^{2\,n}\right)}^p \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x^n)^2*(a + b*x^n + c*x^(2*n))^p,x)
```

```
[Out] int((d + e*x^n)^2*(a + b*x^n + c*x^(2*n))^p, x)
```

3.93 $\int (d+ex^n)\left(a+bx^n+cx^{2n}\right)^p\,dx$

Optimal. Leaf size=288

$$\frac{ex^{1+n}\left(1+\frac{2cx^n}{b-\sqrt{b^2-4ac}}\right)^{-p}\left(1+\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)^{-p}\left(a+bx^n+cx^{2n}\right)^p F_1\left(1+\frac{1}{n};-p,-p;2+\frac{1}{n};-\frac{2cx^n}{b-\sqrt{b^2-}}\right.}{1+n}$$

```
[Out] e*x^(1+n)*(a+b*x^n+c*x^(2*n))^p*AppellF1(1+1/n,-p,-p,2+1/n,-2*c*x^n/(b-(-4*
a*c+b^2)^(1/2)),-2*c*x^n/(b+(-4*a*c+b^2)^(1/2)))/(1+n)/((1+2*c*x^n/(b-(-4*a
*c+b^2)^(1/2)))^p)/((1+2*c*x^n/(b+(-4*a*c+b^2)^(1/2)))^p)+d*x*(a+b*x^n+c*x^
(2*n))^p*AppellF1(1/n,-p,-p,1+1/n,-2*c*x^n/(b-(-4*a*c+b^2)^(1/2)),-2*c*x^n/
(b+(-4*a*c+b^2)^(1/2)))/((1+2*c*x^n/(b-(-4*a*c+b^2)^(1/2)))^p)/((1+2*c*x^n/
(b+(-4*a*c+b^2)^(1/2)))^p)
```

Rubi [A]

time = 0.19, antiderivative size = 288, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.208$, Rules used = {1446, 1362, 440, 1399, 524}

$$dx\left(\frac{2cx^n}{b-\sqrt{b^2-4ac}}+1\right)^{-p}\left(\frac{2cx^n}{\sqrt{b^2-4ac}+b}+1\right)^{-p}\left(a+bx^n+cx^{2n}\right)^p F_1\left(\frac{1}{n};-p,-p;1+\frac{1}{n};-\frac{2cx^n}{b-\sqrt{b^2-4ac}},-\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)+\frac{ex^{n+1}\left(\frac{2cx^n}{b-\sqrt{b^2-4ac}}+1\right)^{-p}\left(\frac{2cx^n}{\sqrt{b^2-4ac}+b}+1\right)^{-p}\left(a+bx^n+cx^{2n}\right)^p F_1\left(1+\frac{1}{n};-p,-p;2+\frac{1}{n};-\frac{2cx^n}{b-\sqrt{b^2-4ac}},-\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)}{n+1}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x^n)*(a + b*x^n + c*x^(2*n))^p,x]
```

```
[Out] (e*x^(1 + n)*(a + b*x^n + c*x^(2*n))^p*AppellF1[1 + n^(-1), -p, -p, 2 + n^(
-1), (-2*c*x^n)/(b - Sqrt[b^2 - 4*a*c]), (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c])
])/((1 + n)*(1 + (2*c*x^n)/(b - Sqrt[b^2 - 4*a*c]))^p*(1 + (2*c*x^n)/(b + S
qrt[b^2 - 4*a*c]))^p) + (d*x*(a + b*x^n + c*x^(2*n))^p*AppellF1[n^(-1), -p,
 -p, 1 + n^(-1), (-2*c*x^n)/(b - Sqrt[b^2 - 4*a*c]), (-2*c*x^n)/(b + Sqrt[b
^2 - 4*a*c])])/((1 + (2*c*x^n)/(b - Sqrt[b^2 - 4*a*c]))^p*(1 + (2*c*x^n)/(b
 + Sqrt[b^2 - 4*a*c]))^p)
```

Rule 440

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, (-b)*(x^n/a), (-d)*(x^n/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 524

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
```

```
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 1362

```
Int[((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^
IntPart[p]*((a + b*x^n + c*x^(2*n))^FracPart[p]/((1 + 2*c*(x^n/(b + Rt[b^2
- 4*a*c, 2])))^FracPart[p]*(1 + 2*c*(x^n/(b - Rt[b^2 - 4*a*c, 2])))^FracPar
t[p])), Int[(1 + 2*c*(x^n/(b + Sqrt[b^2 - 4*a*c])))^p*(1 + 2*c*(x^n/(b - Sq
rt[b^2 - 4*a*c])))^p, x], x] /; FreeQ[{a, b, c, n, p}, x] && EqQ[n2, 2*n] &
& NeQ[b^2 - 4*a*c, 0] &&  !IntegerQ[p]
```

Rule 1399

```
Int[((d_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x
_Symbol] :> Dist[a^IntPart[p]*((a + b*x^n + c*x^(2*n))^FracPart[p]/((1 + 2*
c*(x^n/(b + Rt[b^2 - 4*a*c, 2])))^FracPart[p]*(1 + 2*c*(x^n/(b - Rt[b^2 - 4
*a*c, 2])))^FracPart[p])), Int[(d*x)^m*(1 + 2*c*(x^n/(b + Sqrt[b^2 - 4*a*c]
)))^p*(1 + 2*c*(x^n/(b - Sqrt[b^2 - 4*a*c])))^p, x], x] /; FreeQ[{a, b, c,
d, m, n, p}, x] && EqQ[n2, 2*n]
```

Rule 1446

```
Int[((d_) + (e_.)*(x_)^(n_))*((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_))^(p
_), x_Symbol] :> Int[ExpandIntegrand[(d + e*x^n)*(a + b*x^n + c*x^(2*n))^p,
 x], x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c,
 0]
```

Rubi steps

$$\begin{aligned}\int (d+ex^n)\left(a+bx^n+cx^{2n}\right)^p\,dx &= \int \left(d\left(a+bx^n+cx^{2n}\right)^p+ex^n\left(a+bx^n+cx^{2n}\right)^p\right)\,dx\\ &= d\int \left(a+bx^n+cx^{2n}\right)^p\,dx+e\int x^n\left(a+bx^n+cx^{2n}\right)^p\,dx\\ &= \left(d\left(1+\frac{2cx^n}{b-\sqrt{b^2-4ac}}\right)^{-p}\left(1+\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)^{-p}\left(a+bx^n+cx^2\right.\right.\\ &= \frac{ex^{1+n}\left(1+\frac{2cx^n}{b-\sqrt{b^2-4ac}}\right)^{-p}\left(1+\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)^{-p}\left(a+bx^n+cx^{2n}\right.}{1+n}\end{aligned}$$

Mathematica [A]
time = 0.36, size = 243, normalized size = 0.84

$$\frac{x\left(\frac{b-\sqrt{b^2-4ac}+2cx^n}{b-\sqrt{b^2-4ac}}\right)^{-p}\left(\frac{b+\sqrt{b^2-4ac}+2cx^n}{b+\sqrt{b^2-4ac}}\right)^{-p}\left(a+x^n(b+cx^n)\right)^p\left(ex^nF_1\left(1+\frac{1}{n};-p,-p;2+\frac{1}{n};-\frac{2cx^n}{b+\sqrt{b^2-4ac}},\frac{2cx^n}{-b+\sqrt{b^2-4ac}}\right)+d(1+n)F_1\left(\frac{1}{n};-p,-p;1+\frac{1}{n};-\frac{2cx^n}{b+\sqrt{b^2-4ac}},\frac{2cx^n}{-b+\sqrt{b^2-4ac}}\right)\right)}{1+n}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x^n)*(a + b*x^n + c*x^(2*n))^p,x]
```

```
[Out] (x*(a + x^n*(b + c*x^n))^p*(e*x^n*AppellF1[1 + n^(-1), -p, -p, 2 + n^(-1),
(-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^n)/(-b + Sqrt[b^2 - 4*a*c])] + d
*(1 + n)*AppellF1[n^(-1), -p, -p, 1 + n^(-1), (-2*c*x^n)/(b + Sqrt[b^2 - 4*
a*c]), (2*c*x^n)/(-b + Sqrt[b^2 - 4*a*c])]))/((1 + n)*((b - Sqrt[b^2 - 4*a*
c] + 2*c*x^n)/(b - Sqrt[b^2 - 4*a*c]))^p*((b + Sqrt[b^2 - 4*a*c] + 2*c*x^n)
/(b + Sqrt[b^2 - 4*a*c]))^p)
```

Maple [F]
time = 0.03, size = 0, normalized size = 0.00

$$\int (d+e\,x^{n})\left(a+b\,x^{n}+c\,x^{2n}\right)^{p}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d+e*x^n)*(a+b*x^n+c*x^(2*n))^p,x)
```

```
[Out] int((d+e*x^n)*(a+b*x^n+c*x^(2*n))^p,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+e*x^n)*(a+b*x^n+c*x^(2*n))^p,x, algorithm="maxima")
```

```
[Out] integrate((x^n*e + d)*(c*x^(2*n) + b*x^n + a)^p, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+e*x^n)*(a+b*x^n+c*x^(2*n))^p,x, algorithm="fricas")
```

```
[Out] integral((x^n*e + d)*(c*x^(2*n) + b*x^n + a)^p, x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+e*x**n)*(a+b*x**n+c*x**(2*n))**p,x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+e*x^n)*(a+b*x^n+c*x^(2*n))^p,x, algorithm="giac")
```

```
[Out] integrate((x^n*e + d)*(c*x^(2*n) + b*x^n + a)^p, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int (d+e\,x^n)\,\left(a+b\,x^n+c\,x^{2\,n}\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x^n)*(a + b*x^n + c*x^(2*n))^p,x)
```

```
[Out] int((d + e*x^n)*(a + b*x^n + c*x^(2*n))^p, x)
```

3.94 $\int \frac{(a+bx^n+cx^{2n})^p}{d+ex^n} \, dx$

Optimal. Leaf size=29

$$\text{Int}\left(\frac{(a+bx^n+cx^{2n})^p}{d+ex^n},x\right)$$

`[Out] Unintegrable((a+b*x^n+c*x^(2*n))^p/(d+e*x^n),x)`

Rubi [A]
time = 0.01, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{(a+bx^n+cx^{2n})^p}{d+ex^n} \, dx$$

Verification is not applicable to the result.

`[In] Int[(a + b*x^n + c*x^(2*n))^p/(d + e*x^n),x]`

`[Out] Defer[Int][(a + b*x^n + c*x^(2*n))^p/(d + e*x^n), x]`

Rubi steps

$$\int \frac{(a+bx^n+cx^{2n})^p}{d+ex^n} \, dx = \int \frac{(a+bx^n+cx^{2n})^p}{d+ex^n} \, dx$$

Mathematica [A]
time = 0.20, size = 0, normalized size = 0.00

$$\int \frac{(a+bx^n+cx^{2n})^p}{d+ex^n} \, dx$$

Verification is not applicable to the result.

`[In] Integrate[(a + b*x^n + c*x^(2*n))^p/(d + e*x^n),x]`

`[Out] Integrate[(a + b*x^n + c*x^(2*n))^p/(d + e*x^n), x]`

Maple [A]
time = 0.07, size = 0, normalized size = 0.00

$$\int \frac{(a+b\,x^n+c\,x^{2n})^p}{d+e\,x^n} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*x^n+c*x^(2*n))^p/(d+e*x^n),x)
[Out] int((a+b*x^n+c*x^(2*n))^p/(d+e*x^n),x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^n+c*x^(2*n))^p/(d+e*x^n),x, algorithm="maxima")
[Out] integrate((c*x^(2*n) + b*x^n + a)^p/(x^n*e + d), x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^n+c*x^(2*n))^p/(d+e*x^n),x, algorithm="fricas")
[Out] integral((c*x^(2*n) + b*x^n + a)^p/(x^n*e + d), x)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: HeuristicGCDFailed

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x**n+c*x**(2*n))**p/(d+e*x**n),x)
[Out] Exception raised: HeuristicGCDFailed >> no luck
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^n+c*x^(2*n))^p/(d+e*x^n),x, algorithm="giac")
[Out] integrate((c*x^(2*n) + b*x^n + a)^p/(x^n*e + d), x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{(a + b\,x^n + c\,x^{2\,n})^p}{d + e\,x^n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^n + c*x^(2*n))^p/(d + e*x^n),x)
[Out] int((a + b*x^n + c*x^(2*n))^p/(d + e*x^n), x)
```

3.95 $\int \frac{(a+bx^n+cx^{2n})^p}{(d+ex^n)^2}\,dx$

Optimal. Leaf size=29

$$\operatorname{Int}\left(\frac{(a+bx^n+cx^{2n})^p}{(d+ex^n)^2},x\right)$$

```
[Out] Unintegrable((a+b*x^n+c*x^(2*n))^p/(d+e*x^n)^2,x)
```

Rubi [A]
time = 0.01, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{(a+bx^n+cx^{2n})^p}{(d+ex^n)^2}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(a + b*x^n + c*x^(2*n))^p/(d + e*x^n)^2,x]
[Out] Defer[Int][(a + b*x^n + c*x^(2*n))^p/(d + e*x^n)^2, x]
```

Rubi steps

$$\int \frac{(a+bx^n+cx^{2n})^p}{(d+ex^n)^2}\,dx = \int \frac{(a+bx^n+cx^{2n})^p}{(d+ex^n)^2}\,dx$$

Mathematica [A]
time = 0.23, size = 0, normalized size = 0.00

$$\int \frac{(a+bx^n+cx^{2n})^p}{(d+ex^n)^2}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(a + b*x^n + c*x^(2*n))^p/(d + e*x^n)^2,x]
[Out] Integrate[(a + b*x^n + c*x^(2*n))^p/(d + e*x^n)^2, x]
```

Maple [A]
time = 0.04, size = 0, normalized size = 0.00

$$\int \frac{(a+b\,x^n+c\,x^{2n})^p}{(d+e\,x^n)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*x^n+c*x^(2*n))^p/(d+e*x^n)^2,x)
```

```
[Out] int((a+b*x^n+c*x^(2*n))^p/(d+e*x^n)^2,x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^n+c*x^(2*n))^p/(d+e*x^n)^2,x, algorithm="maxima")
```

```
[Out] integrate((c*x^(2*n) + b*x^n + a)^p/(x^n*e + d)^2, x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^n+c*x^(2*n))^p/(d+e*x^n)^2,x, algorithm="fricas")
```

```
[Out] integral((c*x^(2*n) + b*x^n + a)^p/(2*d*x^n*e + d^2 + x^(2*n)*e^2), x)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: HeuristicGCDFailed

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x**n+c*x**(2*n))**p/(d+e*x**n)**2,x)
```

```
[Out] Exception raised: HeuristicGCDFailed >> no luck
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^n+c*x^(2*n))^p/(d+e*x^n)^2,x, algorithm="giac")
```

```
[Out] integrate((c*x^(2*n) + b*x^n + a)^p/(x^n*e + d)^2, x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{(a + b\,x^n + c\,x^{2\,n})^p}{(d + e\,x^n)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^n + c*x^(2*n))^p/(d + e*x^n)^2,x)
[Out] int((a + b*x^n + c*x^(2*n))^p/(d + e*x^n)^2, x)
```

3.96 $\int \frac{(a+bx^n+cx^{2n})^p}{(d+ex^n)^3} \, dx$

Optimal. Leaf size=29

$$\operatorname{Int}\left(\frac{(a+bx^n+cx^{2n})^p}{(d+ex^n)^3},x\right)$$

```
[Out] Unintegrable((a+b*x^n+c*x^(2*n))^p/(d+e*x^n)^3,x)
```

Rubi [A]
time = 0.01, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{(a+bx^n+cx^{2n})^p}{(d+ex^n)^3} \, dx$$

Verification is not applicable to the result.

```
[In] Int[(a + b*x^n + c*x^(2*n))^p/(d + e*x^n)^3,x]
```

```
[Out] Defer[Int][(a + b*x^n + c*x^(2*n))^p/(d + e*x^n)^3, x]
```

Rubi steps

$$\int \frac{(a+bx^n+cx^{2n})^p}{(d+ex^n)^3} \, dx = \int \frac{(a+bx^n+cx^{2n})^p}{(d+ex^n)^3} \, dx$$

Mathematica [A]
time = 0.60, size = 0, normalized size = 0.00

$$\int \frac{(a+bx^n+cx^{2n})^p}{(d+ex^n)^3} \, dx$$

Verification is not applicable to the result.

```
[In] Integrate[(a + b*x^n + c*x^(2*n))^p/(d + e*x^n)^3,x]
```

```
[Out] Integrate[(a + b*x^n + c*x^(2*n))^p/(d + e*x^n)^3, x]
```

Maple [A]
time = 0.04, size = 0, normalized size = 0.00

$$\int \frac{(a+b\,x^n+c\,x^{2n})^p}{(d+e\,x^n)^3} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*x^n+c*x^(2*n))^p/(d+e*x^n)^3,x)
```

```
[Out] int((a+b*x^n+c*x^(2*n))^p/(d+e*x^n)^3,x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^n+c*x^(2*n))^p/(d+e*x^n)^3,x, algorithm="maxima")
```

```
[Out] integrate((c*x^(2*n) + b*x^n + a)^p/(x^n*e + d)^3, x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^n+c*x^(2*n))^p/(d+e*x^n)^3,x, algorithm="fricas")
```

```
[Out] integral((c*x^(2*n) + b*x^n + a)^p/(3*d^2*x^n*e + d^3 + 3*d*x^(2*n)*e^2 + x
^(3*n)*e^3), x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x**n+c*x**(2*n))**p/(d+e*x**n)**3,x)
```

```
[Out] Timed out
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^n+c*x^(2*n))^p/(d+e*x^n)^3,x, algorithm="giac")
```

```
[Out] integrate((c*x^(2*n) + b*x^n + a)^p/(x^n*e + d)^3, x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{(a+b\,x^n+c\,x^{2\,n})^p}{(d+e\,x^n)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^n + c*x^(2*n))^p/(d + e*x^n)^3,x)
[Out] int((a + b*x^n + c*x^(2*n))^p/(d + e*x^n)^3, x)
```

Chapter 4

Appendix

Local contents

4.1 Download section

The following zip files contain the raw integrals used in this test.

Mathematica format Mathematica_syntax.zip

Maple and Mupad format Maple_syntax.zip

Sympy format SYMPY_syntax.zip

Sage math format SAGE_syntax.zip

4.2 Listing of Grading functions

The following are the current version of the grading functions used for grading the quality of the antiderivative with reference to the optimal antiderivative included in the test suite.

There is a version for Maple and for Mathematica/Rubi. There is a version for grading Sympy and version for use with Sagemath.

The following are links to the current source code.

The following are the listings of source code of the grading functions.

4.2.1 Mathematica and Rubi grading function

```
(* Original version thanks to Albert Rich emailed on 03/21/2017 *)
(* ::Package:: *)

(* Nasser: April 7,2022. add second output which gives reason for the grade *)
(*                       Small rewrite of logic in main function to make it*)
(*                       match Maple's logic. No change in functionality otherwise*)

(* ::Subsection:: *)
(*GradeAntiderivative[result,optimal]*)


(* ::Text:: *)
(*If result and optimal are mathematical expressions, *)
(*         GradeAntiderivative[result,optimal] returns*)
(*  "F" if the result fails to integrate an expression that*)
(*      is integrable*)
(*  "C" if result involves higher level functions than necessary*)
(*  "B" if result is more than twice the size of the optimal*)
(*      antiderivative*)
(*  "A" if result can be considered optimal*)
```

```
GradeAntiderivative[result_,optimal_] := Module[{expnResult,expnOptimal,leafCountResult,leafC
    expnResult  = ExpnType[result];
    expnOptimal = ExpnType[optimal];
    leafCountResult = LeafCount[result];
    leafCountOptimal = LeafCount[optimal];

    (*Print["expnResult=",expnResult," expnOptimal=",expnOptimal];*)
    If[expnResult<=expnOptimal,
        If[Not[FreeQ[result,Complex]], (*result contains complex*)
            If[Not[FreeQ[optimal,Complex]], (*optimal contains complex*)
                If[leafCountResult<=2*leafCountOptimal,
                    finalresult={"A","none"}
                    ,(*ELSE*)
                    finalresult={"B","Both result and optimal contain complex but leaf count
                  ]
            ,(*ELSE*)
                finalresult={"C","Result contains complex when optimal does not."}
            ]
        ,(*ELSE*)(*result does not contains complex*)
            If[leafCountResult<=2*leafCountOptimal,
               finalresult={"A","none"}
            ,(*ELSE*)
               finalresult={"B","Leaf count is larger than twice the leaf count of optimal. $
              ]
        ]
    ,(*ELSE*) (*expnResult>expnOptimal*)
        If[FreeQ[result,Integrate] && FreeQ[result,Int],
            finalresult={"C","Result contains higher order function than in optimal. Order "<
            ,
            finalresult={"F","Contains unresolved integral."}
        ]
    ];

    finalresult
]

(* ::Text:: *)
(*The following summarizes the type number assigned an *)
(*expression based on the functions it involves*)
(*1 = rational function*)
(*2 = algebraic function*)
(*3 = elementary function*)
(*4 = special function*)
(*5 = hyperpergeometric function*)
(*6 = appell function*)
(*7 = rootsum function*)
(*8 = integrate function*)
```

```
(*9 = unknown function*)


ExpnType[expn_] :=
  If[AtomQ[expn],
    1,
  If[ListQ[expn],
    Max[Map[ExpnType,expn]],
  If[Head[expn]===Power,
    If[IntegerQ[expn[[2]]],
      ExpnType[expn[[1]]],
    If[Head[expn[[2]]]===Rational,
      If[IntegerQ[expn[[1]]] || Head[expn[[1]]]===Rational,
        1,
      Max[ExpnType[expn[[1]]],2]],
    Max[ExpnType[expn[[1]]],ExpnType[expn[[2]]],3]]],
  If[Head[expn]===Plus || Head[expn]===Times,
    Max[ExpnType[First[expn]],ExpnType[Rest[expn]]],
  If[ElementaryFunctionQ[Head[expn]],
    Max[3,ExpnType[expn[[1]]]],
  If[SpecialFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],4]],
  If[HypergeometricFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],5]],
  If[AppellFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],6]],
  If[Head[expn]===RootSum,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
  If[Head[expn]===Integrate || Head[expn]===Int,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],8]],
  9]]]]]]]]]]


ElementaryFunctionQ[func_] :=
  MemberQ[{
  Exp,Log,
  Sin,Cos,Tan,Cot,Sec,Csc,
  ArcSin,ArcCos,ArcTan,ArcCot,ArcSec,ArcCsc,
  Sinh,Cosh,Tanh,Coth,Sech,Csch,
  ArcSinh,ArcCosh,ArcTanh,ArcCoth,ArcSech,ArcCsch
},func]


SpecialFunctionQ[func_] :=
  MemberQ[{
  Erf, Erfc, Erfi,
  FresnelS, FresnelC,
```

```
  ExpIntegralE, ExpIntegralEi, LogIntegral,
  SinIntegral, CosIntegral, SinhIntegral, CoshIntegral,
  Gamma, LogGamma, PolyGamma,
  Zeta, PolyLog, ProductLog,
  EllipticF, EllipticE, EllipticPi
},func]


HypergeometricFunctionQ[func_] :=
  MemberQ[{Hypergeometric1F1,Hypergeometric2F1,HypergeometricPFQ},func]


AppellFunctionQ[func_] :=
  MemberQ[{AppellF1},func]
```

4.2.2 Maple grading function

```
# File: GradeAntiderivative.mpl
# Original version thanks to Albert Rich emailed on 03/21/2017

#Nasser 03/22/2017  Use Maple leaf count instead since buildin
#Nasser 03/23/2017  missing 'ln' for ElementaryFunctionQ added
#Nasser 03/24/2017  corrected the check for complex result
#Nasser 10/27/2017  check for leafsize and do not call ExpnType()
#                   if leaf size is "too large". Set at 500,000
#Nasser 12/22/2019  Added debug flag, added 'dilog' to special functions
#                   see problem 156, file Apostol_Problems
#Nasser 4/07/2022   add second output which gives reason for the grade

GradeAntiderivative := proc(result,optimal)
local leaf_count_result,
        leaf_count_optimal,
        ExpnType_result,
        ExpnType_optimal,
        debug:=false;

        leaf_count_result:=leafcount(result);
        #do NOT call ExpnType() if leaf size is too large. Recursion problem
        if leaf_count_result > 500000 then
            return "B","result has leaf size over 500,000. Avoiding possible recursion issues
        fi;

        leaf_count_optimal := leafcount(optimal);
        ExpnType_result  := ExpnType(result);
        ExpnType_optimal := ExpnType(optimal);
```

```
        if debug then
            print("ExpnType_result",ExpnType_result," ExpnType_optimal=",ExpnType_optimal);
        fi;

# If result and optimal are mathematical expressions,
#  GradeAntiderivative[result,optimal] returns
#   "F" if the result fails to integrate an expression that
#       is integrable
#   "C" if result involves higher level functions than necessary
#   "B" if result is more than twice the size of the optimal
#       antiderivative
#   "A" if result can be considered optimal

    #This check below actually is not needed, since I only
    #call this grading only for passed integrals. i.e. I check
    #for "F" before calling this. But no harm of keeping it here.
    #just in case.


    if not type(result,freeof('int')) then
        return "F","Result contains unresolved integral";
    fi;


    if ExpnType_result<=ExpnType_optimal then
        if debug then
             print("ExpnType_result<=ExpnType_optimal");
        fi;
        if is_contains_complex(result) then
            if is_contains_complex(optimal) then
                if debug then
                        print("both result and optimal complex");
                fi;
                if leaf_count_result<=2*leaf_count_optimal then
                    return "A","";
                else
                   return "B",cat("Both result and optimal contain complex but leaf count of re
                                convert(leaf_count_result,string)," vs. $2 (",
                                convert(leaf_count_optimal,string)," ) = ",convert(2*leaf_co
                end if
            else #result contains complex but optimal is not
                if debug then
                        print("result contains complex but optimal is not");
                fi;
                return "C","Result contains complex when optimal does not.";
            fi;
        else # result do not contain complex
```

```
             # this assumes optimal do not as well. No check is needed here.
            if debug then
                   print("result do not contain complex, this assumes optimal do not as well"
            fi;
            if leaf_count_result<=2*leaf_count_optimal then
                if debug then
                    print("leaf_count_result<=2*leaf_count_optimal");
                fi;
                return "A","";
            else
                if debug then
                    print("leaf_count_result>2*leaf_count_optimal");
                fi;
                return "B",cat("Leaf count of result is larger than twice the leaf count of o
                                convert(leaf_count_result,string),"$ vs. $2(",
                                convert(leaf_count_optimal,string),")=",convert(2*leaf_cou
            fi;
        fi;
    else #ExpnType(result) > ExpnType(optimal)
        if debug then
            print("ExpnType(result) > ExpnType(optimal)");
        fi;
        return "C",cat("Result contains higher order function than in optimal. Order ",
                    convert(ExpnType_result,string)," vs. order ",
                    convert(ExpnType_optimal,string),".");
    fi;

end proc:

#
# is_contains_complex(result)
# takes expressions and returns true if it contains "I" else false
#
#Nasser 032417
is_contains_complex:= proc(expression)
  return (has(expression,I));
end proc:

# The following summarizes the type number assigned an expression
# based on the functions it involves
# 1 = rational function
# 2 = algebraic function
# 3 = elementary function
# 4 = special function
# 5 = hyperpergeometric function
# 6 = appell function
# 7 = rootsum function
```

```
# 8 = integrate function
# 9 = unknown function

ExpnType := proc(expn)
  if type(expn,'atomic') then
    1
  elif type(expn,'list') then
    apply(max,map(ExpnType,expn))
  elif type(expn,'sqrt') then
    if type(op(1,expn),'rational') then
       1
    else
       max(2,ExpnType(op(1,expn)))
    end if
  elif type(expn,'`^`') then
    if type(op(2,expn),'integer') then
      ExpnType(op(1,expn))
    elif type(op(2,expn),'rational') then
      if type(op(1,expn),'rational') then
         1
      else
         max(2,ExpnType(op(1,expn)))
      end if
    else
         max(3,ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    end if
  elif type(expn,'`+`') or type(expn,'`*`') then
    max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
  elif ElementaryFunctionQ(op(0,expn)) then
    max(3,ExpnType(op(1,expn)))
  elif SpecialFunctionQ(op(0,expn)) then
    max(4,apply(max,map(ExpnType,[op(expn)])))
  elif HypergeometricFunctionQ(op(0,expn)) then
    max(5,apply(max,map(ExpnType,[op(expn)])))
  elif AppellFunctionQ(op(0,expn)) then
    max(6,apply(max,map(ExpnType,[op(expn)])))
  elif op(0,expn)='int' then
    max(8,apply(max,map(ExpnType,[op(expn)]))) else
  9
  end if
end proc:


ElementaryFunctionQ := proc(func)
  member(func,[
        exp,log,ln,
        sin,cos,tan,cot,sec,csc,
```

```
        arcsin,arccos,arctan,arccot,arcsec,arccsc,
        sinh,cosh,tanh,coth,sech,csch,
        arcsinh,arccosh,arctanh,arccoth,arcsech,arccsch])
end proc:

SpecialFunctionQ := proc(func)
  member(func,[
        erf,erfc,erfi,
        FresnelS,FresnelC,
        Ei,Ei,Li,Si,Ci,Shi,Chi,
        GAMMA,lnGAMMA,Psi,Zeta,polylog,dilog,LambertW,
        EllipticF,EllipticE,EllipticPi])
end proc:

HypergeometricFunctionQ := proc(func)
  member(func,[Hypergeometric1F1,hypergeom,HypergeometricPFQ])
end proc:

AppellFunctionQ := proc(func)
  member(func,[AppellF1])
end proc:

# u is a sum or product.  rest(u) returns all but the
# first term or factor of u.
rest := proc(u) local v;
  if nops(u)=2 then
     op(2,u)
  else
     apply(op(0,u),op(2..nops(u),u))
  end if
end proc:

#leafcount(u) returns the number of nodes in u.
#Nasser 3/23/17  Replaced by build-in leafCount from package in Maple
leafcount := proc(u)
   MmaTranslator[Mma][LeafCount](u);
end proc:
```

4.2.3 Sympy grading function

```
#Dec 24, 2019. Nasser M. Abbasi:
#             Port of original Maple grading function by
#             Albert Rich to use with Sympy/Python
#Dec 27, 2019  Nasser. Added `RootSum`. See problem 177, Timofeev file
#             added 'exp_polar'
from sympy import *

def leaf_count(expr):
   #sympy do not have leaf count function. This is approximation
   return round(1.7*count_ops(expr))

def is_sqrt(expr):
   if isinstance(expr,Pow):
      if expr.args[1] == Rational(1,2):
         return True
      else:
         return False
   else:
      return False

def is_elementary_function(func):
   return func in [exp,log,ln,sin,cos,tan,cot,sec,csc,
         asin,acos,atan,acot,asec,acsc,sinh,cosh,tanh,coth,sech,csch,
         asinh,acosh,atanh,acoth,asech,acsch
      ]

def is_special_function(func):
   return func in [ erf,erfc,erfi,
          fresnels,fresnelc,Ei,Ei,Li,Si,Ci,Shi,Chi,
          gamma,loggamma,digamma,zeta,polylog,LambertW,
          elliptic_f,elliptic_e,elliptic_pi,exp_polar
       ]

def is_hypergeometric_function(func):
   return func in [hyper]

def is_appell_function(func):
   return func in [appellf1]

def is_atom(expn):
   try:
      if expn.isAtom or isinstance(expn,int) or isinstance(expn,float):
        return True
      else:
        return False
```

```
    except AttributeError as error:
        return False

def expnType(expn):
    debug=False
    if debug:
        print("expn=",expn,"type(expn)=",type(expn))

    if is_atom(expn):
       return 1
    elif isinstance(expn,list):
        return max(map(expnType, expn))   #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
            return 1
        else:
            return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn)))
    elif isinstance(expn,Pow):   #type(expn,'`^`')
        if isinstance(expn.args[1],Integer):  #type(op(2,expn),'integer')
            return expnType(expn.args[0])   #ExpnType(op(1,expn))
        elif isinstance(expn.args[1],Rational):  #type(op(2,expn),'rational')
            if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
                return 1
            else:
                return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn)))
        else:
            return max(3,expnType(expn.args[0]),expnType(expn.args[1])) #max(3,ExpnType(op(1,expn)),ExpnT
    elif isinstance(expn,Add) or isinstance(expn,Mul): #type(expn,'`+`') or type(expn,'`*`')
        m1 = expnType(expn.args[0])
        m2 = expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.func):  #ElementaryFunctionQ(op(0,expn))
        return max(3,expnType(expn.args[0]))  #max(3,ExpnType(op(1,expn)))
    elif is_special_function(expn.func): #SpecialFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(4,m1)   #max(4,apply(max,map(ExpnType,[op(expn)])))
    elif is_hypergeometric_function(expn.func): #HypergeometricFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(5,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif is_appell_function(expn.func):
        m1 = max(map(expnType, list(expn.args)))
        return max(6,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif isinstance(expn,RootSum):
        m1 = max(map(expnType, list(expn.args))) #Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
        return max(7,m1)
    elif str(expn).find("Integral") != -1:
```

```
        m1 = max(map(expnType, list(expn.args)))
        return max(8,m1)    #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):

    #print ("Enter grade_antiderivative for sagemath")
    #print("Enter grade_antiderivative, result=",result," optimal=",optimal)

    leaf_count_result  = leaf_count(result)
    leaf_count_optimal = leaf_count(optimal)

    #print("leaf_count_result=",leaf_count_result)
    #print("leaf_count_optimal=",leaf_count_optimal)

    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if str(result).find("Integral") != -1:
        grade = "F"
        grade_annotation =""
    else:
        if expnType_result <= expnType_optimal:
            if result.has(I):
                if optimal.has(I): #both result and optimal complex
                    if leaf_count_result <= 2*leaf_count_optimal:
                        grade = "A"
                        grade_annotation =""
                    else:
                        grade = "B"
                        grade_annotation ="Both result and optimal contain complex but leaf count of result is larger
                else: #result contains complex but optimal is not
                    grade = "C"
                    grade_annotation ="Result contains complex when optimal does not."
            else: # result do not contain complex, this assumes optimal do not as well
                if leaf_count_result <= 2*leaf_count_optimal:
                    grade = "A"
                    grade_annotation =""
                else:
                    grade = "B"
                    grade_annotation ="Leaf count of result is larger than twice the leaf count of optimal. "+str(lea
        else:
            grade = "C"
            grade_annotation ="Result contains higher order function than in optimal. Order "+str(ExpnType_re
```

```
    #print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)

    return grade, grade_annotation
```

4.2.4 SageMath grading function

```
#Dec 24, 2019. Nasser: Ported original Maple grading function by
#           Albert Rich to use with Sagemath. This is used to
#           grade Fricas, Giac and Maxima results.
#Dec 24, 2019. Nasser: Added 'exp_integral_e' and 'sng', 'sin_integral'
#           'arctan2','floor','abs','log_integral'
#June 4, 2022  Made default grade_annotation "none" instead of "" due
#           issue later when reading the file.
#July 14, 2022. Added ellipticF. This is until they fix sagemath, then remove it.

from sage.all import *
from sage.symbolic.operators import add_vararg, mul_vararg

debug=False;

def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    #print("Enter tree_size, expr is ",expr)

    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)

def is_sqrt(expr):
    if expr.operator() == operator.pow:   #isinstance(expr,Pow):
        if expr.operands()[1]==1/2: #expr.args[1] == Rational(1,2):
            if debug: print ("expr is sqrt")
            return True
        else:
            return False
    else:
        return False
```

```
def is_elementary_function(func):
    #debug=False
    m = func.name() in ['exp','log','ln',
            'sin','cos','tan','cot','sec','csc',
            'arcsin','arccos','arctan','arccot','arcsec','arccsc',
            'sinh','cosh','tanh','coth','sech','csch',
            'arcsinh','arccosh','arctanh','arccoth','arcsech','arccsch','sgn',
        'arctan2','floor','abs'
        ]
    if debug:
        if m:
            print ("func ", func , " is elementary_function")
        else:
            print ("func ", func , " is NOT elementary_function")


    return m

def is_special_function(func):
    #debug=False
    if debug:
        print ("type(func)=", type(func))

    m= func.name() in ['erf','erfc','erfi','fresnel_sin','fresnel_cos','Ei',
          'Ei','Li','Si','sin_integral','Ci','cos_integral','Shi','sinh_integral'
          'Chi','cosh_integral','gamma','log_gamma','psi,zeta',
          'polylog','lambert_w','elliptic_f','elliptic_e','ellipticF',
          'elliptic_pi','exp_integral_e','log_integral']

    if debug:
        print ("m=",m)
        if m:
            print ("func ", func ," is special_function")
        else:
            print ("func ", func ," is NOT special_function")


    return m


def is_hypergeometric_function(func):
    return func.name() in ['hypergeometric','hypergeometric_M','hypergeometric_U']

def is_appell_function(func):
    return func.name() in ['hypergeometric']    #[appellf1] can't find this in sagemath
```

```
def is_atom(expn):

   #debug=False
   if debug:
       print ("Enter is_atom, expn=",expn)

   if not hasattr(expn, 'parent'):
      return False


   #thanks to answer at https://ask.sagemath.org/question/49179/what-is-sagemath-equivalent-to-atomic-
   try:
      if expn.parent() is SR:
         return expn.operator() is None
      if expn.parent() in (ZZ, QQ, AA, QQbar):
         return expn in expn.parent() # Should always return True
      if hasattr(expn.parent(),"base_ring") and hasattr(expn.parent(),"gens"):
         return expn in expn.parent().base_ring() or expn in expn.parent().gens()

      return False

   except AttributeError as error:
      print("Exception,AttributeError in  is_atom")
      print ("cought exception" , type(error).__name__ )
      return False


def expnType(expn):

   if debug:
      print (">>>>>Enter expnType, expn=", expn)
      print (">>>>>is_atom(expn)=", is_atom(expn))

   if is_atom(expn):
      return 1
   elif type(expn)==list:   #isinstance(expn,list):
      return max(map(expnType, expn))    #apply(max,map(ExpnType,expn))
   elif  is_sqrt(expn):
      if  type(expn.operands()[0])==Rational:  #type(isinstance(expn.args[0],Rational):
         return 1
      else:
         return max(2,expnType(expn.operands()[0]))   #max(2,expnType(expn.args[0]))
   elif expn.operator() == operator.pow:    #isinstance(expn,Pow)
      if type(expn.operands()[1])==Integer:   #isinstance(expn.args[1],Integer)
         return expnType(expn.operands()[0])    #expnType(expn.args[0])
      elif type(expn.operands()[1])==Rational:   #isinstance(expn.args[1],Rational)
         if type(expn.operands()[0])==Rational:  #isinstance(expn.args[0],Rational)
```

```
            return 1
        else:
            return max(2,expnType(expn.operands()[0]))  #max(2,expnType(expn.args[0]))
    else:
        return max(3,expnType(expn.operands()[0]),expnType(expn.operands()[1])) #max(3,expnType(expn.op
elif expn.operator() == add_vararg or expn.operator() == mul_vararg: #isinstance(expn,Add) or isinstan
    m1 = expnType(expn.operands()[0]) #expnType(expn.args[0])
    m2 = expnType(expn.operands()[1:]) #expnType(list(expn.args[1:]))
    return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
elif is_elementary_function(expn.operator()):  #is_elementary_function(expn.func)
    return max(3,expnType(expn.operands()[0]))
elif is_special_function(expn.operator()): #is_special_function(expn.func)
    m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
    return max(4,m1)   #max(4,m1)
elif is_hypergeometric_function(expn.operator()): #is_hypergeometric_function(expn.func)
    m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
    return max(5,m1)   #max(5,m1)
elif is_appell_function(expn.operator()):
    m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
    return max(6,m1)   #max(6,m1)
elif str(expn).find("Integral") != -1: #this will never happen, since it
        #is checked before calling the grading function that is passed.
        #but kept it here.
    m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
    return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
else:
    return 9

#main function
def grade_antiderivative(result,optimal):


    if debug:
        print ("Enter grade_antiderivative for sagemath")
        print("Enter grade_antiderivative, result=",result)
        print("Enter grade_antiderivative, optimal=",optimal)
        print("type(anti)=",type(result))
        print("type(optimal)=",type(optimal))

    leaf_count_result  = tree_size(result) #leaf_count(result)
    leaf_count_optimal = tree_size(optimal) #leaf_count(optimal)

    #if debug: print ("leaf_count_result=", leaf_count_result, "leaf_count_optimal=",leaf_count_optimal)


    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)
```

```
if debug: print ("expnType_result=", expnType_result, "expnType_optimal=",expnType_optimal)

if expnType_result <= expnType_optimal:
    if result.has(I):
        if optimal.has(I): #both result and optimal complex
            if leaf_count_result <= 2*leaf_count_optimal:
                grade = "A"
                grade_annotation ="none"
            else:
                grade = "B"
                grade_annotation ="Both result and optimal contain complex but leaf count of result is larger t
        else: #result contains complex but optimal is not
            grade = "C"
            grade_annotation ="Result contains complex when optimal does not."
    else: # result do not contain complex, this assumes optimal do not as well
        if leaf_count_result <= 2*leaf_count_optimal:
            grade = "A"
            grade_annotation ="none"
        else:
            grade = "B"
            grade_annotation ="Leaf count of result is larger than twice the leaf count of optimal. "+str(leaf_
else:
    grade = "C"
    grade_annotation ="Result contains higher order function than in optimal. Order "+str(expnType_resu


print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)

return grade, grade_annotation
```